

# Math508 Homework 5

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## Abstract

Simple HMM

## 1 No. 1

Hidden markov model on a simple reflected r.w.  $X_n$  on  $A[0,20]$  starting at 10.  $P(W_1 = \pm 1) = \frac{1}{2}$ .  $Y_n = \min\{\max\{X_n + W_n, 0\}, 20\}$ . See all figures.

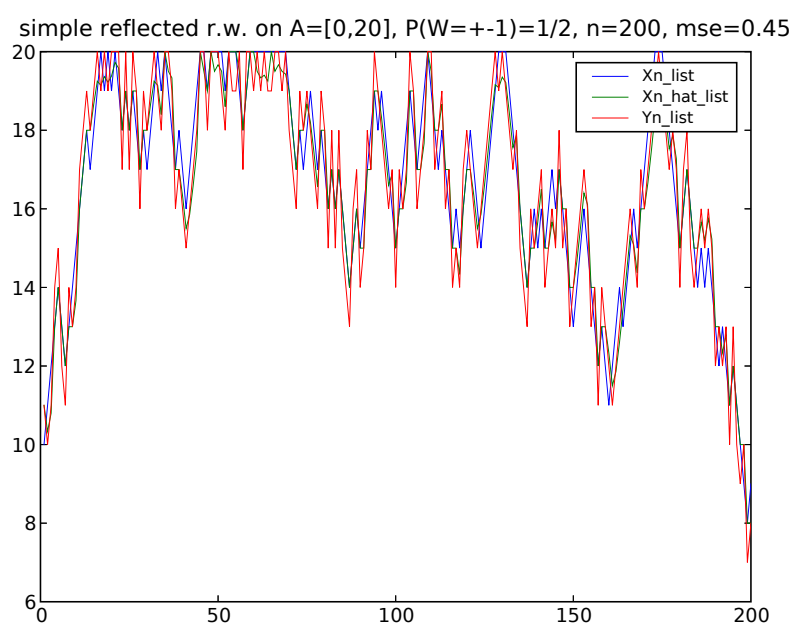


Figure 1:

## 2 No. 2

Tossing of a constant coin with initial coin randomly (uniformly) selected. Given  $Y_{[0,4]} = (H, H, H, T, T)$ , the  $\phi_n^a$  matrix (row is  $n$ , column is the hidden state) is

simple reflected r.w. on  $A=[0,20]$ ,  $P(W=+-3)=1/2$ ,  $n=200$ ,  $mse=0.26$

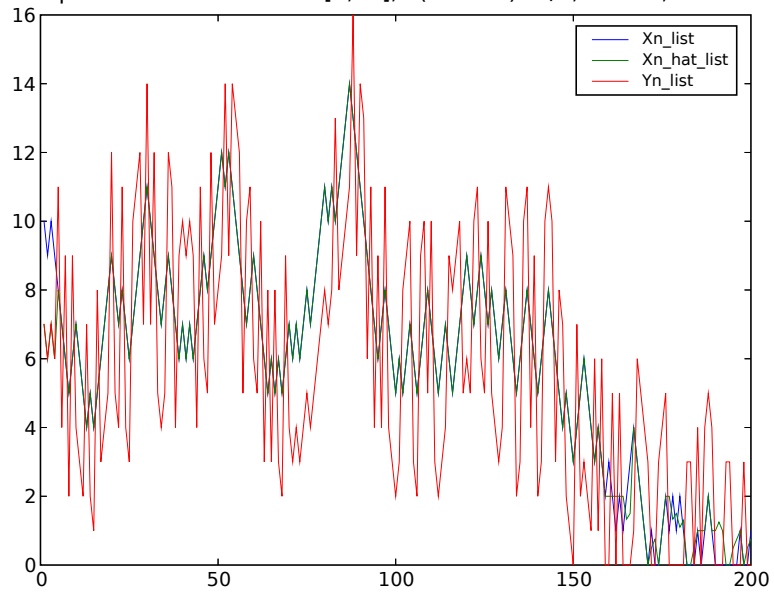


Figure 2:

simple reflected r.w. on  $A=[0,20]$ ,  $P(W=+-5)=1/2$ ,  $n=200$ ,  $mse=0.75$

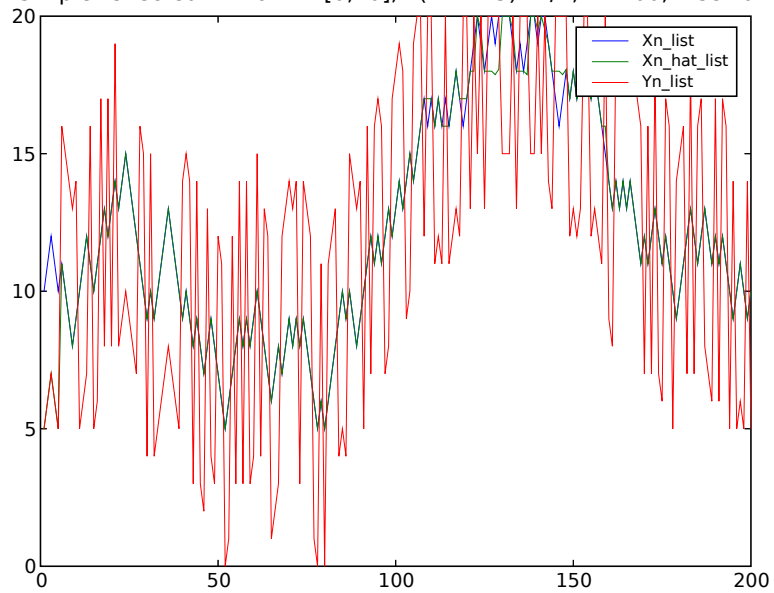


Figure 3:

simple reflected r.w. on  $A=[0,20]$ ,  $P(W=+-10)=1/2$ ,  $n=200$ ,  $mse=0.68$

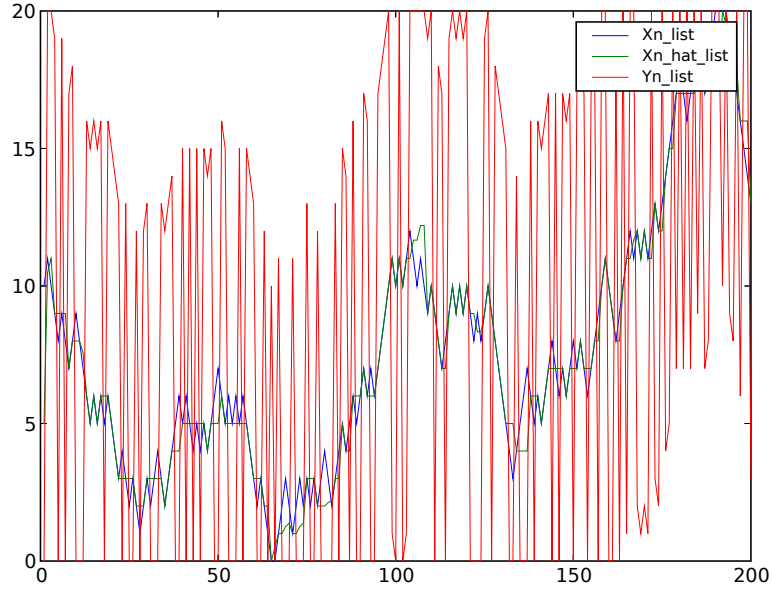


Figure 4:

simple reflected r.w. on  $A=[0,20]$ ,  $P(W=+-15)=1/2$ ,  $n=200$ ,  $mse=4.27$

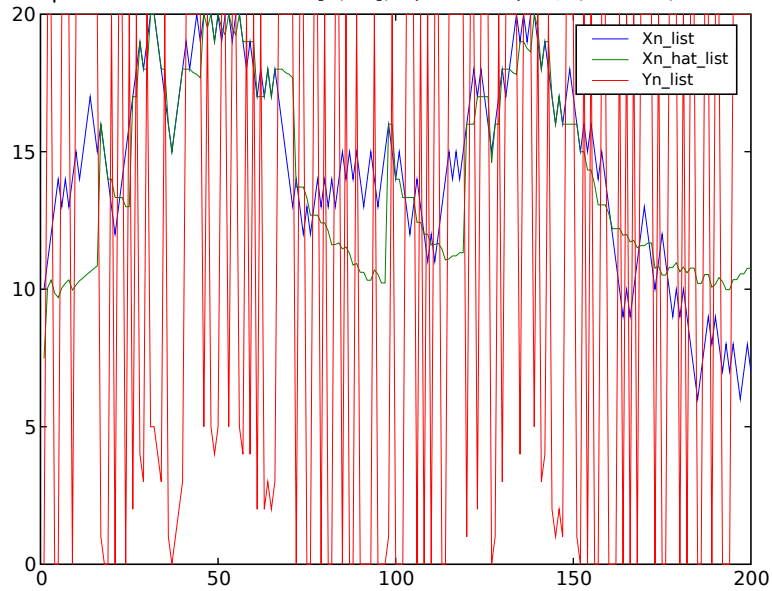


Figure 5:

simple reflected r.w. on  $A=[0,20]$ ,  $P(W=+-18)=1/2$ ,  $n=200$ ,  $mse=6.94$

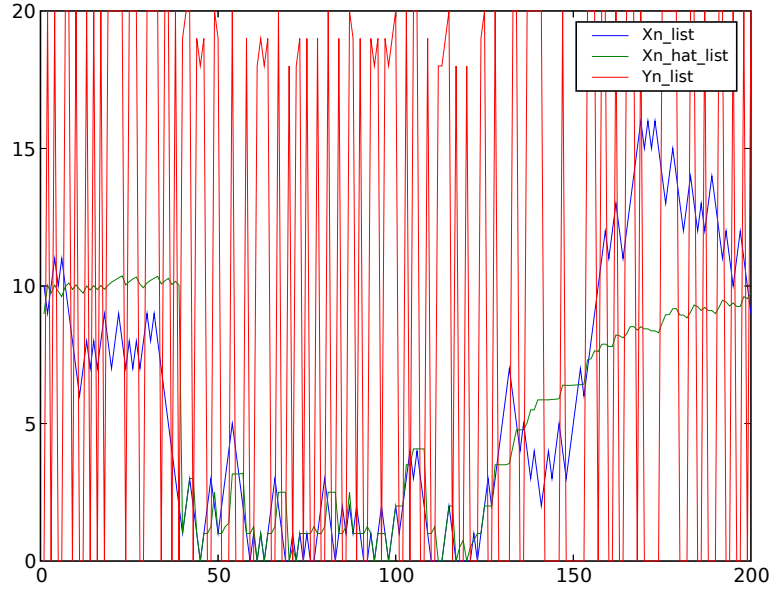


Figure 6:

simple reflected r.w. on  $A=[0,20]$ ,  $P(W=+-20)=1/2$ ,  $n=200$ ,  $mse=19.04$

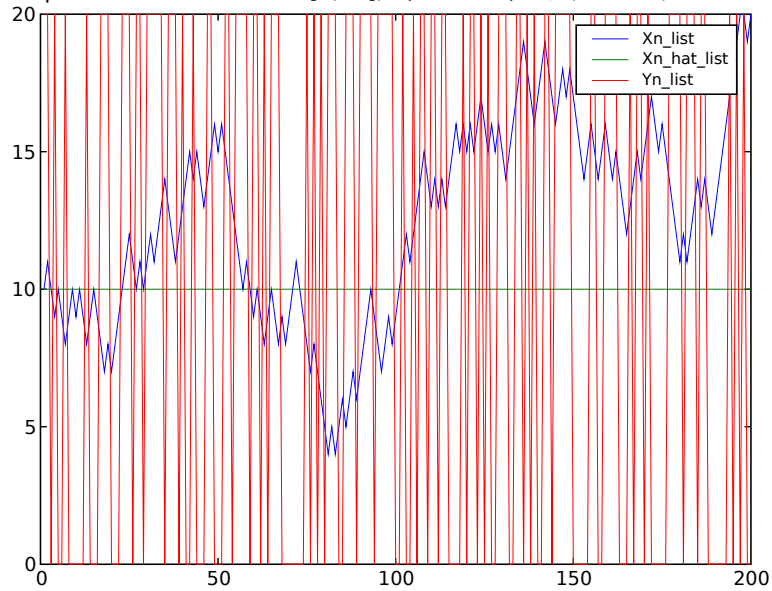


Figure 7:

```
[[ 0.1          0.16666667  0.23333333]
 [ 0.03         0.08333333  0.16333333]
 [ 0.009        0.04166667  0.11433333]
 [ 0.0063       0.02083333  0.0343    ]
 [ 0.00441      0.01041667  0.01029   ]]
```

the posterior  $\pi_n^a$  matrix is

```
[[ 0.2          0.33333333  0.46666667]
 [ 0.10843373  0.30120482  0.59036145]
 [ 0.05454545  0.25252525  0.69292929]
 [ 0.10255019  0.339121    0.55832881]
 [ 0.17558062  0.41473125  0.40968812]]
```

The estimated  $\hat{X}_n$  vector (the state space is  $\{0, 1, 2\}$  is

```
[1.266, 1.482, 1.638, 1.456, 1.234]
```

So it's pretty much the 2nd or 3rd coin.