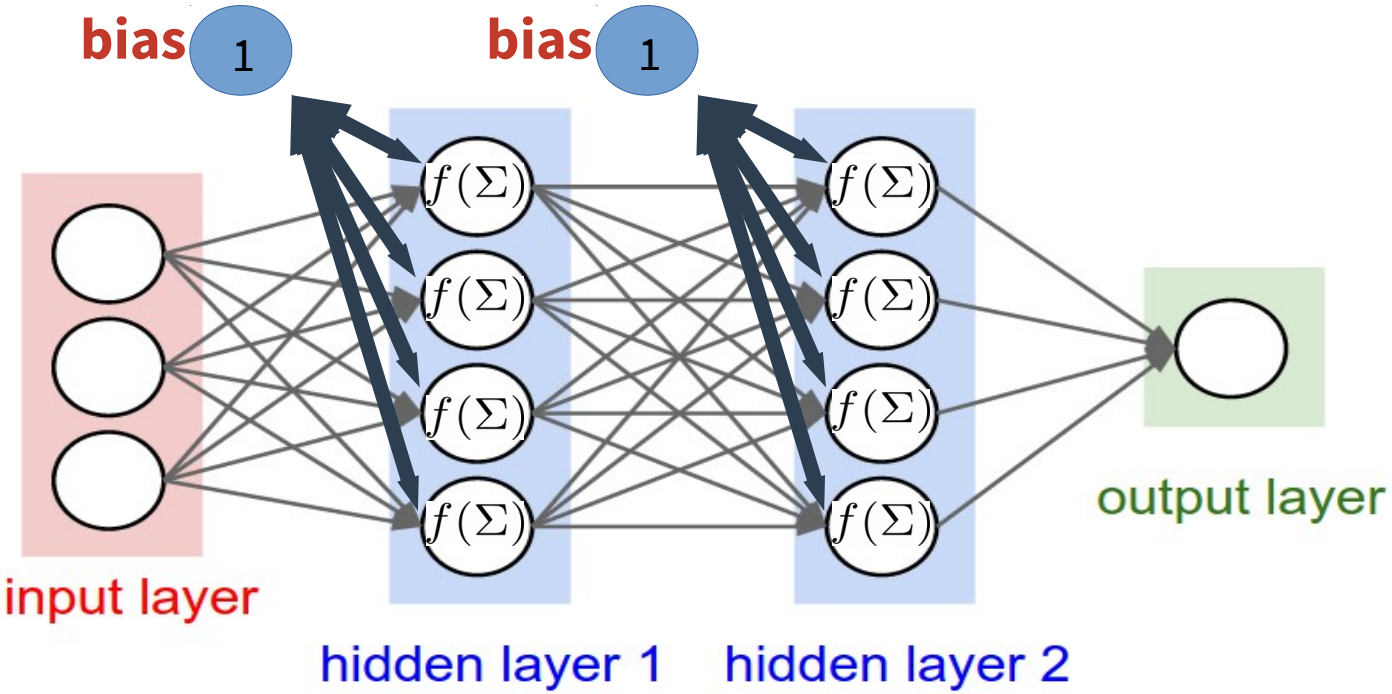
Machine Learning Using Tensorflow

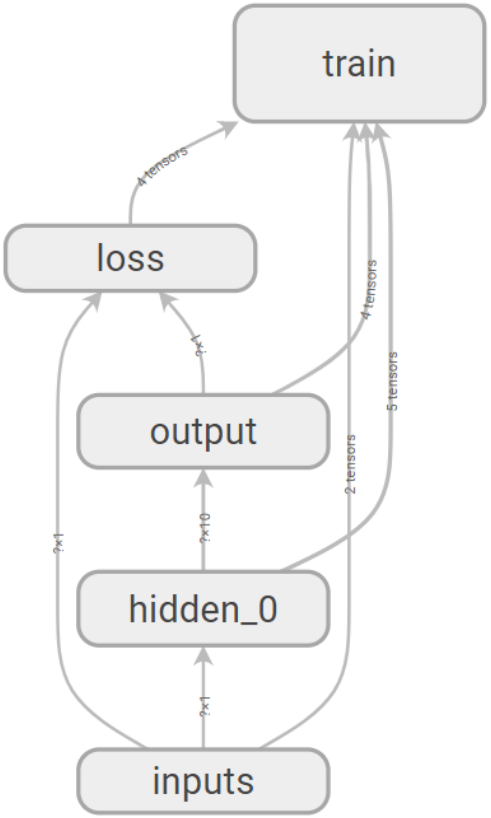
Week4:
Basis of Deep Learning : rethink deep learning

Shu-Ting Pi, PhD
UC Davis

TensorFlow ™

Really understand everything?





Questions

Overfitting

- In general, we can fit “any” data if parameters are many

Dimensionality Curse

- How many data are consider “enough”?

Gradient vanishing and explosion

- How deep can a neural network be?
- Why Relu is better?

Information Theory

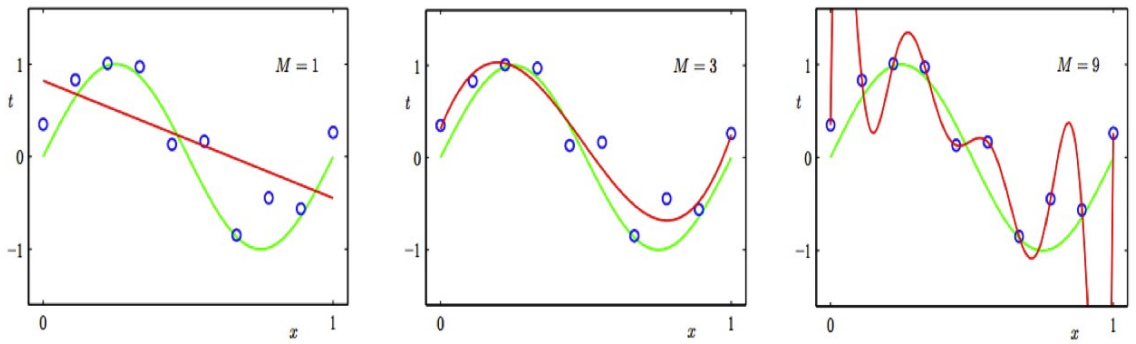
- Ask again: what is deep learning?

Overfitting

Overfitting

Under- and Over-fitting examples

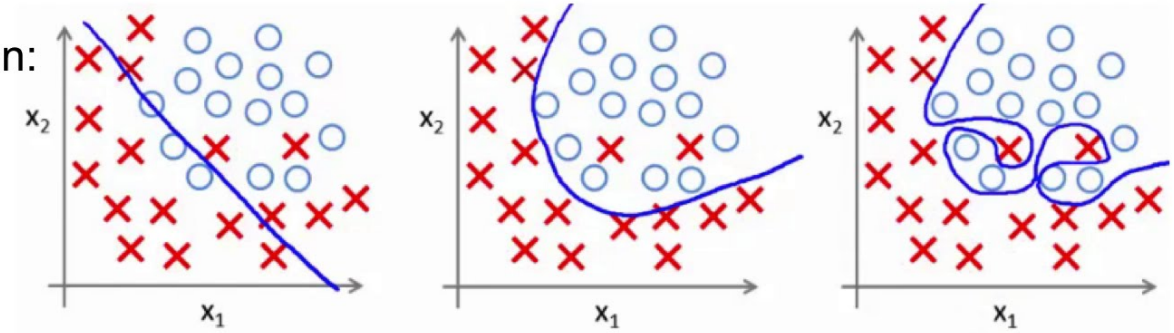
Regression:



predictor too inflexible:
cannot capture pattern

predictor too flexible:
fits noise in the data

Classification:



Regularization

Regularization

When number of parameters in model is high, overfitting is very probable

Solution: add a *regularization term* to the loss function:

$$\mathcal{L} = \frac{1}{N} \sum_i L(x_i, y_i) + \mathcal{L}_{\text{reg}} \rightarrow \min$$

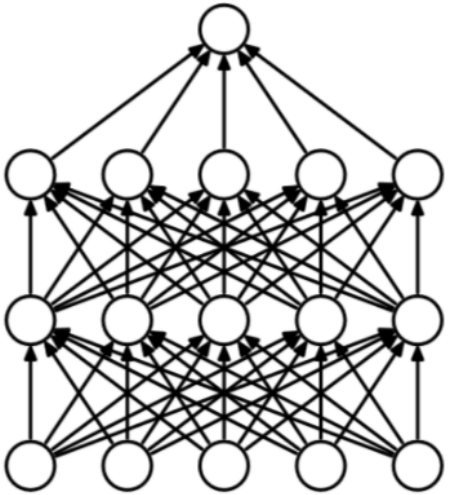
- L_2 regularization : $\mathcal{L}_{\text{reg}} = \alpha \sum_j |w_j|^2$
- L_1 regularization: $\mathcal{L}_{\text{reg}} = \beta \sum_j |w_j|$
- $L_1 + L_2$ regularization: $\mathcal{L}_{\text{reg}} = \alpha \sum_j |w_j|^2 + \beta \sum_j |w_j|$

Hint: Lagrangian constraints

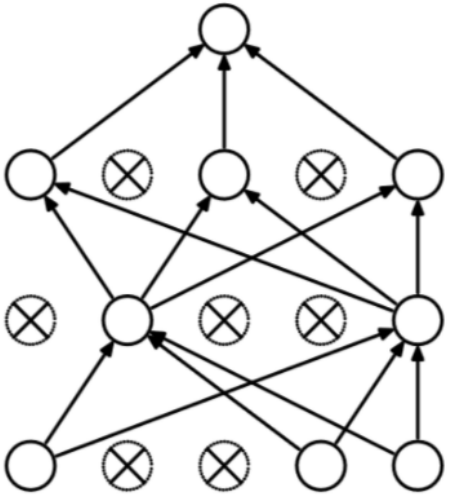
L1: make more W zero, non-derivative
L2: won't make more W zero, derivative

40 / 99

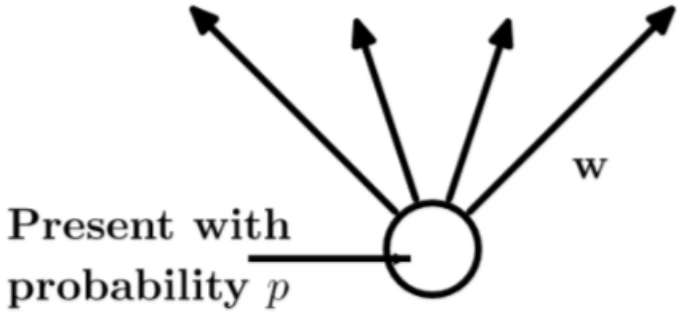
Dropout



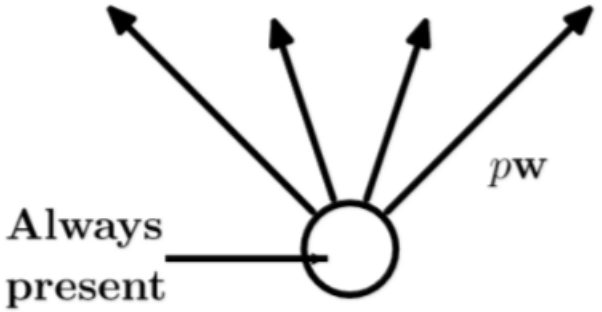
(a) Standard Neural Net



(b) After applying dropout.

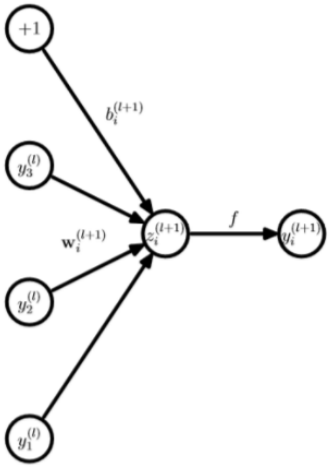


(a) At training time

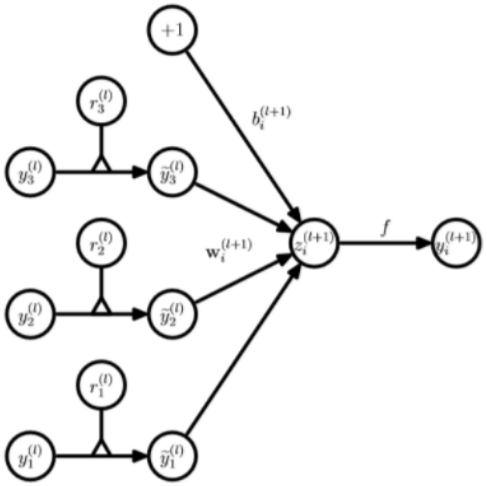


(b) At test time

Dropout



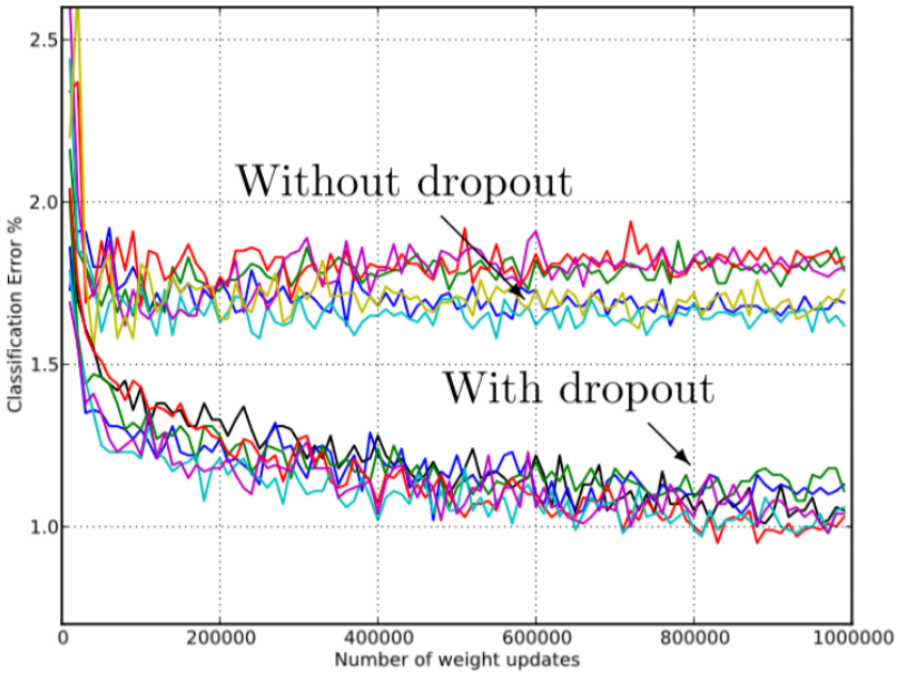
(a) Standard network



(b) Dropout network

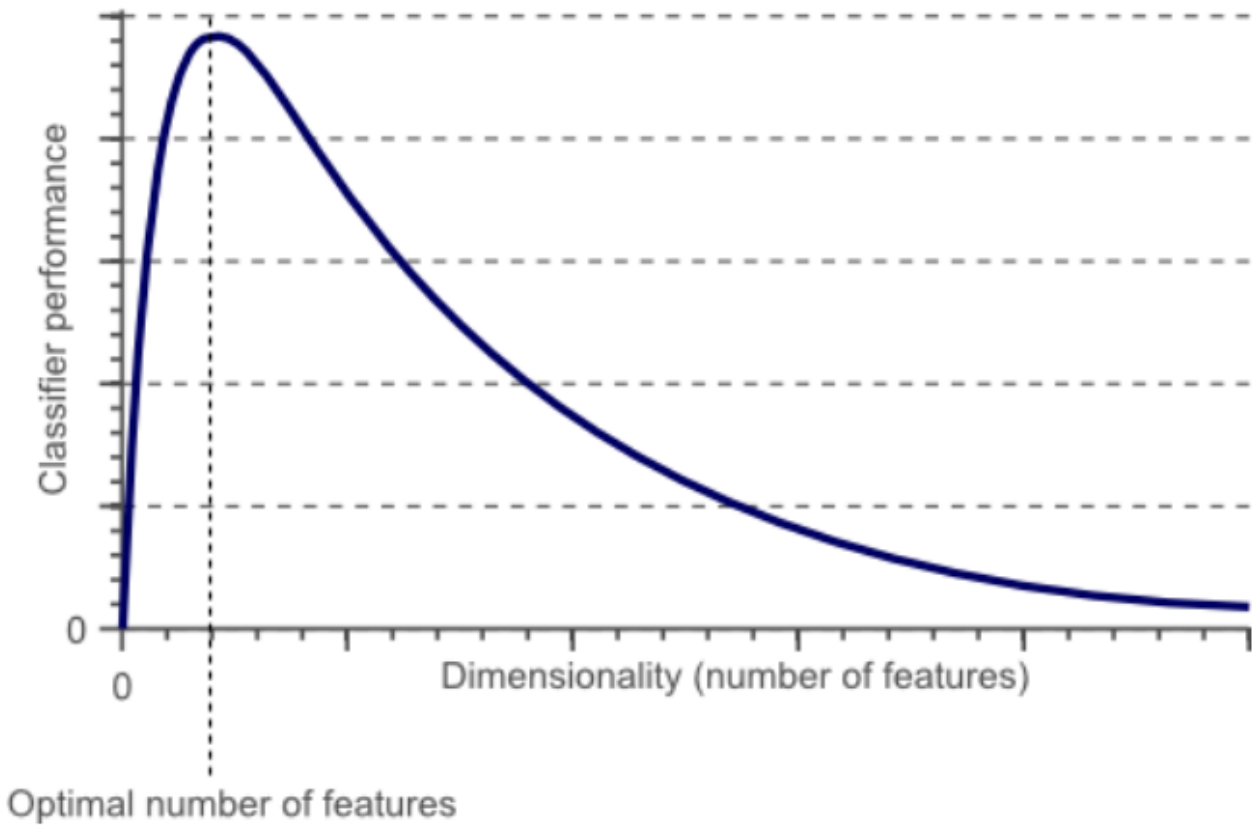
$$
\begin{aligned}
z_i^{(l+1)} &= \mathbf{w}_i^{(l+1)} \mathbf{y}^l + b_i^{(l+1)}, \\
y_i^{(l+1)} &= f(z_i^{(l+1)}),
\end{aligned}
$$

$$
\begin{aligned}
r_j^{(l)} &\sim \text{Bernoulli}(p), \\
\tilde{\mathbf{y}}^{(l)} &= \mathbf{r}^{(l)} * \mathbf{y}^{(l)}, \\
z_i^{(l+1)} &= \mathbf{w}_i^{(l+1)} \tilde{\mathbf{y}}^l + b_i^{(l+1)}, \\
y_i^{(l+1)} &= f(z_i^{(l+1)}).
\end{aligned}
$$



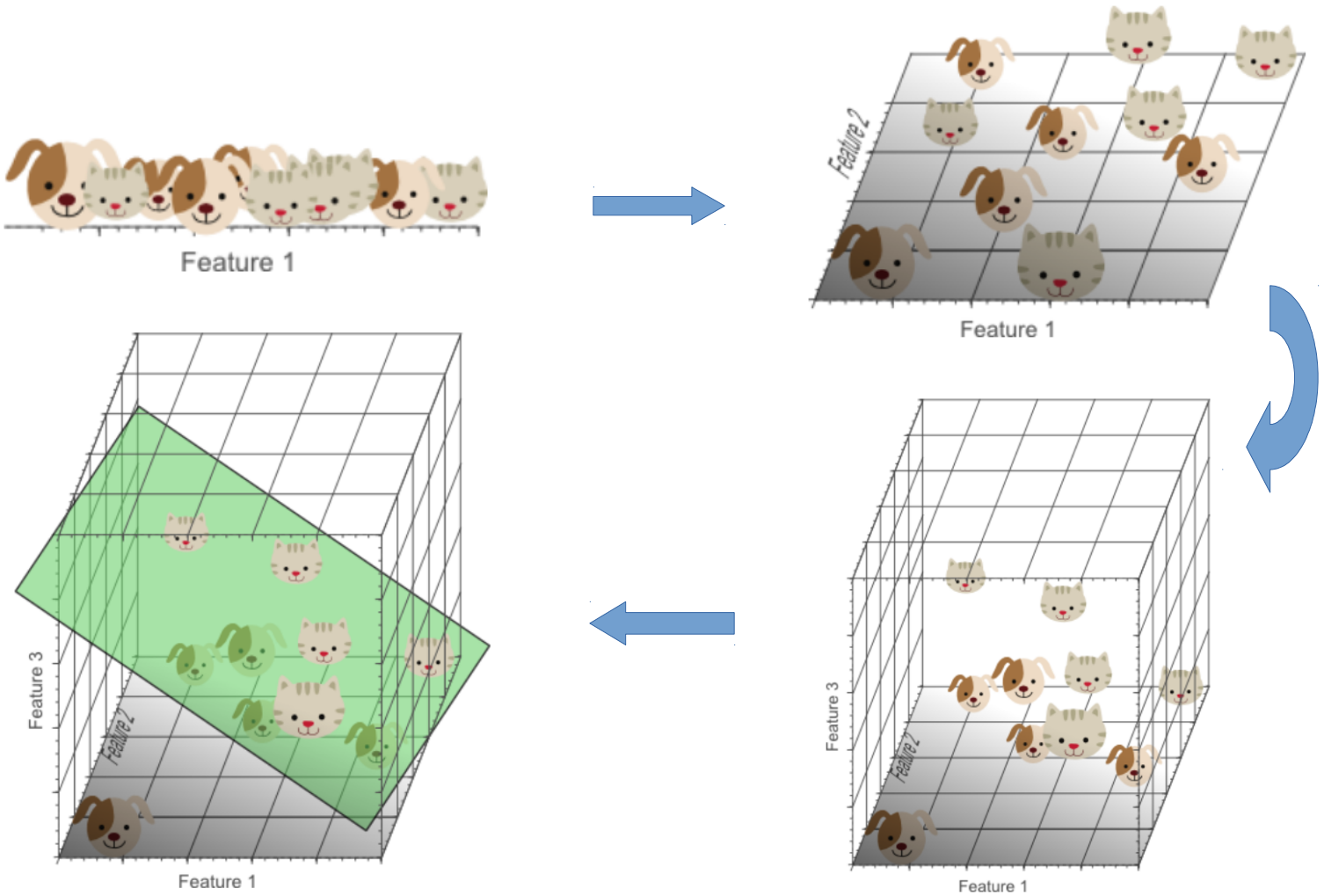
Dimensionality Curse

What is dimensionality curse?

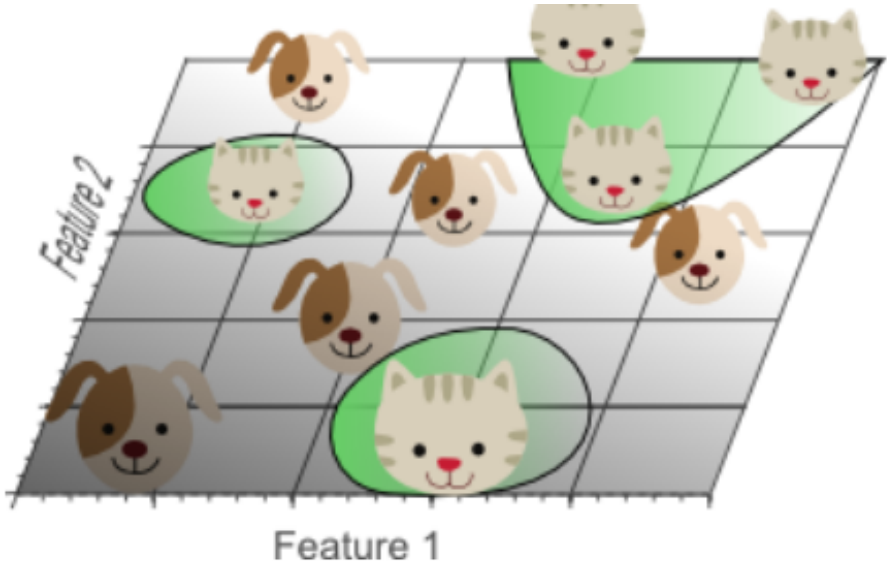


Performance decrease beyond a critical number of features

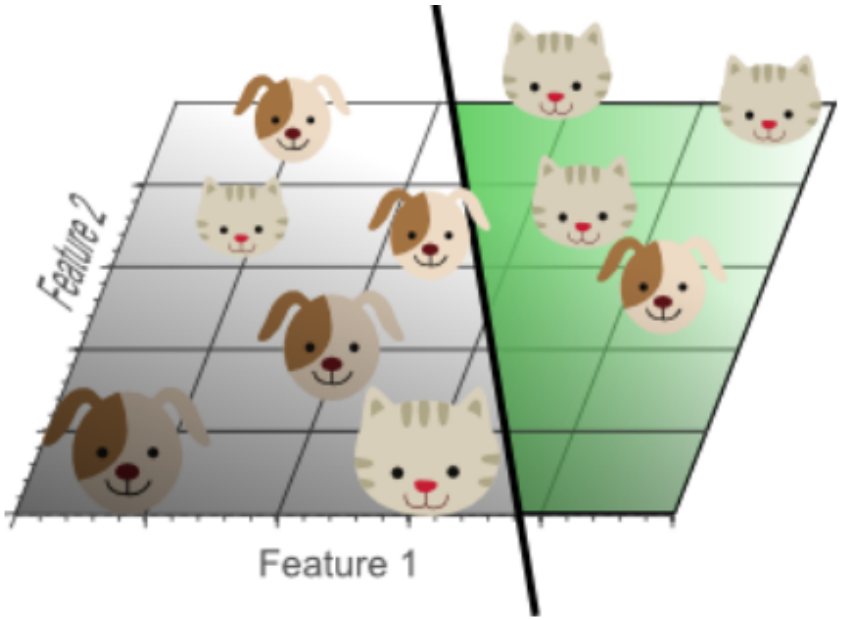
Increase dimension



Which one is better?

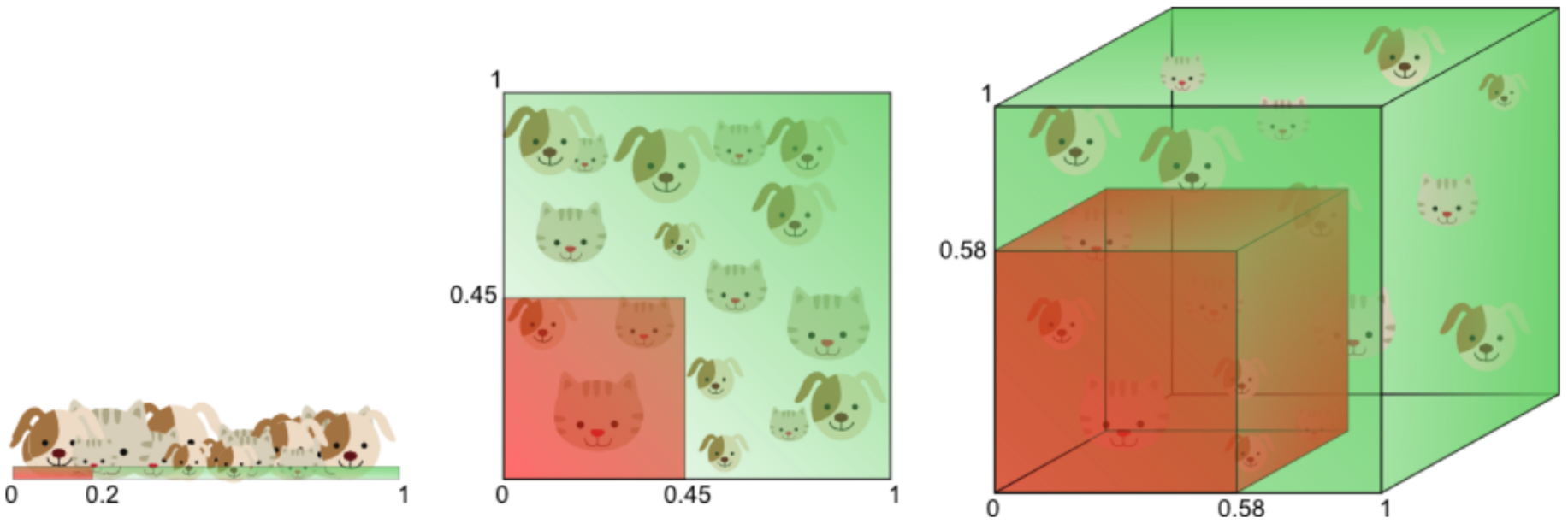


Project 3D linear separation to 2D



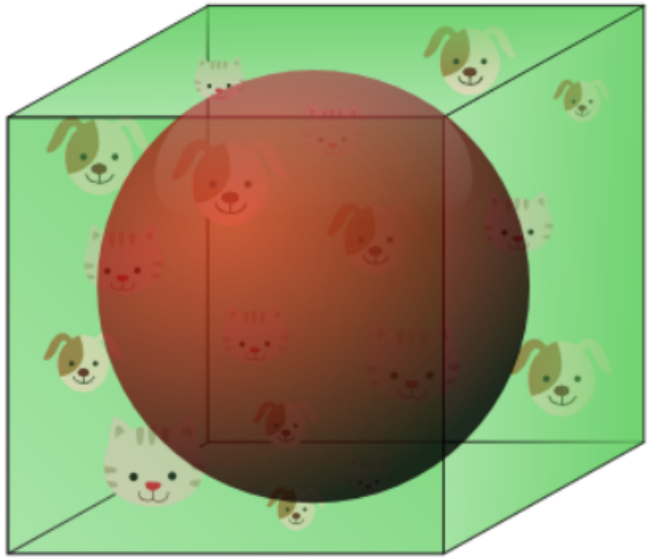
Simple 2D linear separation

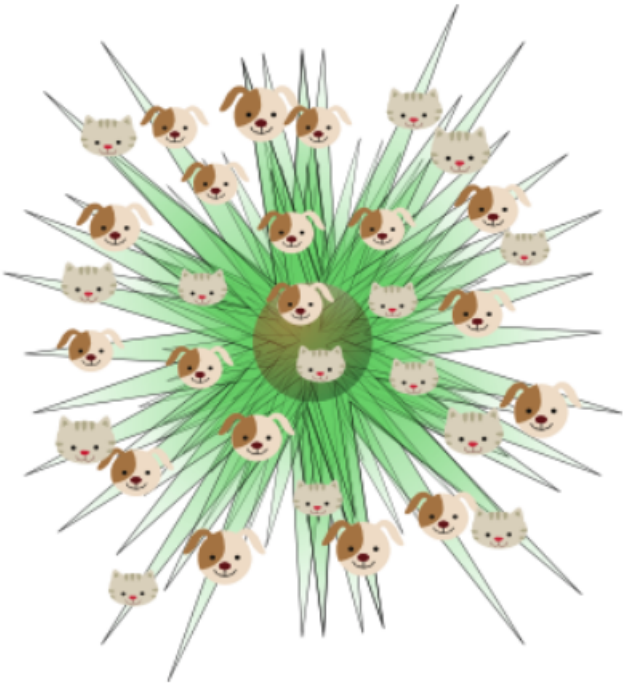
The cubic approach



Want 20% coverage on the value domain:
1D = 0.2 , 2D= 0.45, 3D= 0.58

The corner approach



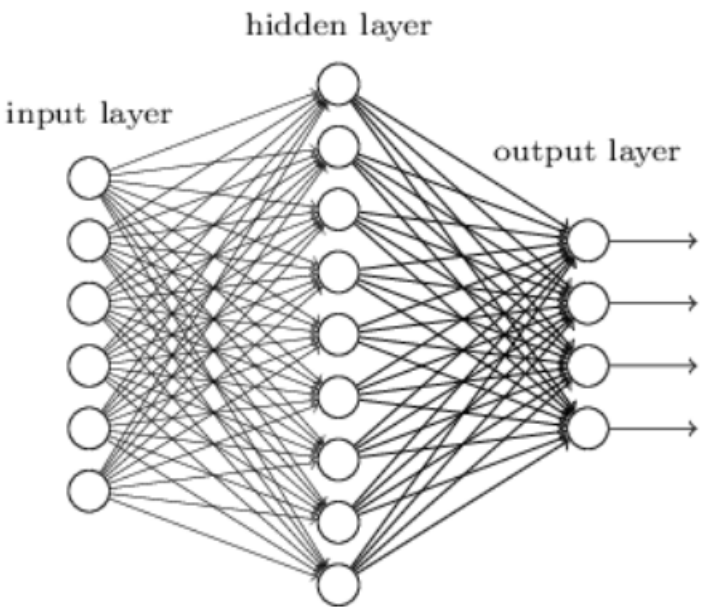


Data around the corner are "sparse" !

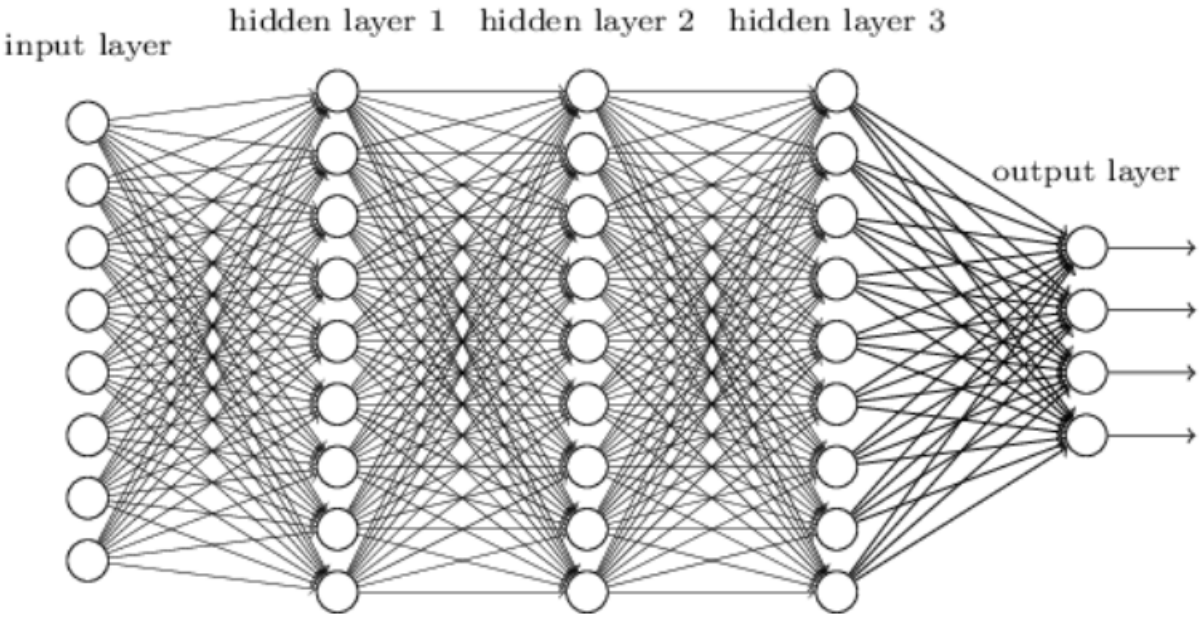
$$V(d) = \frac{\pi^{d/2}}{\Gamma(d/2+1)}(\frac{1}{2})^d \rightarrow \boxed{\frac{V_s}{V_c} = 0.02 \rightarrow lim_{d=\infty}\frac{d_{max}-d_{min}}{d_{min}} = 0}$$

Gradient vanishing & explosion

the deeper the better?



Few-layer NN

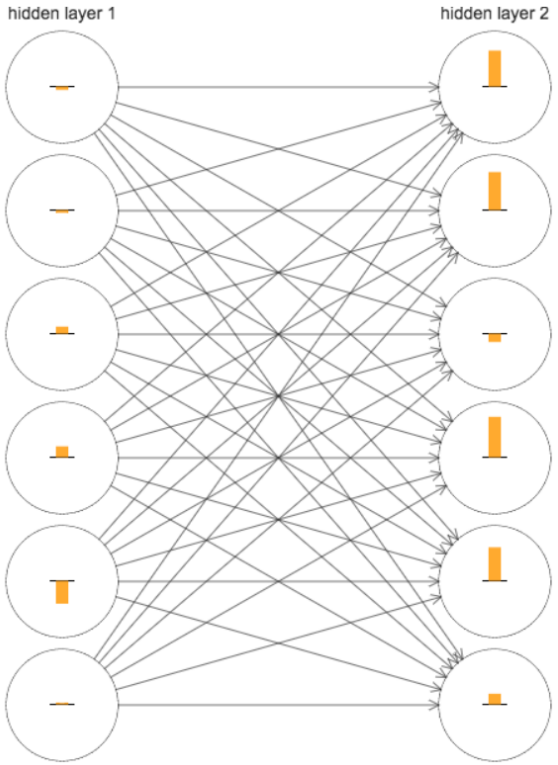


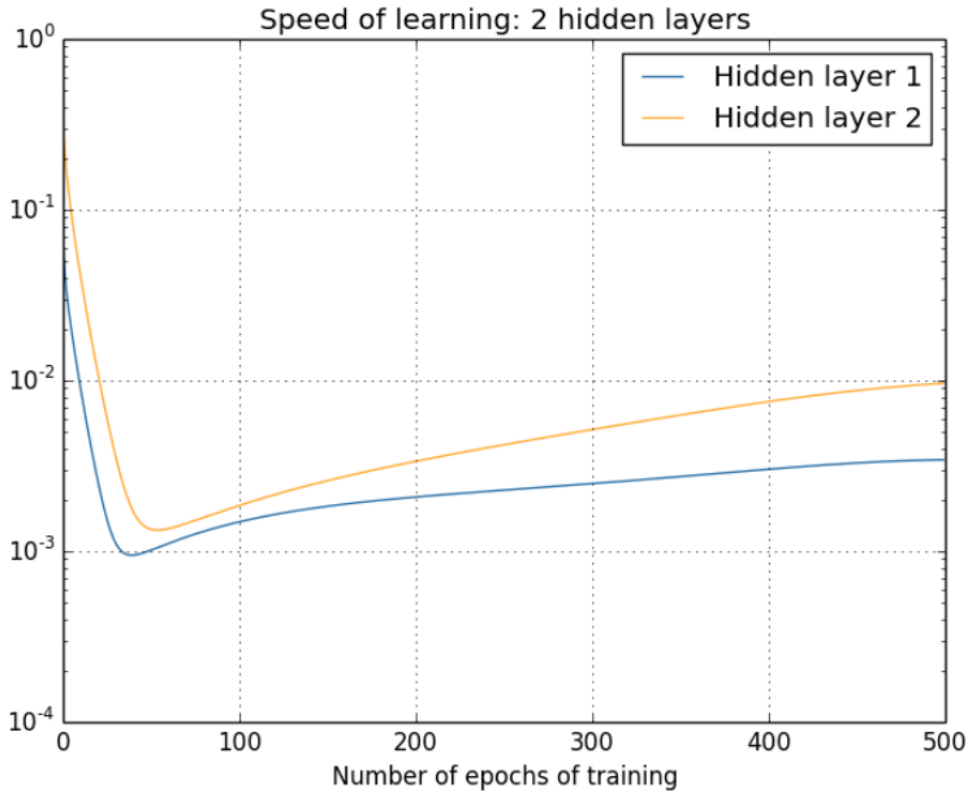
Many-layer NN

More parameters in many-layer NN, so more features can be extract?
Okay, maybe you are right, but how to train it?

What is Gradient Vanising?

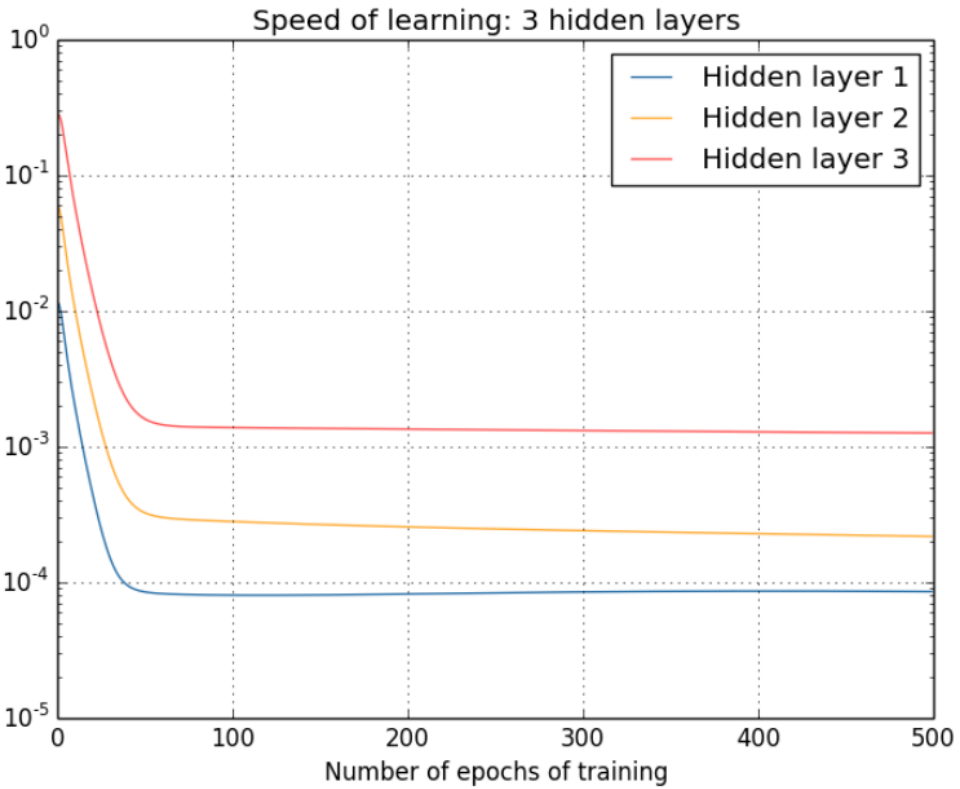
MNIST, sigmoid actvation function

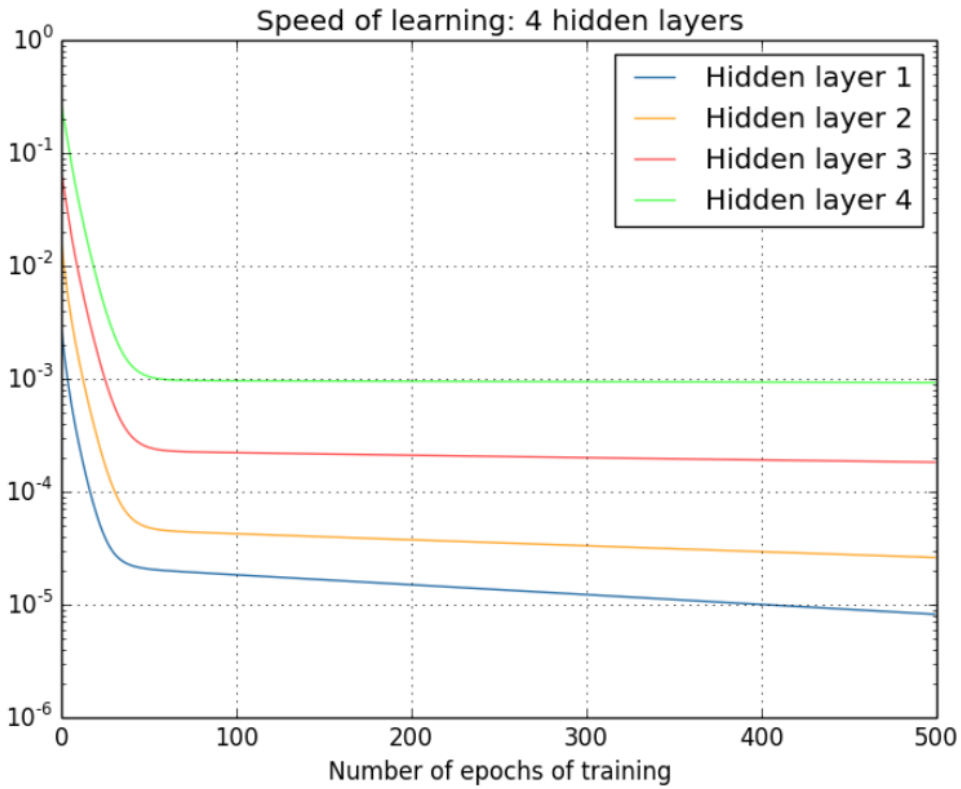




Learning speed: $\frac{\partial S}{\partial b_i}$

Problem becomes worse in multilayer





Why gradient vanishing?

$$\frac{\partial C}{\partial b_1} = \sigma'(z_1) \times w_2 \times \sigma'(z_2) \times w_3 \times \sigma'(z_3) \times w_4 \times \sigma'(z_4) \times \frac{\partial C}{\partial a_4}$$

`dC/db_1`

$$\frac{\partial C}{\partial b_1} \approx \frac{\Delta C}{\Delta b_1}. \tag{114}$$

$$\Delta a_1 \approx \frac{\partial \sigma(w_1 a_0 + b_1)}{\partial b_1} \Delta b_1 \tag{115}$$

$$= \sigma'(z_1) \Delta b_1. \tag{116}$$

$$\Delta z_2 \approx \frac{\partial z_2}{\partial a_1} \Delta a_1 \tag{117}$$

$$= w_2 \Delta a_1. \tag{118}$$

$$\Delta C \approx \sigma'(z_1) w_2 \sigma'(z_2) \ldots \sigma'(z_4) \frac{\partial C}{\partial a_4} \Delta b_1. \tag{120}$$

$$\frac{\partial C}{\partial b_1} = \sigma'(z_1) w_2 \sigma'(z_2) \ldots \sigma'(z_4) \frac{\partial C}{\partial a_4}. \tag{121}$$

$$\frac{\partial C}{\partial b_1} = \sigma'(z_1)\, w_2 \sigma'(z_2)\, w_3 \sigma'(z_3)\, w_4 \sigma'(z_4)\, \frac{\partial C}{\partial a_4}. \tag{122}$$

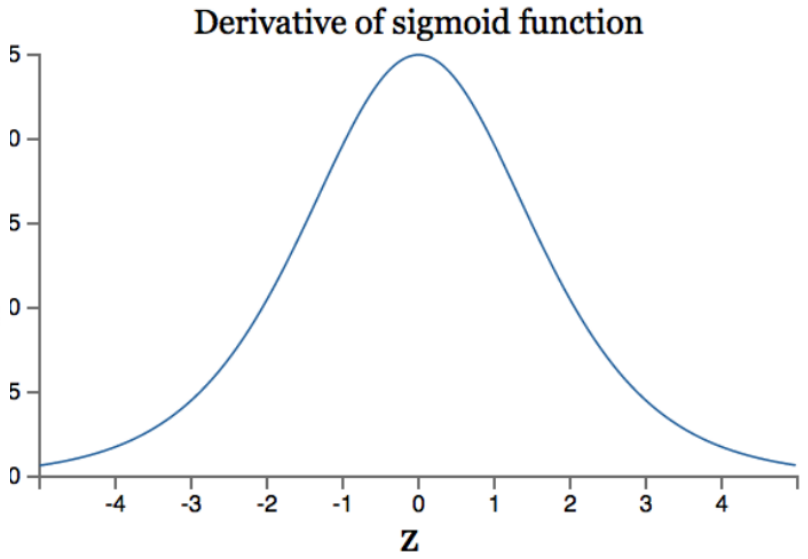
Gradient Vanishing

$$\frac{\partial C}{\partial b_1} = \sigma'(z_1)\, w_2\sigma'(z_2)\, w_3\sigma'(z_3)\, w_4\sigma'(z_4)\, \frac{\partial C}{\partial a_4}.$$

$$\frac{\partial C}{\partial b_1} = \sigma'(z_1)\overbrace{w_2\sigma'(z_2)}^{<\frac{1}{4}}\overbrace{w_3\sigma'(z_3)}^{<\frac{1}{4}}\underbrace{w_4\sigma'(z_4)\frac{\partial C}{\partial a_4}}$$

common terms

$$\frac{\partial C}{\partial b_3} = \sigma'(z_3)\overbrace{w_4\sigma'(z_4)\frac{\partial C}{\partial a_4}}$$



If all W's<1, all layers contribute a value <1/4, exponential decay. Gradient Vanishing !
If all W's are lager, say, > ¼, exponential increase. Gradient explosion !

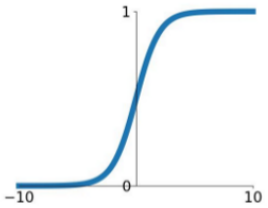
Only 25%, at most, errors can propagate backward !

Deep Learning is Gradient Unstable

Activation Functions

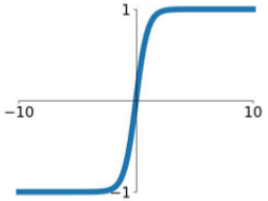
Sigmoid

$\sigma(x) = \frac{1}{1+e^{-x}}$



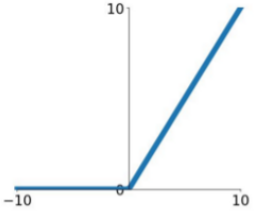
tanh

$\tanh(x)$



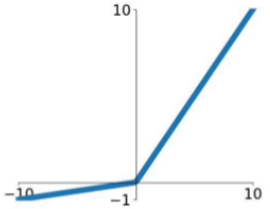
ReLU

$\max(0, x)$



Leaky ReLU

$\max(0.1x, x)$

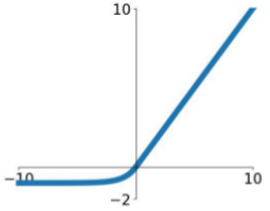


Maxout

$\max(w_1^T x + b_1, w_2^T x + b_2)$

ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



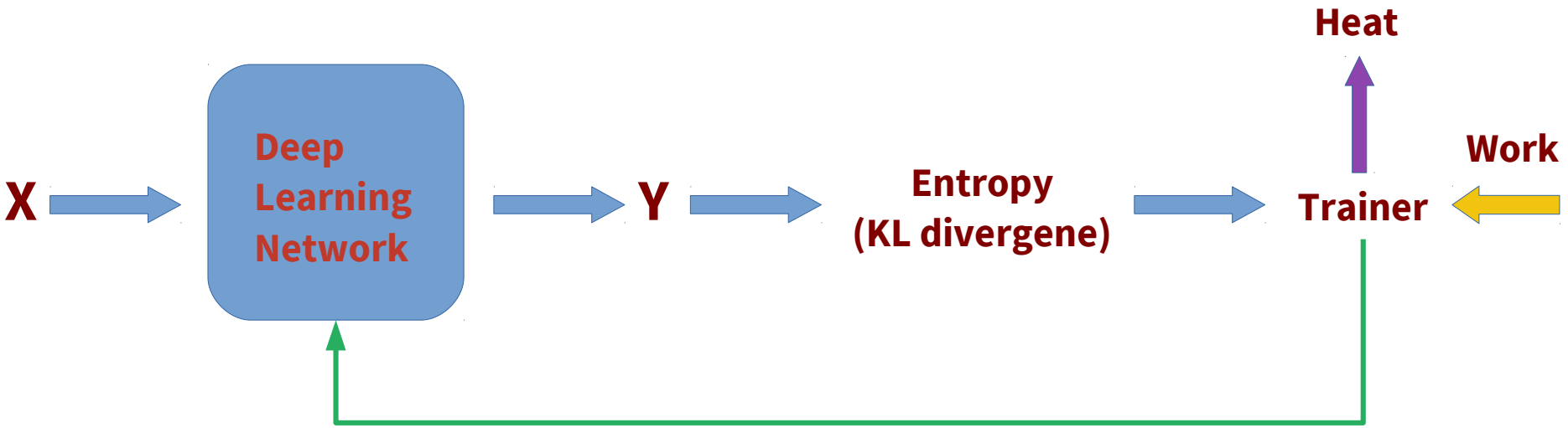
Remember: Deep learning neural network is always "gradient unstable"!

Information Theory

What a neural network really does?

Information

- Any object or event that can reduce “uncertainty”



Deep Learning Network is essential a “Carnot Refrigerator”

Opening the black box of Deep Neural Networks via Information

Ravid Schwartz-Ziv RAVID.ZIV@MAIL.HUJI.AC.IL
Edmond and Lilly Safra Center for Brain Sciences
The Hebrew University of Jerusalem
Jerusalem, 91904, Israel

Naftali Tishby* TISHBY@CS.HUJI.AC.IL
School of Engineering and Computer Science
and Edmond and Lilly Safra Center for Brain Sciences
The Hebrew University of Jerusalem
Jerusalem, 91904, Israel

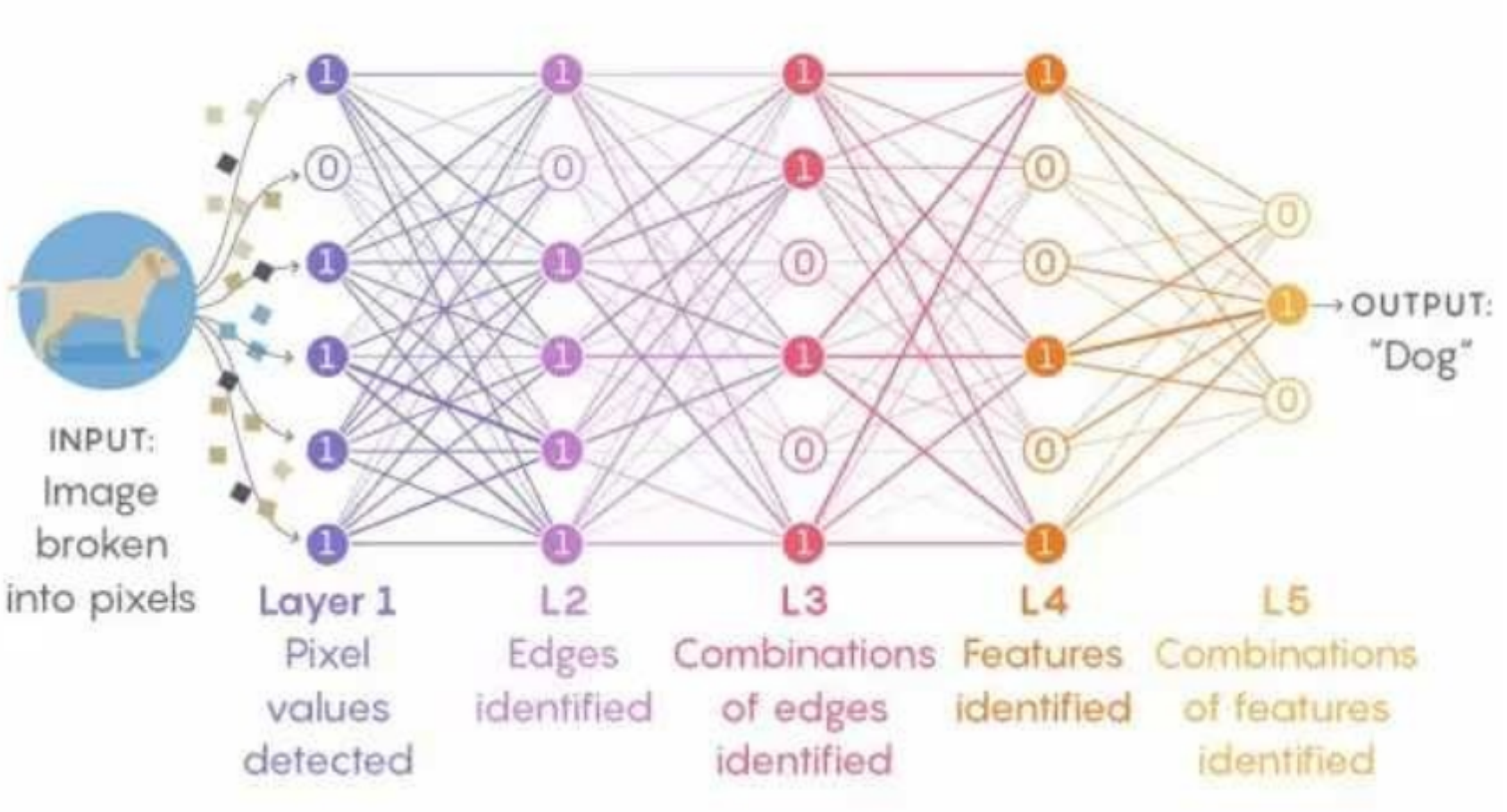
Naftali Tishby
Computer Science & Engineering

Interdisciplinary Center for Neural Computation

intel

How to reduce uncertainty?

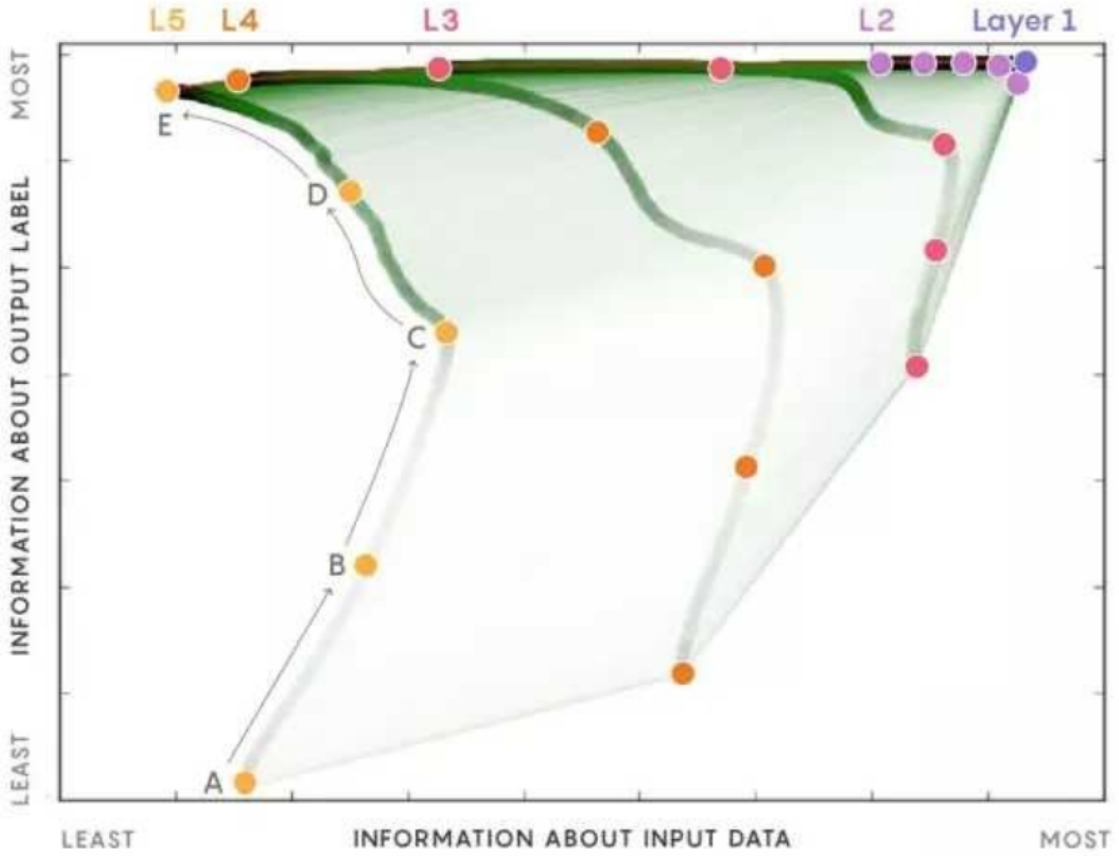
Information Extraction



Inside Deep Learning

Inside Deep Learning

New experiments reveal how deep neural networks evolve as they learn.



A INITIAL STATE: Neurons in Layer 1 encode everything about the input data, including all information about its label. Neurons in the highest layers are in a nearly random state bearing little to no relationship to the data or its label.

B FITTING PHASE: As deep learning begins, neurons in higher layers gain information about the input and get better at fitting labels to it.

C PHASE CHANGE: The layers suddenly shift gears and start to "forget" information about the input.

D COMPRESSION PHASE: Higher layers compress their representation of the input data, keeping what is most relevant to the output label. They get better at predicting the label.

E FINAL STATE: The last layer achieves an optimal balance of accuracy and compression, retaining only what is needed to predict the label.

Why multilayer matters?

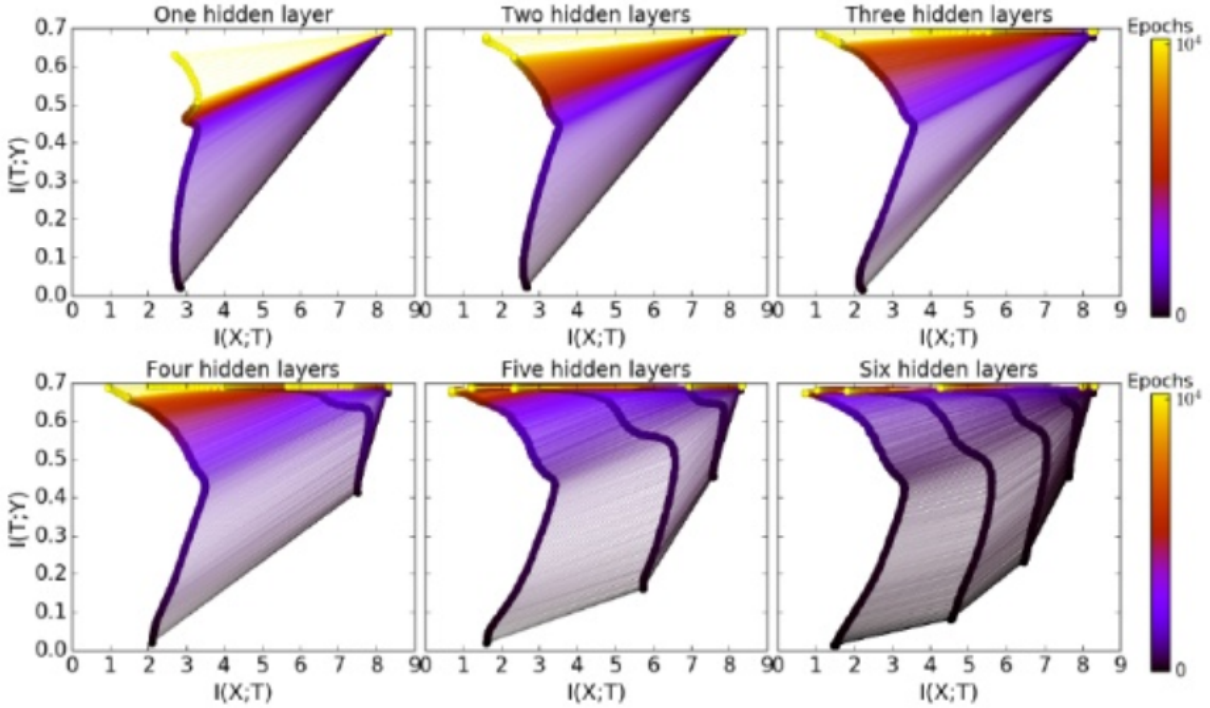


Figure 5: **The layers information paths during the SGD optimization for different architectures.** Each panel is the *information plane* for a network with a different number of hidden layers. The width of the hidden layers start with 12, and each additional layer has 2 fewer neurons. The final layer with 2 neurons is shown in all panels. The line colors correspond to the number of training epochs.

Multilayer can reduce epochs ! But Why?