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Summary

In this paper, we address the problems associated with the shape of toll square. We consider several models in hopes of getting the most comprehensive benefit and the most suitable shape of the plaza.

Through statistical analysis of the toll station traffic flow and accident rate data, we establish two models and a total model.

In the first model, we analyze the throughput model. We need to study the relating factors of the flow of vehicles on the highway. So we base on the speed of high-speed road traffic V and single-lane traffic density K to estimate the throughout Φ. Considering the traffic density K and the rate of decrease of the velocity V, we finally get its model.

In the incident rate model, we conclude almost all the factors including the driver, the service level, and so on. Also we lead into the concept of transition rate. And we get the conclusion of the frequency of traffic accidents is proportional to and toll stations level of service.

The last model is the total model. The model covers the impact of toll plaza shape on throughput, construction cost and accident incidence. The system is a macro framework, and we have an idealized analysis of reality. What's more , we analyze the situation in which case this model can't be applied.

In the end, by solving the model optimization, we can get the maximum benefit . We can get D =300m and the progressive angle TAN Θ=1/17. It is very close to the toll plaza in reality. That is the optimal solution of the shape of tollbooth.

Contents

1 Introduction

Multi-lane divided limited-access toll highways use “ramp tolls” and “barrier tolls” to collect tolls from motorists. A ramp toll is a collection mechanism at an entrance or exit ramp to the highway and these do not concern us here. A barrier toll is a row of tollbooths placed across the highway, perpendicular to the direction of traffic flow. There are usually more tollbooths than there are incoming lanes of traffic. So when exiting the tollbooths in a barrier toll, vehicles must “fan in” from the larger number of tollbooth egress lanes to the smaller number of regular travel lanes. A toll plaza is the area of the highway needed to facilitate the barrier toll, consisting of the fan-out area before the barrier toll, the toll barrier itself, and the fan-in area after the toll barrier.

So, it is of great importance to determine the shape, size, and merging pattern of the area following the toll barrier to maximum interests.

2 Restatement of the Problem

Every highway must have a toll station. When freeway lanes converge to toll booths, we must set up a larger toll window in order to save time and avoid traffic congestion.

However, how to match the number of toll windows and the number of freeway lanes, is worth to explore . As unmanned technology matures, we should also consider this aspect of the factors. In addition, land construction costs is one of the factors we want to test.

New Jersey's high-speed toll station is the difficulty encountered is the need to optimize the vehicle into and out of speed matching. In essence, we need to explore the relationship between the overall shape of the toll booth and its throughput. I need to use mathematical modeling methods to explore an optimal solution. We want to determine the minimum construction cost, the size of the optimal toll station area, the shape, and the form of consolidation. In addition, we need to prevent high accident rates. After establishing the model, we should also analyze its feasibility, and find a better solution than the commonly used program.

3 Preview Work in Traffic Theory

Mark Twain famously remarked, in his disdain for arithmetic, that the answer to all mathematical problems is three. While insightful, the Twain model leaves some room for improvement in addressing the tollbooth conundrum at hand. A five-lane highway will need more than three tollbooths, but Twain wrote great novels.There is a rather substantial literature on models for traffic flow.

Analysis of high-speed toll model, my first thought is the body's blood flow. Just as blood hurtles red blood cells through veins, vehicles pulse down streets toward their destinations. The "average" behavior is assessed, and commonly used variables include steady-state velocity, flux of cars per time, and density of traffic flow.

In addition, we also used a number of traffic science academic terms.

Throughput is the maximum transmission rate that the system can carry under the premise that the system allows the accident rate. It is an important indicator of the highway toll station carrying capacity.

The traffic accident rate is the relative relationship between the number of traffic accidents or the number of injured and killed persons, the number of citizenship (motor) vehicles and the mileage in a certain country or region in a certain period of time.

It not only represents the level of comprehensive management of road traffic, but also the evaluation index of traffic safety. Although there are several kinds of ways to calculate this rate,we can't find the best. China is still comparing the number of accidents, the number of injured, the number of deaths, the amount of direct economic losses of the four indicators with the previous year the same period.

There are also some data requirements for proper terms. Standard lane width for two-way four-lane is 2 * 7.5 meters. Two-way six-lane is 2 * 11.25 meters. Two-way eight-lane is 2 * 15 meters.Freeway tollway lane width is 3.5 meters. The length of wearable cement pavement on both sides of toll station is 100 meters.

4 Properties of a Successful Model

Successful toll station models should achieve the following objectives:

· In ensuring the construction of the toll station under the premise of reliability, we should try to reduce costs.

· The toll station model belongs to the low accident incidence model

· We should guarantee maximum throughput and improve the use of toll stations efficiency.

5 General Assumptions and Definitions

· **There is a toll highway having *L* lanes of travel in each direction and a barrier toll containing *B* tollbooths (*B* > *L*) in each direction.** What we have to do is determining the shape, size, and merging pattern of the area following the toll barrier in which vehicles fan in from *B* tollbooth egress lanes down to *L* lanes of traffic.

· **There is only one type of driver in the system.** In navigating toll plaza traffic, all drivers act according to the same set of rules. Although the individual decisions of any given driver are probabilistic, the associated probabilities are the same for all drivers.

· **All tollbooths offer the same service and vehicles do not distinguish between them.** While several types of tollbooth exist in practice, we have not been charged with distinguishing between them and suggesting their selective use.

· **The number of operating plaza booths remains constant throughout the day.**

6 Main Model

Necessary Conventions :

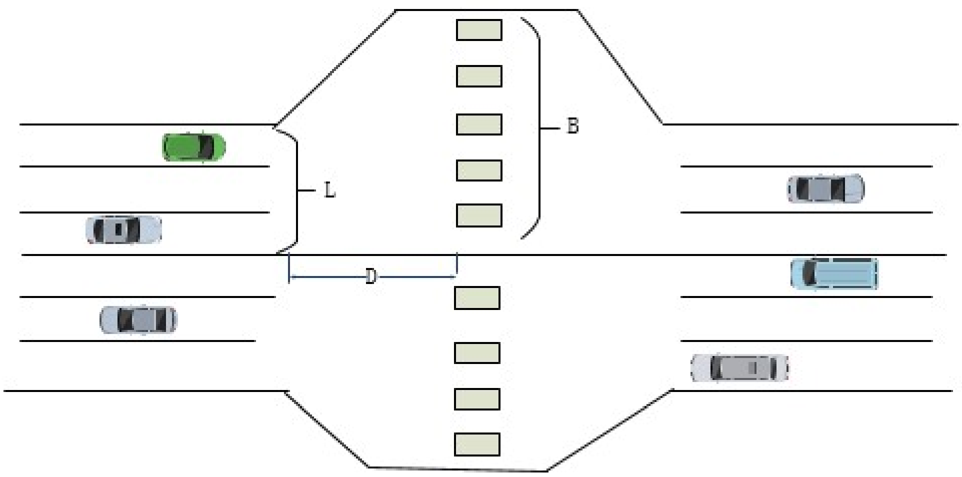
- The width ot toll channel 2.8~3.5m
- Standard lane width 3.75m
- Emergency stop belts 2.5 m
- Central isolation belt 1m
- the width of tollbooth 1.4~1.8m
- the length of tollbooth 2m
- the length of Cement on both sides 200m
- High speed Vf
- Blocking density Kj
- B>L

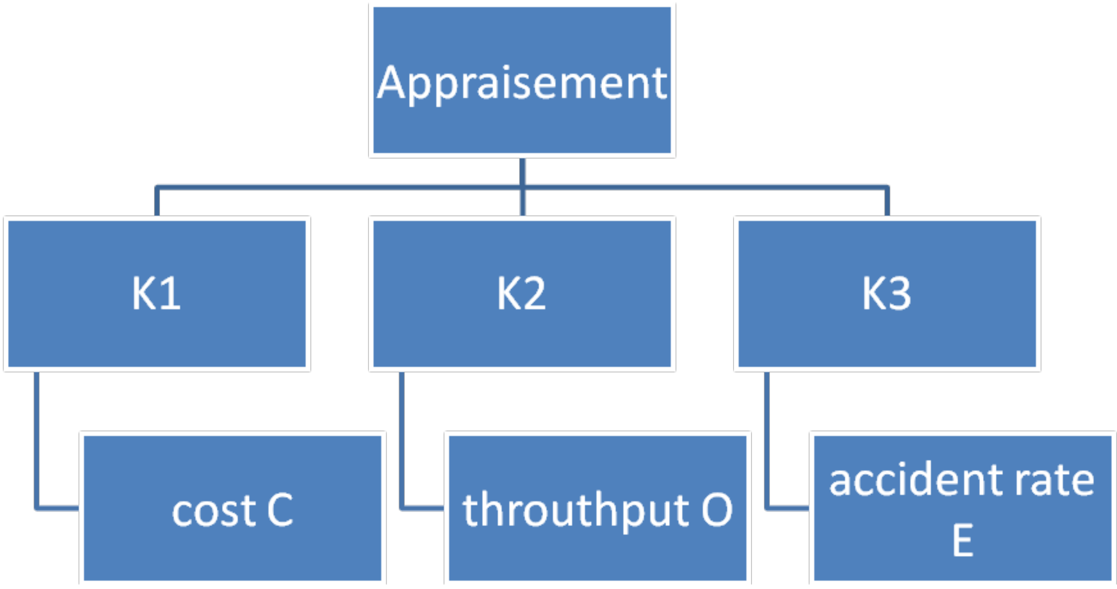
Upper base length of approximately trapezoidal

d2=B*（the width of tollbooth + Standard lane width）

Lower base length of approximately trapezoidal

TANΘ=(d2-d1)/(D-100)





Appraisement E=K1*cost C+throughput o*K2+accident E*K3

K1>0; K2<; K3>0

The appraisement E is as small as possible. (E>0);

From this, we determine the score of D and Θ. We find the smallest score,and define it as the optimal results. The optimal shape is determined by the optimal D and Θ.

The description of the cost

The cost of operating a toll station includes land costs, construction costs, maintenance costs, staff salaries, electricity, water and so on. However, In the background of this problem, we are talking about the shape of the toll plaza, and because the number of toll booths is obtained, the salary of the staff is not considered. The only concern is with the land area of the construction costs and land costs. According to "highway 2016 compensation standards for land acquisition". Land costs is 2000 / square meters, construction costs is 3000 / square meters.

The cost of one side=(d1+d2)*(D+1-100)/2+200*d2

As symmetrical on both sides,

Total cost=(d1+d2)*(D+1-100)+400*d2

Model 1 Throughput Model

In the study of vehicles traveling along the highway, We need to study the relating factors of the flow of vehicles on the highway. And a traffic flow model for expressway is established. Here we are based on the speed of high-speed road traffic V and single-lane traffic density K to estimate the throughout Φ.

We know that the general direction of the highway in more than one lane.

Set the number of lanes B. Here we assume that there is no difference between the lanes, each independent driving and there is no mutual overtaking, crossing the situation. According to the definition of each variable, we can get.

$$\mathbf{O = BKV}$$

In practice, you can see: As the number of vehicles on the road increases, traffic density increases and the driver is forced to lower the speed. When the traffic density from large to small, the speed will increase. This shows that there is a certain relationship between the speed and density, and the speed with the traffic density was decreasing relationship. One of the simplest assumptions is that the vehicle speed decreases linearly with the traffic density. So we can get

$$\frac{dV}{dK} = -c_0$$

Co Is a positive constant

In view of the fact that as the traffic density K increases, the rate of decrease of the velocity V should be increased. Therefore, the above model is modified to be the equation blow.

$$\frac{dV}{dK} = -Kc_0$$

It is easy to know that this model is a quadratic function model.

When the traffic density is zero, that is, K = 0, the speed V can reach the theoretical maximum value, the so-called smooth speed Vf. So we can get

$$V(0) = V_f$$

When the traffic density K reaches the blocking density, the velocity V = 0. The road is in a blocked state. At that time, **V(0)= 0.**

Above all, we can get

$$O= BV_f K[1 - \left(\frac{K}{K_J}\right)^2]$$

If the unit time period up to N two cars into the toll plaza, N take 40, the traffic density K = N / D. According to the traffic data of many sections of the blockage density of 220 / km. So we can be obtained maximum throughput from it.

Model 2 Accident rate model

Traffic accident of the tollbooth is the result of the combined effects of human, vehicle, road environment and other factors . Due to statistical analysis of the causes of toll stations accident, the driver is the direct cause of the accident.

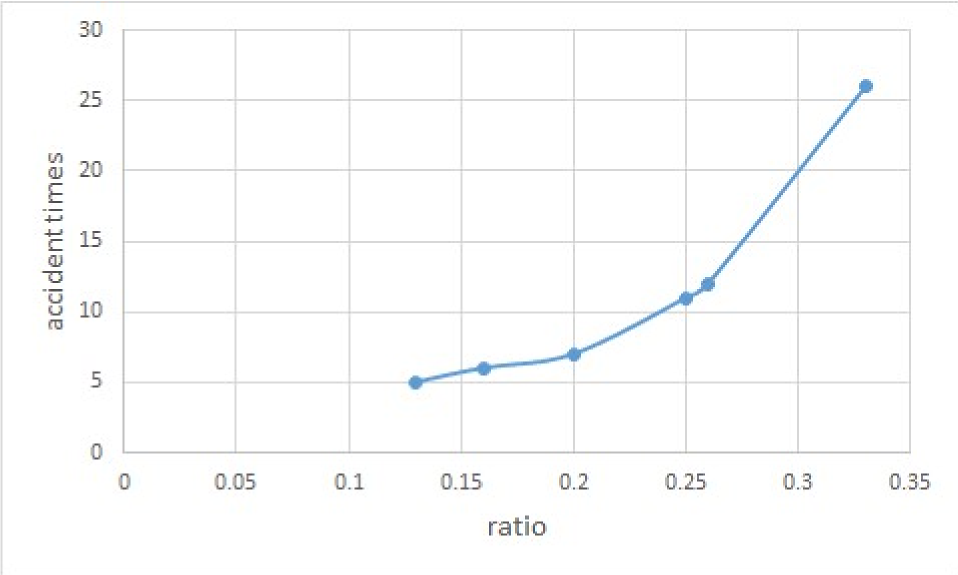
Accidents involving driver factors is as high as 50% or more, which including improper operation of the ratio of about 50% or more. "Improper operation" includes the poor line of sight, road slippage, poor visibility, toll station service level, charging square export section unreasonable and other objective reasons.

So the toll station settings and design parameters and traffic accidents are also related. Which is related to our discussion of the issues is the rate of charging square gradient and charging service level (throughput).

Toll plaza should establish the transition in the entrance and exit. We should guide the vehicle naturally and smoothly out of the toll plaza,so its transition rate should be gently changed which is easy for the vehicle to be in and out.

Otherwise, it’s easy to make the driver a sense of cheap driving, resulting in improper operation and endanger road safety. The transition rate of the toll plaza should equal to TAN Θ=(d2-d1)/(D-100).According to the experience of the United States and Japan, the vehicle drives from a straight line into the other lane whose the skew ratio is of 0.9m / s or so, the driver is generally no car shift sense or discomfort.

Accordingly when the vehicle at the speed of 40km / h and 60km / h, the length of the transition section should be 40m and 55m. Gradual rates are 1/15 and 1/18. In China, the current number of lanes at a toll station is usually determined by capacity and projected traffic volume. Generally, it’s 1 to 3 more than the main lane , the transition length is 25m or so, the gradient can only be 1/5 to 1/7, which is significantly lower than the experience value.



The figure shows the relationship between the highway toll plaza gradient and the traffic accident curve. The figure shows that with the toll plaza gradient rate increasing, the number of traffic accidents will increase and the safety of the toll plaza decreased.

After data regression analysis, we get the correlation between model Toll Plaza fade rate and the number of traffic accidents

$$Y1 = 1.4231e^{8.0664\tan\Theta}$$

Y=The assumption of the number of the traffic accident

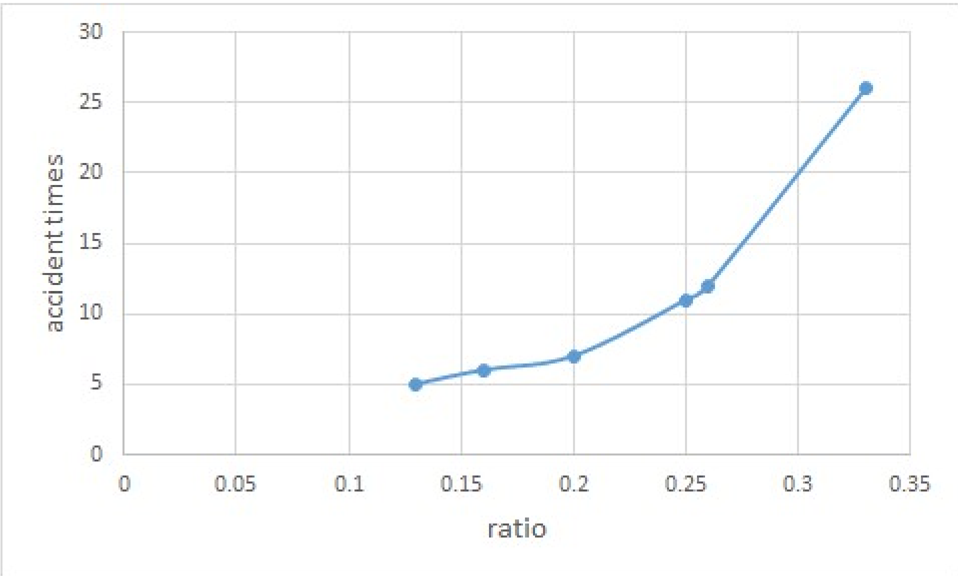
Tan θ=Toll square gradient rate

From the model, we can get .The incidence of traffic accidents and charges square is proportional to the simple rate. When the length of the transition section is too short, or the length of the transition section is too large., the vehicle safety judgment can not meet the security demand of the lane changing into the toll plaza vehicle deceleration, thus leading to the traffic accident.

While another indicator -throughput also indirectly affects the security.

The direct feeling of the driver to the traffic environment of the highway toll station comes from the queuing length of the toll lane. The length of the queue depends on the service level of the toll station (V / C) level. According to the statistical analysis of the time distribution of toll traffic accidents, accident-prone periods coincide with peak traffic periods. It shows that when the traffic volume is small,or the toll station queues the vehicle is not much, the driver can accurately judge and select a lane quickly through.

In the peak traffic volume, the service efficiency of toll station is greatly reduced. When the level of service can not meet the traffic demand, the toll plaza will form a traffic jam and long queues phenomenon. Drivers waiting to pay the station is difficult to determine the shortest possible time through the lane, the driver will be prompted to change psychology or to seize the toll lane and other unsafe traffic behavior in order to be able to quickly enter the shorter queue lane to reduce the rolling time. In this case, the risk of conflicting vehicles within the toll plaza is greatly increased which leads to the traffic accident.



The figure shows the relationship between the toll station service level and the traffic accident curve. After data regression analysis, we can get

$$Y2 = 1.6624.5x^2 - 2964.6x + 1357.9$$

Y-----The assumption of the number of the traffic accident

X-----the toll station service level

In conclusion , the frequency of traffic accidents is proportional to and toll stations level of service. The service efficiency of toll station directly affects the safety performance of toll station

The service level of the toll station is equal to the throughput divided by the number of toll stations. So we can get

$$Y2 = 1.6624.5(\frac{O}{B})^2 - 2964.6(\frac{O}{B}) + 1357.9$$

So the number of accidents E=Y1+Y2

$$E = 1.4231e^{8.0664\tan\Theta} + 1.6624.5(\frac{O}{B})^2 - 2964.6(\frac{O}{B}) + 1357.9$$

Model 3   Total model

Appraisement E= cost C *K2+ throughput O*K2+ accident rate E *K3

It is now necessary to determine the coefficient for the total model, These coefficients are subjectively choice, it may will cause that a different set of coefficients may result in different results.

The cost is almost a construction cost for a one-time investment. So the coefficient can be properly taken small . I assume it takes 0.2. Throughput affects the traffic flow at the toll booth , affect the quality of public transport, and it will also indirectly affect safety. So the coefficient is -0.5. Incident rate is related to the driver's life safety. So it takes 0.3. The final equation is

e=0.2*C-0.5*O+0.3*E

And we can find that

C=(d1+d2)*(D+1-100)+400*d2

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According to the model's simulation, we can get the most suitable D=300m,

Progressive angle TANΘ=1/17

7 Comparsion of Results from the Models

8 The Application of the Models

9 Conclusion

According to the model's simulation, we can get the most suitable D=300m, progressive angle TANΘ=1/17. It is very close to the toll plaza in reality. But what is the main facilities of the construction of the highway tollbooth? It depends on the local terrain, land prices, culture, especially, the local terrain. We assume the toll plaza has symmetrical entrance and exit. But if the land type does not fit, symmetry can not be achieved. What's more, When the toll is built on the slope, the slope will affect the acceleration of the car, thus, the local terrain will affect safety and other factors. This model can not be applied when this unsatisfied condition is encountered. In that case, The model needs to be optimization designed. Everything should be subject to the actual conditions, supplemented by theory.

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11 Non-technical Letter for the New Jersey Turnpike Authority

Highway toll stations and toll plaza is to ensure the surrounding traffic safety and smooth, and they are important facilities to make everything proceed as usual. Highway toll station and the construction of the square, involved in the calculation of the number of toll channels , square geometry and construction scale.

Due to low technical index value, the square construction is too small to cause the toll plaza which just operated a few years become a traffic bottleneck. But on the other hand,due to high technical index value of the toll station, its construction scale is over large and waste much money and resources. Due to improper selection of linear indicators, otherwise, the square is too short, the square is not long enough, it results in high accident rate, and makes huge loss of personal property.

The practical application of the square has bad transition. It has bad effect on its beauty, the most important thing is traffic flow center on the the central lane and makes the outside channel can not be well used. In the peak period, it will even affect the main line. Also make the square useless when completed. Therefore, the development of reasonable toll stations and toll plaza design indicators do good to the construction of toll stations and ensure the smoothness of the toll plaza and traffic safety. Improve the charging efficiency and management level and reduce land acquisition. And it make rational use of funds.

In the main line square on both sides of 50m ~ 150m range of the toll plaza. Ramp toll station on both sides of 30m ~ 100m range., we must use the cement concrete pavement, and shall not use asphalt concrete pavement.The vehicles' frequent brake makes the road bad rutting and shift phenomenon.It will endanger the safety of the square and affect the normal use. 100m is an appropriate range for the mainline toll plaza. Our model consists of several modules. We analyse the relationship between transition rate and throughput, the land cost and accident rate. When the models get different optimal solutions, transition rate change with them. In order to maximize the comprehensive benefits, we have a comprehensive analysis of several variables. In the end, we get the most appropriate transition rate.

D =300m

Progressive angle TANΘ=1/17

Thus, we obtain the given road number L and the number of tolls B, and get most appropriate transition rate. We define the shape of the toll plaza determined by the transition rate as the picture below.