Exercises on Implementation and Complexity of Distributed List Operations

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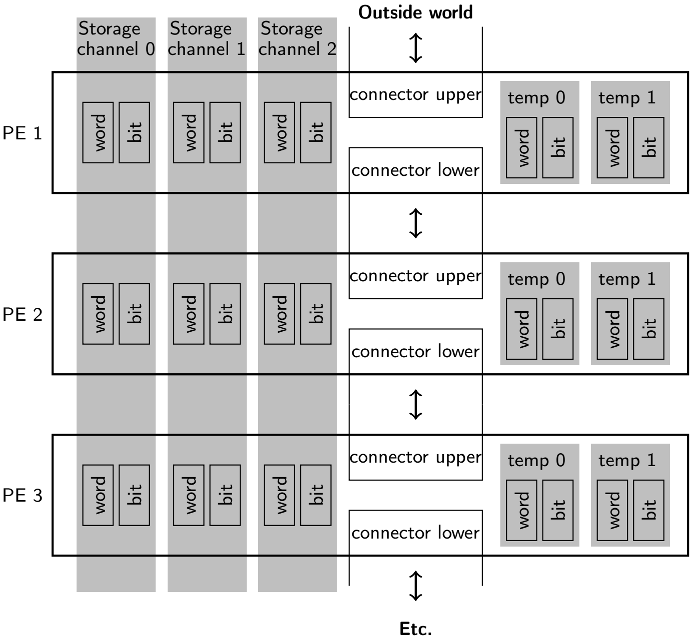


Asynchronous chain

Threads simulate asynchronous processing elements (PEs), each communicating only with its neighbors, via synchronous communication channels.

- A chain of PEs stores a list of words/integers, one integer per PE.
  Control bits tell if the word is non-empty.
  The first empty word terminates the list.
- The chain is infinitely extendable downwards.
  Only the top PE communicates with the outside world.
- Somewhat similar to
  a doubly linked list with access to the front, without storing the length.
- 40 exercises on distributed algorithms for list operations,
  from deque operations to sorting.
- Design, Python implementation, complexity analysis.
- Novel visual tools: dominos and activity diagrams.

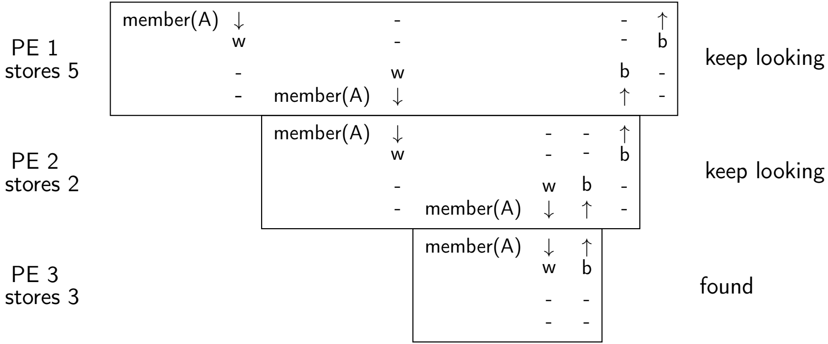
The code written by the students

A sample exercise: test if an integer received via `connectorUpper` is in the stored list. Communication commands with a suffix `_w` are for a word/integer, and `_b` for a bit. `send_o` tells the lower PE to execute the same `member` code.

```
def member(self, channel):
    if not self.bit[channel]: # if storage channel is non-empty
        self.temp_w[0] = self.connectorUpper.receive_w()
        if self.word[channel] == self.temp_w[0]: # found here
            self.connectorUpper.send_b(True)
        else: # keep looking below
            self.connectorLower.send{_o("member", channel)
            self.connectorLower.send_w(self.temp_w[0])
            self.temp_w[0] = self.connectorLower.receive_b()
            self.connectorUpper.send_b(self.temp_w[0])
    else: # if current PE terminates the list
        _ = self.connectorUpper.receive_w()
        self.connectorUpper.send_b(False) # not found anywhere
```

Dominos and activity diagrams designed by students
(in parallel with the design of the algorithm and the code)

While looking for 3 in a list [5,2,3,4,1], PEs execute a sequence of communications of types represented by the **dominos** in this **activity diagram**:



The top and the middle dominos are the same,
except that the top one is stretched horizontally, because of the passage of time.

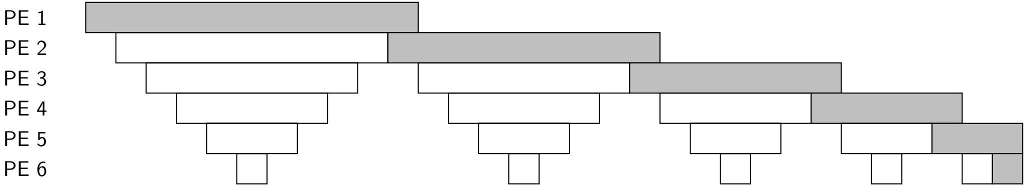
Complexity analysis by students

For the `member` operation on a list of length n, in the worst case,
the activity diagram has width $O(n)$.
Conclusion: overall time complexity of `member` is $O(n)$.

We introduce **top complexity** - the time till the availability of the top PE,
thought of as the width of the top domino.
The top complexity of `member` is $O(n)$.

Another exercise: selection sort

An activity diagram of selection sort on a 5-element distributed list:



The first inverted triangle on the left, brings the smallest value to PE 1.
The second triangle brings the second smallest value to PE 2,

...,

the n-th triangle brings the n-th smallest value to PE n.

From such diagrams, the students judge that
the top complexity of the algorithm is $O(n)$,
and the overall time complexity is $O(n^2)$.

Keywords

- distributed list
- distributed algorithm
- line network
- asynchronous chain
- simulation
- Python 3
- complexity
- domino
- activity diagram
- educational

Conclusion

The asynchronous chain is suitable for practicing design of simple but non-trivial distributed algorithms – on distributed lists.

Students are aided by our visualizations, called dominos and activity diagrams, (which do not adapt to arbitrary network architectures.)

See `github.com/plazajan` for the exercises and a detailed discussion.