1. a) Simulate and plot the trajectory of a simple reflected r.w. $X_n, 0 \leq n \leq N$, with absorption on $[0, 20] \cap \mathbf{Z}$ starting at 5 for $N = 200, N = 1000; N = 2000; N = 20000$;

b) simulate and plot the trajectory of a simple r.w. $X_n, 0 \leq n \leq N$, with absorption on $[0, 20] \cap \mathbf{Z}$ starting at 10 for $N = 50, N = 100; N = 150$.

(Plot $(X_0, X_1, \ldots, X_N)$ against $(0, 1, \ldots, N)$)

2. Assume that Z_n is a Markov Chain on S with transition probabilities $p_{n+1}(c, z), n \geq 1, c, z \in S$. Show that $X_n = (n, Z_n)$ is a time homogeneous Markov chain on
$\tilde{S} = \mathbf{N} \times S$, where $\mathbf{N} = \{0, 1, 2, \ldots\}$.

3. a) Consider a simple reflected r.w. on $[1, d] \cap \mathbf{Z} = \{1, \ldots, d\}$. Find the invariant distribution. Is it unique? Is there an integer m such that P^m has strictly positive entries?

Hint. The transition probabilities are $p_{ij} = p(i, j)$ are given by $p_{11} = p_{12} = 1/2, p_{d-1} = p_{dd} = 1/2$ and $p_{i,i-1} = p_{i,i+1} = 1/2, i = 2, \ldots, d-1$. Write a system of equations and solve it:

$$\pi = \pi P,$$

where $\pi = (\pi_1, \ldots, \pi_d), P = (p_{ij})_{1 \leq i \leq d, 1 \leq j \leq d}$. To answer the question about all positive entries think about the probabilistic *meaning* of the P^m entries)

b) Consider a simple r.w. with absorption on $[1, d] \cap \mathbf{Z} = \{1, \ldots, d\}$. Find all invariant distributions. Is there an integer m such that P^m has strictly positive entries?

Hint. The transition probabilities are $p_{ij} = p(i, j)$ are given by $p_{11} = p_{dd} = 1$, and $p_{i,i-1} = p_{i,i+1} = 1/2, i = 2, \ldots, d-1$. Analyze the system:

$$\pi = \pi P,$$

where $\pi = (\pi_1, \ldots, \pi_d), P = (p_{ij})_{1 \leq i \leq d, 1 \leq j \leq d}$.