3D inversion for estimating total magnetization direction using equivalent layer technique

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ABSTRACT

We have developed a new method for estimating the total magnetization direction of magnetic sources based on equivalent layer technique using total field anomaly data. In this approach, we do not have to impose a strong information about the shape and the depth of the sources, and do not require a regularly spaced data. Usually, this technique is used for processing potential data estimating a 2D magnetic moment distribution over a fictitious layer composed by dipoles below the observation plane. In certain conditions, when the magnetization direction of equivalent sources is almost the same of true body, the estimated magnetic property over the layer is all positve. The methodology uses a positivity constraint to estimate a set of magnetic moment over the layer and a magnetization direction of the layer through a iterative process. Mathematically, the algorithm solve a least squares problem in two steps: the first one solve a linear problem for estimating a magnetic moment and the second solve a non-linear problem for magnetization direction of the layer. We test the methodology applying to synthetic data for different geometries and

magnetization types of sources. Moreover, we applied this method to field data from Goias Alkaline Province (GAP), center of Brazil (FALAR DO RESULTADO DAS APLICAES NAS ANOMALIAS)

METHODOLOGY

Fundamentals of magnetic equivalent layer

Considering a Cartesian coordinate system with x-, y- and z-axis being oriented to north, east and downward, respectively. Let $\Delta T_i \equiv \Delta T(x_i, y_i, z_i)$ be the total field anomaly, at the ith position (x_i, y_i, z_i), produced by a continuos layer located below the observation plane on the depth z_c, where $z_c > z_i$, and $p(x', y', z_c)$ is the distribution of magnetic dipoles per unit area over the layer surface. In this case, the total field anomaly produced by this continuous layer is given by the equation

$$\Delta T_i = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p(x', y', z_c)[\gamma_m \hat{\mathbf{F}}_0^T \mathbf{H}\, \hat{\mathbf{h}}(\mathbf{q})] dx'\, dy', \tag{1}$$

where γ_m is a constant proportional to the free space permeability, $\hat{\mathbf{F}}_0$ is a unit vector with the same direction of the geomagnetic field given by

$$\hat{\mathbf{F}}_0 = \begin{bmatrix} \cos I \cos D \\ \cos I \sin D \\ \sin I \end{bmatrix}, \tag{2}$$

where I and D are the inclination and declination of main field, respectively. The $\mathbf{H}$ is a 3×3 dimensional matrix equal to

$$\mathbf{H} = \begin{bmatrix} \partial_{xx}\phi & \partial_{xy}\phi & \partial_{xz}\phi \\ \partial_{yx}\phi & \partial_{yy}\phi & \partial_{yz}\phi \\ \partial_{zx}\phi & \partial_{zy}\phi & \partial_{zz}\phi \end{bmatrix}, \tag{3}$$

where $\partial_{\alpha\beta}\phi$, $\alpha = x, y, z$, $\beta = x, y, z$, is the second derivative of the function

$$\phi(x-x', y-y', z-z_c) = \frac{1}{[(x-x')^2+(y-y')^2+(z-z_c)^2]^{\frac{1}{2}}}. \tag{4}$$

The $\hat{\mathbf{h}}(\mathbf{q})$ is a unit vector with the magnetization direction of the layer given by

$$\hat{\mathbf{h}}(\mathbf{q}) = \begin{bmatrix} \cos\tilde{\imath}\cos\tilde{d} \\ \cos\tilde{\imath}\sin\tilde{d} \\ \sin\tilde{\imath} \end{bmatrix} \tag{5}$$

and $\mathbf{q}$ is a vector with components given by

$$\mathbf{q} = \begin{bmatrix} \tilde{\imath} \\ \tilde{d} \end{bmatrix}, \tag{6}$$

where $\tilde{\imath}$ and $\tilde{d}$ is the inclination and declination of magnetization of the layer, respectively.

According to the theory, we can reproduce a set of N observed total field anomaly produced by a 3D magnetic source using a bidimensional physical-property distribution. In practical situations, the equivalent layer is composed by a set of M equivalent sources distributed with a constant depth h below the observation plane. It is worth pointing out that, in this work, the equivalent source is represented by a dipole with unit volume. For this reason, the vector $\mathbf{p}$ is the M-dimensional vector defined as parameter vector, whose jth element is the magnetic intensity of the jth equivalent source, and the vector $\mathbf{q}$ contains the inclination and the declination of each equivalent dipole. By discretizing the integrand of equation 1 in a set of points (x_j, y_j, z_c), $j = 1, \ldots, M$, the integral can be given by

$$\Delta T_i(\mathbf{p}, \mathbf{q}) = \sum_{j=1}^{M} p_j g_{ij}(\mathbf{q}) \tag{7}$$

where p_j is the magnetic moment of jth equivalent source and

$$g_{ij}(\mathbf{q}) = \gamma_m \hat{\mathbf{F}}_0^T \mathbf{H}_{ij} \hat{\mathbf{h}}(\mathbf{q}) \tag{8}$$

is a harmonic function that depends on the direction $\mathbf{q}$ of the dipole and the matrix $\mathbf{H}_{ij}$ is formed by the second derivatives of a function ϕ_{ij} that depends on the inverse of the function $r_{ij} = [(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_c)^2]^{1/2}$, analogously to equation 3 and 4.

Equation 7 represents the equivalent layer appproach. It is represented by the sum of the total field anomaly at the observation point (x_i, y_i, z_i) produced by a set of M ficticious equivalent sources, that is in this case a set of dipoles of unit volume, distributed on a horizontal plane at a constant depth z_c, each one with magnetic moment p_j and magnetization direction $\mathbf{q}$. In matrix notation, the equation 7 can be represented as

$$\mathbf{\Delta T}(\mathbf{p}, \mathbf{q}) = \mathbf{G}(\mathbf{q})\mathbf{p} \tag{9}$$

where $\mathbf{G}$ is $N \times M$ matrix composed by the elements g_{ij} of the equation 8.

Iterative process for magnetization estimation

Let $\mathbf{\Delta T}^o$ be an N-dimensional vector whose ith element ΔT_i^o is the total field anomaly observation produced by a magnetic source at the point (x_i, y_i, z_i), $i = 1, \ldots, N$. The estimation of a set of magnetic moments $\mathbf{p}$ and the magnetization direction $\mathbf{q}$ consists to solve a inverse problem of minimizing the difference between the observed total field anomaly $\mathbf{\Delta T}^o$ and the predicted data $\Delta T(\mathbf{p}, \mathbf{q})$ by the equivalent layer (equation 9). In other words, a stable estimates $\mathbf{p}^\sharp$ and $\mathbf{q}^\sharp$ can be obtained by minimizing the objective function given by

$$\Psi(\mathbf{p}, \mathbf{q}) = \| \mathbf{\Delta T}^o - \mathbf{\Delta T}(\mathbf{p}, \mathbf{q}) \|_2^2, \tag{10}$$

where $\Psi(\mathbf{p}, \mathbf{q})$ is the data misfit, which is the Euclidean norm of the difference between the $\mathbf{\Delta T}^o$ and $\mathbf{\Delta T}(\mathbf{p}, \mathbf{q})$.

The procedure of finding a set of magnetic moment $\mathbf{p}^\sharp$ and magnetization direction $\mathbf{q}^\sharp$ which minimize the equation 10 consists to solve an inverse problem for estimating a set of parameters in two steps. Therefore, we split the inverse problem in a mixed solution of two systems of equations. The first one solves a linear system for estimating the part of the magnetic moment. Secondly, the problem is solved through a non-linear process to calculate successive approximations for the part of the magnetization direction at each iteration along the process.

However, at the kth iteration, we impose positivity constraint on the magnetic-moment distribution estimate $\mathbf{p}^k$ by solving the following constrained problem of

$$\begin{aligned} &\text{minimizing} \quad \|\Delta \mathbf{T}^o - \mathbf{G}(\mathbf{q}_{k-1})\mathbf{p}^k\|_2^2 \\ &\text{subject to} \quad \mathbf{p}^k \geqslant 0 \end{aligned} \tag{11}$$

where $\mathbf{G}(\mathbf{q}_{k-1})$ is the $N \times M$ matrix defined in equation 9, $\| \cdot \|_2^2$ represents the squared Euclidean norm and $\mathbf{p}^k \geqslant 0$ means that the magnetic moments of all equivalent sources are positive. This problem is solved by using the nonnegative least squares (NNLS) proposed by (CITAR LAWSON HANSON 1974). In other words, we solve a linear system with positivity constraint at each kth iteration given by the equation

$$\mathbf{p}^k = \left(\mathbf{G}_p^{(k)T} \mathbf{G}_p^{(k)} \right)^{-1} \mathbf{G}_p^{(k)T} \Delta \mathbf{T}^o \tag{12}$$

where $\mathbf{G}_p^{(k)}$ is the magnetic-moment sensitivity matrix at the kth iteration. The elements of this matrix are composed by derivative of equation 7 in relation of jth element of the vector $\mathbf{p}^k$.

After estimating the magnetic-moment distribution $\mathbf{p}^k$ at the kth iteration using the previous estimate $\mathbf{q}_{k-1}$ for the magnetization direction, we estimate a new vector $\mathbf{q}^k$ by solving an unconstrained nonlinear inverse problem of minimizing the squared Euclidean norm of the difference between the observed and predicted total-field anomalies. In this nonlinear inversion we use the Levenberg-Marquardt method (CITAR ASTER). That is, we calculate at each kth iteration the step $\Delta\mathbf{q}^k$ for the magnetization direction by using the equation

$$\Delta\mathbf{q}^k = (\mathbf{G}_q^{(k)T}\mathbf{G}_q^{(k)} + \lambda\mathbf{I})^{-1}\mathbf{G}_q^{(k)T}\mathbf{r}^k \tag{13}$$

where λ is the Marquardt parameter that is updated along the iterative process, $\mathbf{I}$ is a indentity matrix, and the residual at the kth iteration $r^k = \Delta\mathbf{T}^o - \boldsymbol{\Delta}\mathbf{T}(\mathbf{p}^k, \mathbf{q}^{k-1})$. $\mathbf{G}_q^k$ is a sensitivity matrix of the magnetization direction part, whose elements are composed by derivative of equation 7 in relation of each component of the vector $\mathbf{q}^k$, that are the inclination and declination, respectively. The iterative process stops when the squared Euclidean norm of the difference between the observed data $\Delta\mathbf{T}^o$ and predicted data $\boldsymbol{\Delta}\mathbf{T}(\mathbf{p}, \mathbf{q})$ (equation 9) is invariant along succesive iterations.

REFERENCES