Verification and Testing
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Chapter 1

Task 1

This task was solved by following the guide from (Könighofer, 2012). In this example three predicates are used:

- $p = (b == 0)$
- $q = (a < b)$
- $r = (a <= b + 1)$

This is the statement that should be abstracted: $a = a + b + 1;$

As showed in (Könighofer, 2012) the predicates are handled one after another. For each predicate the Hoare rule has to be applied to get the corresponding pre condition. This is the solution for predicate p:

$$\begin{aligned} \{(b &== 0)\} \\ a &= a + b + 1; \\ \{p\} &= \{(b == 0)\} \end{aligned} \tag{1.1}$$

Because of the fact that only the value of a is changed in the assignment predicate $p = (b == 0)$ is not modified. Pre and post conditions are the same. This result can be used to test all combinations of the boolean predicates p, q and r. See table 1.1 for the solution.

The following equation solves the Hoare logic for the predicate q:

$$\begin{aligned} \{(a + b + 1 < b)\} &= \{(a + 1 < 0)\} = \{(a < -1)\} \\ a &= a + b + 1; \\ \{q\} &= \{(a < b)\} \end{aligned} \tag{1.2}$$

Similar to table 1.1 all combinations of p, q and r have to be applied for pre condition $q' = (a < -1)$. Table 1.2 shows the solution.

Cases	post conditions			conclusion	pre condition
	$p = (b == 0)$	$q = (a < b)$	$r = (a \leq b + 1)$		$p' = (b == 0)$
1	0	0	0	$b \neq 0 \wedge$ $\wedge a \geq b \wedge$ $\wedge a > b + 1 =$ $= b \neq 0 \wedge a > b + 1$	0
2	0	0	1	$b \neq 0 \wedge$ $\wedge a \geq b \wedge$ $\wedge a \leq b + 1 =$ $= b \neq 0 \wedge b \leq a \leq b + 1$	0
3	0	1	0	$b \neq 0 \wedge$ $\wedge a < b \wedge$ $\wedge a > b + 1 =$ $= false$	Chosen: 0
4	0	1	1	$b \neq 0 \wedge$ $\wedge a < b \wedge$ $\wedge a \leq b + 1 =$ $= b \neq 0 \wedge a < b$	0
5	1	0	0	$b == 0 \wedge$ $\wedge a \geq b \wedge$ $\wedge a > b + 1 =$ $= b == 0 \wedge a > b + 1$	1
6	1	0	1	$b == 0 \wedge$ $\wedge a \geq b \wedge$ $\wedge a \leq b + 1 =$ $= b == 0 \wedge b \leq a \leq b + 1$	1
7	1	1	0	$b == 0 \wedge$ $\wedge a < b \wedge$ $\wedge a > b + 1 =$ $= false$	Chosen: 1
8	1	1	1	$b == 0 \wedge$ $\wedge a < b \wedge$ $\wedge a \leq b + 1 =$ $= b == 0 \wedge a < b$	1

Table 1.1: All combinations of p, q and r used for pre condition p'

Cases	post conditions			conclusion	pre condition
	$p = (b == 0)$	$q = (a < b)$	$r = (a \leq b + 1)$		$q' = (a < -1)$
1	0	0	0	$b \neq 0 \wedge$ $\wedge a \geq b \wedge$ $\wedge a > b + 1 =$ $= b \neq 0 \wedge a > b + 1$	*
2	0	0	1	$b \neq 0 \wedge$ $\wedge a \geq b \wedge$ $\wedge a \leq b + 1 =$ $= b \neq 0 \wedge b \leq a \leq b + 1$	*
3	0	1	0	$b \neq 0 \wedge$ $\wedge a < b \wedge$ $\wedge a > b + 1 =$ $= false$	Chosen: *
4	0	1	1	$b \neq 0 \wedge$ $\wedge a < b \wedge$ $\wedge a \leq b + 1 =$ $= b \neq 0 \wedge a < b$	*
5	1	0	0	$b == 0 \wedge$ $\wedge a \geq b \wedge$ $\wedge a > b + 1 =$ $= b == 0 \wedge a > b + 1$	0
6	1	0	1	$b == 0 \wedge$ $\wedge a \geq b \wedge$ $\wedge a \leq b + 1 =$ $= b == 0 \wedge b \leq a \leq b + 1$	0
7	1	1	0	$b == 0 \wedge$ $\wedge a < b \wedge$ $\wedge a > b + 1 =$ $= false$	Chosen: 0
8	1	1	1	$b == 0 \wedge$ $\wedge a < b \wedge$ $\wedge a \leq b + 1 =$ $= b == 0 \wedge a < b$	*

Table 1.2: All combinations of p, q and r used for pre condition q'

The same rules have to be applied to predicate r:

$$\begin{aligned} \{(a+b+1 \leq b+1)\} &= \{(a \leq 0)\} \\ a &= a+b+1; \\ \{r\} &= \{(a \leq b+1)\} \end{aligned} \tag{1.3}$$

Again all combinations have to be applied to pre condition $r' = (a \leq 0)$. Table 1.3 shows the solution.

Using the results of tables 1.1, 1.2 and 1.3 these assumptions can be made:

$$\begin{aligned} p &= p?1 : 0 \\ q &= p?(q\&\&r?* : 0) : * \\ r &= p?(q?1 : (r?* : 0)) : * \end{aligned} \tag{1.4}$$

Cases	post conditions			conclusion	pre condition
	$p = (b == 0)$	$q = (a < b)$	$r = (a \leq b + 1)$		$q' = (a \leq 0)$
1	0	0	0	$b \neq 0 \wedge$ $\wedge a \geq b \wedge$ $\wedge a > b + 1 =$ $= b \neq 0 \wedge a > b + 1$	*
2	0	0	1	$b \neq 0 \wedge$ $\wedge a \geq b \wedge$ $\wedge a \leq b + 1 =$ $= b \neq 0 \wedge b \leq a \leq b + 1$	*
3	0	1	0	$b \neq 0 \wedge$ $\wedge a < b \wedge$ $\wedge a > b + 1 =$ $= false$	Chosen: *
4	0	1	1	$b \neq 0 \wedge$ $\wedge a < b \wedge$ $\wedge a \leq b + 1 =$ $= b \neq 0 \wedge a < b$	*
5	1	0	0	$b == 0 \wedge$ $\wedge a \geq b \wedge$ $\wedge a > b + 1 =$ $= b == 0 \wedge a > b + 1$	0
6	1	0	1	$b == 0 \wedge$ $\wedge a \geq b \wedge$ $\wedge a \leq b + 1 =$ $= b == 0 \wedge b \leq a \leq b + 1$	*
7	1	1	0	$b == 0 \wedge$ $\wedge a < b \wedge$ $\wedge a > b + 1 =$ $= false$	Chosen: 1
8	1	1	1	$b == 0 \wedge$ $\wedge a < b \wedge$ $\wedge a \leq b + 1 =$ $= b == 0 \wedge a < b$	1

Table 1.3: All combinations of p, q and r used for pre condition r'

Chapter 2

Task 2

Line sequence for counter example:

- 1 p = False;
- 2 skip;
- 3 while(*) {
- 4 skip;
- 5 p = True;
- 6 skip;
- 3 while(*) {
- 7 }
- 8 assert(!p);

Chapter 3

Task 3

Bibliography

Könighofer, Robert (2012). *Example: Abstraction of a Simple Statement.* URL: `https://verify.iaik.tugraz.at/teaching/vt/pub/Main/AssignmentPage/vt_abstraction.pdf`.