

# Language Technology

<http://cs.lth.se/edan20/>  
Chapter 5, part 2: Word Sequences

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# Word Sequences

Words have specific contexts of use.

Pairs of words like *strong* and *tea* or *powerful* and *computer* are not random associations.

Psychological linguistics tells us that it is difficult to make a difference between *writer* and *rider* without context

A listener will discard the improbable *rider of books* and prefer *writer of books*

A language model is the statistical estimate of a word sequence.

Originally developed for speech recognition

The language model component enables to predict the next word given a sequence of previous words: *the writer of books, novels, poetry*, etc. and not *the writer of hooks, nobles, poultry*, ...



# N-Grams

The types are the distinct words of a text while the tokens are all the words or symbols.

The phrases from *Nineteen Eighty-Four*

*War is peace*

*Freedom is slavery*

*Ignorance is strength*

have 9 tokens and 7 types.

Unigrams are single words

Bigrams are sequences of two words

Trigrams are sequences of three words



# Trigrams

Word	Rank	More likely alternatives
<i>We</i>	9	<i>The This One Two A Three Please In</i>
<i>need</i>	7	<i>are will the would also do</i>
<i>to</i>	1	
<i>resolve</i>	85	<i>have know do...</i>
<i>all</i>	9	<i>the this these problems...</i>
<i>of</i>	2	<i>the</i>
<i>the</i>	1	
<i>important</i>	657	<i>document question first...</i>
<i>issues</i>	14	<i>thing point to...</i>
<i>within</i>	74	<i>to of and in that...</i>
<i>the</i>	1	
<i>next</i>	2	<i>company</i>
<i>two</i>	5	<i>page exhibit meeting day</i>
<i>days</i>	5	<i>weeks years pages months</i>



# Counting Bigrams With Unix Tools

- ❶ `tr -cs 'A-Za-z' '\n' < input_file > token_file`  
Tokenize the input and create a file with the unigrams.
- ❷ `tail +2 < token_file > next_token_file`  
Create a second unigram file starting at the second word of the first tokenized file (+2).
- ❸ `paste token_file next_token_file > bigrams`  
Merge the lines (the tokens) pairwise. Each line of `bigrams` contains the words at index  $i$  and  $i+1$  separated with a tabulation.
- ❹ And we count the bigrams as in the previous script.



# Counting Bigrams in Python

```
bigrams = [tuple(words[inx:inx + 2])  
            for inx in range(len(words) - 1)]
```

The rest of the `count_bigrams` function is nearly identical to `count_unigrams`. As input, it uses the same list of words:

```
def count_bigrams(words):  
    bigrams = [tuple(words[inx:inx + 2])  
                for inx in range(len(words) - 1)]  
    frequencies = {}  
    for bigram in bigrams:  
        if bigram in frequencies:  
            frequencies[bigram] += 1  
        else:  
            frequencies[bigram] = 1  
    return frequencies
```



# Probabilistic Models of a Word Sequence

$$\begin{aligned}P(S) &= P(w_1, \dots, w_n), \\&= P(w_1)P(w_2|w_1)P(w_3|w_1, w_2)\dots P(w_n|w_1, \dots, w_{n-1}), \\&= \prod_{i=1}^n P(w_i|w_1, \dots, w_{i-1}).\end{aligned}$$

The probability  $P(\textit{It was a bright cold day in April})$  from *Nineteen Eighty-Four* corresponds to

$\textit{It}$  to begin the sentence, then  $\textit{was}$  knowing that we have  $\textit{It}$  before, then  $\textit{a}$  knowing that we have  $\textit{It was}$  before, and so on until the end of the sentence.

$$\begin{aligned}P(S) &= P(\textit{It}) \times P(\textit{was}|\textit{It}) \times P(\textit{a}|\textit{It, was}) \times P(\textit{bright}|\textit{It, was, a}) \times \dots \\&\quad \times P(\textit{April}|\textit{It, was, a, bright, \dots, in}).\end{aligned}$$



# Approximations

Bigrams:

$$P(w_i | w_1, w_2, \dots, w_{i-1}) \approx P(w_i | w_{i-1}),$$

Trigrams:

$$P(w_i | w_1, w_2, \dots, w_{i-1}) \approx P(w_i | w_{i-2}, w_{i-1}).$$

Using a trigram language model,  $P(S)$  is approximated as:

$$P(S) \approx P(It) \times P(was|It) \times P(a|It, was) \times P(bright|was, a) \times \dots \\ \times P(April|day, in).$$





# Maximum Likelihood Estimate

Bigrams:

$$P_{MLE}(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i)}{\sum_w C(w_{i-1}, w)} = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})}.$$

Trigrams:

$$P_{MLE}(w_i|w_{i-2}, w_{i-1}) = \frac{C(w_{i-2}, w_{i-1}, w_i)}{C(w_{i-2}, w_{i-1})}.$$



# Conditional Probabilities

A common mistake in computing the conditional probability  $P(w_i|w_{i-1})$  is to use

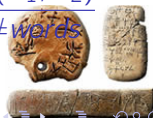
$$\frac{C(w_{i-1}, w_i)}{\# \text{bigrams}}.$$

This is not correct. This formula corresponds to  $P(w_{i-1}, w_i)$ .  
The correct estimation is

$$P_{MLE}(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i)}{\sum_w C(w_{i-1}, w)} = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})}.$$

Proof:

$$P(w_1, w_2) = P(w_1)P(w_2|w_1) = \frac{C(w_1)}{\# \text{words}} \times \frac{C(w_1, w_2)}{C(w_1)} = \frac{C(w_1, w_2)}{\# \text{words}}$$



# Training the Model

The model is trained on a part of the corpus: the **training set**

It is tested on a different part: the **test set**

The vocabulary can be derived from the corpus, for instance the 20,000 most frequent words, or from a lexicon

It can be closed or open

A closed vocabulary does not accept any new word

An open vocabulary maps the new words, either in the training or test sets, to a specific symbol, <UNK>



# Probability of a Sentence: Unigrams

*<s> A good deal of the literature of the past was, indeed, already being transformed in this way </s>*

$w_i$	$C(w_i)$	#words	$P_{MLE}(w_i)$
<s>	7072	–	
<i>a</i>	2482	108140	0.023
<i>good</i>	53	108140	0.00049
<i>deal</i>	5	108140	$4.62 \cdot 10^{-5}$
<i>of</i>	3310	108140	0.031
<i>the</i>	6248	108140	0.058
<i>literature</i>	7	108140	$6.47 \cdot 10^{-5}$
<i>of</i>	3310	108140	0.031
<i>the</i>	6248	108140	0.058
<i>past</i>	99	108140	0.00092
<i>was</i>	2211	108140	0.020
<i>indeed</i>	17	108140	0.00016
<i>already</i>	64	108140	0.00059
<i>being</i>	80	108140	0.00074
<i>transformed</i>	1	108140	$9.25 \cdot 10^{-6}$
<i>in</i>	1759	108140	0.016
<i>this</i>	264	108140	0.0024
<i>way</i>	122	108140	0.0011
</s>	7072	108140	0.065



# Probability of a Sentence: Bigrams

*<s> A good deal of the literature of the past was, indeed, already being transformed in this way </s>*

$w_{i-1}, w_i$	$C(w_{i-1}, w_i)$	$C(w_{i-1})$	$P_{MLE}(w_i w_{i-1})$
<i>&lt;s&gt; a</i>	133	7072	0.019
<i>a good</i>	14	2482	0.006
<i>good deal</i>	0	53	0.0
<i>deal of</i>	1	5	0.2
<i>of the</i>	742	3310	0.224
<i>the literature</i>	1	6248	0.0002
<i>literature of</i>	3	7	0.429
<i>of the</i>	742	3310	0.224
<i>the past</i>	70	6248	0.011
<i>past was</i>	4	99	0.040
<i>was indeed</i>	0	2211	0.0
<i>indeed already</i>	0	17	0.0
<i>already being</i>	0	64	0.0
<i>being transformed</i>	0	80	0.0
<i>transformed in</i>	0	1	0.0
<i>in this</i>	14	1759	0.008
<i>this way</i>	3	264	0.011
<i>way &lt;/s&gt;</i>	18	122	0.148



# Sparse Data

Given a vocabulary of 20,000 types, the potential number of bigrams is  $20,000^2 = 400,000,000$

With trigrams  $20,000^3 = 8,000,000,000,000$

Methods:

- Laplace: add one to all counts
- Linear interpolation:

$$P_{\text{DelInterpolation}}(w_n | w_{n-2}, w_{n-1}) = \lambda_1 P_{MLE}(w_n | w_{n-2} w_{n-1}) + \lambda_2 P_{MLE}(w_n | w_{n-1}) + \lambda_3 P_{MLE}(w_n)$$

- Good-Turing: The discount factor is variable and depends on the number of times a n-gram has occurred in the corpus.
- Back-off



# Laplace's Rule

$$P_{\text{Laplace}}(w_{i+1}|w_i) = \frac{C(w_i, w_{i+1}) + 1}{\sum_w (C(w_i, w) + 1)} = \frac{C(w_i, w_{i+1}) + 1}{C(w_i) + \text{Card}(V)},$$

$w_i, w_{i+1}$	$C(w_i, w_{i+1}) + 1$	$C(w_i) + \text{Card}(V)$	$P_{\text{Lap}}(w_{i+1} w_i)$
<s> a	133 + 1	7072 + 8635	0.0085
a good	14 + 1	2482 + 8635	0.0013
good deal	0 + 1	53 + 8635	0.00012
deal of	1 + 1	5 + 8635	0.00023
of the	742 + 1	3310 + 8635	0.062
the literature	1 + 1	6248 + 8635	0.00013
literature of	3 + 1	7 + 8635	0.00046
of the	742 + 1	3310 + 8635	0.062
the past	70 + 1	6248 + 8635	0.0048
past was	4 + 1	99 + 8635	0.00057
was indeed	0 + 1	2211 + 8635	0.000092
indeed already	0 + 1	17 + 8635	0.00012
already being	0 + 1	64 + 8635	0.00011
being transformed	0 + 1	80 + 8635	0.00011
transformed in	0 + 1	1 + 8635	0.00012
in this	14 + 1	1759 + 8635	0.0014
this way	3 + 1	264 + 8635	0.00045
way </s>	18 + 1	122 + 8635	0.0022



# Good–Turing

Laplace's rule shifts an enormous mass of probability to very unlikely bigrams. Good–Turing's estimation is more effective

Let's denote  $N_c$  the number of  $n$ -grams that occurred exactly  $c$  times in the corpus.

$N_0$  is the number of unseen  $n$ -grams,  $N_1$  the number of  $n$ -grams seen once,  $N_2$  the number of  $n$ -grams seen twice The frequency of  $n$ -grams occurring  $c$  times is re-estimated as:

$$c^* = (c + 1) \frac{E(N_{c+1})}{E(N_c)},$$

Unseen  $n$ -grams:  $c^* = \frac{N_1}{N_0}$  and  $N$ -grams seen once:  $c^* = \frac{2N_2}{N_1}$ .





# Good-Turing for *Nineteen eighty-four*

*Nineteen eighty-four* contains 37,365 unique bigrams and 5,820 bigram seen twice.

Its vocabulary of 8,635 words generates  $8635^2 = 74,563,225$  bigrams whose 74,513,701 are unseen.

New counts:

- Unseen bigrams:  $\frac{37,365}{74,513,701} = 0.0005$ .
- Unique bigrams:  $2 \times \frac{5820}{37,365} = 0.31$ .
- Etc.

Freq. of occ.	$N_c$	$c^*$	Freq. of occ.	$N_c$	$c^*$
0	74,513,701	0.0005	5	719	3.91
1	37,365	0.31	6	468	4.94
2	5,820	1.09	7	330	6.06
3	2,111	2.02	8	250	6.44
4	1,067	3.37	9	179	8.93



# Backoff

If there is no bigram, then use unigrams:

$$P_{\text{Backoff}}(w_i|w_{i-1}) = \begin{cases} \tilde{P}(w_i|w_{i-1}), & \text{if } C(w_{i-1}, w_i) \neq 0, \\ \alpha P(w_i), & \text{otherwise.} \end{cases}$$

Simplified backoff:

$$P_{\text{Backoff}}(w_i|w_{i-1}) = \begin{cases} P_{\text{MLE}}(w_i|w_{i-1}) = \frac{C(w_{i-1}, w_i)}{C(w_{i-1})}, & \text{if } C(w_{i-1}, w_i) \neq 0, \\ P_{\text{MLE}}(w_i) = \frac{C(w_i)}{\# \text{words}}, & \text{otherwise.} \end{cases}$$

The sum of probabilities is not equal to one though.



# Backoff: Example

$w_{i-1}, w_i$	$C(w_{i-1}, w_i)$	$C(w_i)$	$P_{\text{Backoff}}(w_i w_{i-1})$
<s>		7072	—
<s> a	133	2482	0.019
a good	14	53	0.006
good deal	0	5	$4.62 \cdot 10^{-5}$
deal of	1	3310	0.2
of the	742	6248	0.224
the literature	1	7	0.00016
literature of	3	3310	0.429
of the	742	6248	0.224
the past	70	99	0.011
past was	4	2211	0.040
was indeed	0	17	0.00016
indeed already	0	64	0.00059
already being	0	80	0.00074
being transformed	0	1	$9.25 \cdot 10^{-6}$
transformed in	0	1759	0.016
in this	14	264	0.008
this way	3	122	0.011
way </s>	18	7072	0.148

The figures we obtain are not probabilities. We can use the Good-Turing technique to discount the bigrams and then scale the unigram probabilities. This is the Katz backoff.



# Quality of a Language Model (I)

The quality of a language model corresponds to its accuracy in predicting word sequences:  $P(w_1, \dots, w_n)$ : The higher, the better.

We derive the model (the statistics) from a training set and evaluate this quality on a long unseen sequence sequence: The test set.

With the  $n$ -gram approximations, we have:

$$P(w_1, \dots, w_n) = \prod_{i=1}^n P(w_i) \quad \text{Unigrams}$$

$$P(w_1, \dots, w_n) = P(w_1) \prod_{i=2}^n P(w_i | w_{i-1}) \quad \text{Bigrams}$$

$$P(w_1, \dots, w_n) = P(w_1) P(w_2 | w_1) \prod_{i=3}^n P(w_i | w_{i-2}, w_{i-1}) \quad \text{Trigrams}$$

etc.



# Quality of a Language Model (II)

The probability value will depend on the length of the sequence. We take the geometric mean instead to standardize across different lengths:

$$\sqrt[n]{\prod_{i=1}^n P(w_i)} \quad \text{Unigrams}$$

$$\sqrt[n]{P(w_1) \prod_{i=2}^n P(w_i | w_{i-1})} \quad \text{Bigrams}$$

...

In practice, we use the log to compute the per word probability of a word sequence, the entropy rate:

$$H(L) = -\frac{1}{n} \log_2 P(w_1, \dots, w_n).$$

Here the lower, the better

The figures are usually presented with the perplexity metric:

$$PP(p, m) = 2^{H(L)}.$$



# Mathematical Background

Entropy rate:  $H_{rate} = -\frac{1}{n} \sum_{w_1, \dots, w_n \in L} p(w_1, \dots, w_n) \log_2 p(w_1, \dots, w_n),$

Cross entropy:

$$H(p, m) = -\frac{1}{n} \sum_{w_1, \dots, w_n \in L} p(w_1, \dots, w_n) \log_2 m(w_1, \dots, w_n).$$

We have:

$$\begin{aligned} H(p, m) &= \lim_{n \rightarrow \infty} -\frac{1}{n} \sum_{w_1, \dots, w_n \in L} p(w_1, \dots, w_n) \log_2 m(w_1, \dots, w_n), \\ &= \lim_{n \rightarrow \infty} -\frac{1}{n} \log_2 m(w_1, \dots, w_n). \end{aligned}$$

We compute the cross entropy on the complete word sequence of a test set, governed by  $p$ , using a bigram or trigram model,  $m$ , from a training set.



# Other Statistical Formulas

- Mutual information (The strength of an association):

$$I(w_i, w_j) = \log_2 \frac{P(w_i, w_j)}{P(w_i)P(w_j)} \approx \log_2 \frac{N \cdot C(w_i, w_j)}{C(w_i)C(w_j)}.$$

- T-score (The confidence of an association):

$$\begin{aligned} t(w_i, w_j) &= \frac{\text{mean}(P(w_i, w_j)) - \text{mean}(P(w_i))\text{mean}(P(w_j))}{\sqrt{\sigma^2(P(w_i, w_j)) + \sigma^2(P(w_i)P(w_j))}}, \\ &\approx \frac{C(w_i, w_j) - \frac{1}{N}C(w_i)C(w_j)}{\sqrt{C(w_i, w_j)}}. \end{aligned}$$



# T-Scores with Word set

Word	Frequency	Bigram set + word	<i>t</i> -score
<i>up</i>	134,882	5512	67.980
<i>a</i>	1,228,514	7296	35.839
<i>to</i>	1,375,856	7688	33.592
<i>off</i>	52,036	888	23.780
<i>out</i>	12,3831	1252	23.320

Source: Bank of English





# Mutual Information with Word *surgery*

Word	Frequency	Bigram word + surgery	Mutual info
<i>arthroscopic</i>	3	3	11.822
<i>pioneering</i>	3	3	11.822
<i>reconstructive</i>	14	11	11.474
<i>refractive</i>	6	4	11.237
<i>rhinoplasty</i>	5	3	11.085

Source: Bank of English



# Mutual Information in Python

```
def mutual_info(words, freq_unigrams, freq_bigrams):  
    mi = {}  
    factor = len(words) * len(words) / (len(words) - 1)  
    for bigram in freq_bigrams:  
        mi[bigram] = (  
            math.log(factor * freq_bigrams[bigram] /  
                    (freq_unigrams[bigram[0]] *  
                     freq_unigrams[bigram[1]]), 2))  
    return mi
```



# T-Scores in Python

```
def t_scores(words, freq_unigrams, freq_bigrams):  
    ts = {}  
    for bigram in freq_bigrams:  
        ts[bigram] = ((freq_bigrams[bigram] -  
                        freq_unigrams[bigram[0]] *  
                        freq_unigrams[bigram[1]] /  
                        len(words)) /  
                      math.sqrt(freq_bigrams[bigram]))  
  
    return ts
```

