CONCLUSION ~~AND DISCUSSION~~

We have presented a new method for estimating the total magnetization direction of 3D magnetic sources by using the equivalent-layer technique. Our method is formulated as an iterative least-squares problem and imposes a positivity constraint on the magnetic moments over the layer. Prior knowledge about the shape and depth of magnetic sources are not required, neither the use of an ~~envenly~~ spaced data. This methodology can be applied for determining the magnetization direction within multiple sources, considering all of them ~~have~~ the same magnetization direction. Moreover, we ~~show the theoretical proof of the~~ positive ~~characteristic of the~~ equivalent layer ~~when~~ it has the same magnetization direction of the true magnetic source, even if its purely induced or not.

By imposing the positivity constraint ~~for~~ magnetic-moment distribution ~~allows the~~ equivalent-layer ~~to provide informations about~~ the magnetization direction of magnetic sources. The results obtained with the synthetic data produced by unidirectional ~~model~~ have shown the good performance of our method ~~for~~ retrieving the true magnetization direction. Application to field data over the Goias alkaline province (GAP), center of Brazil, has confirmed that our method can be a ~~powerful~~ tool for interpreting real complex geological scenarios ~~as well~~. ~~The application for the complex of~~ Montes Claros suggests the presence of a strong remanent magnetization component, in accordance to previous studies ~~for the same magnetic anomaly~~. Moreover, the all-positive magnetic moment over the layer leads to ~~very plausible~~ RTP anomalies. ~~However, both two synthetic tests and the real data application present a marked residuals in some locations over the map of difference between the observed and predicted data. We consider that this markable feature was caused by~~ shallow ~~interfering~~ sources. ~~After some synthetic tests, regardless the shallow body has the~~

~~same direction of the other sources, it can be produce a data misfit just above the region of an interfering body.~~ Despite reliable ~~results for~~ magnetization direction ~~estimation,~~ we cannot infer if the shallow source has the same direction of other bodies ~~or not. For this reason, it is necessary a further analysis for interpreting the anomalies caused by shallow interferences.~~

APPENDIX A

CONSEQUENCES OF HIGH-LATITUDE ESTIMATION

~~One critical~~ limitation for estimating the magnetization direction ~~results of the case of estimating magnetization direction in~~ high-latitudes. In this appendix, we prove the existence of high latitude difficulties providing a theoretical basis and testing it on extreme cases.

The process of estimating magnetization direction using equivalent-layer technique is divided into two inversions. One ~~by solving~~ a ~~constrained~~ linear ~~problem~~ using positivity ~~for~~ magnetic-moment distribution and the ~~other~~ an unconstrained nonlinear ~~estimation for~~ magnetization direction. A way to extract ~~infomations~~ about the rank-deficiency and ill-~~conditioning of~~ linear systems is exploring the singular value decomposition. This procedure allows an $N \times M$ matrix can be rewritten as the product of three matrices, one of them formed by ~~the~~ set of singular values arranged in ~~order of decreasing size~~. ~~The context of decomposing a matrix in singular values is to obtain pieces of information that can be estimated. It can also be useful to dictate which rows or columns of the matrix are linear independent.~~ ~~The fact is that,~~ if a row or a column of a matrix ~~is all null, it impacts directly on the linear dependence~~. Consequently, it leads to a rank-deficiency ~~and~~ an ill-~~conditioning of a~~ linear system~~. Moreover, it is associated with~~ zero singular values. ~~Thus, the construction of~~ the $N \times 2$ sensitivity matrix $\mathbf{G}_p^k$ is explicitly given by

$$\mathbf{G}_p^k = \begin{bmatrix} \mathbf{p}^{k^T} \partial_{\tilde{\imath}} \mathbf{g}_1(\mathbf{q}^k) & \mathbf{p}^{k^T} \partial_{\tilde{d}} \mathbf{g}_1(\mathbf{q}^k) \\ \vdots & \vdots \\ \mathbf{p}^{k^T} \partial_{\tilde{\imath}} \mathbf{g}_N(\mathbf{q}^k) & \mathbf{p}^{k^T} \partial_{\tilde{d}} \mathbf{g}_N(\mathbf{q}^k) \end{bmatrix}, \tag{A-1}$$

in which $\mathbf{p}^{k^T} \partial_\alpha \mathbf{g}_i(\mathbf{q}^k)$, $\alpha = \tilde{\imath}, \tilde{d}$, is a derivative ~~in relation to~~ inclination (first column) and the declination (second column) evaluated at the ith observation point. ~~That is, by calculating the derivative of equation 17 in relation to magnetization direction is in fact the derivative of the vector $\hat{\mathbf{m}}(\mathbf{q})$ (equation 5). The derivative of the vector $\hat{\mathbf{m}}(\mathbf{q})$ in relation to inclination is given by~~

$$\cancel{\frac{\partial \hat{\mathbf{m}}(\mathbf{q})}{\partial \tilde{\imath}}} = \begin{bmatrix} -\sin\tilde{\imath}\cos\tilde{d} \\ -\sin\tilde{\imath}\sin\tilde{d} \\ \cos\tilde{\imath} \end{bmatrix}, \tag{A-2}$$

and ~~analogously for the declination is equal to~~

$$\cancel{\frac{\partial \hat{\mathbf{m}}(\mathbf{q})}{\partial \tilde{d}}} = \begin{bmatrix} -\cos\tilde{\imath}\sin\tilde{d} \\ \cos\tilde{\imath}\cos\tilde{d} \\ 0 \end{bmatrix}. \tag{A-3}$$

Hence, for the case whose ~~the~~ magnetization direction of the true magnetic source has inclination 90° and declination 0°, the equation A-2 and A-3 is, respectively, equal to

$$\cancel{\frac{\partial \hat{\mathbf{m}}(\mathbf{q})}{\partial \tilde{\imath}}} = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \tag{A-4}$$

and

$$\cancel{\frac{\partial \hat{\mathbf{m}}(\mathbf{q})}{\partial \tilde{d}}} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \tag{A-5}$$

We can notice ~~from all these reasoning~~ that the second ~~row~~ of the sensitivity matrix A-1 ~~is all null for a vertical~~ magnetization direction. ~~Consequently, it leads to a rank-deficiency and the ill-conditioning of a linear system in this situation. However, its different for the extreme case of low latitude in which the magnetization direction has the inclination 0° and declination 0°.~~

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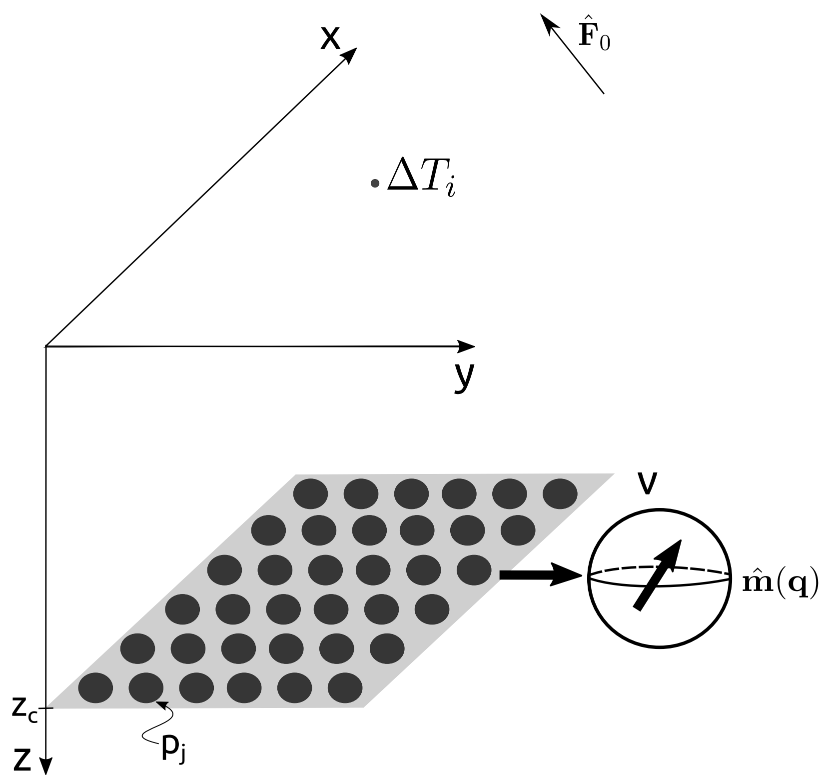

Figure 1: Schematic representation of an equivalent layer. The layer is positioned over the horizontal plane at a depth of $z = z_c$. $\Delta T_i = f_i(\mathbf{s})$ is the predicted total-field anomaly at the point (x_i, y_i, z_i) produced by the set of M equivalent sources (black dots). Each source is located at the point (x_j, y_j, z_c), $j = 1, \ldots, M$, and represented by a dipole with unity volume υ with magnetization direction $\hat{\mathbf{m}}(\mathbf{q})$ and magnetic moment p_j.

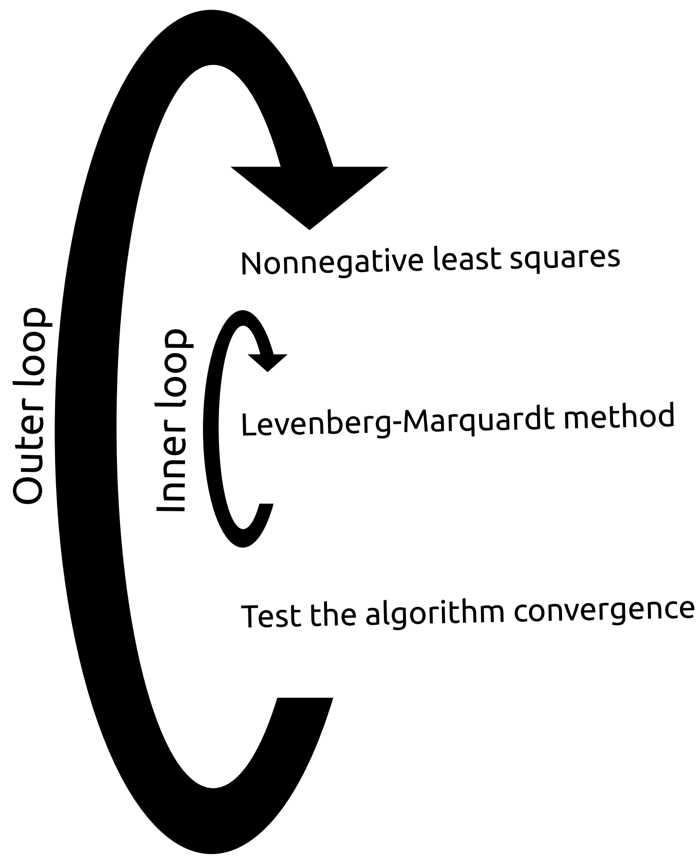

Figure 2: Iterative scheme overview for NNLS and Levenberg-Marquardt method for estimating magnetization direction. The outer loop is the nonnegative solution for magnetic-moment distribution and the inner loop calculates the magnetization direction correction using Levenberg-Marquardt method.

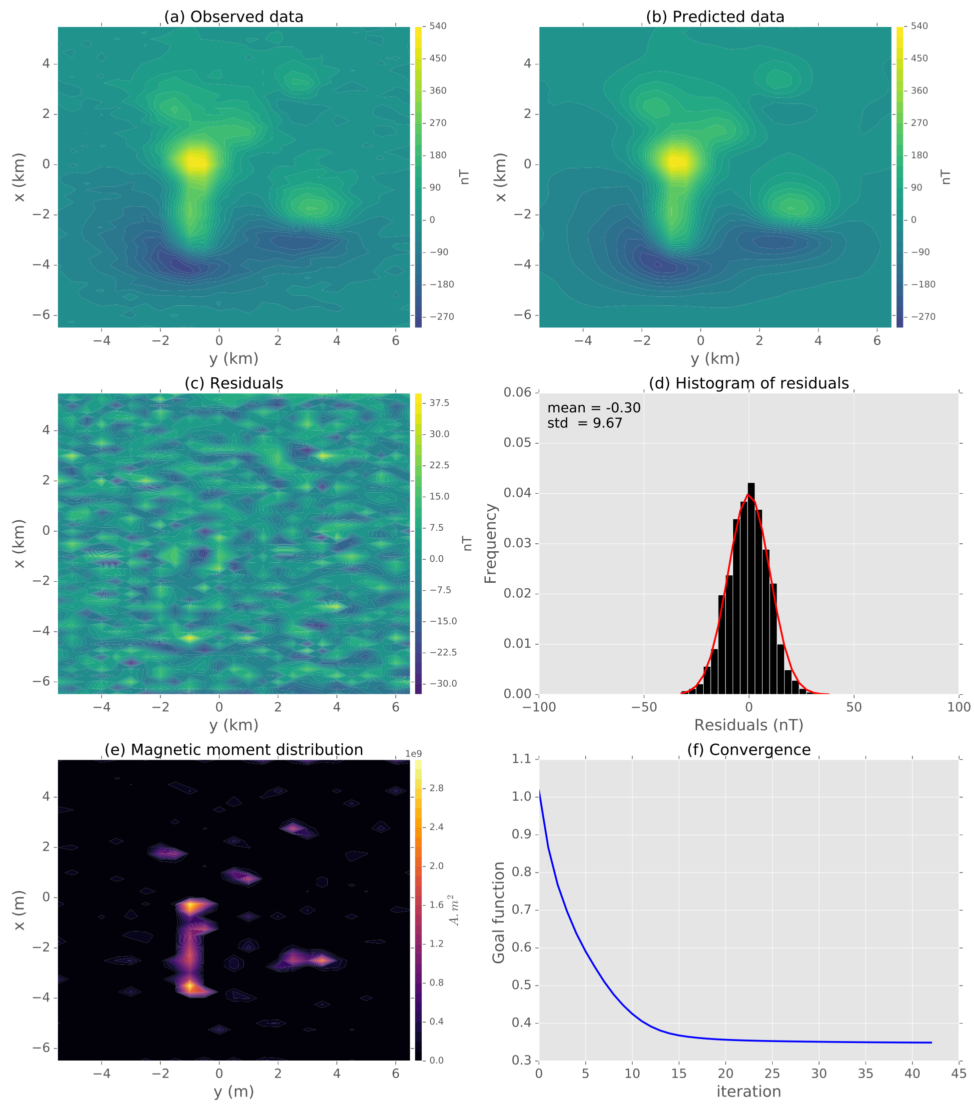

Figure 3: ~~Application to synthetic data for~~ unidirectional ~~model.~~ (a) Noise-corrupted data. (b) Predicted data produced by equivalent layer. (c) Difference between the data shown in panels ~~(a) and (b).~~ (d) Histogram of residuals. (e) All-positive magnetic~~-~~moment distribution. (f) Goal function value (equation 19a) per iteration showing the convergence.

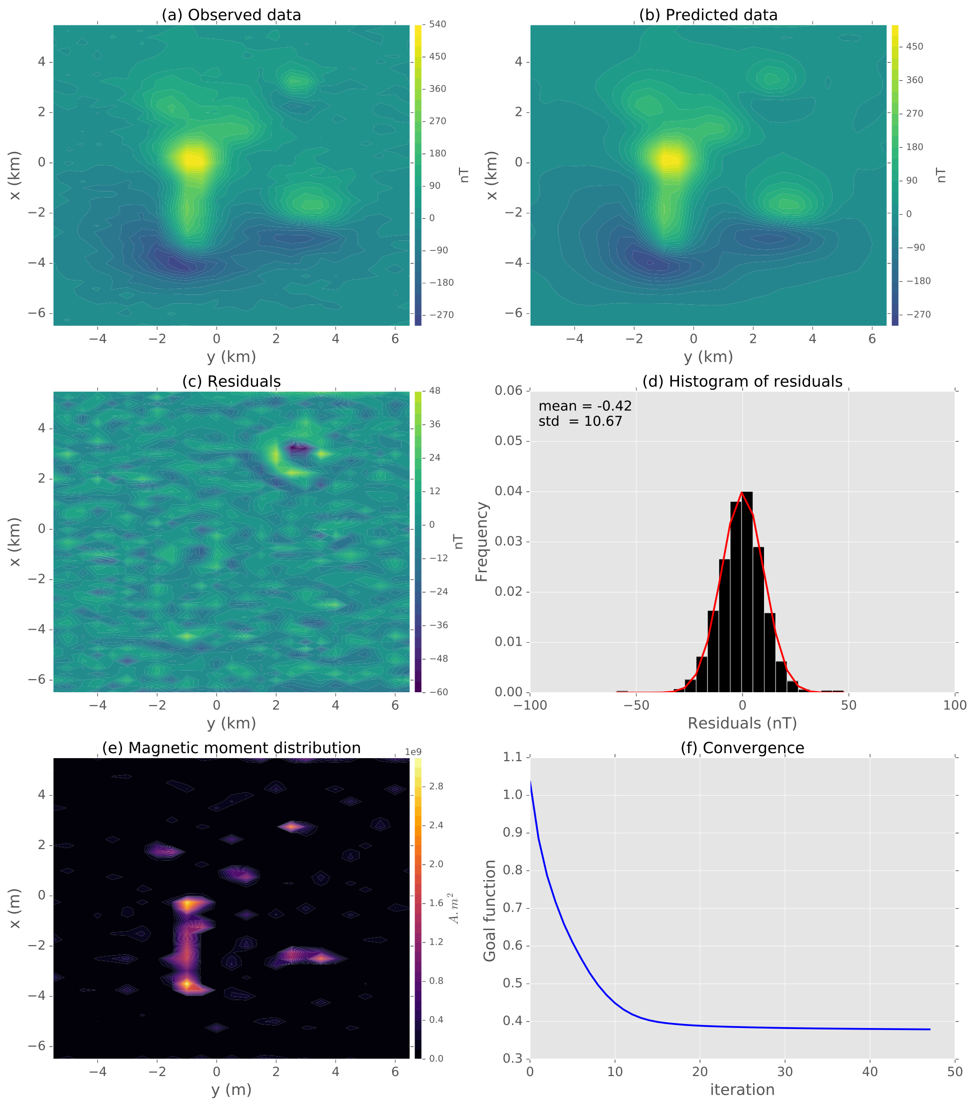

Figure 4: ~~Application to synthetic data with a shallow interfering source.~~ (a) Noise-corrupted ~~data~~. (b) Predicted data produced by equivalent layer. (c) Difference between the data shown in panels ~~(a) and (b).~~ (d) Histogram of residuals. (e) All-positive magnetic moment distribution. (f) Goal function value (equation 19a) per iteration showing the convergence.

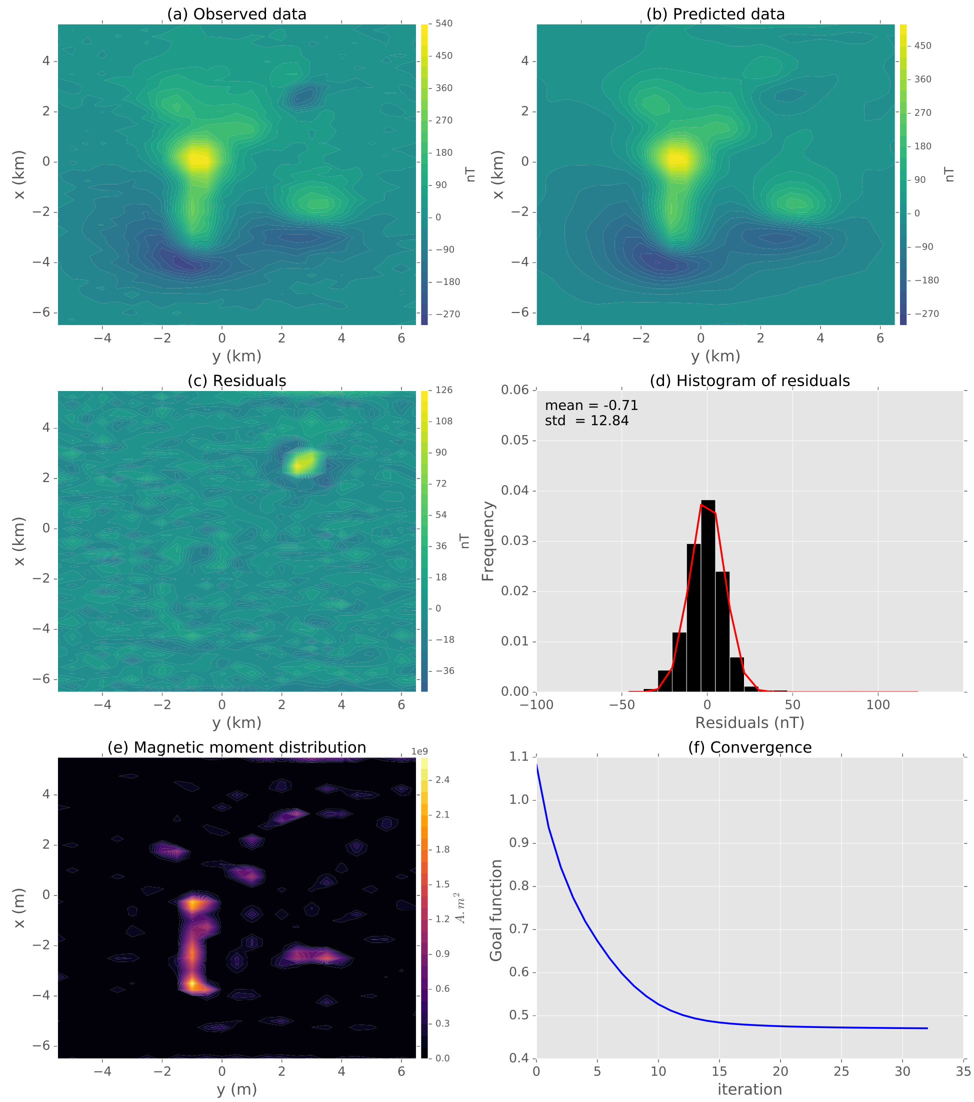

Figure 5: ~~Application to synthetic data with a shallow interfering source with different magnetization direction.~~ (a) Noise-corrupted ~~data.~~ (b) Predicted data produced by equivalent layer. (c) Difference between the data shown in panels ~~(a) and (b).~~ (d) Histogram of residuals. (e) All-positive magnetic moment distribution. (f) Goal function value (equation 19a) per iteration showing the convergence.

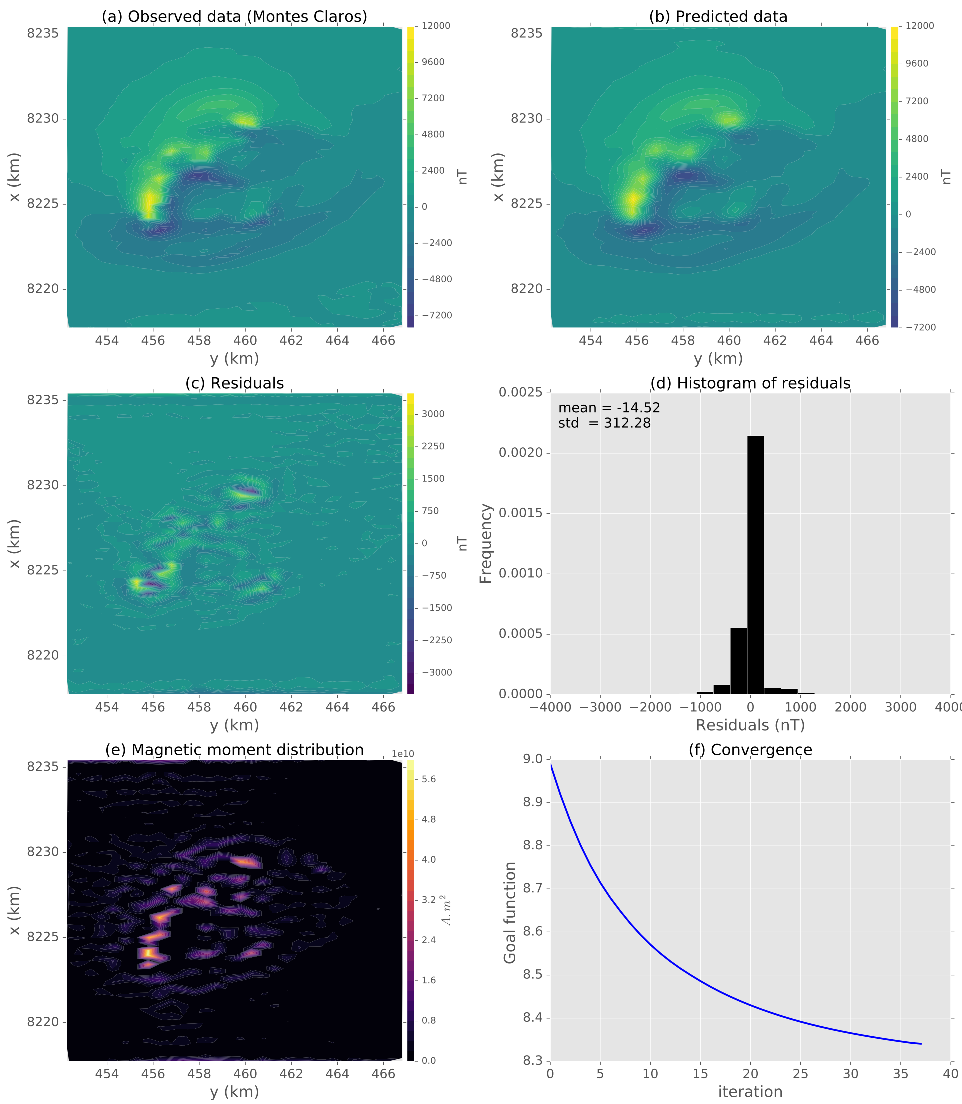

Figure 6: Application to field data ~~located in complex of~~ Montes Claros. (a) ~~Observation data.~~ (b) Predicted data produced by equivalent layer. (c) Difference between the data shown in panels ~~(a) and (b).~~ (d) Histogram of residuals. (e) All-positive magnetic moment distribution. (f) Goal function value (equation 19a) per iteration showing the convergence.

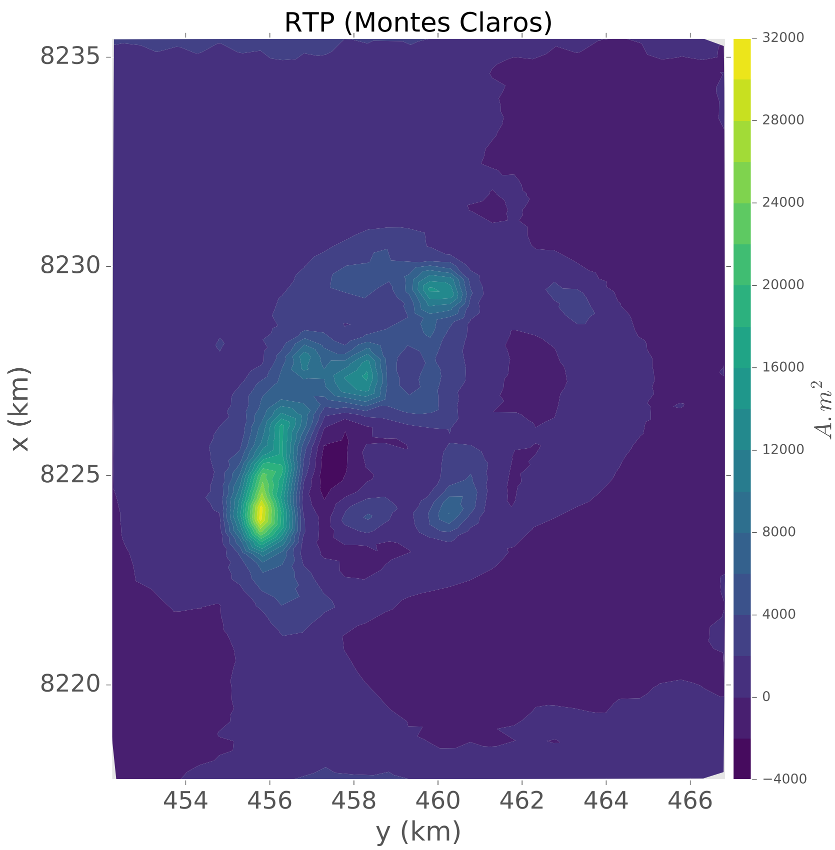

Figure 7: Application to field data ~~located in~~ complex of Montes Claros. RTP anomaly computed by using the estimated magnetization distribution shown in figure 6e.