Assume $V_1, V_2, \ldots$ is a sequence of i.i.d. independent of another sequence of i.i.d. $W_0, W_1, \ldots,$ $\mathbf{E}V_1 = \mathbf{E}W_0 = 0, \mathbf{E}\left(V_1^2\right) = \mathbf{E}\left(W_0^2\right) = 1$. Consider

$$X_n = aX_{n-1} + \varepsilon V_n, n \geq 1$$

and

$$Y_n = X_n + \delta W_n, n \geq 0.$$

1. Assume $X_0 \sim N(0,1)$, $W_0, W_1, \ldots,$ $V_1, V_2, \ldots$ are independent, $V_1 \sim N(0,1), W_0 \sim N(0,1)$. Simulate $X_n, Y_n, \hat{X}_n, 0 \leq n \leq N$, for

a) $N = 200, a = 0.9, \varepsilon = 0.3, \delta = 1$; b) $N = 200, a = 0.8, \varepsilon = 0.9, \delta = 2$;

Plot on the same graph $X_n, Y_n, \hat{X}_n, 0 \leq n \leq N$;

Plot the error $P_n, 0 \leq n \leq N$.

2. Assume X_0, $W_0, W_1, \ldots,$ $V_1, V_2, \ldots$ are independent and discreet, $\mathbf{P}\left(V_1 = \pm 1\right) = \mathbf{P}\left(W_1 = \pm 1\right) = \frac{1}{2}, \mathbf{P}\left(X_0 = 1\right) = 1$. Simulate $X_n, Y_n, \hat{X}_n, 0 \leq n \leq N$, for

$N = 200, a = 0.9, \varepsilon = 0.3, \delta = 1$;

Plot on the same graph $X_n, Y_n, \hat{X}_n, 0 \leq n \leq N$;

Plot the error $P_n, 0 \leq n \leq N$.