Programming Language Syntax: Top-down Parsing

Read: Scott, Chapter 2.3.2 and 2.3.3

Lecture Outline

- Top-down parsing (also called LL parsing)
  - LL(1) parsing table
  - FIRST, FOLLOW, and PREDICT sets
  - LL(1) grammars
- Bottom-up parsing (also called LR parsing)
  - A brief overview, no detail

LL(1) Parsing Table

- One dimension: nonterminal to expand
- Other dimension: lookahead token

	a
A	α

- E.g., entry "nonterminal *A* on terminal **a**" contains production $A \rightarrow \alpha$
- Meaning: when parser is at nonterminal *A* and lookahead token is **a**, then parser expands *A* by production $A \rightarrow \alpha$

LL(1) Parsing Table

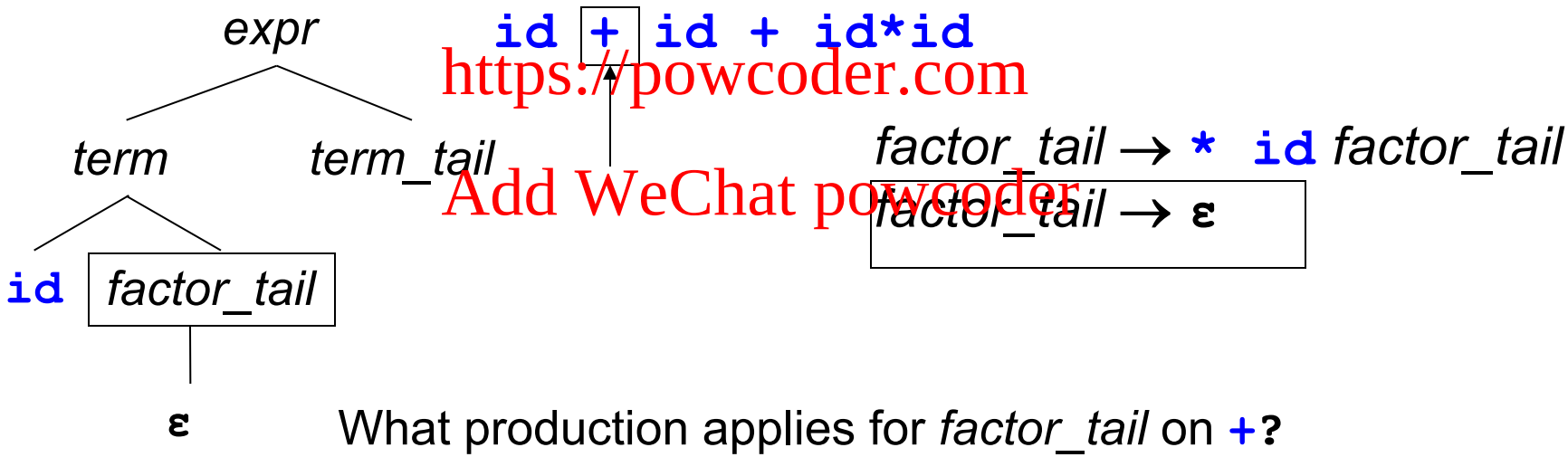
start → *expr* **$$**
expr → *term term_tail*
term → **id** *factor_tail*
term_tail → **+** *term term_tail* | **ε**
factor_tail → ***** **id** *factor_tail* | **ε**

	id	**+**	*****	**$$**
start	*expr* **$$**	-	-	-
expr	*term term_tail*	-	-	-
term_tail	-	**+** *term term_tail*	-	**ε**
term	**id** *factor_tail*	-	-	-
factor_tail	-	**ε**	***** **id** *factor_tail*	**ε**

Intuition

expr → *term term_tail*
term_tail → **+** *term term_tail* | **ε**
term → **id** *factor_tail*
factor_tail → ***** **id** *factor_tail* | **ε**

- Top-down parsing
  - Parse tree is built from the top to the leaves
  - Always expand the leftmost nonterminal

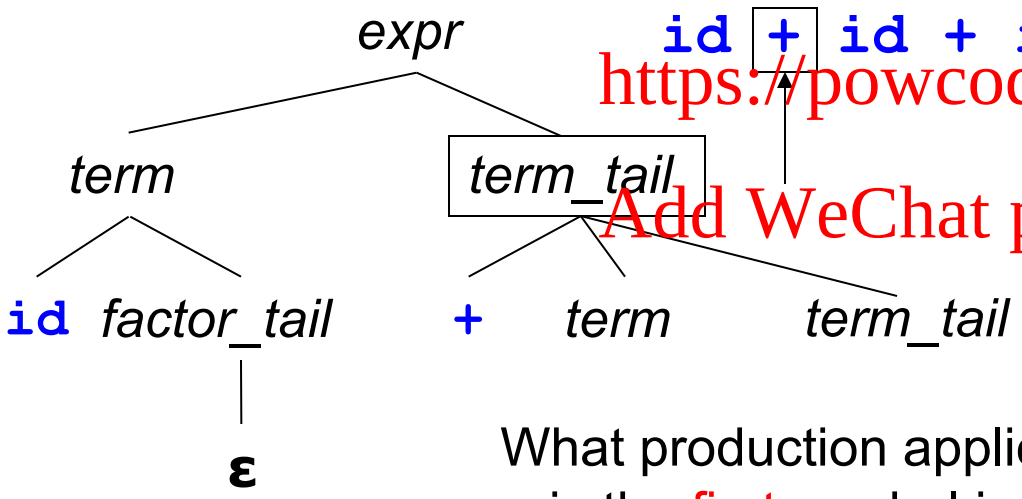


What production applies for *factor_tail* on **+**?
+ does not belong to an expansion of *factor_tail.* However, *factor_tail* has an epsilon production and **+** belongs to an expansion of *term_tail* which follows *factor_tail.* Thus, predict the epsilon production.

Intuition

expr → *term term_tail*
term_tail → **+** *term term_tail* | **ε**
term → **id** *factor_tail*
factor_tail → ***** **id** *factor_tail* | **ε**

- Top-down parsing
  - Parse tree is built from the top to the leaves
  - Always expand the leftmost nonterminal



term_tail → **+** *term term_tail*
term_tail → **ε**

What production applies for *term_tail* on **+**?
+ is the first symbol in expansions of **+** *term term_tail.*

Thus, predict production *term_tail* → **+** *term term_tail*

LL(1) Tables and LL(1) Grammars

- We can construct an LL(1) parsing table for any context-free grammar
  - In general, the table will contain multiply-defined entries. That is, for some nonterminal and lookahead token, more than one production applies.

- A grammar whose LL(1) parsing table has no multiply-defined entries is said to be LL(1) grammar
  - LL(1) grammars are a very special subclass of context-free grammars. Why?

FIRST and FOLLOW sets

- Let α be any sequence of nonterminals and terminals
  - FIRST(α) is the set of terminals **a** that begin the strings derived from α. E.g., *expr* ⇒* **id**…, thus **id** in FIRST(*expr*)
  - If there is a derivation α ⇒* ε, then ε is in FIRST(α)

- Let *A* be a nonterminal
  - FOLLOW(*A*) is the set of terminals **b** (including special end-of-input marker **$$**) that can appear immediately to the right of *A* in some sentential form:

    start ⇒* …*A***b**… ⇒*…

Computing FIRST

Notation:
α is an arbitrary sequence
of terminals and nonterminals

- Apply these rules until no more terminals or **ε** can be added to any FIRST(α) set

  (1) If α starts with a terminal **a**, then FIRST(α) = { **a** }

  (2) If α is a nonterminal X, where $X \rightarrow \varepsilon$, then add **ε** to FIRST(α)

  (3) If α is a nonterminal $X \rightarrow Y_1 Y_2 \ldots Y_k$ then add **a** to FIRST(X) if for some i, **a** is in FIRST(Y_i) and **ε** is in all of FIRST(Y_1), … FIRST(Y_{i-1}). If **ε** is in all of FIRST(Y_1), … FIRST(Y_k), add **ε** to FIRST(X).

  - Everything in FIRST(Y_1) is surely in FIRST(X)
  - If Y_1 does not derive **ε**, then we add nothing more;

    Otherwise, we add FIRST(Y_2), and so on

  Similarly, if α is $Y_1 Y_2 \ldots Y_k$, we'll repeat the above

Warm-up Exercise

start → *expr* **$$**
expr → *term term_tail*
term → **id** *factor_tail*
term_tail → **+** *term term_tail* | **ε**
factor_tail → ***** **id** *factor_tail* | **ε**

FIRST(*term*) = { **id** }

FIRST(*expr*) =

FIRST(*start*) =

FIRST(*term_tail*) =

FIRST(**+** *term term_tail*) =

FIRST(*factor_tail*) =

Exercise

start → *S* **$$**	*B* → **z** *S* | **ε**
S → **x** *S* | *A* **y**	*C* → **v** *S* | **ε**
A → *BCD* | **ε**	*D* → **w** *S*

Compute FIRST sets:

FIRST(**x** *S*) =

FIRST(*S*) =

FIRST(*A* **y**) =

FIRST(*A*) =

FIRST(*BCD*) =

FIRST(*B*) =

FIRST(**z** *S*) =

FIRST(*C*) =

FIRST(**v** *S*) =

FIRST(*D*) =

FIRST(**w** *S*) =

Computing FOLLOW

Notation:
A,B,S are nonterminals.
α,β are arbitrary sequences of terminals and nonterminals.

- Apply these rules until nothing can be added to any FOLLOW(*A*) set

  (1) If there is a production $A \rightarrow \alpha B \beta$, then everything in FIRST(β) except for **ε** should be added to FOLLOW(*B*)

  (2) If there is a production $A \rightarrow \alpha B$, or a production $A \rightarrow \alpha B \beta$, where FIRST(β) contains **ε**, then everything in FOLLOW(*A*) should be added to FOLLOW(*B*)

Warm-up

start → *expr* **$$**
expr → *term term_tail*
term → **id** *factor_tail*
term_tail → **+** *term term_tail* | **ε**
factor_tail → ***** **id** *factor_tail* | **ε**

FOLLOW(*expr*) = { **$$** }
FOLLOW(*term_tail*) =
FOLLOW(*term*) =
FOLLOW(*factor_tail*) =

Exercise

start → *S* **$$**	*B* → **z** *S* | **ε**
S → **x** *S* | *A* **y**	*C* → **v** *S* | **ε**
A → *BCD* | **ε**	*D* → **w** *S*

Compute FOLLOW sets:

FOLLOW(*A*) =

FOLLOW(*B*) =

FOLLOW(*C*) =

FOLLOW(*D*) =

FOLLOW(*S*) =

PREDICT Sets

$$\text{PREDICT}(A \rightarrow \alpha) = \begin{cases} \text{FIRST}(\alpha) & \text{if } \alpha \text{ does not derive } \varepsilon \\ (\text{FIRST}(\alpha) - \{\varepsilon\}) \cup \text{FOLLOW}(A) & \text{if } \alpha \text{ derives } \varepsilon \end{cases}$$

Constructing LL(1) Parsing Table

- Algorithm uses PREDICT sets:

foreach production $A \rightarrow \alpha$ in grammar G

  foreach terminal **a** in PREDICT($A \rightarrow \alpha$)

    add $A \rightarrow \alpha$ into entry parse_table[A,**a**]

- If each entry in parse_table contains at most one production, then G is said to be LL(1)

Exercise

start → *S* **$$**	*B* → **z** *S* | **ε**
S → **x** *S* | *A* **y**	*C* → **v** *S* | **ε**
A → *BCD* | **ε**	*D* → **w** *S*

Compute PREDICT sets:

PREDICT(*S* → **x** *S*) =

PREDICT(*S* → *A* **y**) =

PREDICT(*A* → *BCD*) =

PREDICT(*A* → **ε**) =

… etc…

Writing an LL(1) Grammar

- Most context-free grammars are not LL(1) grammars
- Obstacles to LL(1)-ness
  - Left recursion is an obstacle. Why?

$$expr \rightarrow expr \texttt{ + } term \mid term$$
$$term \rightarrow term \texttt{ * id} \mid \texttt{id}$$

  - Common prefixes are an obstacle. Why?

$$stmt \rightarrow \texttt{if b then } stmt \texttt{ else } stmt \mid \texttt{if b then } stmt \mid \texttt{a}$$

Removal of Left Recursion

- Left recursion can be removed from a grammar mechanically
- Started from this left-recursive expression grammar:

expr → *expr* **+** *term* | *term*
term → *term* ***** **id** | **id**

- After removal of left recursion we obtain this equivalent grammar, which is LL(1):

expr → *term term_tail*
term_tail → **+** *term term_tail* | **ε**
term → **id** *factor_tail*
factor_tail → ***** **id** *factor_tail* | **ε**

Removal of Common Prefixes

- Common prefixes can be removed mechanically as well, by using left-factoring
- Original if-then-else grammar:

stmt → **if b then** *stmt* **else** *stmt* |
if b then *stmt* |
a

- After left-factoring:

stmt → **if b then** *stmt else_part* | **a**
else_part → **else** *stmt* | **ε**

Exercise

start → *stmt* **$$**
stmt → **if b then** *stmt else_part* | **a**
else_part → **else** *stmt* | **ε**

- Compute FIRSTs:

FIRST(*stmt* **$$**), FIRST(**if b then** *stmt else_part*), FIRST(**a**), FIRST(**else** *stmt*)

- Compute FOLLOW:

FOLLOW(*else_part*)

- Compute PREDICT sets for all 5 productions
- Construct the LL(1) parsing table. Is this grammar an LL(1) grammar?

Exercise

start → *stmt* **$$**
stmt → **if b then** *stmt else_part* | **a**
else_part → **else** *stmt* | **ε**

- Compute FIRSTs:

FIRST(*stmt* **$$**) =

FIRST(**if b then** *stmt else_part*) =

FIRST(**a**) =

FIRST(**else** *stmt*) =

Exercise

start → *stmt* **$$**
stmt → **if b then** *stmt else_part* | **a**
else_part → **else** *stmt* | **ε**

- Compute FOLLOW:

FOLLOW(*else_part*) =

Exercise

start → *stmt* **$$**
stmt → **if b then** *stmt else_part* | **a**
else_part → **else** *stmt* | **ε**

- Construct the LL(1) parsing table

- Is this grammar an LL(1) grammar?

Exercise

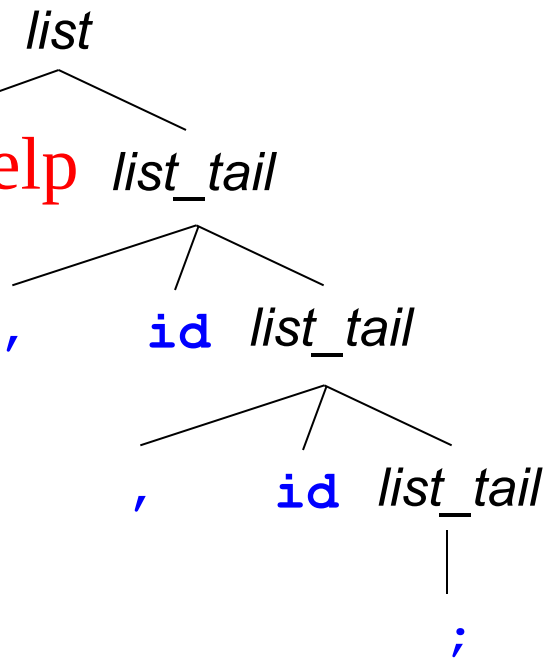
Lecture Outline

- Top-down parsing (also called LL parsing)
  - LL(1) parsing table
  - FIRST, FOLLOW, and PREDICT sets
  - LL(1) grammars
- Bottom-up parsing (also called LR parsing)
  - A brief overview, no detail

Bottom-up Parsing

- Terminals are seen in the order of appearance in the token stream:

`id , id , id ;`



- Parse tree is constructed
  - From the leaves to the top
  - A right-most derivation in reverse

list → **id** *list_tail*
list_tail → **,** **id** *list_tail* | **;**

Bottom-up Parsing

list → **id** *list_tail*
list_tail → **,** **id** *list_tail* | **;**

Stack	Input	Action
	id,id,id;	shift
id	**,id,id;**	shift
id,	**id,id;**	shift
id,id	**,id;**	shift
id,id,	**id;**	shift
id,id,id	**;**	shift
id,id,id;		reduce by *list_tail*→**;**

Bottom-up Parsing

list → **id** *list_tail*
list_tail → **,** **id** *list_tail* | **;**

Stack	Input	Action
id,id,id *list_tail*		reduce by *list_tail* → **,id** *list_tail*
id,id *list_tail*		reduce by *list_tail* → **,id** *list_tail*
id *list_tail*		reduce by *list* → **id** *list_tail*
list		ACCEPT

Bottom-up Parsing

- Also called LR parsing
- LR parsers work with LR(k) grammars
  - L stands for "left-to-right" scan of input
  - R stands for "rightmost" derivation
  - k stands for "need k tokens of lookahead"
- We are interested in LR(0) and LR(1) and variants in between
- LR parsing is better than LL parsing!
  - Accepts larger class of languages
  - Just as efficient!

LR Parsing

- The parsing method used in practice
  - LR parsers recognize virtually all PL constructs
  - LR parsers recognize a much larger set of grammars than predictive parsers
  - LR parsing is efficient
- LR parsing variants
  - SLR (or Simple LR)
  - LALR (or Lookahead LR) – **`yacc`**/**`bison`** generate LALR parsers
  - LR (Canonical LR)
  - SLR < LALR < LR

Main Idea

- Stack ← Input
- Stack: holds the part of the input seen so far
  - A string of both terminals and nonterminals
- Input: holds the remaining part of the input
  - A string of terminals
- Parser performs two actions
  - Reduce: parser pops a "suitable" production right-hand-side off top of stack, and pushes production's left-hand-side on the stack
  - Shift: parser pushes next terminal from the input on top of the stack

Example

- Recall the grammar

expr → *expr* **+** *term* | *term*
term → *term* ***** **id** | **id**

  - This is not LL(1) because it is left recursive
  - LR parsers can handle left recursion!

- Consider string

```
id + id * id
```

id + id*id

Stack	Input	Action
	id+id*id	shift **id**
id	**+id*id**	reduce by *term* → **id**
term	**+id*id**	reduce by *expr* → *term*
expr	**+id*id**	shift **+**
*expr***+**	**id*id**	shift **id**
*expr***+id**	***id**	reduce by *term* → **id**

expr → *expr* **+** *term* | *term*
term → *term* ***** **id** | **id**

id + id*id

Stack	Input	Action
expr + *term*	* id	shift *
expr + *term* *	id	shift id
expr + *term* * id		reduce by *term* → *term* * id
expr + *term*		reduce by *expr* → *expr* + *term*
expr		ACCEPT, SUCCESS

expr → *expr* + *term* | *term*
term → *term* * id | id

id + id*id

Sequence of reductions performed by parser

id+id*id

term+**id*id**

expr+**id*id**

expr+*term***id*

expr+*term*

expr

- A rightmost derivation in reverse
- The stack (e.g., *expr*) concatenated with remaining input (e.g., **+id*id**) gives a sentential form (*expr*+**id*id**) in the rightmost derivation

expr → *expr* **+** *term* | *term*
term → *term* ***** **id** | **id**

The End