Lecture Notes in Category Theory

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December 8, 2019

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Chapter 1

Category

1.1 Definition

Definition 1 *We call* category *any tuple* $(\mathrm{Ob}, \mathrm{Arr}, \mathrm{dom}, \mathrm{cod}, \mathrm{id}, \circ)$ *where:*

$$
\begin{array}{ll}
(1) & \mathrm{Ob} \textit{ is a collection with equality} \\
(2) & \mathrm{Arr} \textit{ is a collection with equality} \\
(3) & \mathrm{dom} : \mathrm{Arr} \to \mathrm{Ob} \textit{ is a map} \\
(4) & \mathrm{cod} : \mathrm{Arr} \to \mathrm{Ob} \textit{ is a map} \\
(5) & \mathrm{id} : \mathrm{Ob} \to \mathrm{Arr} \textit{ is a map} \\
(6) & \circ : \mathrm{Arr} \times \mathrm{Arr} \to \mathrm{Arr} \textit{ is a partial map}
\end{array}
$$

with the following properties, given $f, g, h \in \mathrm{Arr}$ *and* $a \in \mathrm{Ob}$*:*

$$
\begin{array}{ll}
(1) & g \circ f \textit{ is defined } \Leftrightarrow \mathrm{cod}(f) = \mathrm{dom}(g) \\
(2) & \mathrm{cod}(f) = \mathrm{dom}(g) \ \Rightarrow \ \mathrm{dom}(g \circ f) = \mathrm{dom}(f) \\
(3) & \mathrm{cod}(f) = \mathrm{dom}(g) \ \Rightarrow \ \mathrm{cod}(g \circ f) = \mathrm{cod}(g) \\
(4) & \mathrm{dom}\,(\,\mathrm{id}(a)\,) = a = \mathrm{cod}\,(\,\mathrm{id}(a)\,) \\
(5) & \mathrm{dom}(f) = a \ \Rightarrow \ f \circ \mathrm{id}(a) = f \\
(6) & \mathrm{cod}(f) = a \ \Rightarrow \ \mathrm{id}(a) \circ f = f \\
(7) & \mathrm{cod}(f) = \mathrm{dom}(g) \wedge \mathrm{cod}(g) = \mathrm{dom}(h) \ \Rightarrow \ (h \circ g) \circ f = h \circ (g \circ f)
\end{array}
$$

Chapter 2

Functor

Chapter 3

Natural Transformation

Chapter 4

Adjunction

4.1 Definition

Definition 2 *We call* adjunction *an ordered pair* (F, G) *where* F *is a functor* $F : \mathcal{C} \to \mathcal{D}$ *and* G *is a functor* $G : \mathcal{D} \to \mathcal{C}$ *while* $\mathcal{C}$ *and* $\mathcal{D}$ *are two locally-small categories for which there exists a natural isomorphism:*

$$\alpha \; : \; \mathcal{D} \circ (F \times I_{\mathcal{D}}) \; \Rightarrow \; \mathcal{C} \circ (I_{\mathcal{C}^{op}} \times G)$$

in the functor category $[\,\mathcal{C}^{op} \times \mathcal{D}\,,\, \mathcal{S}et\,]$, *where* F *also denotes* $F : \mathcal{C}^{op} \to \mathcal{D}^{op}$.