

# Generalized positivity constraint on magnetic equivalent layers

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## ABSTRACT

It is known from potential theory that a continuous and planar layer of dipoles can exactly reproduce the total-field anomaly produced by arbitrary 3D sources. We show that, if the magnetization direction of this layer is the same as that of the true sources, its magnetic-moment distribution is all positive. This property holds true regardless of whether the magnetization of the true sources is purely induced or not. By using this generalized positivity constraint, we present a new iterative method for estimating the total magnetization direction of 3D magnetic sources based on the equivalent-layer technique. Our method does not impose a priori information either about the shape or depth of the sources, does not require regularly spaced data and presumes that the sources have a uniform magnetization direction. At each iteration, our method performs two steps. The first one solves a constrained linear inverse problem to estimate a positive magnetic-moment distribution over a discrete equivalent layer of dipoles. We consider that the equivalent sources are located on a plane and have an uniform and fixed magnetization direction. The second step uses the estimated magnetic-moment distribution and solves a nonlinear inverse problem for estimating a new magnetization direction for the dipoles. The algorithm stops when the equivalent layer yields a total-field anomaly that fits the observed data. Tests with synthetic data simulating different geological scenarios show that the final estimated magnetization direction

successfully approximates that of the true sources. We applied our method to a field data from the Goiás Alkaline Province (GAP), over the Montes Claros complex, center of Brazil. The result suggests the presence of intrusions with remarkable remanent magnetization, in agreement with the current literature for this region.

## INTRODUCTION

Most of magnetic methods require knowledge of the magnetization direction, otherwise they yield unsatisfactory interpretations of the exploration targets. This fact has been propelled the development of several techniques for estimating magnetization direction over the last 50 years. The strategies for estimating this quantity can be divided into two main groups. The first one comprises the methods which presume a prior information about the shape of geological sources. The iterative method presented by Bhattacharyya (1966) presumes that the magnetic source has a rectangular prismatic shape. Emilia and Massey (1974) approximate a seamount by a set of stacked prisms with uniform magnetization direction and variable magnetization intensity. Parker et al. (1987) proposed a method for estimating de magnetization direction formulating as an optimization problem by imposing an uniformly magnetization distribution [QUAL A PREMISSA DESTE MTODO?]. Ryuji and Uchiyama (2005) also approximates a seamount by a set of juxtaposed prisms, but estimate a magnetization direction for each one. Finally, Oliveira Jr et al. (2015) approximate the magnetic sources by spherical bodies with known centers for estimating their magnetization directions. The second group is formed by methods which do not presume any information about the shape of the magnetic sources. Fedi et al. (1994), for example, proposed a method that determines the best magnetization direction among a set a tentative values used to perform successive reductions-to-the-pole on Fourier domain. Medeiros and Silva (1995) presented an interpretation method that estimates the total magnetization direction and the spatial orientation of the source based on the multipole expansion. [ACHO QUE ESTE MTODO PERTENCE AO PRIMEIRO GRUPO PORQUE PRESUME QUE O CORPO TEM TRS PLANOS DE SIMETRIA MUTAMENTE ORTOGONAIS]. Phillips (2005) uses the Helbig's integral for estimating the components of the magnetic-moment vector. Tontini

and Pedersen (2008) extended the Phillips’ method by using the same Helbig’s integral to estimate the magnetization direction and its magnitude, also providing information about the position of the center of magnetization distribution. Lelièvre and Oldenburg (2009) developed a method for estimating the magnetization direction in complex geological scenarios. Their method approximates the subsurface by a grid of juxtaposed prisms and estimates the components of the magnetization vector for each prism. In addition, there are methods based on the correlation of potential-field quantities (e.g., Dannemiller and Li, 2006; Gerovska et al., 2009; Liu et al., 2015; Zhang et al., 2018).

Estimating the magnetization direction is extremely important not only for interpretation, but also for processing the total-field anomaly data. One technique in spatial domain commonly used for processing potential-field data is the equivalent layer. It was first introduced in exploration geophysics by Dampney (1969) and Emilia and Massey (1974) for processing gravity and magnetic data, respectively. After these pioneer works, this technique has been widely used for computing interpolation (Cordell, 1992; Mendonça and Silva, 1994; Barnes and Lumley, 2011; Siqueira et al., 2017), upward (or downward) continuation (Hansen and Miyazaki, 1984; Li and Oldenburg, 2010), reduction to the pole (Silva, 1986; Leão and Silva, 1989; Guspí and Novara, 2009; Oliveira Jr. et al., 2013), the amplitude of anomalous field (Li and Li, 2014) and for denoising gradient data (Martinez and Li, 2016). The equivalent-layer technique consists in approximating the observed data by that produced by a layer of discrete sources (e.g., prisms, dipoles or point masses), which are commonly known as equivalent sources. The data produced by this fictitious layer (the equivalent layer) are commonly called predicted data.

In scanning magnetic microscopy, the equivalent-layer technique is generally used for interpreting the magnetic-moment distribution within thin planar sections of rock samples.

Notice that, in this case, the equivalent layer resembles the true source (a thin section of rock). Weiss et al. (2007) presented one of the first works using the equivalent-layer technique in scanning magnetic microscopy. They pointed out without proof that the estimated magnetic-moment distribution on the layer is all-positive if the magnetization direction of the equivalent sources is equal to the one used for artificially magnetizing the rock sample. Baratchart et al. (2013) show mathematically that, assuming a uniform magnetization direction within the thin section, the inverse problem of estimating the magnetic-moment distribution has uniqueness. Lima et al. (2013) proposed a method on the frequency domain to investigate solutions having a uniform magnetization direction equal to that of a thin section of geological sample. They show empirically that, in this case, the estimated magnetic-moment distribution on the layer is entirely positive.

In the geophysical exploration, the equivalent-layer technique is predominantly used for processing potential-field data. Under this perspective, there is no relationship between the physical-property distribution on the equivalent layer and the true geologic sources. Hence, the layer is just a mathematical abstraction devoid of geological meaning. Few authors in geophysical exploration literature have been addressed the use of the equivalent-layer technique for interpreting the geologic sources. Pedersen (1991), for example, discussed the relationship between potential field and equivalent source. Medeiros and Silva (1996) and Dias et al. (2007) estimated an apparent-magnetization map on a layer by using Tikhonov and entropic regularizations respectively. Siqueira et al. (2017) established a relationship between the excess of mass estimated over the equivalent layer and the true one. Li et al. (2014) proved, by using an approach in the Fourier domain, the existence of an all-positive magnetic-moment distribution over the layer and use this to overcome the low-latitude instability. However, these authors considered only the particular case in which the magnetic

sources have a purely induced magnetization.

Here, we prove mathematically that the all-positive magnetic-moment distribution within an equivalent layer exists even in the presence of remanent magnetization of the true geologic sources. This all-positive magnetic-moment distribution holds true for all cases in which the magnetization direction of the equivalent sources has the same direction as that of the true geologic sources, regardless of whether the magnetization of the true sources is purely induced or not. Grounded on this generalized positivity constraint, we present a new iterative method that uses the equivalent-layer technique for estimating the uniform magnetization direction of arbitrary sources by inverting total-field anomaly data. Our method does not presume any information about the shape of the sources. At each iteration, our method solves (i) a linear inverse problem, subjected to a positivity constraint, for estimating the magnetic-moment distribution within a planar equivalent layer of dipoles, and (ii) a non-linear inverse problem for estimating the uniform magnetization direction of the equivalent sources. Tests with synthetic data generated by different geological scenarios show that the estimated magnetization direction converges to that of the true sources. We also applied our method to field data from the Goiás alkaline province (GAP), over the Montes Claros complex, center of Brazil. Our result is in agreement with that obtained independently by Zhang et al. (2018) at the same area, suggests the presence of a remarkable remanent magnetization and shows the good performance of our method in interpreting a complex geological scenario.

## METHODOLOGY

### Fundamentals of magnetic equivalent layer and the positive magnetic-moment distribution

Considering a Cartesian coordinate system with  $x$ -,  $y$ - and  $z$ -axis being oriented to north, east and downward, respectively. Let  $\Delta T_i \equiv \Delta T(x_i, y_i, z_i)$  be the total-field anomaly, at the  $i$ th position  $(x_i, y_i, z_i)$ , produced by a continuous layer located below the observation plane at a depth equal to  $z_c$ , where  $z_c > z_i$ , and  $p(x', y', z_c)$  is the distribution of magnetic dipoles moment per unit area over the layer. The total-field anomaly produced by this continuous layer is given by

$$\Delta T_i = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p(x', y', z_c) [\gamma_m \hat{\mathbf{F}}_0^T \mathbf{M}(x_i, y_i, z_i, x', y', z_c) \hat{\mathbf{m}}(\mathbf{q})] dx' dy', \quad (1)$$

where  $\gamma_m$  is a constant proportional to the vacuum permeability,  $\hat{\mathbf{F}}_0$  is a unit vector with the same direction of the main geomagnetic field given by

$$\hat{\mathbf{F}}_0 = \begin{bmatrix} \cos I_0 \cos D_0 \\ \cos I_0 \sin D_0 \\ \sin I_0 \end{bmatrix}, \quad (2)$$

where  $I_0$  and  $D_0$  are, respectively, the inclination and declination and  $\mathbf{M}(x_i, y_i, z_i, x', y', z_c)$  is a  $3 \times 3$  dimensional matrix (Oliveira Jr et al., 2015) equal to

$$\mathbf{M}(x_i, y_i, z_i, x', y', z_c) = \begin{bmatrix} \partial_{xx}\phi & \partial_{xy}\phi & \partial_{xz}\phi \\ \partial_{yx}\phi & \partial_{yy}\phi & \partial_{yz}\phi \\ \partial_{zx}\phi & \partial_{zy}\phi & \partial_{zz}\phi \end{bmatrix}, \quad (3)$$

where  $\partial_{\alpha\beta}\phi$ ,  $\alpha = x, y, z$  and  $\beta = x, y, z$ , is the second derivative of the scalar function

$$\phi(x_i, y_i, z_i, x', y', z_c) = \frac{1}{[(x_i - x')^2 + (y_i - y')^2 + (z_i - z_c)^2]^{\frac{1}{2}}}. \quad (4)$$

with respect to the Cartesian coordinates  $x_i$ ,  $y_i$  and  $z_i$  of the observation points. The  $\hat{\mathbf{m}}(\mathbf{q})$  is a unit vector with the magnetization direction of the dipoles over layer given by

$$\hat{\mathbf{m}}(\mathbf{q}) = \begin{bmatrix} \cos I \cos D \\ \cos I \sin D \\ \sin I \end{bmatrix} \quad (5)$$

and  $\mathbf{q}$  is a  $2 \times 1$  vector with components given by

$$\mathbf{q} = \begin{bmatrix} I \\ D \end{bmatrix}, \quad (6)$$

where  $I$  and  $D$  are the inclination and declination of the magnetization direction of the dipoles on the layer, respectively. We can also notice that the vector defined in equation 5 has a single and uniform magnetization direction of all dipoles on the layer. For convenience, this unit vector can be rewritten as follows

$$\hat{\mathbf{m}}(\mathbf{q}) = \mathbf{R}\hat{\mathbf{h}}, \quad (7)$$

where  $\hat{\mathbf{h}}$  defines the uniform magnetization direction of an arbitrary magnetic source and  $\mathbf{R}$  is a  $3 \times 3$  matrix obtained from Euler's rotation theorem. This theorem states that any rotation can be parametrized by using three parameters called Euler angles (H. Goldstein and Safko, 1980). That is, if all dipoles that set up the equivalent layer have the same magnetization direction  $\hat{\mathbf{m}}(\mathbf{q})$  and this direction is the same as the true magnetic source  $\hat{\mathbf{h}}$ ,



then the matrix  $\mathbf{R}$  (equation 7) is equal to identity. For this reason, the total-field anomaly produced by equivalent layer at the  $i$ th position  $(x_i, y_i, z_i)$  (equation 1) can be rewritten as

$$\Delta T_i = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p(x', y', z_c) [\gamma_m \hat{\mathbf{F}}_0^T \mathbf{M}(x_i, y_i, z_i, x', y', z_c) \hat{\mathbf{h}}] dx' dy', \quad (8)$$

which represents the total-field anomaly produced by continuous layer with the same direction of the arbitrary magnetic source. Thus, the reduction to the pole (RTP) field  $\Delta T_i^{PL}$  produced by equivalent layer at the point  $(x_i, y_i, z_i)$  is equal to

$$\Delta T_i^{PL} = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p(x', y', z_c) [\gamma_m \partial_{zz} \phi(x_i, y_i, z_i, x', y', z_c)] dx' dy', \quad (9)$$

where  $\partial_{zz} \phi(x_i, y_i, z_i, x', y', z_c)$  is the second derivative of the inverse of distance (equation 4) with respect of  $z_i$ , evaluated at the point  $(x_i, y_i, z_i)$ . However, by considering the RTP field  $\Delta T_i^{PS}$  that would be produced by an arbitrary uniformly magnetized source at the pole, we have

$$\Delta T_i^{PS} = \gamma_m \partial_{zz} \Gamma(x_i, y_i, z_i) m, \quad (10)$$

where  $m$  is the magnetization intensity of the magnetic source. The  $\partial_{zz} \Gamma(x_i, y_i, z_i)$  is the second derivative in relation to  $z_i$  of a scalar function  $\Gamma(x_i, y_i, z_i)$

$$\Gamma(x_i, y_i, z_i) = \iiint_v \frac{dv}{[(x_i - \alpha)^2 + (y_i - \beta)^2 + (z_i - \gamma)^2]^{\frac{1}{2}}} \quad (11)$$

in which  $\alpha$ ,  $\beta$  and  $\gamma$  are the Cartesian coordinates of an infinitesimal element inside the volume  $v$  of the magnetic source. From equation 9 and 10, we obtain

$$m \partial_{zz} \Gamma(x_i, y_i, z_i) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p(x', y', z_c) \partial_{zz} \phi(x_i, y_i, z_i, x', y', z_c) dx' dy'. \quad (12)$$

We can notice that equation 12 can be calculated differentiating the following equation

$$m \partial_z \Gamma(x_i, y_i, z_i) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{p(x', y', z_c)(z_c - z_i)}{[(x_i - x')^2 + (y_i - y')^2 + (z_i - z_c)^2]^{\frac{3}{2}}} dx' dy' \quad (13)$$

with respect to the vertical component  $z_i$ , where  $z_c > z_i$ .

From potential-field theory, we can highlight the classical upward continuation integral

$$U(x_i, y_i, z_i) = \frac{(z_c - z_i)}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \frac{U(x', y', z_c)}{[(x_i - x')^2 + (y_i - y')^2 + (z_i - z_c)^2]^{\frac{3}{2}}} dx' dy', \quad (14)$$

where the function  $U(x_i, y_i, z_i)$  is an hamornic function at all  $(x_i, y_i, z_i)$  and  $U(x', y', z_c)$  is the same harmonic function at  $(x', y', z_c)$  (Blakely, 1996). In this case, if this harmonic function represents the total-field anomaly at the point  $(x_i, y_i, z_i)$ , it can be mathematically interpreted as the convolution between its values  $U(x', y', z_c)$  and the vertical derivative with respect to  $z_i$  of the equation 4, evaluated on the horizontal plane  $z_i = z_c$ . Therefore, according to the classical upward continuation function (equation 14), the magnetic-moment distribution  $p(x', y', z_c)$  in equation 13 assumes the form

$$p(x', y', z_c) = \frac{m}{2\pi} \partial_z \Gamma(x', y', z_c), \quad (15)$$

where  $\partial_z \Gamma(x', y', z_c)$  is the derivative of the scalar function 11 in relation to  $z_i$  evaluated over the equivalent layer. The most interesting aspect of equation 15 is that the magnetic-moment distribution is defined as the product of a positive constant  $\frac{m}{2\pi}$  and the function  $\partial_z \Gamma(x', y', z_c)$ , which is all positive at all points  $(x', y', z_c)$  over the equivalent layer. This

relation is similar to that presented by Pedersen (1991) and ?. In the wavenumber domain, these authors determined the magnetic-moment distribution within a continuous equivalent layer with the same magnetization direction as the local-geomagnetic field at the pole. They also considered a planar equivalent layer located below and parallel to a horizontal plane containing the observed total-field anomaly. They assume a magnetic source having a purely induced magnetization. Under these assumptions, Pedersen (1991) and ? concluded that the magnetic-moment distribution within the continuous equivalent layer is proportional to the pseudogravity anomaly produced by the source on the plane of the equivalent layer. By following different approaches using magnetic microscopy data, Baratchart et al. (2013) and Lima and Weiss (2016) pointed out by imposing a nonnegativity constraint, the solution to the inverse problem is a unique magnetic-moment distribution. Here, we do not follow the same wavenumber-domain reasoning used by all these authors. Moreover, equation 15 generalizes this positivity condition because (1) it holds true for all cases in which the magnetization of the equivalent layer has the same direction as the true magnetization of the sources, whether it is purely induced or not, (2) does not require that the observed total-field anomaly data be on a plane.

## Parametrization and forward problem

In practical situations, it is not possible to determine a continuous magnetic-moment distribution  $p(x', y', z_c)$  over the layer as shown in equation 1. For this reason, the layer has to be approximated by a discrete set of dipoles with unit volume located at a constant depth  $z = z_c$ . By discretizing the integrand of equation 1, the predicted total-field anomaly at the point  $(x_i, y_i, z_i)$  is given by

$$\Delta T_i = f_i(\mathbf{s}), \quad (16)$$

where  $\Delta T_i$  is the  $i$ th element of the  $N$ -dimensional vector of predicted total-field anomaly  $\Delta \mathbf{T}(\mathbf{s})$ . The function  $f_i$  maps the unknown parameters onto the data, in which the parameter vector  $\mathbf{s}$  is formed by an  $M$ -dimensional vector  $\mathbf{p}$  whose  $j$ th element  $p_j$  is the magnetic moment of the  $j$ th dipole with a single magnetization direction  $\mathbf{q}$  (equation 6) assigned to all dipoles (Figure 1). Explicitly, the function  $f_i$  is described as

$$f_i(\mathbf{s}) = \sum_{j=1}^M g_{ij}(\mathbf{q}) p_j = \mathbf{g}_i^T(\mathbf{q}) \mathbf{p}, \quad (17)$$

where  $\mathbf{g}_i(\mathbf{q})$  is a  $M$ -dimensional vector whose  $ij$ th element is given by

$$g_{ij}(\mathbf{q}) = \gamma_m \hat{\mathbf{F}}_0^T \mathbf{M}_{ij} \hat{\mathbf{m}}(\mathbf{q}) \quad (18)$$

and it is a harmonic function representing the total-field anomaly at the  $i$ th position  $(x_i, y_i, z_i)$  yielded by a  $j$ th dipole located at  $(x_j, y_j, z_c)$  with unit magnetic-moment intensity. The matrix  $\mathbf{M}_{ij}$  is formed by the second derivatives of a function  $\phi_{ij}$  that depends on the inverse of the scalar function  $r_{ij} = [(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_c)^2]^{1/2}$ , analogously to equations 3 and 4. From equations 16-18, we can notice that the predicted total-field anomaly  $\Delta \mathbf{T}(\mathbf{s})$  has a linear relation with the magnetic moment  $\mathbf{p}$  and a nonlinear relation with the magnetization direction  $\mathbf{q}$ .

## Inverse problem

Let  $\Delta \mathbf{T}^o$  be an  $N$ -dimensional vector whose  $i$ th element  $\Delta T_i^o$  is the total-field anomaly observation produced by a magnetic source at the point  $(x_i, y_i, z_i)$ ,  $i = 1, \dots, N$ . In order to

estimate the magnetization direction, we have to formulate an inverse problem by imposing a positivity constraint on the magnetic-moment distribution. Here, it is accomplished by solving the following constrained problem of

$$\text{minimizing} \quad \Psi(\mathbf{s}) = \|\Delta \mathbf{T}^o - \Delta \mathbf{T}(\mathbf{s})\|_2^2 + \mu f_0 \|\mathbf{p}\|_2^2 \quad (19a)$$

$$\text{subject to} \quad \mathbf{p} \geq 0. \quad (19b)$$

On the right side of equation 19a the first and second terms are the data-misfit function and the zeroth-order Tikhonov regularization function,  $\mu$  is the regularizing parameter,  $\|\cdot\|_2^2$  represents the squared Euclidean norm and  $f_0$  is a normalizing factor. In the inequality 19b,  $\mathbf{0}$  is a null vector and the inequality sign is applied element by element. This inequality imposes positivity constraints on the estimated magnetic moments of all dipoles, which is solved by using the nonnegative least squares (NNLS) proposed by Lawson and Hanson (1974).

Minimizing the goal function shown in equation 19a starts with an initial approximation  $\mathbf{s}^k$  to the parameter vector and then solving a sequence of linear problem of estimating a correction  $\Delta \mathbf{s}^k$  at each  $k$ th iteration. The procedure is repeated until a minimum of goal function (equation 19a) is reached. This procedure is determined by using a gradient-based iterative optimization method like Gauss-Newton (Aster et al., 2005). Mathematically, the correction  $\Delta \mathbf{s}^k$  is expressed as a second-order expansion of the goal function given by

$$\Psi(\mathbf{s}^k + \Delta \mathbf{s}^k) \approx \Psi(\mathbf{s}^k) + \mathbf{J}^{kT}(\mathbf{s}^k) \Delta \mathbf{s}^k + \frac{1}{2} \Delta \mathbf{s}^{kT} \mathbf{H}^k(\mathbf{s}^k) \Delta \mathbf{s}^k \quad (20)$$

in which  $\mathbf{J}^k(\mathbf{s}^k)$  and  $\mathbf{H}^k(\mathbf{s}^k)$  are, respectively, the gradient vector and the Hessian matrix of equation 19a. Thus, taking the gradient with respect to the parameter perturbation

vector  $\Delta \mathbf{s}^k$  of the expanded equation 20 and setting the result equal to the null vector, it is obtained by solving the linear system

$$\mathbf{H}^k(\mathbf{s}^k) \bar{\Delta} \mathbf{s}^k = -\mathbf{J}^k(\mathbf{s}^k), \quad (21)$$

where the estimate  $\bar{\Delta} \mathbf{s}^k$  is a single step of the Gauss-Newton method required to attain the minimum of the expanded function (equation 20). The linear system given by equation 21 can be rewritten as

$$\left[ \begin{array}{c|c} \mathbf{H}_{pp}^k & \mathbf{H}_{pq}^k \\ \hline \mathbf{H}_{qp}^k & \mathbf{H}_{qq}^k \end{array} \right] \begin{bmatrix} \Delta \mathbf{p}^k \\ \Delta \mathbf{q}^k \end{bmatrix} = - \begin{bmatrix} \mathbf{J}_p^k \\ \mathbf{J}_q^k \end{bmatrix}, \quad (22)$$

in which  $\mathbf{J}_\alpha^k$  and  $\mathbf{H}_{\alpha\beta}^k$ , where  $\alpha = p, q$  and  $\beta = p, q$ , are the gradient vector and the Hessian matrix calculated in relation to each element of the magnetic-moment vector  $\mathbf{p}$  and the magnetization direction vector  $\mathbf{q}$ , respectively. Besides, in order to simplify the equation 22, we consider null cross-derivatives.

However, The gradient vector and the Hessian matrix relative to the part of magnetic moments are, respectively,

$$\mathbf{J}_p^k = -2\mathbf{G}_p^{kT} [\Delta \mathbf{T}^o - \Delta \mathbf{T}(\mathbf{s}^k)] + 2\mu f_0^k \mathbf{p}^k \quad (23)$$

and

$$\mathbf{H}_{pp}^k = 2\mathbf{G}_p^{kT} \mathbf{G}_p^k + 2\mu f_0^k \mathbf{I} \quad (24)$$

where  $\mathbf{G}_p^k$  is the  $N \times M$  sensitivity matrix at the  $k$ th iteration, whose  $ij$ th element is given by equation 18,  $\mathbf{I}$  is an identity matrix and the normalizing factor  $f_0^k$  is equal to

$$f_0^k = \frac{\text{tr}(\mathbf{G}_p^{kT} \mathbf{G}_p^k)}{M}, \quad (25)$$

where  $\text{tr}$  is denotaded as the trace of the matrix  $\mathbf{G}_p^{kT} \mathbf{G}_p^k$  and  $M$  is the total number of dipoles composing the layer. From the correction  $\bar{\Delta} \mathbf{p}^k = \bar{\mathbf{p}}^{k+1} - \bar{\mathbf{p}}^k$ , we can conclude that the linear system to be solved is given by

$$\left[ \mathbf{G}_p^{kT} \mathbf{G}_p^k + \mu f_0^k \mathbf{I} \right] \bar{\mathbf{p}}^{k+1} = \mathbf{G}_p^{kT} \Delta \mathbf{T}^o. \quad (26)$$

Owing to nonlinear relation of the magnetization direction  $\mathbf{q}^k$  with the predicted total-field anomaly, the gradient vector and the Hessian matrix at the  $k$ th for this case are, respectively,

$$\mathbf{J}_q^k = -2\mathbf{G}_q^{kT} [\Delta \mathbf{T}^o - \Delta \mathbf{T}(\mathbf{s}^k)] \quad (27)$$

and

$$\mathbf{H}_{qq}^k \approx 2\mathbf{G}_q^{kT} \mathbf{G}_q^k \quad (28)$$

in which  $\mathbf{G}_q^k$  is a  $N \times 2$  sensitivity matrix, whose elements are composed by derivative of equation 17 in relation of each element of the vector  $\mathbf{q}^k$ , that are the inclination and declination. Nevertheless, to calculate the correction  $\bar{\Delta} \mathbf{q}^k$  at the  $k$ th iteration, we use the Levenberg-Marquardt method (Aster et al., 2005) by solving the linear system

$$\left[ \mathbf{G}_q^{kT} \mathbf{G}_q^k + \lambda^k \mathbf{I} \right] \bar{\Delta} \mathbf{q}^k = \mathbf{G}_q^{kT} [\Delta \mathbf{T}^o - \Delta \mathbf{T}(\mathbf{s}^k)], \quad (29)$$

where  $\lambda$  is the Marquardt parameter that is updated along the iterative process and  $\mathbf{I}$  is an identity matrix. After estimating the parameter correction  $\bar{\Delta} \mathbf{q}^k$  at the  $k$ th iteration, we

update the magnetization direction such that

$$\bar{\mathbf{q}}^{k+1} = \bar{\mathbf{q}}^k + \Delta \bar{\mathbf{q}}^k. \quad (30)$$

### Iterative process for magnetization estimation

The magnetic moments  $\bar{\mathbf{p}}$  and the magnetization direction  $\bar{\mathbf{q}}$  are obtained through an inverse problem of minimizing the difference between the observed data  $\Delta \mathbf{T}^o$  and the predicted data  $\Delta \mathbf{T}(\mathbf{s})$ . As we can notice, the solution of the inversion is given by solving two linear systems shown in equation 26 and equation 29. For this reason, we propose a nested algorithm for solving the inverse problem in two steps.

Our iterative algorithm starts with an initial guess for the magnetization direction  $\mathbf{q}_0$ . At the  $k$ th iteration, we estimate a set of magnetic moment  $\bar{\mathbf{p}}^{k+1}$  by imposing a positivity constraint on equation 26. After estimating the magnetic-moment distribution  $\bar{\mathbf{p}}^{k+1}$  within the equivalent layer at the  $k$ th iteration using the previous estimate  $\bar{\mathbf{q}}^k$  for the magnetization direction, we estimate the corrections of magnetization direction vector  $\Delta \bar{\mathbf{q}}^k$  by solving the unconstrained nonlinear inverse problem (equation 29) and update the magnetization direction by equation 30. The iterative process stops when the goal function (equation 19a) is invariant along successive iterations. An overview of the algorithm is shown in Figure 2 and in the algorithm 1.



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**Algorithm 1:** Nested NNLS and Levenberg-Marquardt method

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**Input** :  $\Delta \mathbf{T}^o, \mathbf{q}_0$

**Output:**  $\bar{\mathbf{p}}, \bar{\mathbf{q}}$

**while** (*not converge*) or ( $i < i_{max}$ ) **do**

**step 1:** Solve equation 26 using NNLS ;

**step 2:** Compute goal function (equation 19a) ;

**while** (*not converge*) or ( $j < j_{max}$ ) **do**

**step 3:** Initialize the Levenberg-Marquardt algorithm ;

**step 4:** Compute goal function (equation 19a);

**while**  $k < k_{marq}$  **do**

**step 5:** Solve equation 29 ;

**step 6:** Update the magnetization direction estimate (equation 30) ;

**step 7:** Compute goal function (equation 19);

**end**

**step 8:** Analysis of convergence for inner loop.

**end**

**step 9:** Analysis of convergence for outer loop.

**end**

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### The choice of layer depth $z_c$ and regularization parameter $\mu$

The procedure for the use of our methodology for estimating the total magnetization require the choice of two main parameters. The first one is the layer depth  $z_c$  as shown in Figure 1 and the second is the regularization parameter  $\mu$  shown in equation 26.

The method of the choice of layer is based on a classical approach proposed by Dampney

(1969). The author pointed out that the layer depth should satisfy an interval from 2.5 to 6 times the grid spacing. It should be notice that this rule was applied on an evenly spaced data. However, the choice for applying our method should correspond to an interval from 2 to 3 times the greater grid spacing. It is necessarily to point out that this range of values was found empirically due to the application on a irregular grid data.

To solve the equation 26 we have to choose a reliable regularization parameter  $\mu$ . For this purpose, we use the L-curve method (Hansen and O’Leary, 1993). This approach is widely used in the literature to find a regularizing parameter, which filtering out enough noise without losing too much information in the final solution. The procedure of finding the parameter plot a curve of optimal values between the solution norm and residual norm. The corner of the curve is the optimal regularization parameter which gives a threshold between the regularizing function and the data misfit.

## APPLICATION TO SYNTHETIC DATA

We applied the proposed method to three synthetic data sets simulating different geological scenarios. The first one is generated by a model containing a set of multiple sources with different geometries, all of them with the same magnetization direction. The second data set is generated by a set of multiple magnetic bodies, but one them being a shallow-seated source with the same magnetization direction. In the third test, we violate the hypothesis of unidirectional magnetization by simulating a shallow-seated source with different magnetization direction from the other bodies.

In all tests, the simulated data were computed on a regular grid of  $49 \times 25$  points (with a total of  $N = 1225$  observations) at a constant height of 100 m. We assume an observation area extending 12 km along the x- and y-axis, resulting a grid spacing of 250 m and 500 m on x- and y-axes, respectively. The data were contaminated with pseudorandom Gaussian noise with zero mean and 10 nT standard deviation. The geomagnetic field direction simulated was  $I_0 = -40^\circ$  and  $D_0 = -22^\circ$  for the inclination and declination, respectively. In the inversion, we use an equivalent layer composed by a grid of  $49 \times 25$  dipoles (with a total of  $M = 1225$  equivalent sources) positioned at a depth of  $z_c = 1150$  m below the observation plane (2.5 times the greater grid spacing). We use the L-curve to choose the regularizing parameter ( $\mu$ ). Our algorithm starts with an initial guess  $\mathbf{q}_0 = (-10^\circ, -10^\circ)$  for inclination and declination, respectively.

### Unidirectional magnetization sources

We generate a 3D prism with polygonal cross-section whose the top is positioned at a depth of 450 m and the bottom 3150 m with magnetization intensity of 4 A/m. We also generate

two spheres with magnetization intensity equal to 3 A/m and radius equal to 500 m. The coordinates of the spheres centers  $x_c = 1800$  m,  $y_c = -1800$  m and  $z_c = 1000$  m and  $x_c = 800$  m,  $y_c = 800$  m and  $z_c = 1000$  m. We produce two rectangular prisms with 2.5 A/m of magnetization intensity. The smaller prism has the top at a depth of 450 m and side lengths of 1000 m, 700 m and 500 m along x-,y- and z-axes, respectively. The greater prism has the top at a depth of 500 m and side lengths of 1000 m, 2000 m and 1550 m along x-,y- and z-axes. All simulated sources have inclination  $-25^\circ$  and declination  $30^\circ$ . The noise-corrupted data is shown in Figure 3a.

Figure 3b shows the predicted data produced by equivalent layer. Figure 3c shows the residuals defined as the difference between the simulated data (Figure 3a) and the predicted data (Figure 3b). The residuals appear normally distributed with a mean of  $-0.30$  nT and a standard deviation of  $9.67$  nT as shown in Figure 3d. The estimated magnetization direction  $\bar{\mathbf{q}}$  has inclination  $-28.6^\circ$  and declination  $30.8^\circ$  which are very close to the true one. Figure 3e shows the estimated magnetic-moment distribution  $\bar{\mathbf{p}}$ . The convergence of the algorithm is shown in Figure 3f. These results show that the all-positive magnetic-moment distribution and the estimated magnetization direction produce an acceptable data fitting.

### Unidirectional magnetization with shallow-seated source

Here, we test the methodology performance when a shallow-seated source exists. The model seems the previous test except for the smaller prism, whose the top is 150 m deep while maintaining its volume. The magnetization intensity of this shallow prism is equal to 1.5 A/m. The magnetization direction of all sources is  $-25^\circ$  inclination and declination  $30^\circ$ , respectively. The synthetic data is shown in Figure 4a.

Figure 4b shows the predicted total-field anomaly produced by equivalent layer. Figure 4c shows the residuals defined as the difference between the simulated data (Figure 4a) and the predicted data (Figure 4b). The residuals appear normally distributed with a mean of  $-0.42$  nT and a standard deviation of  $10.67$  nT as shown in Figure 4d. Figure 4e shows the estimated magnetic-moment distribution  $\bar{\mathbf{p}}$ . The convergence of the algorithm is shown in Figure 4f. Despite the large residual located above the shallow-seated source, we consider that the methodology produced a reliable result because the estimated magnetization direction  $\bar{\mathbf{q}}$  has inclination  $-28.7^\circ$  and declination  $31.7^\circ$  and its very close to the corresponding true magnetization direction, and the all-positive magnetic-moment distribution produces an acceptable data fitting.

### Shallow-seated source with different magnetization direction

In this test, we simulate the presence of a shallow-seated body with different magnetization direction from the other magnetic sources. The shallow prism has the dimension and magnetization intensity equal to the previous test. However, the magnetization direction of the shallow prism is  $20^\circ$  of inclination and  $-30^\circ$  of declination, while the other sources has inclination  $-25^\circ$  and declination  $30^\circ$ . The noise-corrupted data is shown in Figure 5a.

Figure 5b shows the predicted total-field anomaly. Figure 5c shows the residuals defined as the difference between the simulated data (Figure 5a) and the predicted data (Figure 5b). The residuals have a mean of  $-0.71$  nT and a standard deviation of  $10.67$  nT as shown in Figure 5d. The estimated magnetization direction  $\bar{\mathbf{q}}$  has inclination  $-30.4^\circ$  and declination  $27.6^\circ$ . Figure 5e shows the estimated magnetic-moment distribution  $\bar{\mathbf{p}}$ . The convergence of the algorithm is shown in Figure 5f. Despite the slight difference from

the true magnetization direction, the estimated magnetic-moment distribution produces an acceptable data fit. With the exception of the small area exactly above the small-seated prism most of the residuals are closely 0 nT.

## APPLICATION TO FIELD DATA

The Goiás alkaline province (GAP) is a region in the central part of Brazil that there are occurrences of mafic-ultramafic alkaline magmatism. This region presents a variety of rocks with an extensive petrographic types. Throughout the area there are mafic-ultramafic complexes (plutonic intrusions), subvolcanic alkaline intrusions (diatremes) and volcanic products (kamafugite lava flows) with several dikes. Some of the main alkaline complexes of GAP are: the Montes Claros de Goiás, Diorama, Corrego dos Bois, Morro do Macaco and Fazenda Buriti. These alkaline intrusions are surrounded by a Precambrian basement and the Phanerozoic sedimentary rocks of the Paraná Basin. (Junqueira-Brod et al., 2005; Carlson et al., 2007; Marangoni and Mantovani, 2013; Dutra et al., 2014). Recent studies indicate the existence of a remarkable remanent magnetization component within these intrusions (Marangoni and Mantovani, 2013; Oliveira Jr et al., 2015; Marangoni et al., 2016; Zhang et al., 2018).

The aeromagnetic survey has a flight pattern with north-south spaced from  $\sim 500$  m and  $\sim 8$  m along each line, and a constant height of 100 m from the terrain. The geomagnetic field direction for this area was  $-19.5^\circ$  and  $-18.5^\circ$  for inclination and declination, respectively. We invert the total-field anomaly (Figure 6a) from the alkaline complex of Montes Claros. To speed up data processing and inversion, we downsampled the data along the flight lines, resulting a grid of  $55 \times 32$  points (a total of  $N = 1787$  observations). This new set up results an approximately 320 m and 470 m grid spacing along the x- and y-axes, respectively. We use an equivalent layer composed by a grid of  $55 \times 32$  dipoles (a total of  $M = 1787$  equivalent sources) positioned at a depth of 840 m below the observation plane ( $\sim 2$  times the greater grid spacing). The algorithm starts with an initial guess of  $-70^\circ$

and  $50^\circ$  for the inclination and declination, respectively. Figure 6b shows the predicted data produced by equivalent layer. Figure 6c shows the residuals defined as the difference between the observed data (Figure 6a) and the predicted data (Figure 6b). Notice that, two small places in Figure 6d where large residuals are clearly apparent may indicate the existence of shallow-seated geologic sources with different magnetization direction. However, the histogram of residuals (Figure 6d) is acceptable with its mean of  $-14.52$  nT ( $\sim 0.1\%$  of the maximum value of total-field anomaly data) and standard deviation of  $312.28$  nT ( $\sim 2\%$  of the maximum value of total-field anomaly data). The estimated magnetization direction  $\bar{\mathbf{q}}$  has inclination  $-50.2^\circ$  and declination  $34.9^\circ$ . Figure 6e and 6f shows the estimated magnetic-moment distribution  $\bar{\mathbf{p}}$  and the convergence of the algorithm. We check the quality of the estimated magnetization direction by computing the reduction-to-pole of the observed total-field anomaly. We can notice that the RTP field anomaly (Figure 7) exhibits predominantly positive values and decays to zero towards the borders of study area. For this reason, we consider that the estimated magnetization direction led to a satisfactory RTP anomaly. We conclude with these results that the all-positive magnetic moment distribution and the estimated magnetization direction produce an acceptable data fitting. The estimated magnetization direction also confirms the existence of remarkable remanent magnetization.



## CONCLUSION

We have shown mathematically that the total-field anomaly data caused by a set of magnetic sources with uniform magnetization direction can be exactly reproduced by a continuous and planar layer of dipoles having an all-positive magnetic-moment distribution. This theoretical property holds true for the case in which the layer has the same magnetization direction as that of the true sources, regardless of whether they have a purely induced magnetization or not. By using this generalized positivity constraint, we have presented a new iterative method for estimating the total magnetization direction of 3D magnetic sources based on the equivalent-layer technique. At each iteration, we impose a positivity constraint on the estimated magnetic-moment distribution of the layer and solves a non-linear inverse problem for estimating the magnetization direction of the equivalent sources. Prior knowledge about the shape and depth of magnetic sources are not required, neither the use of an evenly spaced data set. This methodology can be applied for determining the magnetization direction of multiple sources, considering all of them with the same magnetization direction. Results obtained with synthetic data produced by multiple sources have shown that the estimated magnetization direction obtained by our iterative method successfully retrieves the true one. Tests with synthetic data have also illustrated how the presence of a relatively shallow-seated source affects the result obtained by our method for the cases in which it has a magnetization direction equal to and different from that of the other sources. They have shown that, in the second case, the shallow source may negatively impact the estimated magnetization direction. In both cases, the equivalent layer yielded large data misfits above the shallow source. An application to field data over the Goiás alkaline province (GAP), center of Brazil, has confirmed that our method can be a reliable tool for interpreting complex geological scenarios. The result over the Montes

Claros complex suggests the presence of a strong remanent magnetization component and corroborates a previous study conducted independently at the same area. The estimated magnetic-moment distribution over the layer has led to a very acceptable reduction to the pole, but have also produced large data-misfits at some isolated regions. We presume that these locally large data-misfits are due to shallow sources, however we cannot infer if they have the same magnetization direction of the other bodies.

## APPENDIX A

### CONSEQUENCES OF HIGH-LATITUDE ESTIMATION

A limitation for estimating the magnetization direction using our method is its application to high-latitude anomalies. In this appendix, we prove the existence of high latitude difficulties providing a theoretical basis and testing it on extreme cases.

The process of estimating magnetization direction using equivalent-layer technique is divided into two inversions. One solves a constraining linear inversion using positivity and minimum-norm constraints on the magnetic-moment distribution and the second one comprises an unconstrained nonlinear inversion for estimating the magnetization direction. A way to extract information about the rank-deficiency and ill-conditioned linear systems is exploring the singular value decomposition. This procedure allows an  $N \times M$  matrix can be rewritten as the product of three matrices, one of them formed by a set of singular values arranged in descending order. From linear algebra, if there is a row or a column of a matrix entirely formed by zeros then it means that the linear equations are linearly dependent. Consequently, it leads to a rank-deficiency, ill-conditioned linear system and zero singular values. Here, the ill-posed system of linear equations is given in equation 29 for estimating the correction of the magnetization direction ( $\bar{\Delta \mathbf{q}}^k$ ). This system is based on 2 variables (inclination and declination) where the  $N \times 2$  sensitivity matrix  $\mathbf{G}_q^k$  is explicitly given by

$$\mathbf{G}_q^k = \begin{bmatrix} \partial_I \mathbf{g}_1^{kT} \bar{\mathbf{p}}^k & \partial_D \mathbf{g}_1^{kT} \bar{\mathbf{p}}^k \\ \vdots & \vdots \\ \partial_I \mathbf{g}_N^{kT} \bar{\mathbf{p}}^k & \partial_D \mathbf{g}_N^{kT} \bar{\mathbf{p}}^k \end{bmatrix}, \quad (\text{A-1})$$

in which  $\bar{\mathbf{p}}^{k^T}$  is the estimated magnetic-moment distribution on the equivalent layer by solving the linear system given in equation 26 with positivity constraint and  $\partial_\alpha \mathbf{g}_i(\mathbf{q}^k)$ , where  $\alpha = I, D$ , a derivative of equation 17 with respect to inclination (first column) and the declination (second column) evaluated at the  $i$ th observation point, respectively. That is, by calculating the derivative of equation 17 in relation to magnetization direction is in fact the derivative of the equation 18 in relation to each element of the vector  $\hat{\mathbf{m}}(\mathbf{q})$  (equation 5). That is, one element of the vector  $\partial_\alpha \mathbf{g}_i(\mathbf{q}^k)$  is equal to

$$\partial_I g_{ij}(\mathbf{q}) \equiv \gamma_m \hat{\mathbf{F}}_0^T \mathbf{M}_{ij} \partial_I \hat{\mathbf{m}}(\mathbf{q}) = \gamma_m \hat{\mathbf{F}}_0^T \mathbf{M}_{ij} \begin{bmatrix} -\sin I \cos D \\ -\sin I \sin D \\ \cos I \end{bmatrix}, \quad (\text{A-2})$$

and

$$\partial_D g_{ij}(\mathbf{q}) \equiv \gamma_m \hat{\mathbf{F}}_0^T \mathbf{M}_{ij} \partial_D \hat{\mathbf{m}}(\mathbf{q}) = \gamma_m \hat{\mathbf{F}}_0^T \mathbf{M}_{ij} \begin{bmatrix} -\cos I \sin D \\ \cos I \cos D \\ 0 \end{bmatrix}. \quad (\text{A-3})$$

Hence, for the case whose magnetization direction of the true magnetic source has inclination  $90^\circ$  and declination  $0^\circ$ , the equation A-2 and A-3 is, respectively, equal to

$$\partial_I g_{ij}(\mathbf{q}) \equiv \gamma_m \hat{\mathbf{F}}_0^T \mathbf{M}_{ij} \partial_I \hat{\mathbf{m}}(\mathbf{q}) = \gamma_m \hat{\mathbf{F}}_0^T \mathbf{M}_{ij} \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \quad (\text{A-4})$$

and

$$\partial_D g_{ij}(\mathbf{q}) \equiv \gamma_m \hat{\mathbf{F}}_0^T \mathbf{M}_{ij} \partial_D \hat{\mathbf{m}}(\mathbf{q}) = \gamma_m \hat{\mathbf{F}}_0^T \mathbf{M}_{ij} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \quad (\text{A-5})$$

We can notice in this case that the second column of the sensitivity matrix in equation A-1 has all zero elements when the magnetization direction is vertical, leading to ill-posed inversion.

## REFERENCES

- Aster, R. C., B. Borchers, and C. H. Thurber, 2005, Parameter estimation and inverse problems (international geophysics): Academic Press.
- Baratchart, L., D. P. Hardin, E. A. Lima, E. B. Saff, and B. P. Weiss, 2013, Characterizing kernels of operators related to thin-plate magnetizations via generalizations of Hodge decompositions: *Inverse Problems*, **29**, 015004(29pp).
- Barnes, G., and J. Lumley, 2011, Processing gravity gradient data: *GEOPHYSICS*, **76**, I33–I47.
- Bhattacharyya, B. K., 1966, A method for computing the total magnetization vector and the dimensions of a rectangular block shaped from magnetic anomalies: *GEOPHYSICS*, **31**, 74–96.
- Blakely, R. J., 1996, *Potential theory in gravity and magnetic applications*: Cambridge University press.
- Carlson, R. W., A. L. N. Araujo, T. C. Junqueira-Brod, J. C. Gaspar, J. A. Brod, I. A. Petrinovic, M. H. B. M. Hollanda, M. M. Pimentel, and S. Sichel, 2007, Chemical and isotopic relationships between peridotite xenoliths and maficultrapotassic rocks from southern brazil: *Chemical Geology*, **242**, 415 – 434.
- Cordell, L., 1992, A scattered equivalent-source method for interpolation and gridding of potential-field data in three dimensions: *GEOPHYSICS*, **57**, 629–636.
- Dampney, C. N. G., 1969, The equivalent source technique: *GEOPHYSICS*, **34**, 39–53.
- Dannemiller, N., and Y. Li, 2006, A new method for determination of magnetization direction: *GEOPHYSICS*, **71**, L69–L73.
- Dias, F. J. S., V. C. Barbosa, and J. B. Silva, 2007, 2d gravity inversion of a complex interface in the presence of interfering sources: *GEOPHYSICS*, **72**, I13–I22.

- Dutra, A. C., Y. Marangoni, and R. I. F. Trindade, 2014, Aeromagnetic and physical-chemical properties of some complexes from Goiás Alkaline Province: Brazilian Journal of Geology, **44**, 361 – 373.
- Emilia, D. A., and R. L. Massey, 1974, Magnetization estimation for nonuniformly magnetized seamounts: GEOPHYSICS, **39**, 223–231.
- Fedi, M., G. Florio, and A. Rapolla, 1994, A method to estimate the total magnetization direction from a distortion analysis of magnetic anomalies1: Geophysical Prospecting, **42**, 261–274.
- Gerovska, D., M. J. Araúzo-Bravo, and P. Stavrev, 2009, Estimating the magnetization direction of sources from southeast bulgaria through correlation between reduced-to-the-pole and total magnitude anomalies: Geophysical Prospecting, **57**, 491–505.
- Guspí, F., and I. Novara, 2009, Reduction to the pole and transformations of scattered magnetic data using newtonian equivalent sources: GEOPHYSICS, **74**, L67–L73.
- H. Goldstein, C. P. P. J., and J. L. Safko, 1980, Classical mechanics: Addison-Wesley.
- Hansen, P., and D. O’Leary, 1993, The use of the l-curve in the regularization of discrete ill-posed problems: SIAM Journal on Scientific Computing, **14**, 1487–1503.
- Hansen, R. O., and Y. Miyazaki, 1984, Continuation of potential fields between arbitrary surfaces: GEOPHYSICS, **49**, 787–795.
- Junqueira-Brod, T. C., J. C. Gaspar, J. A. Brod, and C. V. Kafino, 2005, Kamafugitic diatremes: their textures and field relationships with examples from the goiás alkaline province, brazil: Journal of South American Earth Sciences, **18**, 337 – 353. (Volcanic rocks in Brazil through time and different tectonic settings).
- Lawson, C. L., and R. J. Hanson, 1974, Solving least squares problems: SIAM.
- Leão, J. W. D., and J. B. C. Silva, 1989, Discrete linear transformations of potential field

- data: *GEOPHYSICS*, **54**, 497–507.
- Lelièvre, P. G., and D. W. Oldenburg, 2009, A 3d total magnetization inversion applicable when significant, complicated remanence is present: *GEOPHYSICS*, **74**, L21–L30.
- Li, S.-L., and Y. Li, 2014, Inversion of magnetic anomaly on rugged observation surface in the presence of strong remanent magnetization: *GEOPHYSICS*, **79**, J11–J19.
- Li, Y., M. Nabighian, and D. W. Oldenburg, 2014, Using an equivalent source with positivity for low-latitude reduction to the pole without striation: *GEOPHYSICS*, **79**, J81–J90.
- Li, Y., and D. W. Oldenburg, 2010, Rapid construction of equivalent sources using wavelets: *GEOPHYSICS*, **75**, L51–L59.
- Lima, E. A., and B. P. Weiss, 2016, Ultra-high sensitivity moment magnetometry of geological samples using magnetic microscopy: *Geochemistry, Geophysics, Geosystems*, **17**, 3754–3774.
- Lima, E. A., B. P. Weiss, L. Baratchart, D. P. Hardin, and E. B. Saff, 2013, Fast inversion of magnetic field maps of unidirectional planar geological magnetization: *J. Geophys. Res.: Solid Earth*, **118**, 2723–2752.
- Liu, S., X. Hu, Y. Xi, T. Liu, and S. Xu, 2015, 2d sequential inversion of total magnitude and total magnetic anomaly data affected by remanent magnetization: *GEOPHYSICS*, **80**, K1–K12.
- Marangoni, Y., H. Zhang, and H. Ferreira, 2016, Gravity and magnetic integrated data interpretation of the córrego dos bois complex, goiás alkaline province, central brazil: *Revista Brasileira de Geofísica*, **33**, 599–610.
- Marangoni, Y. R., and M. S. Mantovani, 2013, Geophysical signatures of the alkaline intrusions bordering the paraná basin: *Journal of South American Earth Sciences*, **41**, 83 – 98. (Alkaline Magmatism and the Lithospheric Mantle : a special issue in honour of the



- work of Celso de Barros Gomes on the occasion of his 77th birthday).
- Martinez, C., and Y. Li, 2016, Denoising of gravity gradient data using an equivalent source technique: *GEOPHYSICS*, **81**, G67–G79.
- Medeiros, W. E., and J. B. C. Silva, 1995, Simultaneous estimation of total magnetization direction and 3-d spatial orientation: *GEOPHYSICS*, **60**, 1365–1377.
- , 1996, Geophysical inversion using approximate equality constraints: *GEOPHYSICS*, **61**, 1678–1688.
- Mendonça, C. A., and J. B. C. Silva, 1994, The equivalent data concept applied to the interpolation of potential field data: *GEOPHYSICS*, **59**, 722–732.
- Oliveira Jr., V. C., V. C. F. Barbosa, and L. Uieda, 2013, Polynomial equivalent layer: *GEOPHYSICS*, **78**, G1–G13.
- Oliveira Jr, V. C., D. P. Sales, V. C. F. Barbosa, and L. Uieda, 2015, Estimation of the total magnetization direction of approximately spherical bodies: *Nonlinear Processes in Geophysics*, **22**, 215–232.
- Parker, R. L., L. Shure, and J. A. Hildebrand, 1987, The application of inverse theory to seamount magnetism: *Reviews of Geophysics*, **25**, 17–40.
- Pedersen, L. B., 1991, Relations between potential fields and some equivalent sources: *GEO-PHYSICS*, **56**, 961–971.
- Phillips, D. J., 2005, Can we estimate total magnetization directions from aeromagnetic data using helbig’s integrals?: *Earth, Planets and Space*, **57**, 681–689.
- Ryuji, K., and A. Uchiyama, 2005, Three-dimensional magnetization vector inversion of a seamount: *Earth, Planets and Space*, **57**, 691–699”.
- Silva, J. B. C., 1986, Reduction to the pole as an inverse problem and its application to low-latitude anomalies: *GEOPHYSICS*, **51**, 369–382.

- Siqueira, F. C. L., V. C. Oliveira Jr., and V. C. F. Barbosa, 2017, Fast iterative equivalent-layer technique for gravity data processing: A method grounded on excess mass constraint: *GEOPHYSICS*, **82**, G57–G69.
- Tontini, F. C., and L. B. Pedersen, 2008, Interpreting magnetic data by integral moments: *Geophysical Journal International*, **174**, 815–824.
- Weiss, B. P., E. A. Lima, L. E. Fong, and F. J. Baudenbacher, 2007, Paleomagnetic analysis using squid microscopy: *Journal of Geophysical Research: Solid Earth*, **112**, n/a–n/a. (B09105).
- Zhang, H., D. Ravat, Y. R. Marangoni, G. Chen, and X. Hu, 2018, Improved total magnetization direction determination by correlation of the normalized source strength derivative and the rtp fields: *GEOPHYSICS*, **0**, 1–45.

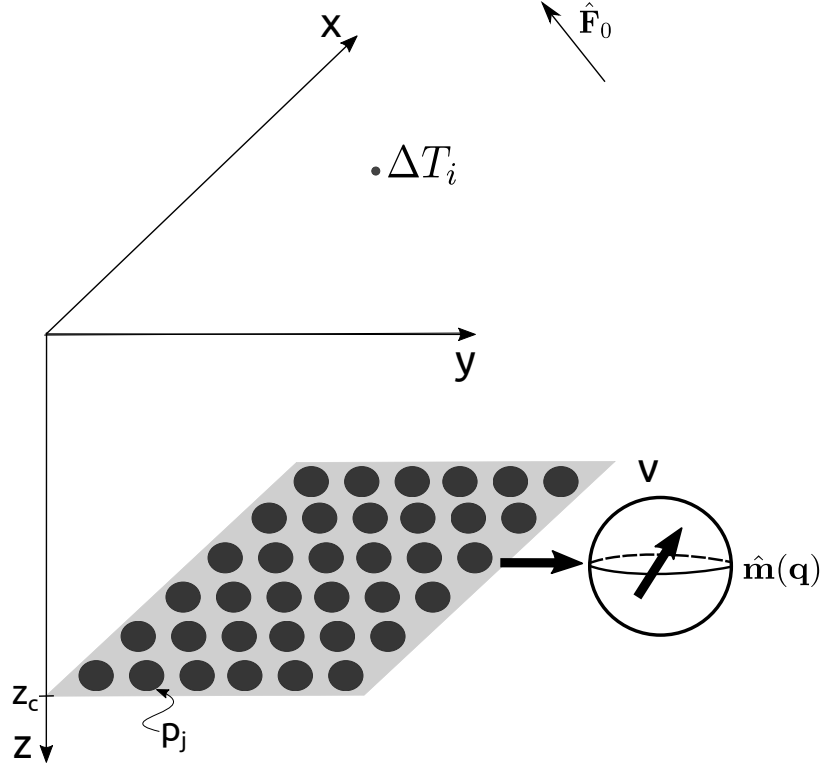


Figure 1: Schematic representation of an equivalent layer. The layer is positioned over the horizontal plane at a depth of  $z = z_c$ .  $\Delta T_i = f_i(\mathbf{s})$  is the predicted total-field anomaly at the point  $(x_i, y_i, z_i)$  produced by the set of  $M$  equivalent sources (black dots). Each source is located at the point  $(x_j, y_j, z_c)$ ,  $j = 1, \dots, M$ , and represented by a dipole with unity volume  $v$  with magnetization direction  $\hat{\mathbf{m}}(\mathbf{q})$  and magnetic moment  $p_j$ .

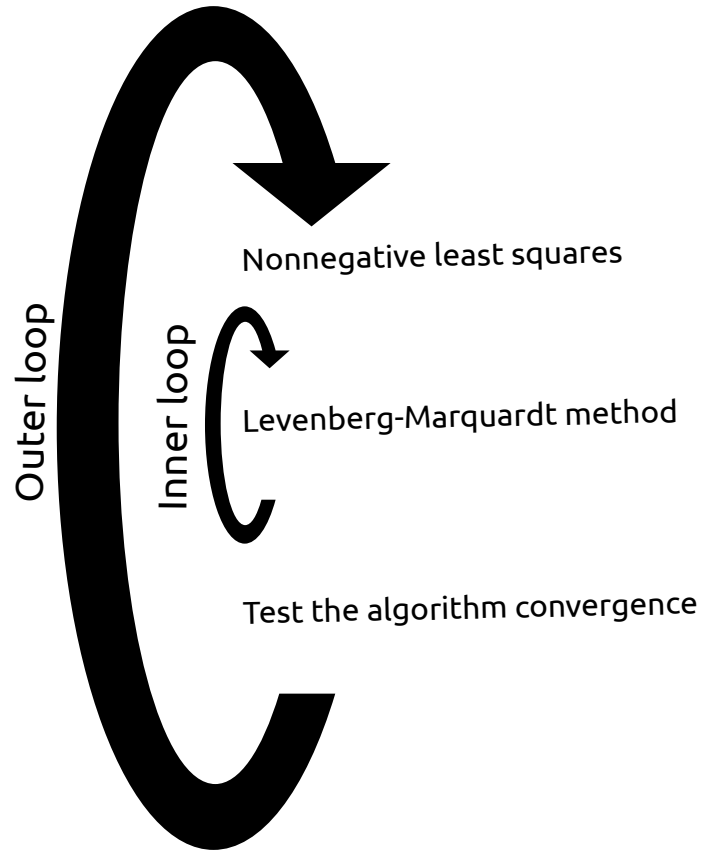


Figure 2: Iterative scheme overview for NNLS and Levenberg-Marquardt method for estimating magnetization direction. The outer loop is the nonnegative solution for magnetic-moment distribution and the inner loop calculates the magnetization direction correction using Levenberg-Marquardt method.

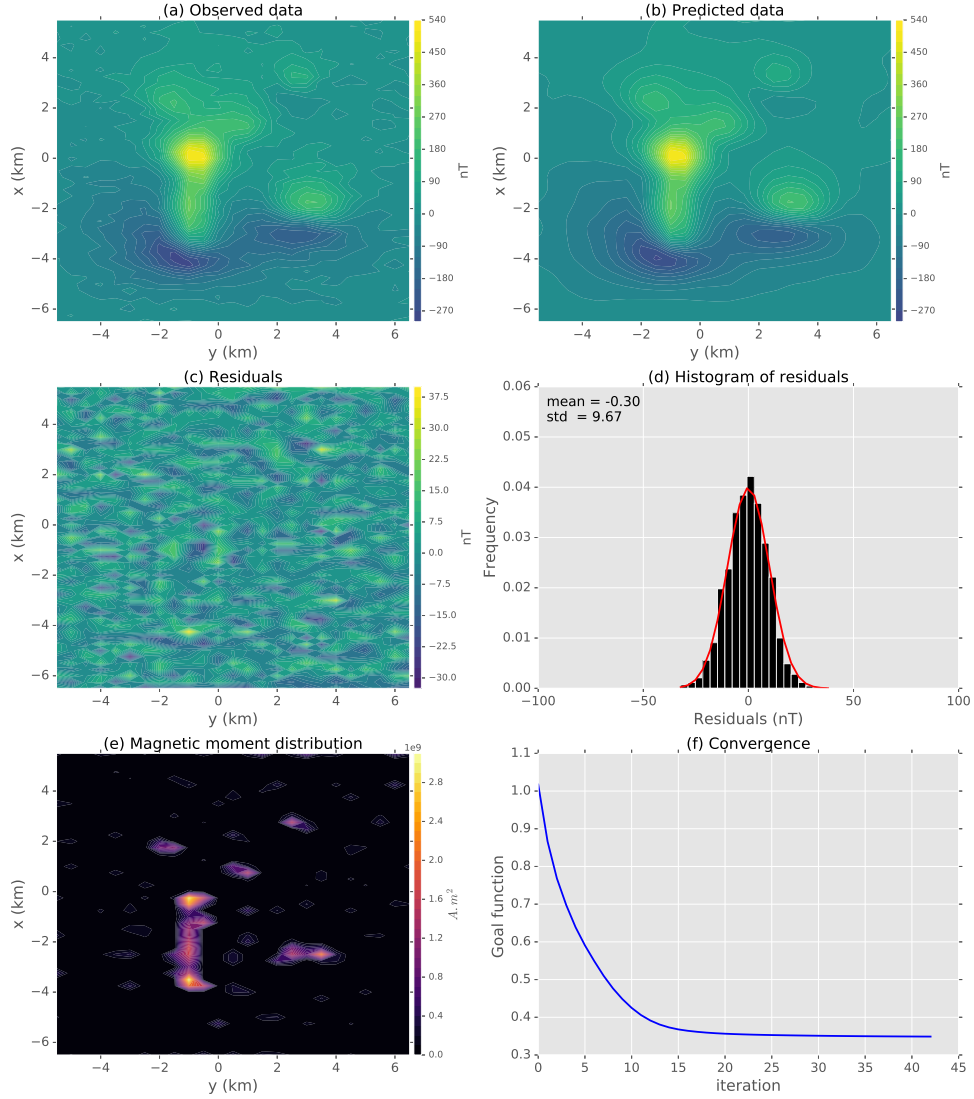


Figure 3: Application to synthetic data for multiple sources with unidirectional magnetization. (a) Noise-corrupted total-field anomaly. (b) Predicted data produced by equivalent layer. (c) Difference between the data shown in panels a and b. (d) Histogram of residuals. (e) All-positive magnetic-moment distribution. (f) Goal function value (equation 19a) per iteration showing the convergence.

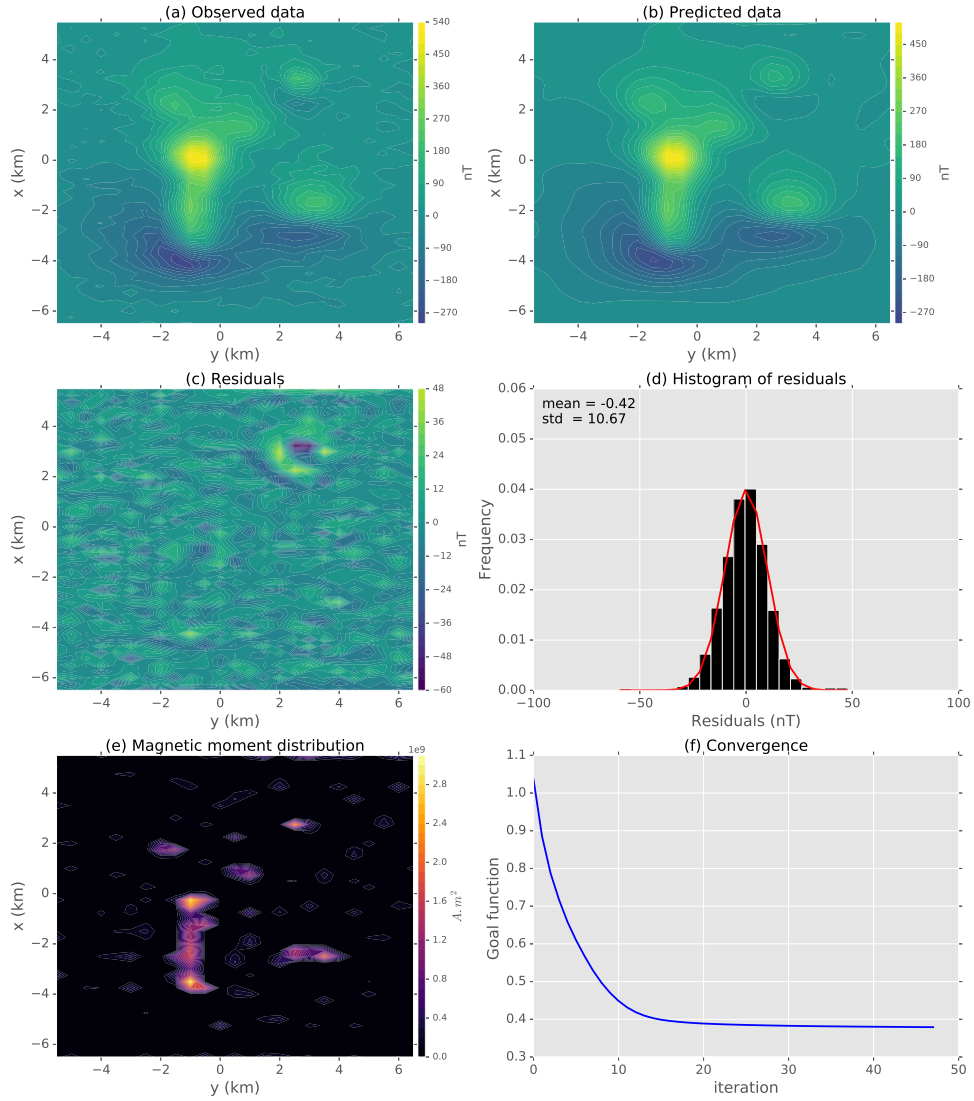


Figure 4: Multiple synthetic sources with a shallow-seated body under unidirectional magnetization. (a) Noise-corrupted total-field anomaly. (b) Predicted data produced by equivalent layer. (c) Difference between the data shown in panels a and b. (d) Histogram of residuals. (e) All-positive magnetic-moment distribution. (f) Goal function value (equation 19a) per iteration showing the convergence.

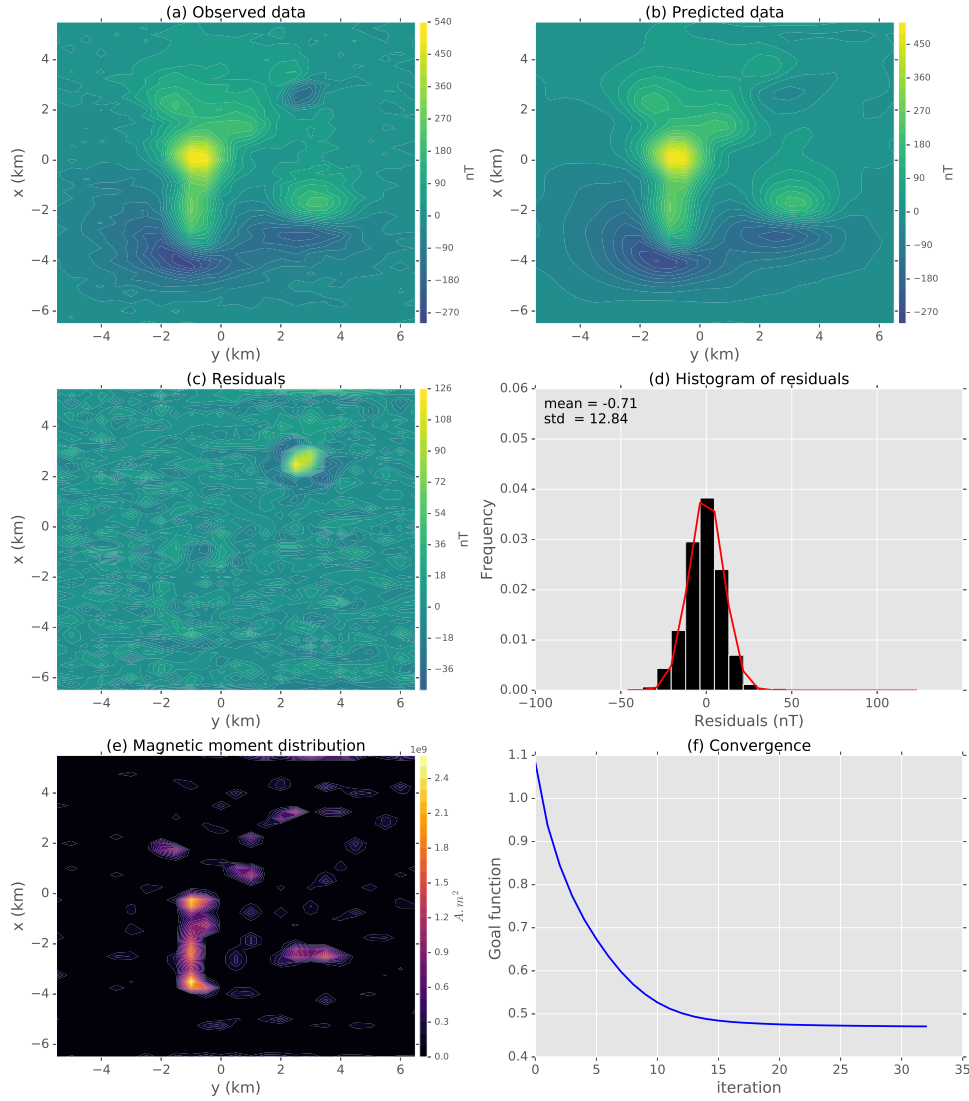


Figure 5: Multiple synthetic sources with a shallow-seated body under different magnetization directions. (a) Noise-corrupted total-field anomaly. (b) Predicted data produced by equivalent layer. (c) Difference between the data shown in panels a and b. (d) Histogram of residuals. (e) All-positive magnetic-moment distribution. (f) Goal function value (equation 19a) per iteration showing the convergence.

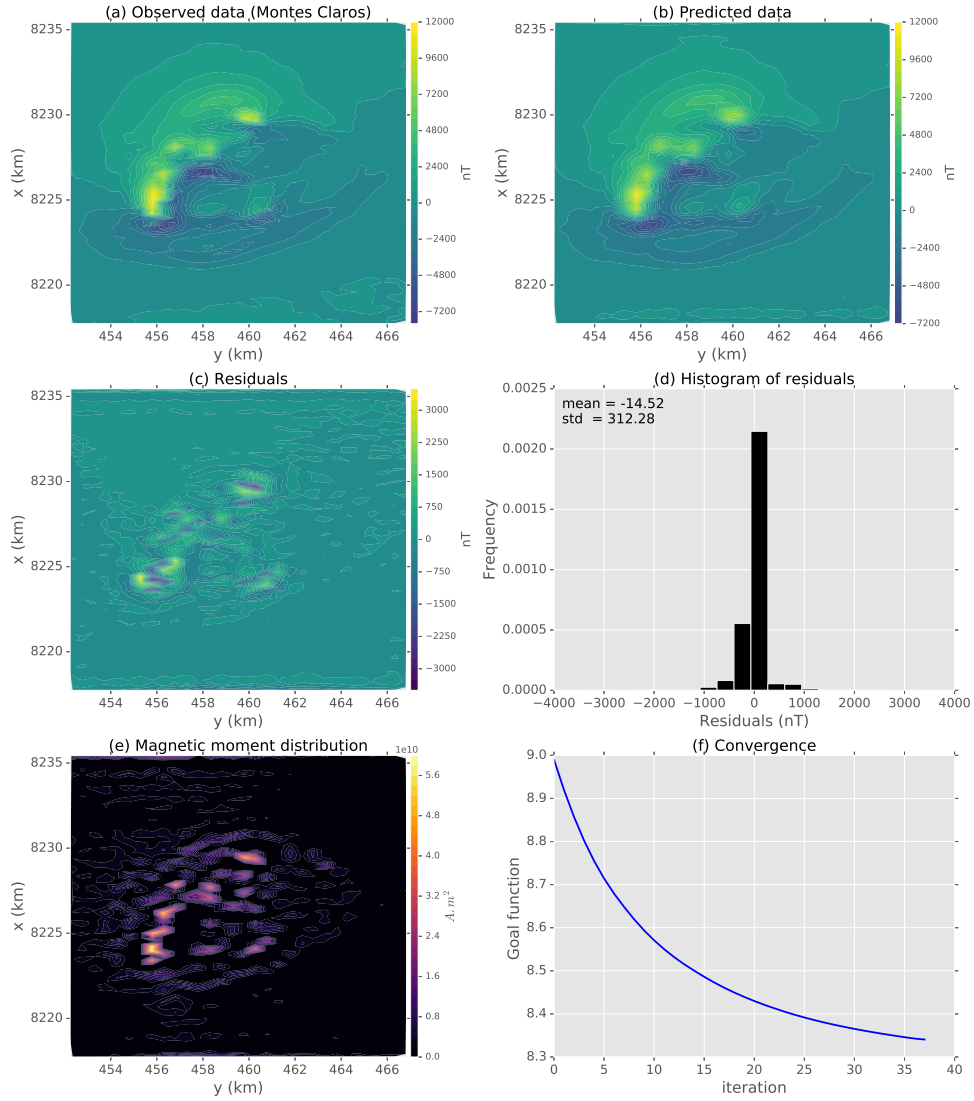


Figure 6: Application to field data from alkaline complex of Montes Claros (Brazil). (a) Observed total-field anomaly. (b) Predicted data produced by equivalent layer. (c) Difference between the data shown in panels a and b. (d) Histogram of residuals. (e) All-positive magnetic-moment distribution. (f) Goal function value (equation 19a) per iteration showing the convergence.



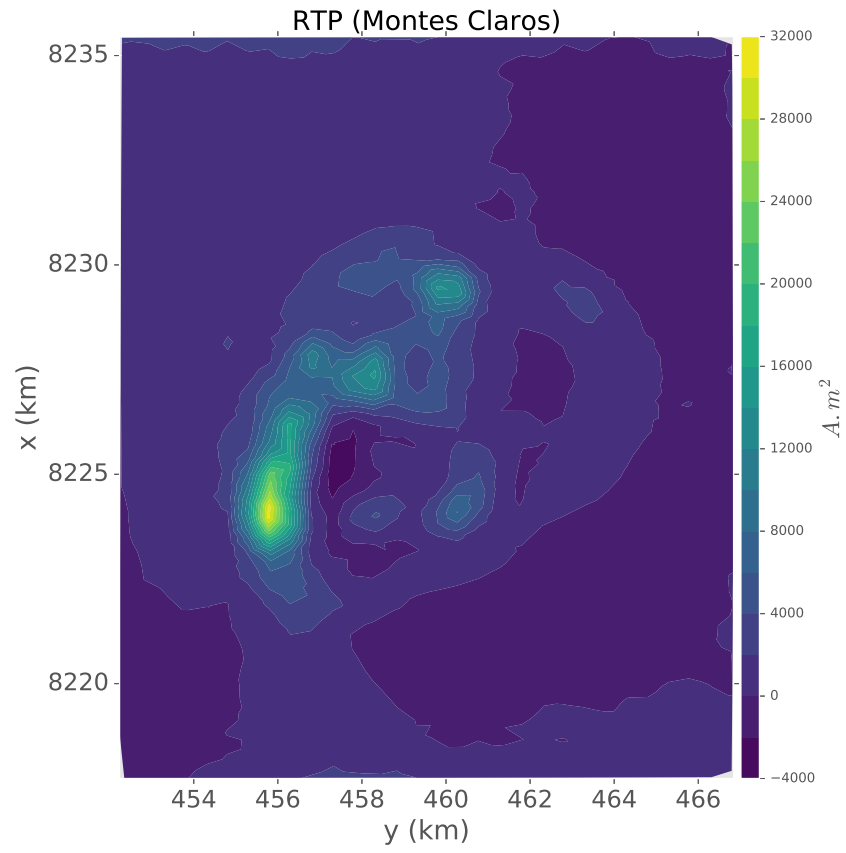


Figure 7: Application to field data from alkaline complex of Montes Claros (Brazil). RTP anomaly computed by using the estimated magnetization distribution shown in figure 6e.