MID TERM TEST
CSE4101/5101: Advanced Data Structures

- This Test is closed book and lasts from 5:30 pm to 7:00 pm.
- Read all questions before deciding in what order to answer them.
- Do not use any electronic/mechanical computation/communication devices.
- This booklet contains 6 pages including this cover page.
- You may use back side of pages for scratch work.

Name:_____

Student Number:_____

Problem	Points Received	Points Worth
1		30
2		30
3		40
TOTAL		100

Problem 1. [30%] Fill in the underlined blanks with the most appropriate and simplified answer.

(a) If we perform a sequence of n INCREMENT operations on a binary counter with initial value C (NOT ZERO), the total number of bit flips is at most _____ .

(b) Consider maintaining a dynamic table T with expansion and contraction. One such strategy that yields $O(1)$ amortized time per insert and delete and keeps the table load factor at least 80% is:
contraction policy: _____,
expansion policy: _____.

(c) True or False: Among all off-line strategies on sequential linear lists with free/paid exchanges, the decreasing frequency count method is the optimum, i.e., minimum cost strategy. Answer: _____ .

(d) Given a 2-3-4 tree consisting of n keys, we can output those n keys in sorted order in optimal time $\Theta($_____$)$.

(e) In an arbitrary n-node Red-Black tree, the (exact, not asymptotic) maximum number of rotations used in a single **bottom-up** insertion is _____, and this happens when _____ _____ .

(f) For the amortized analysis of the Splay Trees we defined the potential function to be _____ .
The amortized cost of an insert operation is _____ .

(g) A self-adjusting version of the Dynamic Order Statistic tree can be implemented by augmenting Splay Trees by the addition of a _____ field to each node. In addition to the usual dictionary operations, such a data structure supports the order statistics operations _____ and _____ in _____ amortized time per operation.

(h) True or False: Let x and y be two arbitrary external nodes in a binary tree T, such that x appears before y in the inorder sequence of (internal/external) nodes of T. If T is leftist, then $depth(x) \geq depth(y)$.
Answer: _____ .

(i) True or False: Depth of the rightmost external node in an n-node skew heap is $O(\log n)$.
Answer: _____ .

(j) True or False: In a DeleteMin operation on a skew heap, the number of right-heavy nodes of the resulting skew heap is always less than or equal to the number of right-heavy nodes of the input skew heap.
Answer: _____ .

Problem 2. [30%] Consider the following sequence of keys:

(5, 16, 22, 45, 2, 10, 18, 30, 50, 12, 1).

Consider the **bottom-up** insertion of items with this set of keys, in the order given, into an initially empty:

a) 2-3-4 tree T_1.

b) Red-Black Tree T_2.

c) Splay Tree T_3.

Draw T_1 and T_2 and T_3 after each insertion.

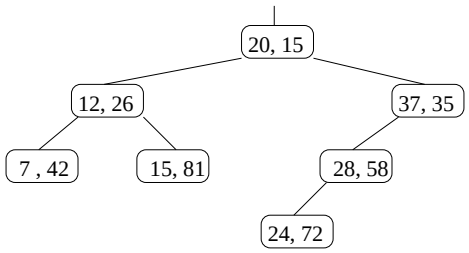
Problem 3. [40%] We are given a set $P = \{p_1, p_2, \cdots, p_n\}$ of n points in the plane. Assume each point is given by its x and y coordinates, $p_i = (x_i, y_i)$, for $i = 1..n$. We wish to store P in an n node binary tree T with the following properties:

(i) Each node of T holds a distinct point of P,

(ii) T appears as a Binary Search Tree with respect to the x-coordinates of its stored points,

(iii) T appears as a min-heap with respect to the y-coordinates of its stored points.

We call such a data structure a *Search-Heap Tree (SHT)*. The figure below shows an example.



(a) [5%] Show the SHT of the following set of points: $P = \{(12, 31), (24, 15), (4, 23), (18, 5), (14, 53), (16, 7)\}$.

(b) [9%] If all points in P have distinct x coordinates and distinct y coordinates, show that SHT of P exists and is unique. What happens to the existence or uniqueness of SHT if we remove the coordinate distinctness assumption?

(c) [9%] Let T represent the SHT of the point set P. Suppose the y-coordinate of a point of P stored at a **given** node v of T is **decreased** to y_{new}. How would you update this y-coordinate and use *rotation* to restore the SHT property of T?

(d) [17%] The problem is to construct the SHT of P, where P is a set of n points in the plane, given in **sorted** order of their x-coordinates. Give a detailed design and analysis of the most efficient algorithm you can for this problem. *[Hint: Use incremental construction and amortized analysis. Less efficient algorithms may get partial credit.]* **[You may continue your answer on the next page.]**