n10_mcs_cva_swap

May 15, 2017

1 Introduction to Monte Carlo Simulation in Finance

1.1 Derivatives CVA calculation example Monte-Carlo with python

Here we'll show an example of code for CVA calculation (Credit Valuation Adjustment) using python with simple Monte-Carlo method with portfolio consisting just of a single interest rate swap.It's easy to generalize code to include more financial instruments.

CVA calculation algorithm:

1) Simulate yield curve at future dates
2) Calculate your derivatives portfolio NPV (net present value) at each time point for each scenario
3) Calculate CVA as sum of Expected Exposure multiplied by probability of default at this interval

$$CVA = (1 - R) \int DF(t)EE(t)dQ_t$$

where R is the Recovery Rate (normally set to 40%) $EE(t)$ is the expected exposure at time t and dQ_t the survival probability density, $DF(t)$ is the discount factor at time t.

Outline

1. In this simple example we will use a modified version of Hull White model to generate future yield curves. In practice many banks use some yield curve evolution models based on this model. As you can see in the slides, in Hull White model the short rate r_t is distributed normally with known mean and variance.

2. For each point of time we will generate whole yield curve based on short rate. Then we will price our interest rate swap on each of these curves;

3. to approximate CVA we will use BASEL III formula for regulatory capital charge approximating default probability [or survival probability] as $exp(-S_T/(1-R))$ so we get

$$CVA = (1 - R) \sum_i \frac{EE(T_i)^\star + EE(T_{i-1}^\star}{2} \left(e^{-S(T_{i-1})/(1-R)} - e^{-S(T_i)/(1-R)}\right)$$

where $EE^\star$ is the discounted Expected Exposure of portfolio.

Details For this first example we'll take 2% flat forward yield curve.

Additive Factor Gaussian Model for future yield curve simulations The model is given by dynamics (Brigo & Mercurio p. 143):

$$r(t) = x(t) + \phi(t)$$

where

$$dx(t) = -ax(t)dt + \sigma dW_t \quad x(0) = 0$$

and ϕ is a deterministic shift which is added in order to fit exactly the initial zero coupon curve So the short rate $r(t)$ is distributed normally with mean and variance given by (Brigo & Mercurio p.144 equations 4.6 with $\eta = 0$)

$$E(r_t|r_s) = x(s)e^{-a(t-s)} + \phi(t)$$

$$Var(r_t|r_s) = \frac{\sigma^2}{2a}\left(1 - e^{-2a(t-s)}\right)$$

where $\phi(T) = f^M(0,T) + \frac{\sigma^2}{2a}\left(1 - e^{-aT}\right)^2$ and $f^M(0,T)$ is the market instantaneous forward rate at time t as seen at time 0.

Model discount factors are calculated as in Brigo & Mercurio (section 4.2):

$$P(t,T) = \frac{P^M(0,T)}{P^M(0,t)}\exp\left(\mathcal{A}(t,T)\right) \quad (BM\,eq.\,4.14)$$

$$\mathcal{A}(t,T) = \frac{1}{2}\left[V(t,T) - V(0,T) + V(0,t)\right] - \frac{1 - e^{-a(T-t)}}{a}x(t) \quad (BM\,eq.\,4.14)$$

where

$$V(t,T) = \frac{\sigma^2}{a^2}\left[T - t + \frac{2}{a}e^{-a(T-t)} - \frac{1}{2a}e^{-2a(T-t)} - \frac{3}{2a}\right] \quad (BM\,eq.\,4.10)$$

The calculation of the model discount factor is incapsulated into the class *GPPDiscountCurve* which is a specialization of the *DiscountCurve* class.

```
In [1]: import datetime
        from math              import exp, sqrt
        from qfin_calendar     import *
        from qfin_ir_classes   import *
        from qfin_pocket       import *
```

```
module qfin_ir_classes imported
```

```
In [2]: class GPPDiscountCurve():
            def __init__(self,  dc, a, sigma, x_t, simulDate):
                self.__dc             = dc
                self.__a              = a
                self.__sigma          = sigma
                self.__simulDate      = simulDate
                self.__x_t            = x_t
                #
                self.__year_fract     = YearFractionFactory.create_istance(day_count_basis.basis_lin_act_
                self.__t              = self.__year_fract(self.__dc.obsdate(), simulDate)

                # The instantaneous forward rate is calculated approximately assuming a temporal lag of
                # discount function at t = simulated date
                PM_0T                 = dc.df(simulDate)
```

```
        # discount function at t = simulated date + 1 day
        PM_OT1                = dc.df(simulDate + addTimeInterval(1, 'd'))
        # tau = 1 day = 1/365 on annual basis
        tau                   = 1/365.0
        f_MOT                 = - (PM_OT1/PM_OT - 1)/tau
        # Phi function, Brigo & Mercurio p. 146 eq (4.12) with eta = 0
        self.__phi            = f_MOT + (sigma**2 / (2*a))*(1.0 - exp(-a*self.__t))**2


    def update(self, x, simulDate):
        self.__t            = self.__year_fract(self.__dc.obsdate(), simulDate)
        self.__x_t          = x
        self.__simulDate    = simulDate
        self.__PM_0t        = self.__dc.df(simulDate)

    def V(self,t,T):
        #
        # Computation of V(t,T) function as in Brigo & Mercurio p. 145 equation 4.10 with eta =
        #
        sigma = self.__sigma
        a     = self.__a
        v     = ((sigma/a)**2)*(T-t+(2.0/a)*exp(-a*(T-t))-(1.0/(2.0*a))*exp(2*a*(T-t))-(3.0/(2.
        return v

    def df(self, date_T):
        #
        # Pricing of a Zero Coupon Bond
        # Brigo & Mercurio p. 146 equation 4.14 without y(t)
        #
        index = (self.__simulDate, date_T)
        x_t         = self.__x_t
        T           = self.__year_fract(self.__dc.obsdate(), date_T)
        PM_0T       = self.__dc.df(date_T)
        PM_0t       = self.__dc.df(self.__simulDate)
        t           = self.__t
        a           = self.__a
        sigma       = self.__sigma
        AtT         = 0.5 *(self.V(t,T)  - self.V(0,T) + self.V(0,t)) - x_t*(1.0 - exp(-a*(T-t))
        PtT         = (PM_0T / PM_0t) * exp(AtT)

        return  PtT

    def phi(self):
        return self.__phi

    def obsdate(self):
        return self.__simulDate
```

Valuing portfolio On each of this curve we will value swap using the Swap class defined in *qfin_swap* module. For our example portfolio we'll take one interest rate swap EUR 1MM notional receiving 2% every 6m, TARGET calendar, with 5 years maturity. Actual/360 daycounter for both legs.

```
In [3]: today            = datetime.date(2014,10,24)
        #
```

```
# create evaluation date discount curve
#
# starting value for short rate
spot_rate       =   0.02
fardate         = today + addTimeInterval(10.0, 'y')
year_fraction   = YearFractionFactory.create_istance(day_count_basis.basis_lin_act_365)
df              = 1.0 / ((1.0 + spot_rate / 365.0) **(year_fraction(today, fardate) * 365.0))
crv_today       = DiscountCurve(today, [fardate], [df])
#
gpp_k           = 0.1
gpp_sigma       = 0.005

mc_runs         = 1000

year_fraction   = YearFractionFactory.create_istance(day_count_basis.basis_lin_act_365)
#
# initial values
#
libor_schedule  = LiborScheduler(fixing_lag = 2, day_count = day_count_basis.basis_lin_act_360,
gpp_dc          = GPPDiscountCurve(crv_today,
                                   gpp_k,
                                   gpp_sigma,
                                   0,
                                   today)
libor_calc      = LiborForwardCalculator(gpp_dc, libor_schedule)
libor_indx      = Libor(today, 0.0, '6m', libor_calc, 'EURIBOR 6M')
#
# create test swap
#
swap            = Swap(today,
                       libor_indx,
                       '5y',                 # maturity
                       0.02,                 # strike rate
                       notional = 1000000,
                       pay_fix = +1,
                       pay_float = -1,
                       float_tenor='6m',
                       fixed_tenor='6m')

fixing_dates    = [f.fixing_date() for l in swap.legs() for f in l.flows() if f.index() != None]
#
# maturities set generator
#
time_range      = xrange(1, 12 * 5 + 6, 1)
sim_dates       = [today] + [today + addTimeInterval(x, 'm') for x in time_range]
#
# add fixing dates to simulation dates
#
sim_dates       = set(sim_dates)
sim_dates.update(fixing_dates)
sim_dates       = sorted(sim_dates)
#
# calculation of simulation times according to the year fraction convention chosen
#
```

```
sim_times       = [year_fraction(today, d) for d in sim_dates]
sim_times       = np.array(sim_times)
```

To generate future values of r_t first we will generate matrix of standard normal variables using numpy library function *numpy.random.standard_normal()*. After that we will apply transform $mean + variance \cdot Normal(0,1)$ to get r_t after getting the value of r_t, we will construct yield curve using model discount factors and store each yield curve in matrix *fwdMat*.

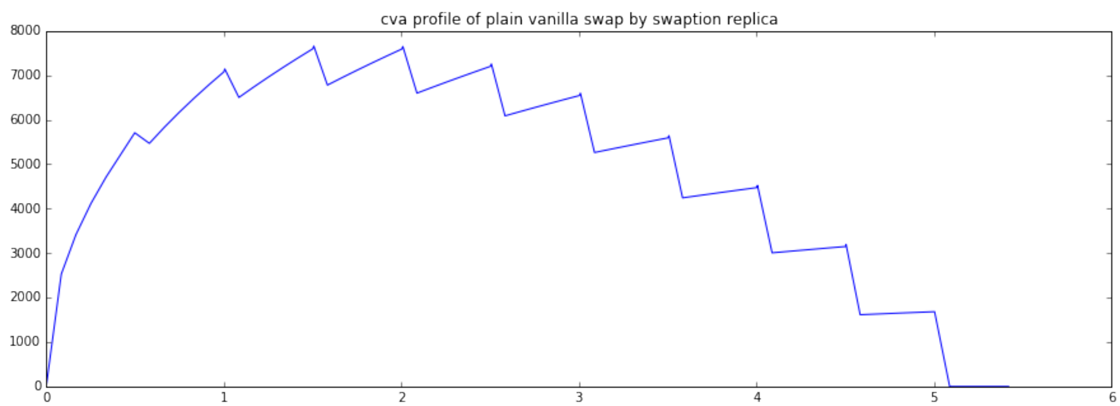
```
In [4]: #
        #--------------------------------------------------------------------------------------------------
        # we generate a matrix Nsim x len(T) of standard normal random numbers
        # seed is fixed just to get always the same result
        #
        np.random.seed(1)
        stdnorm = np.random.standard_normal(size=(mc_runs,len(sim_times)-1))
        #
        fwdMat      =   np.zeros(shape=len(sim_times))
        fwdMat[0]   =   gpp_dc.phi()
        numeraire   =   np.zeros(shape=len(sim_times))
        #
        # npv matrix
        npvMat= [ [ 0 for i in xrange(len(sim_times)) ] for nSim in xrange(mc_runs) ]
        # short rate and discount curve simulation loop
        r = gpp_dc.phi()
        var_factor = 0.5*(gpp_sigma*gpp_sigma/gpp_k)
        # simulation loop
        for nSim in xrange(mc_runs):
            x_s = 0
            for nT in xrange(1,len(sim_times)):
                # ref. B&M eq. (3.35), (3.37) pg. 73
                t           = sim_times[nT]
                s           = sim_times[nT-1]
                drift       = x_s * exp(-gpp_k*(t-s))
                var         = var_factor*(1-exp(-2*gpp_k*(t-s)))
                x_t         = drift + stdnorm[nSim,nT-1] * sqrt(var)
                # updating curve
                gpp_dc.update(x_t, sim_dates[nT])
                fwdMat[nT]  = libor_indx.forward(sim_dates[nT])
                # computing numeraire
                numeraire[nT] = numeraire[nT-1] + r * (t - s)
                #
                table = Table1D(sim_dates, fwdMat)
                fixed = {'EURIBOR 6M':table}
                price = swap.price(fixed, gpp_dc, sim_dates[nT])
                npvMat[nSim][nT] = price['npv'] * np.exp(-numeraire[nT])
                #
                r = x_t + gpp_dc.phi()
                x_s = x_t
```

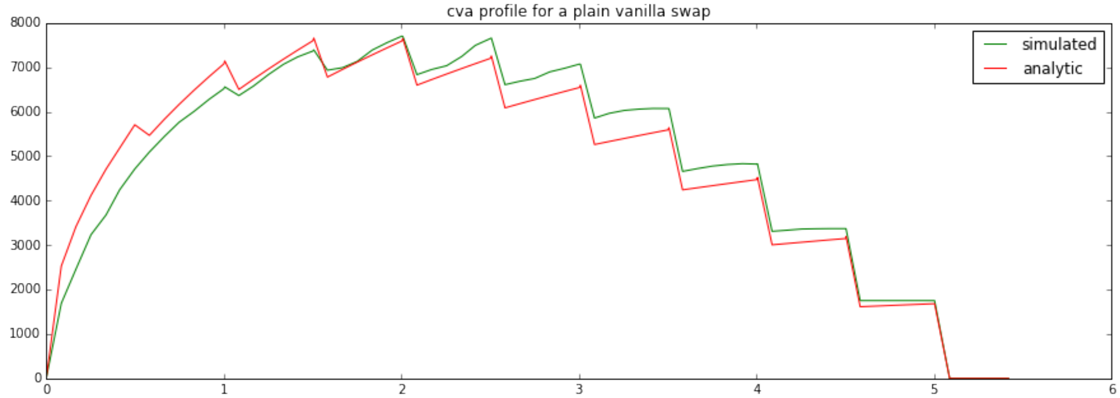
CVA calculation After getting matrix of all NPV at each point for each simulation we will replace negative values with 0. Then we average each column of the matrix (corresponding to averaging for each time point) to get Expected Exposure. and finally calculate CVA as sum as in BASEL 3 formula. Here we'll take 500bps flat CDS spread.

Finally we plot the profile

```
In [5]: npvMat=np.array(npvMat)
        npvMat[npvMat<0]=0
        EPE = np.mean(npvMat,axis=0)

        cds_spread      = 0.05
        recovery_rate   = 0.4

        # calculate CVA
        sum_cva=0
        for i in xrange(len(sim_times)-1):
            sum_cva=sum_cva + EPE[i] *                               \
             (exp(-cds_spread*sim_times[i]  /(1.0-recovery_rate))- \
              exp(-cds_spread*sim_times[i+1]/(1.0-recovery_rate))  \
             )

        CVA = (1.0-recovery_rate)*sum_cva
```

```
In [6]: %matplotlib inline

        import matplotlib
        import matplotlib.pyplot as plt

        from n09_cva_swap import EPE_Swaption

        plt.figure(figsize=(15,5))
        plt.title("cva profile for a plain vanilla swap")
        plt.plot(sim_times,EPE,          color = 'green', label='simulated')
        plt.plot(sim_times,EPE_Swaption, color = 'red',   label='analytic')
        plt.legend()
        plt.show()
```




cva profile for a plain vanilla swap
simulated
analytic
8000
7000
6000
5000
4000
3000
2000
1000
0
0
1
2
3
4
5
6

```
In [ ]:
```