1. a) Assume Z_n is a sequence of Gaussian r.v. and

$$\lim_{n\to\infty} \mathbf{E}e^{irZ_n} \text{ exists for all } n.$$

Show that $\lim_{n\to\infty} \mathbf{E}(Z_n)$ and $\lim_{n\to\infty} \mathbf{E}\left(Z_n^2\right)$ exist.

b) Use a) to show that $L_\delta = (W_{t+\delta} - W_t)/\delta$ can not have any limit as $\delta \to 0$ (here $t \geq 0, W_t$ is a standard Wiener process.

2. Assume

$$X_t = X_0 + \int_0^t a_s ds + \int_0^t \sigma_s dW_s,\ t \geq 0,$$

where a_s and σ_s are $\mathcal{F}_s$-adapted and

$$\int_0^t |a_s| ds + \int_0^t |\sigma_s|^2 ds < \infty \text{ a.s.}$$

for all $t \geq 0$. Apply Ito formula to derive a SDE for $\alpha_t = e^{X_t}, t \geq 0$.