Math508 Homework 6

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Abstract

Simple HMM in Smoothing and Prediction

1 No. 1

Hidden markov model on a simple reflected r.w. X_n on A[0,20] starting at 10. $P(W_1 = \pm L) = \frac{1}{2}$. $Y_n = min\{max\{X_n + W_n, 0\}, 20\}$. See all figures.

1.1 No.1 Part a

It's about the smoothing estimates of X_{100}: $\hat{X_{100,T}}$ for $T = 100, ..., 200$. Plot on the same graph X_{100} (as a constant) and $\hat{X_{100,T}}$. See Figure 1 with $L = 5$. X_100 and $\hat{X_{100,T}}$ are overlapped.

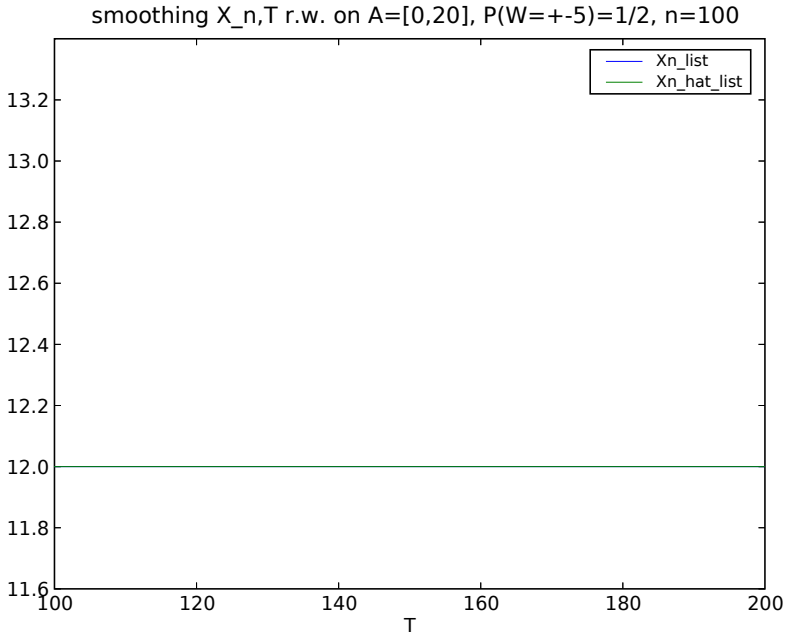

Figure 1:

1.2 No.1 Part b

Get the prediction estimates of X_{200}: $\hat{X_{200,n}}$ for $n = 100, ..., 200$. Plot on the same graph X_{200} and $\hat{X_{200,n}}$. For $L = 1$, check Figure 2; for $L = 3$, check Figure 3; for $L = 5$, check Figure 4; for $L = 10$, check Figure 5; for $L = 15$, check Figure 6; for $L = 18$, check Figure 7; for $L = 20$, check Figure 8.

2 No. 2

Tossing of coins with initial coin randomly(uniformly) selected. Tossing continues possibly changing the coin with probability 1/3. Assume we observed $Y_{[0,10]} = HHTHHTHTTTT$.

2.1 No. 2 Part a, Smoothing

Find $P(X_5 = i|Y_{[0,8]})$ and $P(X_5 = i|Y_{[0,10]})$, $i = 1, 2, 3$. Here is the answer. The 1st row is $Y_{[0,8]}$ and 2nd row is $Y_{[0,10]}$. The columns correspond to $i = 1, 2, 3$. It looks like future information doesn't help in the smoothing. The reason is that the backward equation $\mu_n^i(b_{[n+1,T]})$ has same value for different i regardless of n.

```
[[ 0.46666667  0.33333333  0.2       ]
 [ 0.46666667  0.33333333  0.2       ]]
```

2.2 No. 2 Part b, Prediction

Find $P(X_{10} = i|Y_{[0,5]})$ and $P(X_{10} = i|Y_{[0,6]})$, $i = 1, 2, 3$. Here is the answer. The 1st row is $P(X_{10} = i|Y_{[0,5]})$ and 2nd row is $P(X_{10} = i|Y_{[0,6]})$. The columns correspond to $i = 1, 2, 3$. It seems the observation has no power in terms of prediction. The reason is that the transition matrix of X, $P^n(X_i, X_j)$ doesn't change with n.

```
[[ 0.33333333  0.33333333  0.33333333]
 [ 0.33333333  0.33333333  0.33333333]]
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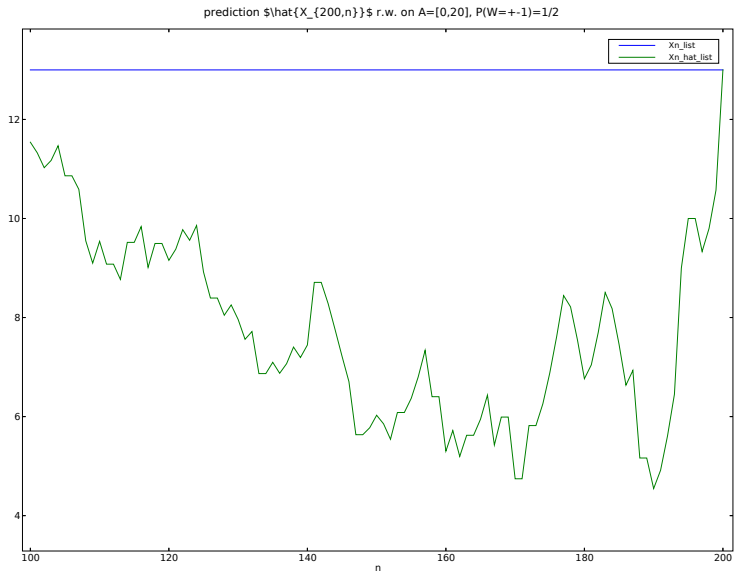

Figure 2:

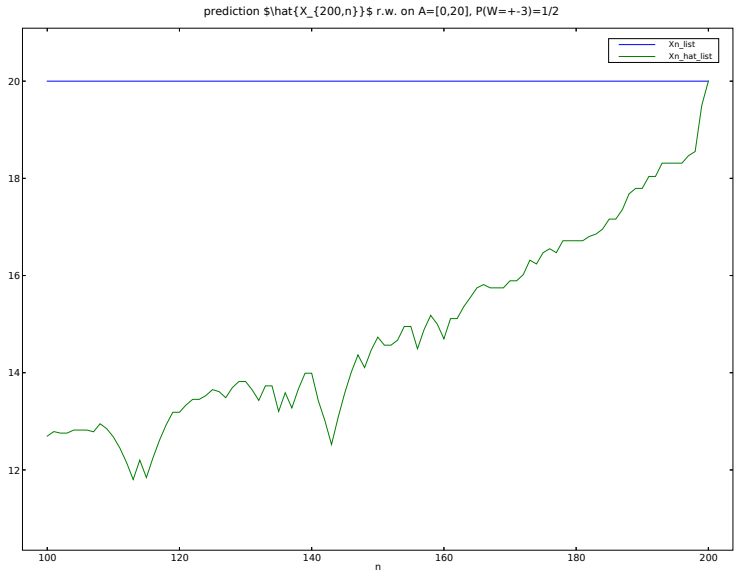

Figure 3:

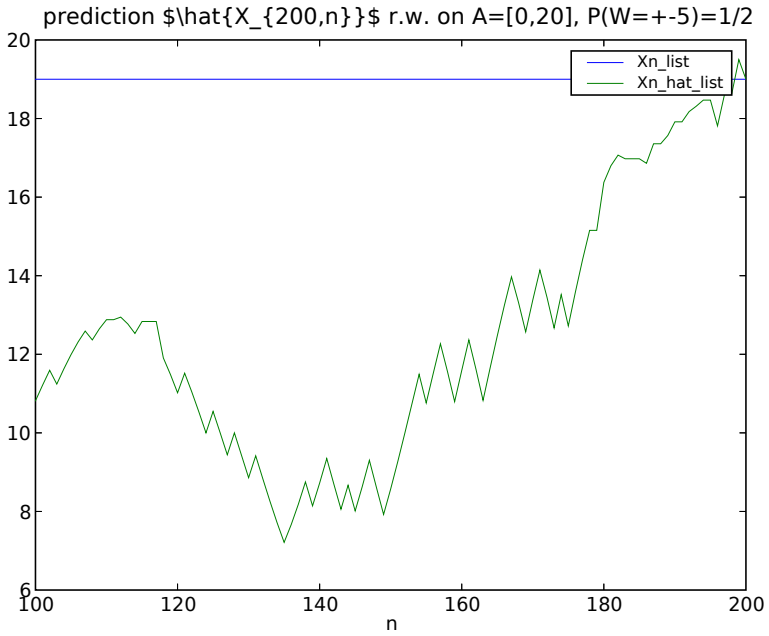

Figure 4:

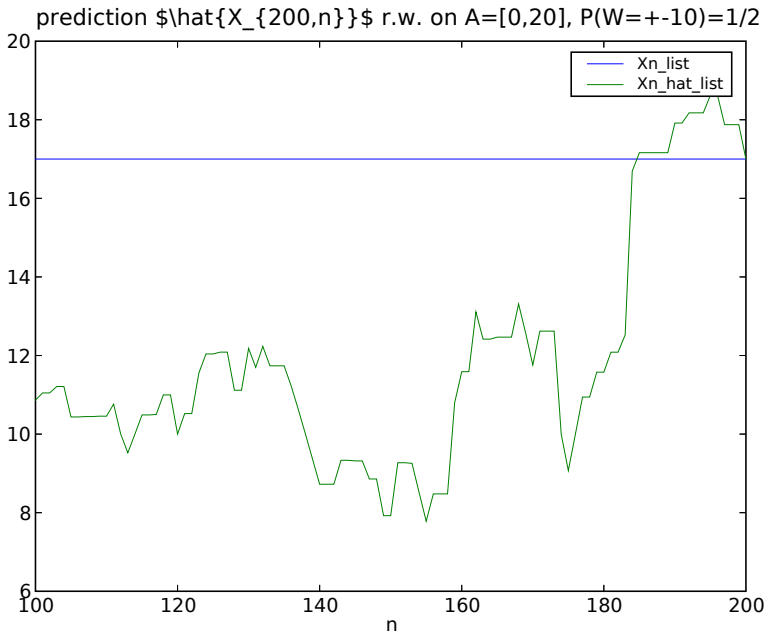

Figure 5:

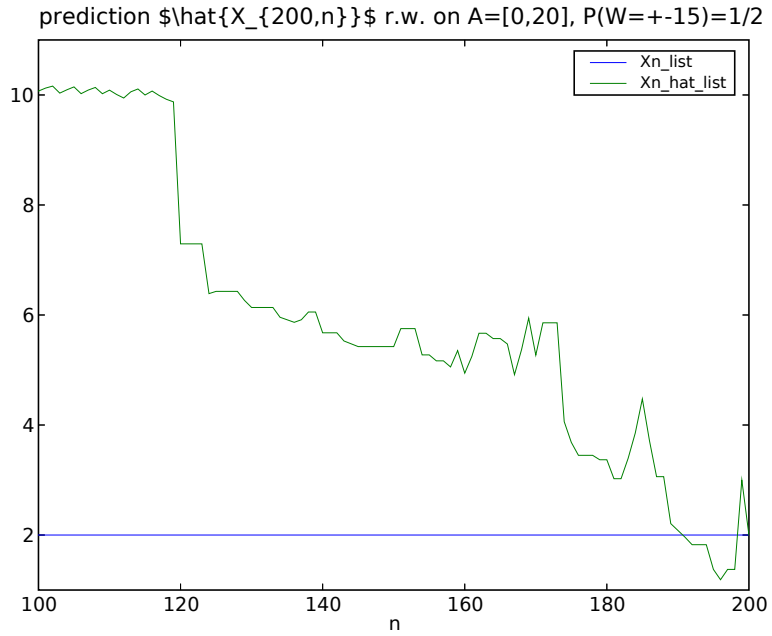

Figure 6:

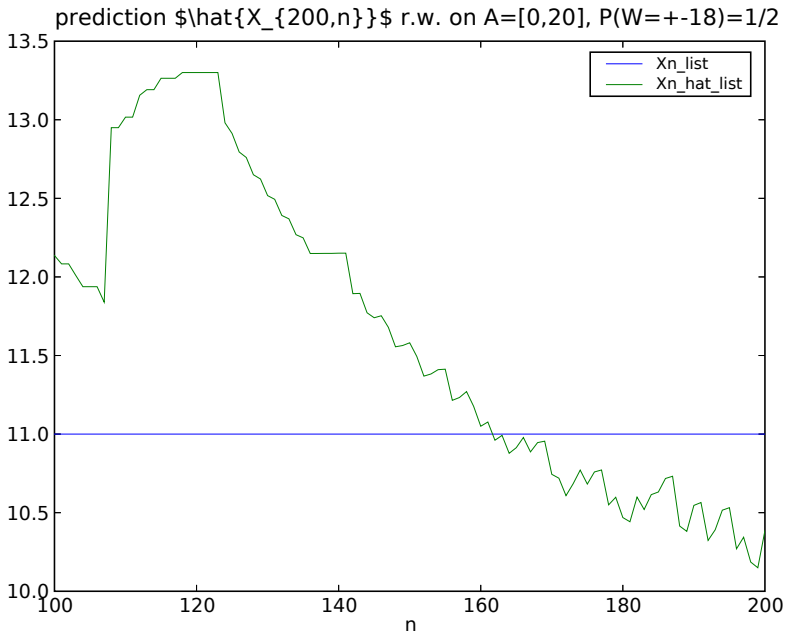

Figure 7:

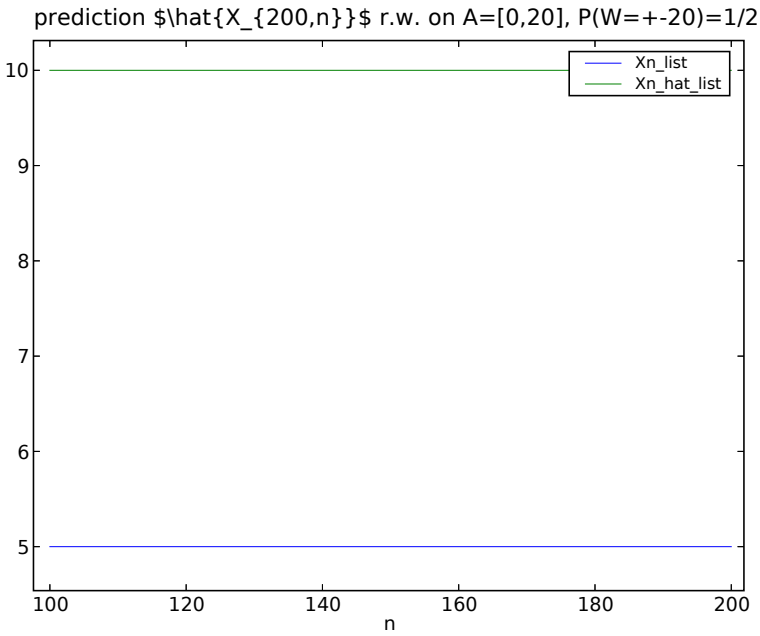
prediction $\hat{X_{200,n}}$ r.w. on A=[0,20], P(W=+-20)=1/2
Xn_list
Xn_hat_list
10
9
8
7
6
5
100
120
140
160
180
200
n

Figure 8: