Introduction to Monte Carlo in Finance

5 - Variance Reduction Methods

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Outline

Variance Reduction Methods

- In this section we briefly discuss techniques for improving on the speed and efficiency of a simulazion, usually called *variance reduction techniques*;
- If we do nothing about efficiency, the number of MC replications we need to achieve acceptable pricing acccuracy may be surprisingly large;
- As a result in many cases variance reduction techiques are a practical requirement;
- From a general point of view these methods are based on two principal strategies for reducing variance:
  - Taking advantage of tractable features of a model to adjust or correct simulation output
  - Reducing the variability in simulation input

Variance Reduction Methods

- From the first section we remember that the variance of the estimator is

$$var\left(\tilde{I}_n\right) = \frac{var(f(U_i))}{n}$$

- So, the standard error of the sample mean is the standard deviation or

$$SE\left(\tilde{I}_n\right) = \frac{\sigma_f}{\sqrt{n}}$$

where $\sigma_f^2 = var(f(U_i))$

Variance Reduction Methods

- The most commonly used strategies for variance reduction are the following:
    - **Antithetic variates**
    - **Moment Matching**
    - Control variates
    - **Stratified Sampling**
    - Importance Sampling
    - Low-discrepancy sequences

Subsection 1

Antithetic Variables

Variance Reduction Methods - Antithetic Variates

- In this case we construc the estimator by using two brownian trajectories that are mirror images of each other;
- This causes cancellation of dispersion;
- This method tends to reduce the variance modestly but it is extremely easy to implement and as a result very commonly used;
- For the antithetic method to work we need V^+ and V^- to be negatively correlated;
- this will happen if the payoff function is a monotonic function of Z;

Variance Reduction Methods - Antithetic Variates

- To apply the antithetic variate technique, we generate standard normal random numbers Z and define two set of samples of the undelying price

$$S_T^+ = S_0 e^{(r-\sigma^2/2)T+\sigma\sqrt{T}Z} \quad S_T^- = S_0 e^{(r-\sigma^2/2)T+\sigma\sqrt{T}(-Z)}$$

- Similarly we define two sets of discounted payoff samples ...

$$V_T^+ = \max[S^+(T) - K, 0] \quad V_T^- = \max[S^-(T) - K, 0]$$

- ... and at last we construct our mean estimator by averaging these samples

$$\bar{V}_0 = \frac{1}{n} \sum_{j=1}^{n} \frac{1}{2} \left(V_j^+ + V_j^- \right)$$

Subsection 2

Moment Matching

Variance Reduction Methods - Moment Matching

- Let $z_i, i = 1, ..., n$, denote an independent standard normal random vector used to drive a simulation.
- The sample moments will not exactly match those of the standard normal. The idea of moment matching is to transform the z_i to match a finite number of the moments of the underlying population.
- For example, the first and second moment of the normal random number can be matched by defining

$$\tilde{z}_i = (z_i - \tilde{z})\frac{\sigma_z}{s_z} + \mu_z, i = 1, ...n \tag{1}$$

where $\tilde{z}$ is the sample mean of the z_i and σ_z is the population standard deviation, s_z is the sample standard deviation of z_i, and μ_z s the population mean.

Notebook

- **GitHub :** polyhedron-gdl;
- **Notebook :** n03_mcs;