Estimating total magnetization direction using equivalent-layer technique

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ABSTRACT

We have developed a new method for estimating the total magnetization direction of magnetic sources based on equivalent layer technique using total field anomaly data. In this approach, we do not have to impose a strong information about the shape and the depth of the sources, and do not require a regularly spaced data. Usually, this technique is used for processing potential data estimating a 2D magnetic moment distribution over a fictitious layer composed by dipoles below the observation plane. In certain conditions, when the magnetization direction of equivalent sources is almost the same of true body, the estimated magnetic property over the layer is all positve. The methodology uses a positivity constraint to estimate a set of magnetic moment over the layer and a magnetization direction of the layer through a iterative process. Mathematically, the algorithm solve a least squares problem in two steps: the first one solve a linear problem for estimating a magnetic moment and the second solve a non-linear problem for magnetization direction of the layer. We test the methodology applying to synthetic data for a complicated scenarios and geometries of sources. Moreover, we applied the method to field data from Goias Alkaline Province (GAP), center of Brazil, over Montes Claros complex. The result suggests the area is composed by intrusions with remarkable strong remanent magnetization component, being in agreement with the current literature for this region.

METHODOLOGY

Fundamentals of magnetic equivalent layer and the positive magnetic-moment distribution

Considering a Cartesian coordinate system with x-, y- and z-axis being oriented to north, east and downward, respectively. Let $\Delta T_i \equiv \Delta T(x_i, y_i, z_i)$ be the total-field anomaly, at the ith position (x_i, y_i, z_i), produced by a continuos layer located below the observation plane at a depth equal to z_c, where $z_c > z_i$, and $p(x', y', z_c)$ is the distribution of magnetic dipoles per unit area over the layer. The total-field anomaly produced by this continuous layer is given by

$$\Delta T_i = \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} p(x', y', z_c)[\gamma_m \hat{\mathbf{F}}_0^T \mathbf{M}(x_i, y_i, z_i, x', y', z_c)\, \hat{\mathbf{m}}(\mathbf{q})] dx'\, dy', \tag{1}$$

where γ_m is a constant proportional to the vacuum permeability, $\hat{\mathbf{F}}_0$ is a unit vector with the same direction of the main geomagnetic field given by

$$\hat{\mathbf{F}}_0 = \begin{bmatrix} \cos I \cos D \\ \cos I \sin D \\ \sin I \end{bmatrix}, \tag{2}$$

where I and D are, respectively, the inclination and declination and $\mathbf{M}(x_i, y_i, z_i, x', y', z_c)$ is a 3×3 dimensional matrix (PAPER LADY DAI) equal to

$$\mathbf{M}(x_i, y_i, z_i, x', y', z_c) = \begin{bmatrix} \partial_{xx}\phi & \partial_{xy}\phi & \partial_{xz}\phi \\ \partial_{yx}\phi & \partial_{yy}\phi & \partial_{yz}\phi \\ \partial_{zx}\phi & \partial_{zy}\phi & \partial_{zz}\phi \end{bmatrix}, \tag{3}$$

where $\partial_{\alpha\beta}\phi$, $\alpha = x, y, z$ and $\beta = x, y, z$, is the second derivative of the scalar function

$$\phi(x_i, y_i, z_i, x', y', z_c) = \frac{1}{[(x_i - x')^2 + (y_i - y')^2 + (z_i - z_c)^2]^{\frac{1}{2}}}. \tag{4}$$

with respect to the Cartesian coordinates x_i, y_i and z_i of the observation points. The $\hat{\mathbf{m}}(\mathbf{q})$ is a unit vector with the magnetization direction of the dipoles over layer given by

$$\hat{\mathbf{m}}(\mathbf{q}) = \begin{bmatrix} \cos \tilde{\imath} \cos \tilde{d} \\ \cos \tilde{\imath} \sin \tilde{d} \\ \sin \tilde{\imath} \end{bmatrix} \tag{5}$$

and $\mathbf{q}$ is a 2×1 vector with components given by

$$\mathbf{q} = \begin{bmatrix} \tilde{\imath} \\ \tilde{d} \end{bmatrix}, \tag{6}$$

where $\tilde{\imath}$ and $\tilde{d}$ are the inclination and declination of the magnetization direction of the dipoles on the layer, respectively. We can also notice that the vector defined in equation 5 has a single and uniform magnetization direction of all dipoles on the layer. For convenience, this unit vector can be rewritten as follows

$$\hat{\mathbf{m}}(\mathbf{q}) = \mathbf{R}\hat{\mathbf{h}}\,, \tag{7}$$

where $\hat{\mathbf{h}}$ defines the uniform magnetization direction of an abitrary magnetic source and $\mathbf{R}$ is a $3{\times}3$ matrix obtained from Euler's rotation theorem. This theorem states that any rotation can be parametrized by using three parameters called Euler angles (CITAR GOLDSTEIN). That is, if all dipoles that set up the equivalent layer have the same magnetization direction $\hat{\mathbf{m}}(\mathbf{q})$ and this direction is the same as the true magnetic source $\hat{\mathbf{h}}$, then the matrix $\mathbf{R}$

(equation 7) is equal to identity. For this reason, the total-field anomaly produced by equivalent layer at the ith position (x_i, y_i, z_i) (equation 1) can be rewritten as

$$\Delta T_i = \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} p(x', y', z_c)[\gamma_m \hat{\mathbf{F}}_0^T \mathbf{M}(x_i, y_i, z_i, x', y', z_c)\, \hat{\mathbf{h}}]dx'\, dy', \tag{8}$$

which represents the total-field anomaly produced by continuous layer with the same direction of the arbitrary magnetic source. Thus, the RTP field ΔT_i^{PL} produced by equivalent layer at the point (x_i, y_i, z_i) is equal to

$$\Delta T_i^{PL} = \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} p(x', y', z_c)[\gamma_m \partial_{zz}\phi(x_i, y_i, z_i, x', y', z_c)]dx'\, dy', \tag{9}$$

where $\partial_{zz}\phi(x_i, y_i, z_i, x', y', z_c)$ is the second derivative of the inverse of distance (equation 4) with respect of z_i, evaluated at the point (x_i, y_i, z_i). However, by considering the RTP field ΔT_i^{PS} produced by an arbitrary uniformly magnetized source, we have

$$\Delta T_i^{PS} = \gamma_m \partial_{zz}\Gamma(x_i, y_i, z_i)\, m, \tag{10}$$

which represents the total-field anomaly produced at the pole, where m is the magnetization intensity of the magnetic source. The $\partial_{zz}\Gamma(x_i, y_i, z_i)$ is the second derivative in relation to z_i of a scalar function $\Gamma(x_i, y_i, z_i)$ that depends on source geometry and the observation point (x_i, y_i, z_i). From equation 9 and 10, we obtain

$$m\, \partial_{zz}\Gamma(x_i, y_i, z_i) = \int\limits_{-\infty}^{+\infty}\int\limits_{-\infty}^{+\infty} p(x', y', z_c)\partial_{zz}\phi(x_i, y_i, z_i, x', y', z_c)dx'\, dy'. \tag{11}$$

We can notice that equation 11 can be calculated differentiating the following equation

$$m\,\partial_z\Gamma(x_i, y_i, z_i) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{p(x', y', z_c)(z_c - z_i)}{[(x_i - x')^2 + (y_i - y')^2 + (z_i - z_c)^2]^{\frac{3}{2}}} dx'\, dy', \tag{12}$$

where $z_c > z_i$, with respect to the vertical component z_i. From potential-field theory, we can highlight the classical upward continuation integral

$$U(x_i, y_i, z_i) = \frac{(z_c - z_i)}{2\pi} \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \frac{U(x', y', z_c)}{[(x_i - x')^2 + (y_i - y')^2 + (z_i - z_c)^2]^{\frac{3}{2}}} dx'\, dy', \tag{13}$$

where the function $U(x_i, y_i, z_i)$ is an hamornic function at all (x_i, y_i, z_i) (CITAR BLAKELY). In this case, this function represents the total-field anomaly at the point (x_i, y_i, z_i) which can be mathematically interpreted as the convolution between its values $U(x', y', z_c)$ and the vertical derivative in relation to z_i of the equation 4, evaluated on the horizontal plane $z_i = z_c$. Therefore, according the classical upward continuation function (equation 13), the magnetic-moment distribution $p(x', y', z_c)$ in equation 12 assumes the form

$$p(x', y', z_c) = \frac{m}{2\pi}\,\partial_z\Gamma(x', y', z_c), \tag{14}$$

where $\partial_z\Gamma(x', y', z_c)$ is the derivative of the scalar function $\partial_z\Gamma(x_i, y_i, z_i)$ in relation to z_i evaluated over the equivalent layer. The most interesting aspect of equation 14 is that the magnetic-moment distribution is defined as the product of a positive constant $\frac{m}{2\pi}$ and the function $\partial_z\Gamma(x', y', z_c)$, which is all positive at all points (x', y', z_c) over the equivalent layer. This relation is simular to that presented by CITAR PEDERSEN (1991) and LI (2014). In the wavenumber domain, these authors determined the magnetic-moment distribution within a continuous equivalent layer with the same magnetization direction as the local-geomagnetic field at the pole. They also considered a planar equivalent layer located below and parallel to a horizontal plane containing the observed total-field anomaly. They assume a magnetic source having a purely induced magnetization. Under these assumptions, CITAR PEDERSEN (1991) and LI (2014) concluded that the magnetic moment distribution within the continuous equivalent layer is proportional to the pseudogravity anomaly produced by the source on the plane of the equivalent layer. By following different approaches, CITAR BARATCHART (2013) and Lima (2016) pointed out that, by imposing a nonnegativity constraint, the solution to the inverse problem is a unique distribution for the magnetic equivalent sources. Here, we do not follow the same wavenumber-domain reasoning used by these authors. Equation 14, however, generalizes this positivity condition because it (1) holds true for all cases in which the magnetization of the equivalent layer has the same direction as the true magnetization of the sources, whenever it is purely induced or not, (2) does not require that the observed total-field anomaly data be on a plane and, (3) does not require a planar equivalent layer.

Forward problem and iterative process for magnetization estimation

However, in practical situations, its not possible to determine a continuous magnetic-moment distribution $p(x', y', z_c)$ over the layer as shown in equation 1. For this reason, the continuous equivalent layer have to be approximated by a discrete set of M dipoles with unit volume located at a constant depth $z = z_c$. Let $\mathbf{p}$ be an M-dimensional parameter vector, whose jth element p_j is the magnetic moment of the jth dipole and $\mathbf{q}$ be a vector containing the inclination $\tilde{i}$ and declination $\tilde{d}$ of all dipole, analogously to equation 6. Mathematically, by discretizing the integrand of equation 1, the total-field anomaly produced by equivalent layer at the point (x_i, y_i, z_i) is given by

The choice of layer depth $\mathbf{z_c}$ and regularization parameter μ

The procedure for the use of our methodology for estimating the total magnetization require the choice of two main parameters. The first one is the layer depth z_c as shown in figure 1 and the second is the regularization parameter μ shown in equation **??**.

The method of the choice of layer is based on a classical approach proposed by (CITAR DAMPNEY). The author pointed out that the layer depth should satisfy an interval from 2.5 to 6 times the grid spacing below the observation plane. It should be notice that the rule proposed by (CITAR DAMPNEY) was applied on an evenly spaced data. However, the choice for applying our method should correspond to an interval from 2 to 3 times to the greater grid spacing. It is necessarily to point out that this range of values was found empirically.

To solve the equation **??** we have to choose a reliable regularization parameter μ. For this purpose, we use the L-curve method proposed by (CITAR HANSEN 1992). This approach is widely used in the literature to find a regularization parameter which filtering out enough noise whithout loosing to much information in the final solution. The procedure of finding the parameter is basically to plot a curve of optimal values between the solution norm and residual norm. The corner of the curve is the final result which gives a threshold between the regularization function and the data misfit.

APPENDIX A

CONSEQUENCES OF HIGH-LATITUDE ESTIMATION

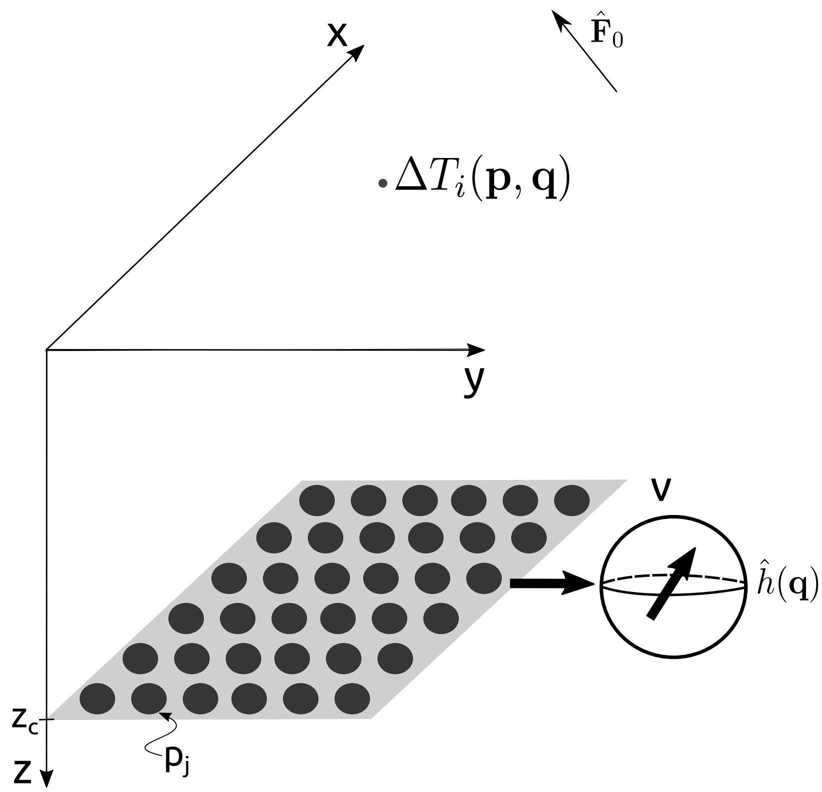

Figure 1: Schematic representation of an equivalent layer. The layer is positioned over the horizontal plane at a depth of $z = z_c$. $\Delta T_i(\mathbf{p}, \mathbf{q})$ is the predicted total-field anomaly at the point (x_i, y_i, z_i) produced by the set of M equivalent sources (black dots). Each source is located at the point (x_j, y_j, z_c), $j = 1, \ldots, M$, and represented by a dipole with unity volume v with magnetization direction $\hat{\mathbf{h}}(\mathbf{q})$ and magnetic moment p_j.

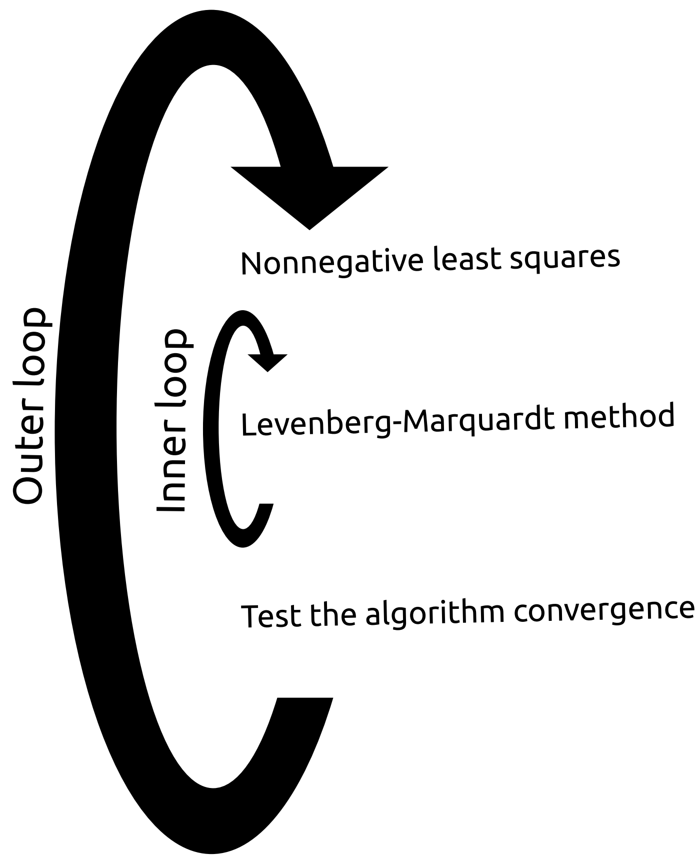

Figure 2: Iterative scheme overview for NNLS and Levenberg-Marquardt method for estimating magnetization direction. The outer loop is the nonnegative solution for magnetic-moment distribution and the inner loop calculates the magnetization direction using Levenberg-Marquardt method.