

# Uni IT Security Notes

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## Uni IT Security Notes

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# Uni IT Security Notes

## Basics

# Security Mindset

- ▶ Focus on weaknesses, not on features
- ▶ Don't rely on the "good case"
- ▶ Anticipate what an attacker could do to a system
- ▶ Weight security against user experience and privacy

# Security Objectives

- ▶ **Confidentiality**/conf
  - ▶ Nobody but the legitimate receiver can read a message
  - ▶ Third party cannot gain access to communication patterns
- ▶ **Integrity**/int: The contents of communication can't be changed
- ▶ **Authenticity**/authN
  - ▶ **Entity Authentication**: Communication partners can prove their respective identity to one another
  - ▶ **Message Authentication**: It can be verified that a message is authentic (unaltered and sent by the correct entity)
- ▶ **Authorization**/authZ
  - ▶ Service or information is only available to those who have correct access rights
  - ▶ Depends on authentication being set up
- ▶ **Non-Repudiation**/nRep: A sender cannot deny having sent a message or used a service
- ▶ **Availability**/avail: Service is available with sufficient performance
- ▶ **Access Control**/ac: Access to services and information is

# Attacks, Threats and Vulnerabilities

- ▶ **Attacker:** A person who has the skill and motivation to carry out an attack: The steps needed to carry out an attack
- ▶ **Vulnerability:** Some characteristics of the target that can result in a security breach
- ▶ **Threat:** Combination of an attacker, an attack vector and a vulnerability
- ▶ **Attack:** A threat that has been realized and has caused a security breach

# Threat Identification

- ▶ Define **system boundaries**: What is part of your system, what is not?
- ▶ Define **security objectives**: What is important for your system to be secure?
- ▶ **List all threats** you can think of: Brainstorming and discussion with experts
- ▶ Use **conventions**:
  - ▶ Similar threat models
  - ▶ Requirement specifications
  - ▶ How to break or circumvent the specifications
  - ▶ Note security assumptions of the system
  - ▶ Be careful with perimeter security: What if perimeter has been breached?
  - ▶ Note *possible*, but not yet exploitable vulnerabilities



## Security Frameworks

# Network Specific Threat Examples

- ▶ Remote Attacks
- ▶ Eavesdropping: Sniffing of information
- ▶ Altering information
- ▶ Spoofing
- ▶ DoS
- ▶ Session hijacking
- ▶ Viruses attacking clients
- ▶ Spam
- ▶ Phishing
- ▶ Data trails/privacy leaks

# STRIDE: Attacks on a Multi-User System

- ▶ **S**poofing of Identity
- ▶ **T**ampering with Information
- ▶ **R**epudiation
- ▶ **I**nformation Disclosure
- ▶ **D**oS
- ▶ **E**scalation of Privileges

# Security policies

- ▶ Classification of system states into “allowed” and “forbidden” states
- ▶ Secure system: Is only in allowed states
- ▶ Breached system: Is in forbidden state

Malware

# Malware

- ▶ Performs unwanted functions
- ▶ Often runs without user's consent
- ▶ Telemetry (often hidden in proprietary software behind EULAs)
- ▶ Backdoors

## Networking

# TCP Overview

- ▶ Characteristics
  - ▶ Reliable
  - ▶ Connection-Oriented
  - ▶ Full-Duplex
  - ▶ Layer atop IP
  - ▶ Connection management: Setup, Release and Abort
  - ▶ Ordered delivery (package sequence control)
  - ▶ Repetition of lost packets
  - ▶ End-to-End ACKs
  - ▶ Checksum in header
- ▶ Identified by a 5-tuple
  - ▶ Source IP
  - ▶ Destination IP
  - ▶ Transport Protocol
  - ▶ Source Port
  - ▶ Destination Port



# TCP Connection Establishment

- ▶ Virtual connection between two systems
- ▶ 3-Way-Handshake with connection states

An example connection from the client to the server:

<Client>

<Server>

[Closed]

[Closed]

SEQ=x      CTL=SYN      =>

[SYN Sent]

<=    SEQ=y      CTL=SYN+ACK      ACK=x+1

[SYN Received]

SEQ=x+1      CTL=ACK      ACK=y+1    =>

[Established]

[Established]

# IP Security Issues

- ▶ IP header doesn't have confidentiality or integrity protection
  - ▶ Faking the sender address is easy to do
  - ▶ Traffic can be analyzed by sniffing packet headers
- ▶ IP payload doesn't have confidentiality or integrity protection
  - ▶ Eavesdropping is possible by sniffing packets
- ▶ Loose coupling with lower layers:
  - ▶ Easy to divert traffic
  - ▶ Availability can be easily attacked
  - ▶ Confidentiality and integrity can't be guaranteed
- ▶ Unprotected error signaling via ICMP: Fake error messages can affect availability
- ▶ DNS is insecure; i.e. DNS spoofing

# TCP Security Issues

- ▶ TCP header doesn't have confidentiality or integrity protection
- ▶ Session hijacking
  - ▶ When sniffing session details, attacker can impersonate a peer in a TCP connection
  - ▶ Attackers can guess session details and attack remotely using spoofed IP addresses
- ▶ RST attack: Attackers can reset/abort attacks by injecting packets with the RST flag
- ▶ Port scanning
  - ▶ Find out open ports
  - ▶ Determine software running on port
- ▶ SYN flooding
  - ▶ Overload system resources by initializing many connections and not pursuing them

# Port Scanning

- ▶ Objective: **Collect information**
  - ▶ Installed services
  - ▶ Software versions
  - ▶ OS
  - ▶ Firewall
- ▶ Enumeration based on port
  - ▶ Well-known ports (i.e. SSH → 22)
  - ▶ Invalid connection requests: Different way of error handling can be used to fingerprint the OS
- ▶ Possible scanning methods
  - ▶ TCP connect scan
  - ▶ Half-open scan
  - ▶ SYN-ACK scan
  - ▶ ACK scan

# TCP Protection Mechanisms

- ▶ SYN flood protection
  - ▶ Limit rate of SYN packets
  - ▶ SYN cookies (RFC 4987)
    - ▶ Limit resources
    - ▶ Half-open connections are not stored in the connection table but instead as a hash in the ISN
    - ▶ Only if the 3rd ACK handshake packet matches the sequence number, the connection is added to the connection table
    - ▶ Server does not need to maintain any state information on half-open connections: Resources can't be exhausted
- ▶ Connections are only accepted if the sequence numbers are within a certain range of acceptable values (attackers would have to sniff sequence numbers or guess them)

# Session Hijacking

- ▶ Attacker takes over existing connection between two peers
- ▶ Requirement: Attacker has to sniff or guess sequence numbers of the connection correctly

# RST Attacks (In-Connection DoS)

Inject packet with RST flag into ongoing connection: Connection has to be aborted immediately

## Blind IP Spoofing

Firewall is configured to only allow one source IP address and destination IP address ( $A \rightarrow B$ ).

To circumvent this restriction:

1. Attacker starts DoS attack on A to prevent A from sending RST packets to B
2. Attacker sends TCP connection setup packet with A's source IP address to B
3. B sends SYN+ACK packet to A, but can't respond due to DoS
4. Attacker sends TCP connection ACK packet to B with ACK matching the initial sequence number chosen by B (which has to be guessed, as B sent the SYN+ACK packet to A, not the attacker)

Only works if B uses a predictable algorithm for its ISN and packet filters aren't in place.



## Perimeter Defense in Practice

# Architecture Recommendations

- ▶ Known from medieval cities, castles etc.
- ▶ Definition of system boundary between “inside” and “outside”
- ▶ Different threat models for inside and outside
  - ▶ **Inside:** Trusted
  - ▶ **Outside:** Untrusted
- ▶ Objectives
  - ▶ Create said boundary
  - ▶ Only a defined set of communication relations is allowed
  - ▶ Special security checks
  - ▶ Limited number of interconnection points
  - ▶ Simpler to manage and audit than a completely open architecture
- ▶ Problems
  - ▶ Requires intelligent selection of system boundaries
  - ▶ May require multiple levels of perimeters
  - ▶ No system/user in the “trusted inside” can truly be trusted

# Application in Networking

- ▶ Installing security devices at the network border
- ▶ Separation of network areas into inside/outside
- ▶ Prevent sensitive information from being sent to the outside (view the system in the inside as the potential, probably unintentional attacker)
- ▶ Multiple levels can increase security
- ▶ But: Perimeter security is not sufficient on its own!
  - ▶ There will probably be additional non-secured paths into the network (i.e. `ssh -R`)
  - ▶ Some malicious traffic might look like “normal” traffic and can pass

# Stateless Packet Filter

- ▶ Access Control List (ACL): Applies set of rules to each incoming packets
- ▶ Discards (denies, blocks) or forwards (allows, permits) packets based on ACL
- ▶ Typically configured by IP and TCP/UDP header fields
- ▶ Stateless inspection: Established connections can only be detected with the ACK control flag
- ▶ Can be easy to misconfigure by forgetting essential protocols
  - ▶ DNS
  - ▶ ICMP
- ▶ Advantages
  - ▶ Fast/High throughput
  - ▶ Simple to realize
  - ▶ Software-based, can be added as a package
  - ▶ Simple to configure
- ▶ Disadvantages
  - ▶ Inflexible
  - ▶ Many attacks can only be detected using stateful filtering
  - ▶ Rules and their priorities can easily get confusing

# Stateful Packet Filters

- ▶ Store connection states
- ▶ Can make decisions based on
  - ▶ TCP connections
  - ▶ UDP replies to previous outgoing packet with same IP:Port relation (“UDP Connection”)
  - ▶ Application protocol states
- ▶ Similar to application layer gates/proxy firewalls, but less intruding in communication
- ▶ Rules can be more specific than in stateless packet filters
- ▶ Rules are easier to enforce, i.e. incoming TCP packets don't have to be allowed in because they have ACK set

# Stateful Firewalls

- ▶ Tries to fix the problems of stateless inspection
  - ▶ Too many packets have to be allowed by default (ACK → No SYN-scanning protection)
  - ▶ Protocols like FTP or SIP, which dynamically allocate port numbers, can't be filtered securely
- ▶ Create state per TCP or UDP flow
  - ▶ Source and Destination IP:Port
  - ▶ Protocol
  - ▶ Connection state
- ▶ A packet which is not associated with a state is dropped immediately
- ▶ Packets which belong to a previously established TCP/UDP "connection" are allowed to pass without further checks
- ▶ State tables have to be cleaned up periodically to prevent resource starvation

# Application Layer Proxies

- ▶ Protected host during connection establishment
- ▶ Different kinds
  - ▶ Application level
  - ▶ Circuit level
  - ▶ Forward proxy (client-side)
  - ▶ Reverse proxy (server-side)

# Application Level Gateways

- ▶ Conversion between different application layer protocols
- ▶ Evaluation up to OSI layer 7
  - ▶ Protocol verification
  - ▶ Authentication
  - ▶ Malware scanning
  - ▶ Spam filtering
  - ▶ Attack pattern filtering
- ▶ Advantage: Security policies can be enforced at application level
- ▶ Disadvantage: Computing and memory performance requirements



## Demilitarized Zone (DMZ)

- ▶ **Outside world:** Global Internet
- ▶ **Outside router:** Routes packet to and from bastion host
- ▶ **Bastian host:** Proxy server and relay host
- ▶ **Inside router:** Routes packets only to and from bastion host
- ▶ **Inside (protected):** Intranet

The DMZ creates 2/3 lines of defense by the use of a stub network.

Multi-Level DMZs can create even more secure perimeter defenses:

Global Internet → Access Router and Packet Filter → Public Services Host (offers i.e. public Web services) → Screening Router and Packet filter (prevents IP spoofing) → Mail host (for external mail communication) → Bastion host (i.e. proxy for FTP and Web access) → Intranet

# Web Application Firewalls (WAFs)

- ▶ Acts on the application layer
- ▶ Is a reverse proxy
- ▶ Can protect the web server from “evil” client input
  - ▶ Cross-Site scripting
  - ▶ SQL injection: Filters out JS or SQL commands in client input by removing special symbols (i.e. <, ' etc)
  - ▶ Cookie poisoning: Stores the hash values of sent cookies
  - ▶ HTML manipulation: Encrypts URL parameters

# Intrusion Detection Systems (IDS)

- ▶ Security product that is specialized on detecting anomalies during live operation of networks and computers
  - ▶ Virus/Botnet activity
  - ▶ Suspicious network activity (malware phoning home)
- ▶ Basic Approaches
  - ▶ **Signature based:** Use attack signatures/known malicious communication activity patterns
  - ▶ **Anomaly based:** Significant deviation from previously recorded baseline activity
  - ▶ **Rule based:** Define allowed by behaviour by app-specific set of legitimate actions
- ▶ Actions
  - ▶ Send ut alarm
  - ▶ Logging
  - ▶ Blocking of known patterns
- ▶ Realization
  - ▶ Appliance
  - ▶ Integration in firewall
  - ▶ Integration into host

## Symmetric Encryption

# Symmetric Encryption Overview

## **Alice:**

1. Creates message
2. Chooses key
3. Computes ciphertext
4. Send ciphertext to Bob

## **Eve (Attacker):**

1. Copies ciphertext
2. Tries to guess the key

## **Bob:**

1. Receives ciphertext
2. Uses key
3. Computes plaintext
4. Reads message

# Kerckhoffs' Principle

- ▶ From “La Cryptographie Militaire”
- ▶ Most important point: **The security of a crypto system must lie in the non-disclosure of the key but not in the non-disclosure of the algorithm**
- ▶ Implementation
  - ▶ Keep secret which function you used for encryption
  - ▶ But a disclosure of the set of functions should not create a problem

# Strong Algorithms

- ▶ There is no attack that can break it with less effort than a brute force attack (“complete enumeration”)
- ▶ There are so many keys that a complete search of key space is infeasible

# Crypto Attack Classes

- ▶ **Active** attacks
  - ▶ Most relevant for cryptographic protocols
  - ▶ Active interference (modification, insertion or deletion of messages)
  - ▶ Man in the middle (MITM) can receive messages and modify them on the way to the receiver
- ▶ **Passive** attacks: Pure eavesdropping, without interference with communication



# Perfect Security

Ciphertext does not give any information you don't already have about the plaintext

# One-Time-Pad

- ▶ **Vernam Cypher:** Create ciphertext by XOR addition of secret key and plaintext
- ▶ **Mauborgne:** Random key, never re-use key (“one time”)
- ▶ **Shannon:** OTP is unbreakable if key is ...
  - ▶ Truly random
  - ▶ As large
  - ▶ Never reused
  - ▶ Kept secret

# Stream Cyphers

Encryption like one-time-pad, but using pseudo-random bits instead of true random (using a **Cryptographically Secure Pseudo-Random Number Generator (CSPRNG)**)

# Cryptographically Secure Pseudo-Random Number Generators (CSPRNG)

A CSPRNG must ...

- ▶ Be unpredictable
- ▶ Be computationally infeasible to compute the next outputs

... when the initial state of the CSPRNG is not known

# Design Principles for Block Cyphers

Two methods for frustrating a statistical analysis:

- ▶ **Confusion:** The ciphertext should depend on the plaintext in such a complicated way that an attacker cannot gain any information from the ciphertext (redundancy should not be visible anymore in the ciphertext)
- ▶ **Diffusion:** Each plaintext and key bit should influence as many ciphertext bits as possible
  - ▶ Changing one bit in plaintext → Many pseudo-random changes in ciphertext
  - ▶ Changing one bit in the key → Many pseudo-random changes in ciphertext

# Feistel Networks

- ▶ Described by Horst Feistel
- ▶ Algorithm
  - ▶ Plaintext block B is divided in 2 halves
  - ▶ Derive  $r$  round key keys from key
  - ▶ Feed one half through round function  $F$
  - ▶ Then XOR the result with the other half
  - ▶ Exchange halves
- ▶ Repeat  $r$  times

# DES (Tripple DES)

- ▶ Single DES breakable in less than 24h (complete search of key space)
- ▶ Tripple DES is still secure
- ▶ Three steps of DES on each data block using up to three keys
- ▶ Decryption in reverse sequence
- ▶ 3 independend keys are the most secure
- ▶ Three same keys can be used for (insecure) DES compatibility

# AES Key Features

- ▶ FIPS standard 197
- ▶ Key length: 128/192/256 bit
- ▶ Block size: 128 bit
- ▶ Iterative rounds of substitutions and permutation, but no Feistel structure
- ▶ 10, 12 or 14 rounds
- ▶ Blocks of 16 bytes arranged in 4x4 state matrix
- ▶ Components of the round function are invertible and independent of key
  - ▶ **Substitute Bytes:** Non-linear substitution of bytes in state
  - ▶ **Shift Rows:** Cyclic shifting of rows
  - ▶ **Min Columns:** Multiplication of state elements with a fixed 4x4 matrix M



# Modes of Operation for Block Cyphers

- ▶ Objective: Encrypt multiple plaintext blocks with the same block cypher
- ▶ Straightforward solution: blockwise encryption (“Electronic Codebook Mode”)
- ▶ Problem: Patterns in the distribution of plaintext blocks remain visible

# Cypher Block Chaining (CBC)

- ▶ Avoids telltale patterns in ciphertext
- ▶ Decryption fails if a data block is missing or corrupted
- ▶ Each data block is encrypted in relation to the previous block

# Counter Mode (CTR)

- ▶ Simple and efficient
- ▶ Random access still possible
- ▶ No issues if data block is missing
- ▶ Incrementing counter is involved in randomization per data block

# Padding

- ▶ Plaintext needs to be a full number of blocks
- ▶ If plaintext does not fill the last block completely, it must be padded before encryption
  - ▶ In order to facilitate safe decryption, the last block is always padded: For example for a block size of  $n$  bytes, there are  $1 \dots n$  bytes added to the plaintext before encryption
  - ▶ Decryption can check last bytes and strip them off correspondingly
- ▶ Always need to pad with at least one byte!
- ▶ Common methods
  - ▶ Pad with bytes of the same value as the number of padding bytes (PKCS#5; i.e. if there are three bytes to be padded, add 0x03 0x03 0x03)
  - ▶ Pad with 0x80 followed by 0x00 bytes
  - ▶ Pad with zeroes except for the last byte that indicates the number of padding bytes
  - ▶ Pad with zeroes
  - ▶ Pad with space characters (0x20)

# Key Length Considerations

- ▶ Cryptography is always a matter of complexity
  - ▶ With enough time and/or space, all schemes can theoretically be broken
  - ▶ “brute force” attacks
  - ▶ Example: 56bit keys DES can be broken in <24h since 1999
- ▶ Meanwhile
  - ▶ 128bit keys have to be replaced in the coming years
  - ▶ 192bit keys are secure in medium term
  - ▶ 256bit keys are hard to crack due to physical boundaries
- ▶ Quantum computers might be able to crack keys much more quickly
- ▶ Numbers refer to unbroken algorithms in symmetric cryptography
  - ▶ Broken algorithm is one where an  $n$  bit key can be determined trying out significantly less than  $2^n$  keys

## Message Authentication

# Message Authentication Codes (MACs)

- ▶ Objectives
  - ▶ **Integrity protection:** Prevent unauthorized manipulation of data
  - ▶ **Message authentication:** Prevent unauthorized origination on behalf of others
- ▶ Idea: Compute a cryptographic checksum (MAC)
- ▶ Required Properties
  - ▶ Cannot be counterfeited; without having the sender's secret, it is too complex to ...
    - ▶ Find another message matching the same MAC
    - ▶ Construct a suitable MAC for another message
  - ▶ Even smallest changes to message cause a big change of the MAC

# General Scenario

## Alice:

1.  $m = \text{"I love you. Alice"}$
2. Select secret key  $K$
3. Compute  $MAC_K(m)$

## Bob:

1. Receives  $m'$
2. Selects secret key  $K$
3. Computes  $MAC_K(m')$
4. Compares computed MAC with received MAC  $\rightarrow$  Matches!

**Assertion:** If computed MAC equals the MAC included in the received message, an owner of the key (Alice) really sent this message and it was not changed on the way.



## Scenario with Modified Message

**Alice:** Same as in General Scenario

**Mallory:**

►  $m = \text{"It's all over! Alice."}$

**Bob**

1. Receives  $m'$
2. Selects secret key  $K$
3. Computes  $MAC_K(m')$
4. Compares computed MAC with received MAC → Doesn't match!
5. Ignore  $m$

# MAC Computation

- ▶ Requirements
  - ▶ Shared key  $k$  between sender and receiver
  - ▶ Hash function to create a code that changes if the message has been altered
- ▶ Using **block cypher**  $f_k$  and **hash function**  $hash$ :  
 $MAC(m) = f_k(hash(m))$
- ▶ Using a **key dependent cryptographic hash function**  
 $hash(k, m)$ :  $MAC(m) = hash(k, m)$

# Hash Function Requirements

- ▶ Weak **collision resistance**: For a given message and hash it is impossible/to complex to find another message such that the hashes match
- ▶ **One-way** property
  - ▶ Easy to compute hash
  - ▶ Impossible to find message from hash

## Asymmetric Encryption

# Public Key Cryptography

## Alice:

1. Generates key pair  $(PK_{Alice}, SK_{Alice})$
2. Published  $PK_{Alice}$  at Trent's
3.  $c$  received  $\rightarrow$  decrypts  $m = D_{SK_{Alice}}(C)$

## Trent:

- ▶ Stores public keys
- ▶ Provides public keys on request

## Bob:

1. Wants to send  $m$  to Alice confidentially
2. Obtains  $PK_{Alice}$  from Trent
3. Computes  $c = E_{PK_{Alice}}(m)$
4. Sends  $c$  to Alice

# RSA Key Generation

1. Alice chooses 2 large prime numbers  $p, q$  and computes  $n = p \cdot q$ ,  $\phi(n) = (p - 1)(q - 1)$
2. Alice chooses an integer  $e$  with  $1 < e < \phi(n)$  that is relatively prime to  $\phi(n)$
3. Alice computes an integer  $d$  with  $1 < d < \phi(n)$  and  $d \cdot e = k \cdot \phi(n) + 1$
4. Alice publishes her public key  $PK_{Alice} = (e, )$
5. Alice keeps her private key  $SK_{Alice} = d$  and  $p, q, \phi(n)$  secret

# RSA Encryption

1. Bob obtains  $PK_{Alice} = (e, n)$
2. Bob composes plaintext  $m \in M = \{1, 2, \dots, n - 1\}$
3. Bob computes the ciphertext  $c = E_{PK_{Alice}}(m) = m^e \bmod n$
4. Bob sends  $c$  to Alice

# RSA Decryption

Alice can obtain the plaintext message  $m$  by computing

$$m = D_{SK_{Alice}}(c) = c^d \bmod n = m^{ed} \bmod n$$



# RSA Security

- ▶ **RSA problem:** Given  $e$ ,  $n$  and  $c = m^e \bmod n$ , find  $m$ 
  - ▶ Most efficient approach to solve the RSA problem is currently the integer factorization of  $n$ : An upper limit to the complexity of the problem; can be used to derive the private key from the prime factors
  - ▶ Quantum computers will be more efficient in doing integer factorization (Shor's algorithm)
  - ▶ RSA problem and integer factorization still lack mathematical proof for their complexity
- ▶ **Organizational properties**
  - ▶ **Authenticity** of the public key  $(e, n)$
  - ▶ **Confidentiality** of the secret key  $(d, p, q)$
- ▶ **Mathematical properties**
  - ▶ **Complexity of factoring** the modulus  $n$
  - ▶ **Complexity of solving** the RSA problem
- ▶ Failure of any properties will compromise the security of the method!

# Hybrid Method

Combination of asymmetric and symmetric key methods.

## Alice:

1. Generates key pair  $(PK_{Alice}, SK_{Alice})$
2. Publishes  $PK_{Alice}$  at Trent's
3.  $c$  received  $\rightarrow$  Decrypts  $K = D_{SK_{Alice}}(c)$
4. Alice and bob switch over to the symmetric key algorithm with key  $K$

## Trent:

- ▶ Stores public keys
- ▶ Provides public keys on request

## Bob:

1. Obtains  $PK_{Alice}$  from Trent
2. Generates symmetric key  $K$
3. Computes  $c = E_{PK_{Alice}}(K)$
4. Sends  $c$  to Alice

# Discrete Logarithms

**Primitive element:** Let  $p$  be a prime number. An element  $g \leq p - 1$  is called primitive element  $\bmod p$  if for each  $A \in \{1, 2, \dots, p - 1\}$  there is an  $x$  such that  $A = g^x \bmod p$

**Discrete logarithm:** Let  $p$  be a prime number and let  $g \leq p - 1$  be a primitive element  $\bmod p$ . Then an element  $x$  is called discrete logarithm of  $A$  to base  $g \bmod p$  if  $A = g^x \bmod p$ .

**Discrete logarithm problem:** Given  $A, g, p$ , find  $x \leq p - 1$  with  $A = g^x \bmod p$

# One-Way Functions

- ▶ “Trap-door” functions
- ▶ Easy to compute in one direction (i.e.  $f(x) = g^x \bmod p$ )
- ▶ Hard to invert
  - ▶ Ideally only possible using complete enumeration of all possible inputs
  - ▶ I.e. for a given  $y$  you need to try out all possible values  $x = 0, 1, \dots, p - 1$  to find one  $x_0 : f(x_0) = y$
- ▶ Definition of complexity often of the P and NP complexity classes
  - ▶ **P**: Answer of a problem can be found in polynomial time ( $b$  bits of problem size  $\rightarrow$  algorithm takes time  $b^k$ )
  - ▶ **NP**: Answer of problem cannot be found in polynomial time ( $b$  bits of problem size  $\rightarrow$  algorithm takes time  $k^b$ ), but the correctness of given answer can be checked in polynomial time

# Diffie-Hellman Key Exchange Protocol

**Purpose:** Allow communication partners without prior knowledge of another to establish a shared secret key over an insecure communication channel

1. Alice and Bob agree publicly on prime number  $p$  and a primitive element  $g \leq p - 1$
2. Alice randomly chooses  $\alpha \in \{2, \dots, p - 2\}$  and computes  $A = g^\alpha \mod p$
3. Bob randomly chooses  $\beta \in \{2, \dots, p - 2\}$  and computes  $B = g^\beta \mod p$
4. Alice and Bob publicly exchange  $A$  and  $B$
5. Alice and Bob hold a common secret key  $K$ :
  - 5.1  $K_B = A^\beta \mod p = g^{\alpha\beta} \mod p$
  - 5.2  $K_A = B^\alpha \mod p = g^{\alpha\beta} \mod p = K_B$

# Diffie-Hellman Key Exchange Protocol Security

It depends on three properties which can't be relaxed:

- ▶ **Discrete logarithm problem:** There is no efficient inversion for integer exponentiation
- ▶ **Authenticity** of exchanged messages: No protection against MITM attacks!
- ▶ **Diffie-Hellman problem complexity:** Given  $g, p, A = g^x \bmod p, B = g^y \bmod p$  find  $K = g^{xy} \bmod p$

# Digital Signatures

- ▶ Requirements
  - ▶ **Tamper-proof**
  - ▶ **Unambiguous attribution** of signature to signing person/identity
  - ▶ **Inseparable connection** between signature and signed document
  - ▶ **Non-repudiability** of signature
- ▶ Typical approach
  - ▶ Encrypt hash of document with secret key
  - ▶ Signature can be verified using the public key

## Alice:

1. Generates key pair  $(PK_{Alice}, SK_{Alice})$
2. Publishes  $PK_{Alice}$  at Trent's
3. Computes  $sig_{Alice}(m) = E_{SK_{Alice}}(hash(m))$

## Trent:

- ▶ Stores public keys
- ▶ Provides public keys on request

# RSA Signatures

- ▶ Conventions
  - ▶  $PK_{Alice} = (e, n)$
  - ▶  $SK_{Alice} = d$
  - ▶  $m$  is the message to be signed
  - ▶  $h$  is the secure hash function
- ▶ **Computation** of signature:  $sig_{Alice}(m) = (h(m))^d \mod n$
- ▶ **Verification** of signature
  - ▶ Bob receives  $(m', sig')$
  - ▶ Bob computes  $h(m')$  and  $(sig')^e \mod n$
  - ▶ If both match, the signature is verified