Random number generators

The purpose of this exercise is to learn commands related to generating random numbers. I will display Mathematica code. You should do these exercises in your software.

Problems

1: Generate a set of 10 random numbers in the interval [0,1]
2: Generate a set of 10 random integers in the interval [0,100]
3: Generate a random 5x3 matrix with elements in [-1,1]
4: Generate a random 5x3x2 tensor with elements in [-1,1]
5: Set the seed of your random number generator
6: Construct a set of five seeds such that they produce “independent” sequences
7: Construct random vectors using two different random number generators. Compute their correlation.

Solutions in Mathematica

Generate a set of 10 random numbers in the interval [0,1]

```
RandomReal[{0, 1}, 10]
```

```
{0.667533, 0.260779, 0.632094, 0.874639, 0.474332, 0.764076, 0.458275, 0.218798, 0.905521, 0.838295}
```

Generate a set of 10 random integers in the interval [0,100]

```
RandomInteger[{0, 100}, 10]
```

```
{16, 53, 59, 59, 76, 55, 81, 71, 10, 86}
```

Generate a random 5x3 matrix with elements in [-1,1]

```
RandomReal[{-1, 1}, {5, 3}] // MatrixForm
```

$$\begin{pmatrix} 0.926696 & -0.42096 & -0.59172 \\ -0.529402 & -0.0425862 & 0.169546 \\ 0.219476 & 0.906842 & 0.824725 \\ 0.966591 & -0.744897 & 0.187096 \\ 0.174689 & 0.0733133 & -0.613631 \end{pmatrix}$$

Generate a random 5x3x2 tensor with elements in [-1,1]

```
RandomReal[{-1, 1}, {5, 3, 2}] // MatrixForm
```

$$\begin{pmatrix} \begin{pmatrix} 0.558505 \\ 0.510557 \end{pmatrix} & \begin{pmatrix} -0.592045 \\ -0.402522 \end{pmatrix} & \begin{pmatrix} 0.0771919 \\ -0.99609 \end{pmatrix} \\ \begin{pmatrix} 0.431604 \\ -0.660808 \end{pmatrix} & \begin{pmatrix} 0.716534 \\ -0.575687 \end{pmatrix} & \begin{pmatrix} -0.391103 \\ -0.320514 \end{pmatrix} \\ \begin{pmatrix} -0.146219 \\ -0.363517 \end{pmatrix} & \begin{pmatrix} -0.967186 \\ -0.202442 \end{pmatrix} & \begin{pmatrix} 0.997106 \\ -0.51907 \end{pmatrix} \\ \begin{pmatrix} 0.436583 \\ -0.306418 \end{pmatrix} & \begin{pmatrix} 0.457372 \\ -0.528844 \end{pmatrix} & \begin{pmatrix} -0.909521 \\ 0.199616 \end{pmatrix} \\ \begin{pmatrix} 0.720445 \\ -0.875065 \end{pmatrix} & \begin{pmatrix} 0.591866 \\ -0.130155 \end{pmatrix} & \begin{pmatrix} 0.236105 \\ 0.365533 \end{pmatrix} \end{pmatrix}$$

Set the seed of your random number generator

Set the seed to 111

```
SeedRandom[111]
```

Generate 10 random numbers:

```
RandomReal[{0, 1}, 10]
```

```
{0.183729, 0.779252, 0.755279, 0.203977, 0.298739, 0.538596, 0.00195476, 0.715802, 0.169596, 0.858267}
```

Generate another 10 random numbers:

```
RandomReal[{0, 1}, 10]
```

```
{0.212156, 0.304449, 0.339743, 0.42689, 0.318241, 0.016407, 0.398779, 0.998553, 0.240465, 0.718291}
```

The second set is different from the first. Suppose you want to reproduce the first one. To do that, you reset the random seed and repeat the

command to produce 10 random numbers:

```
SeedRandom[111]
RandomReal[{0, 1}, 10]
```

```
{0.183729, 0.779252, 0.755279, 0.203977, 0.298739, 0.538596, 0.00195476, 0.715802, 0.169596, 0.858267}
```

Learning how to set the seed is important if you want to reproduce your results.

Construct a set of five seeds such that they produce "independent" sequences

Suppose that you want to run multiple independent simulations using multiple cores. You need to worry about the simulations using the same random numbers. The simplest way to proceed is to collect the k'th random number for large k (which is 1,000,000 here):

```
seeds = Table[RandomInteger[{0, 10^15}, 1 000 000] // First, {5}]
```

```
{238 866 718 082 033, 768 116 715 466 487, 81 204 824 699 566, 27 744 378 941 541, 40 675 798 548 130}
```

Generate five sequences using these seeds

```
Do[sequence[i] = (SeedRandom[seeds[[i]]]; RandomReal[{-1, 1}, 10 000 000]), {i, 1, 5}]
```

Test the means of the sequences

```
Table[Mean[sequence[i]], {i, 1, 5}]
```

```
{-0.0000360529, -0.000397505, 0.00015519, -0.000133556, -0.000268851}
```

Next we check the correlation of the sequences to see if the different sequences are uncorrelated.

```
Table[Correlation[sequence[i], sequence[j]], {i, 1, 5}, {j, 1, 5}] // MatrixForm
```

$$\begin{pmatrix} 1. & -0.0000773604 & 0.000336033 & -0.000625089 & 0.000615243 \\ -0.0000773604 & 1. & 0.0000993751 & -0.000175511 & 0.000439943 \\ 0.000336033 & 0.0000993751 & 1. & -0.000393792 & 0.000078347 \\ -0.000625089 & -0.000175511 & -0.000393792 & 1. & -0.0000147805 \\ 0.000615243 & 0.000439943 & 0.000078347 & -0.0000147805 & 1. \end{pmatrix}$$

Construct random vectors using two different random number generators. Compute their correlation.

You should know your RNG. Mathematica has several options. I illustrate here the linear congruential method and MersenneTwister options. By the way, Mersenne Twister is, as far as I know, the best available algorithm.

```
SeedRandom[2020, Method -> "Congruential"];
cong = RandomReal[{-1, 1}, 1 000 000];
```

```
Mean[cong]
```

```
0.000391582
```

```
StandardDeviation[cong]
```

```
0.577303
```

```
SeedRandom[2020, Method -> "MersenneTwister"];
Mers = RandomReal[{-1, 1}, 1 000 000];
```

```
Mean[Mers]
```

```
-0.000192635
```

```
StandardDeviation[Mers]
```

```
0.576972
```

```
Correlation[cong, Mers]
```

```
-0.00134177
```