1. (20 points, tracking a reflected r.w.) Assume the signal X_n is a simple reflected r.w. on $A = [0, 20] \cap \mathbf{Z}$ straiten at 10. Let $W_0, W_1, \ldots$ be a sequence of i.i.d. taking two values with prob. 1/2:

$$\mathbf{P}\left(W_1 = \pm L\right) = 1/2,$$

The observation Y_n is defined by

$$Y_n = \min\left\{\max\left\{X_n + W_n, 0\right\}, 20\right\}.$$

(It is hidden Markov model with $\gamma(x, w) = \min\left\{\max\left\{x + w, 0\right\}, 20\right\}$)

Simulate X_n, Y_n, and the best estimate

$$\hat{X}_n = \frac{\sum_{a \in A} a\phi_n^a}{\sum_{r \in A} \phi_n^r}$$

in time interval $[0, 200]$, for $L = 1$; $L = 3$; $L = 5$; $L = 10$; $L = 15$; $L = 18$; $L = 20$.

Plot X_n and $\hat{X}_n$ and also $X_n, \hat{X}_n, Y_n,\ 0 \leq n \leq 200$.

2. (5 points). Assume we have three coins $\{1, 2, 3\}$ with the following probabilities

	1	2	3
$P(H)$	0.3	0.5	0.7
$P(T)$	0.7	0.5	0.3

At $n = 0$ a coin is randomly selected and tossed, then the tossing of the same coin continues. Assume we observed $Y_{[0,4]} = (H, H, H, T, T)$.

The "signal" $X_n \in \{1, 2, 3\}$ is the chosen coin (it has a constant "trajectory" only initial distribution is random: it is 1,2 or 3 with prob. 1/3)

Find $\phi_n^a = \phi_n^a\left(Y_{[0,n]}\right)$, and $\pi_n^a = \pi_n^a\left(Y_{[0,n]}\right)$, $a \in \{1, 2, 3\}, n = 0, 1, 2, 3, 4$.