3D inversion for estimating total magnetization direction using equivalent layer technique

André L. A. Reis * and Vanderlei C. Oliveira Jr.*

* *Observatório Nacional, Rio de Janeiro, Brazil*

e-mails: decoluisreis@gmail.com, vanderlei@on.br

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ABSTRACT

We have developed a new methor for estimating the total magnetization direction of magnetic sources based on equivalent layer technique using total field anomaly data. In this approach, we do not have to impose a strong information about the shape and the depth of the sources, and do not require a regularly spaced data. Usually, this technique is used for processing potential data estimating a 2D magnetic moment distribution over a ficticious layer composed by dipoles below the observation plane. In certain conditions, when the magnetization direction of equivalent sources is almost the same of true body, the estimated magnetic property over the layer is all positve. The methodology uses a positivity constraint to estimate a set of magnetic moment and a magnetization direction of the layer through a iterative process. Mathematically, the algorithm solve a least squares problem in two steps: the first one solve a linear problem for estimating a magnetic moment and the second solve a non-linear problem for magnetization direction of the layer. We test the methodology applying to synthetic data for different geometries and magnetization types

of sources. Moreover, we applied this method to field data from Goias Alkaline Province (GAP), center of Brazil, showing that the methodology can be a good tool for estimating the magnetization component of the alkaline intrusion complex in the Diorama region. The result for this complex suggests that this source has a remarkable strong remanent magnetization component. The magnetization direction estimated for this complex is $-47°$ and $-111°$ for inclination and declination, respectivelly.

METHODOLOGY

Fundamentals of magnetic equivalent layer

Considering a Cartesian coordinate system with x-, y- and z-axis being oriented to north, east and downward, respectively. Let $\Delta T_i \equiv \Delta T(x_i, y_i, z_i)$ be the total field anomaly, at the i-th position (x_i, y_i, z_i), produced by a continuos layer located below the observation plane on the depth z_c, where $z_c > z_i$, and $p(x', y', z_c)$ is the distribution of magnetic dipoles per unit area over the layer surface. In this case, the total field anomaly produced by this continuous layer is given by the equation

$$\Delta T_i = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p(x', y', z_c)[\gamma_m \hat{\mathbf{F}}_0^T \mathbf{H} \, \hat{\mathbf{h}}(\mathbf{q})] dx' \, dy', \tag{1}$$

where γ_m is a constant proportional to the vaccum permeability, $\hat{\mathbf{F}}_0$ is a unit vector with the same direction of the geomagnetic field $\mathbf{F}_0$ and $\mathbf{H}$ is a 3×3 matrix equal to

$$\mathbf{H} = \begin{bmatrix} \partial_{xx}\phi & \partial_{xy}\phi & \partial_{xz}\phi \\ \partial_{yx}\phi & \partial_{yy}\phi & \partial_{yz}\phi \\ \partial_{zx}\phi & \partial_{zy}\phi & \partial_{zz}\phi \end{bmatrix}, \tag{2}$$

where $\partial_{\alpha\beta}\phi$, $\alpha = x, y, z$, $\beta = x, y, z$, is the second derivative of the function

$$\phi(x - x', y - y', z - z_c) = \frac{1}{r}, \tag{3}$$

where $r = [(x-x')^2 + (y-y')^2 + (z-z_c)^2]^{1/2}$ and $\hat{\mathbf{h}}(\mathbf{q})$ is a unit vector with the magnetization direction of the layer that depends on the vector $\mathbf{q}$ given by

$$\mathbf{q} = \begin{bmatrix} i \\ \\ d \end{bmatrix}, \tag{4}$$

where i and d is the inclination and declination, respectively.

According the theory, we can reproduce a set of N observed total field anomaly produced by a 3D magnetic source using a bidimensional physical-property distribution. In practical situations, the equivalent layer is composed by a set of M equivalent sources distributed with a constant depth h below the observation plane. It is worth pointing out that, in this work, the equivalent source is represented by a dipole with unit volume. For this reason, the vector $\mathbf{p}$ is the M-dimensional vector defined as parameter vector, whose jth element is the magnetic intensity of the jth equivalent source, and the vector $\mathbf{q}$ contains the inclination and the declination of each equivalent dipole. By discretizing the integrand of equation 1 in a set of points (x_j, y_j, z_c), $j = 1, \ldots, M$, the integral can be given by

$$\Delta T_i(\mathbf{p}, \mathbf{q}) = \sum_{j=1}^{M} p_j g_{ij}(\mathbf{q}) \tag{5}$$

where p_j is the magnetic moment of jth equivalent source and

$$g_{ij}(\mathbf{q}) = \gamma_m \hat{\mathbf{F}}_0^T \mathbf{H}_{ij} \hat{\mathbf{h}}(\mathbf{q}) \tag{6}$$

is a harmonic function that depends on the direction $\mathbf{q}$ of the dipole and the matrix $\mathbf{H}_{ij}$ is formed by the second derivatives of a function ϕ_{ij} that depends on $r_{ij} = [(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_c)^2]^{1/2}$, analogously to equation 2 and 3.

Equation 5 represents the equivalent layer appproach. It is represented by the sum of the total field anomaly at the observation point (x_i, y_i, z_i) produced by a set of M

ficticious equivalent sources, that is in this case a set of dipoles of unit volume, distributed on a horizontal plane at a constant depth z_c, each one with magnetic moment p_j and magnetization direction $\mathbf{q}$. In matrix notation, the equation 5 can be represented as

$$\mathbf{\Delta T}(\mathbf{p}, \mathbf{q}) = \mathbf{G}(\mathbf{q})\mathbf{p} \tag{7}$$

where $\mathbf{G}$ is $N \times M$ matrix composed by the elements g_{ij} of the equation 6.

Iterative process for magnetization estimation

Let $\mathbf{\Delta T}^o$ be an N-dimensional vector whose ith element ΔT_i^o is the total field anomaly observation produced by a magnetic source at the point (x_i, y_i, z_i), $i = 1, \ldots, N$. The estimation of a set of magnetic moments $\mathbf{p}$ and the magnetization direction $\mathbf{q}$ consists to solve a inverse problem of minimizing the difference between the observed total field anomaly $\mathbf{\Delta T}^o$ and the predicted data $\Delta T(\mathbf{p}, \mathbf{q})$ by the equivalent layer (equation 7). In other words, a stable estimates $\mathbf{p}^\sharp$ and $\mathbf{q}^\sharp$ can be obtained by minimizing the function given by

$$\Psi(\mathbf{p}, \mathbf{q}) = \parallel \mathbf{\Delta T}^o - \mathbf{\Delta T}(\mathbf{p}, \mathbf{q}) \parallel_2^2, \tag{8}$$

where $\Psi(\mathbf{p}, \mathbf{q})$ is the data misfit, which is the Euclidean norm of the difference between the $\mathbf{\Delta T}^o$ and $\mathbf{\Delta T}(\mathbf{p}, \mathbf{q})$.

The procedure to find a set of $\mathbf{p}^\sharp$ and $\mathbf{q}^\sharp$ which minimize the equation 8 consists to solve an inverse problem iteratively from an initial step. To accomplish this process, a initial approximation for the magnetic moment... (PAREI AQUI)

REFERENCES