CKForms - Manual for version 2.x

Notation and convention

This instruction descibes the functionality associated with the following article (Bochenski, Jastrzebski, and Tralle 2021)

We use the notation of the theory of real Lie algebras from CoReLG Package, (Dietrich, Faccin, and Graaf 2014).

Function for real Lie algebras

`RealRank(g)`

The input is a real Lie algebra $\mathfrak{g}$ (as an Lie algebra object). The output is the real rank of $\mathfrak{g}$ (the dimension of the Cartan subalgebra of $\mathfrak{g}$).

`AHypRank(g)`

The input is a real Lie algebra $\mathfrak{g}$ (as an Lie algebra object). The output is the a-hyperbolic rank of $\mathfrak{g}$.

Example:

```
gap> g:=RealFormById("A",5,6);
<Lie algebra of dimension 35 over SqrtField>
gap> NameRealForm(g);
"sl(6,R)"
gap> RealRank(g);
5
gap> AHypRank(g);
3
```

Main procedure

As in Theorem 6 in (Bochenski, Jastrzebski, and Tralle 2021) we are checking three conditions:

- L_0 - Calabi–Markus phenomenon, $\mathrm{rank}_{\mathbb{R}}\,\mathfrak{g} = \mathrm{rank}_{\mathbb{R}}\,\mathfrak{h}$
- L_1 - $\mathrm{rank}_{\text{a-hyp}}(\mathfrak{g}) = \mathrm{rank}_{\text{a-hyp}}(\mathfrak{h})$
- L_2 - $\mathrm{rank}_{\text{a-hyp}}(\mathfrak{g}) > \mathrm{rank}_{\mathbb{R}}\,\mathfrak{h}$
- L_3 - none of the above conditions is met

```
gap> CheckRankConditions("A",5,6);
g=sl(6,R) | real rank(g)=5 | a-hyp rank(g)=3
----------------------------
#1: h=sl(3,R)+sl(3,R) + a torus of  1 non-compact dimensions | real rank(h)=5 | ahyp rank(h)=2
 | L0-true | L1-false | L2-false | L3-false
----
#2: h=sl(3,C) + a torus of 1 compact dimensions | real rank(h)=2 | ahyp rank(h)=1
 | L0-false | L1-false | L2-true | L3-false
----
#3: h=sl(2,R)+sl(4,R) + a torus of  1 non-compact dimensions | real rank(h)=5 | ahyp rank(h)=3
 | L0-true | L1-true | L2-false | L3-false
----
```

```
#4: h=sl(5,R) + a torus of  1 non-compact dimensions | real rank(h)=5 | ahyp rank(h)=2
 | L0-true | L1-false | L2-false | L3-false
----
#5: h=sl(3,R) | real rank(h)=2 | ahyp rank(h)=1
 | L0-false | L1-false | L2-true | L3-false
----
#6: h=sl(2,R)+sl(3,R) | real rank(h)=3 | ahyp rank(h)=2
 | L0-false | L1-false | L2-false | L3-true
----
#7: h=su(4) | real rank(h)=0 | ahyp rank(h)=0
 | L0-false | L1-false | L2-true | L3-false
----
#8: h=su(2,2) | real rank(h)=2 | ahyp rank(h)=2
 | L0-false | L1-false | L2-true | L3-false
----
#9: h=sl(2,H) | real rank(h)=1 | ahyp rank(h)=1
 | L0-false | L1-false | L2-true | L3-false
----
#10: h=sl(4,R) | real rank(h)=3 | ahyp rank(h)=2
 | L0-false | L1-false | L2-false | L3-true
----
#11: h=sp(3,R) | real rank(h)=3 | ahyp rank(h)=3
 | L0-false | L1-true | L2-false | L3-false
----
```

The next possible `h` are consecutive maximal subalgebras of `g` generated by the function `MaximalReductiveSubalgebras` from (Dietrich, Faccin, and Graaf 2014).

All calculations are being done in the "Database - v2" section.

Next if `L3` is equal to `true`, then we check the condition for orbits as in Theorem 5 in (Bochenski, Jastrzebski, and Tralle 2021).

```
gap> CheckProperSL2RAction("A",5,6,6);
proper
```

The last argument of the function `CheckProperSL2RAction` is an index of the maximal subalgebra (`subalgs` output `MaximalReductiveSubalgebras`). # References

Bochenski, Maciej, Piotr Jastrzebski, and Aleksy Tralle. 2021. "Homogeneous Spaces of Real Simple Lie Groups with Proper Actions of Non Virtually Abelian Discrete Subgroups: A Calculational Approach." http://arxiv.org/abs/2106.05777.

Dietrich, H., P. Faccin, and W. A. de Graaf. 2014. "CoReLG, Computation with Real Lie Groups, Version 1.20." http://users.monash.edu/~heikod/corelg/.