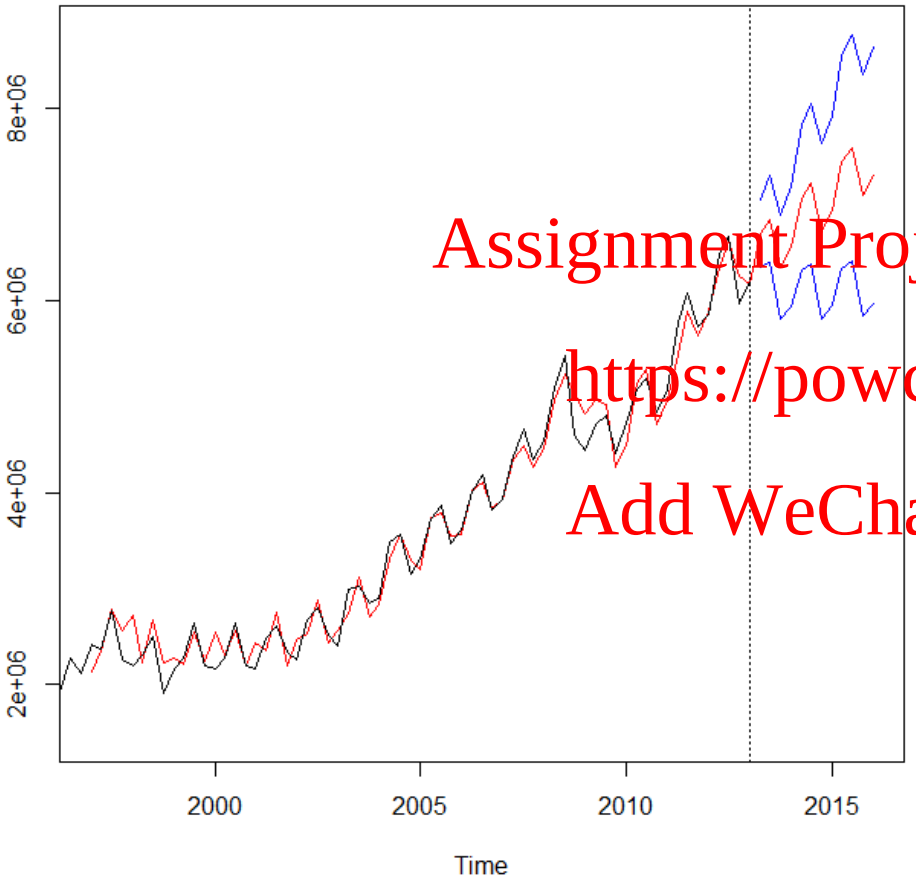
# Growth Models

# Agenda

| Start | End | Item |
|---|---|---|
| | | Gompertz Curves |
| | | Bass Models |
| | | SIR |
| | | |
| | | |
| | | |
| | | |

# Growth Models are a form of time series forecasting






## Model Based Methods

- Holt Winters
- Time Series Decomposition
- Book Chapter 17
  - Regression Based Forecasting

## Data Driven Methods

- Forecasted value at period "n" is equal to some interactions of x-variable inputs

# Model Based

Model Based approaches do not have x-variable inputs.

Use statistical, mathematical or other scientific model to approximate a data series.



Parameters of the model are learned in training then used to generate forecasts or more specifically the behavior of the phenomena.



*Can be used with few data points*

Remember how we only had the time series to get level, trend & seasonality?

# When to use model vs data driven forecasting

- Data Driven
  - Needs trustworthy historical data
  - Variables are engineered as inputs i.e. month dummy variables lagged x days as inputs
  - Apply a machine learning method like XGB




Data driven works when there is historical pattern, & data can be trusted. Model based approaches work when there is no/limited/lots of noise in the data.

# When to use model vs data driven forecasting

- Model Driven
  - Needs fewer data points
  - Lots of variability that is not easily quantifiable
  - Assumptions can be made for curve parameters

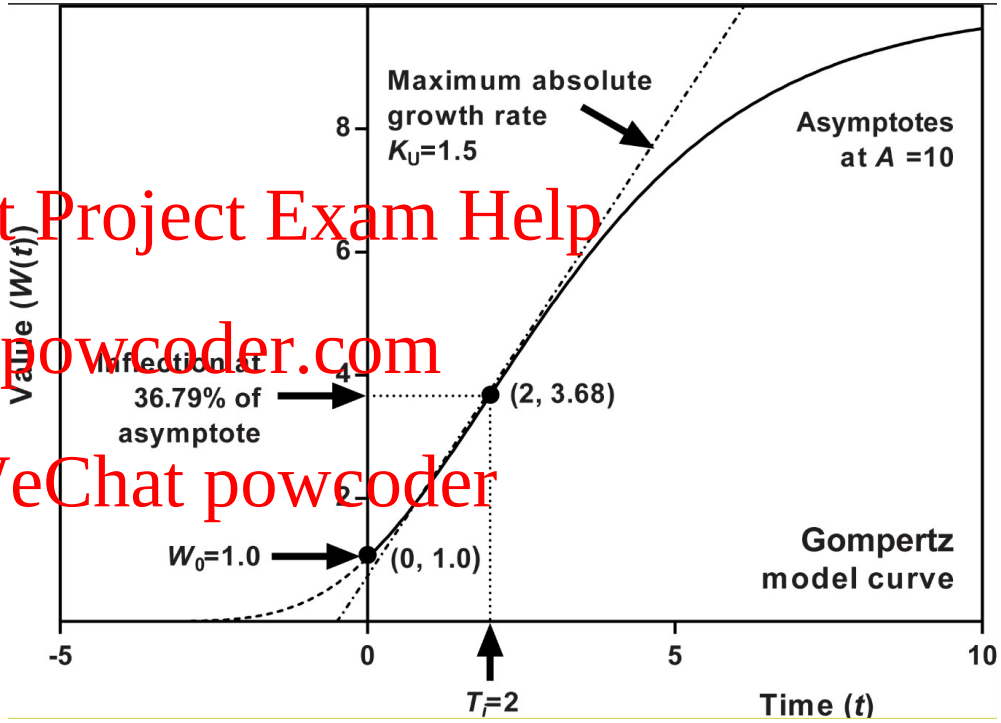



Data driven works when there is historical pattern, & data can be trusted. Model based approaches work when there is no/limited/lots of noise in the data.

# Gompertz Growth Curves

## Gompertz Specifics

- New Product Forecasting
- Tumor Growth Modeling
- Assumes the population doesn't change (no natural birth/death rates)






Can help with planning on completely new item forecasting with only a few starting points & an upper limit to the population (market size potential)

# Gompertz Growth Curves

$$f(t) = ae^{-be^{-ct}}$$



*Double exponential*



Where:
a = asymptote
b = displacement on x-axis
c = maximum growth rate
e = 2.71828

# Gompertz Growth Curves

$$f(t) = ae^{-be^{-ct}}$$



*Double exponential*



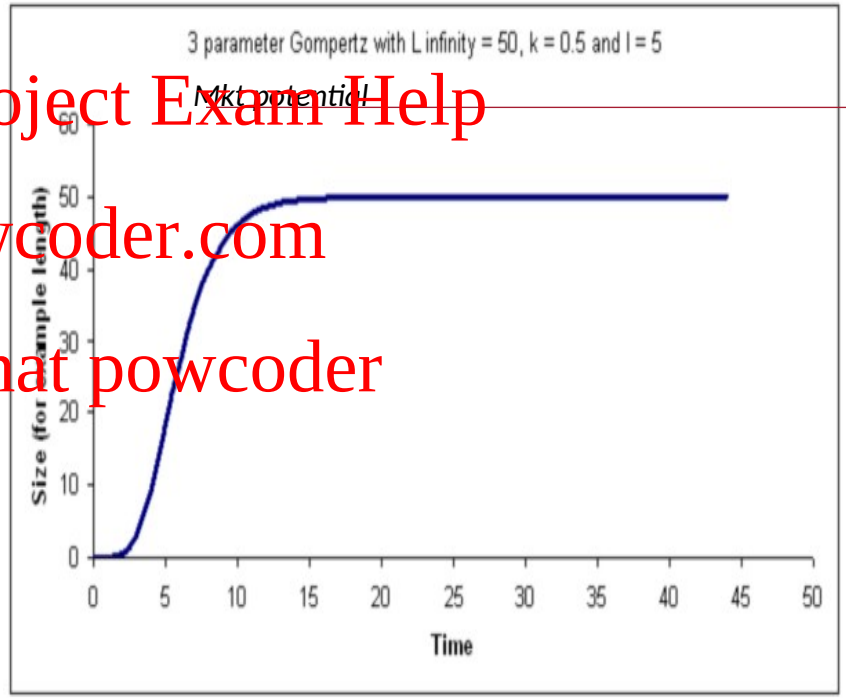
Where:
a = asymptote
b = displacement on x-axis
c = growth rate
e = 2.71828

Biz:
a = market potential
b = how stretched out is the curve
c = slope
e = 2.71828

# Gompertz Growth Curves

## Gompertz Specifics

Biz:
a = 50

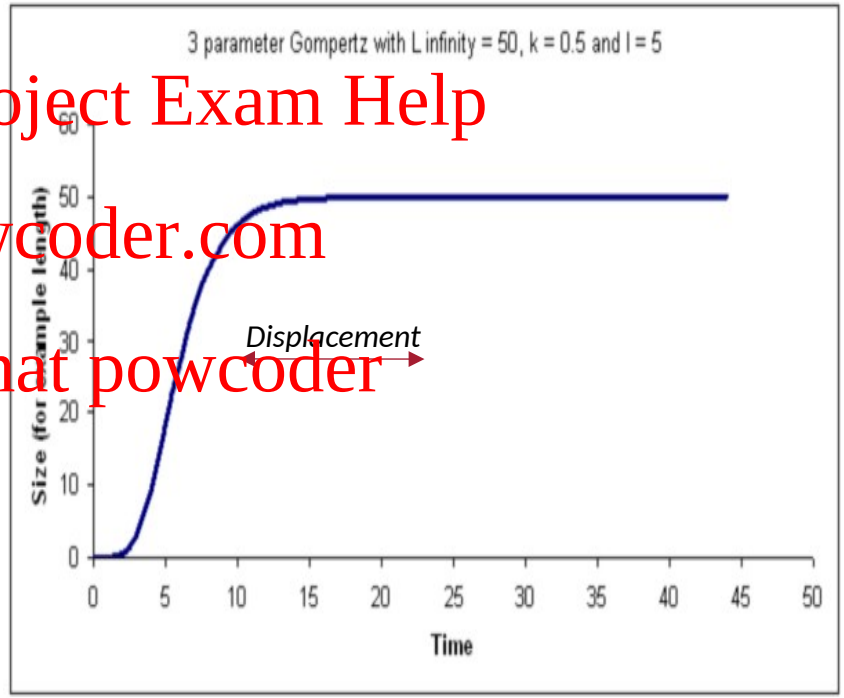


# Gompertz Growth Curves

## Gompertz Specifics

Biz:
a = 50
b = 5

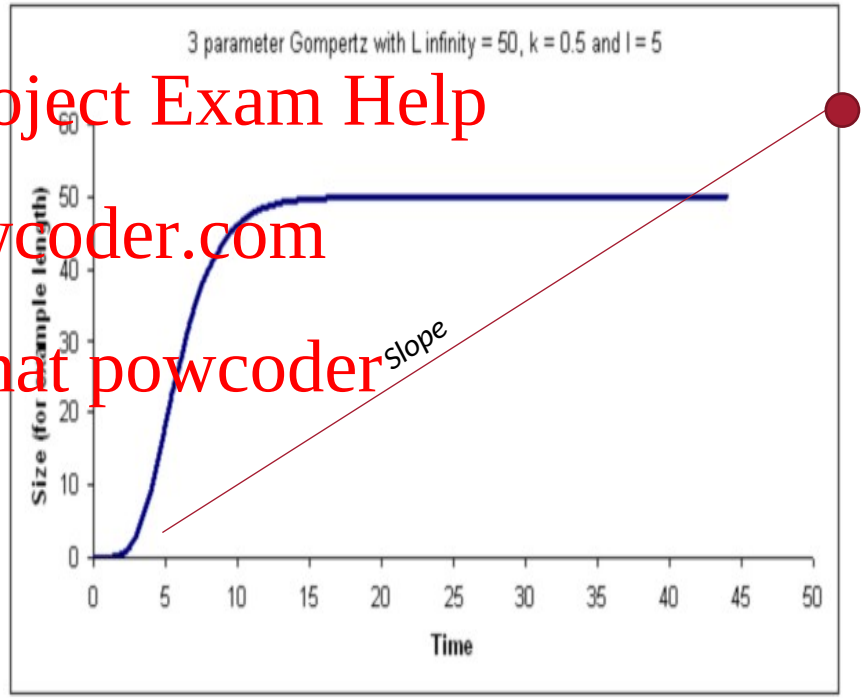




# Gompertz Growth Curves

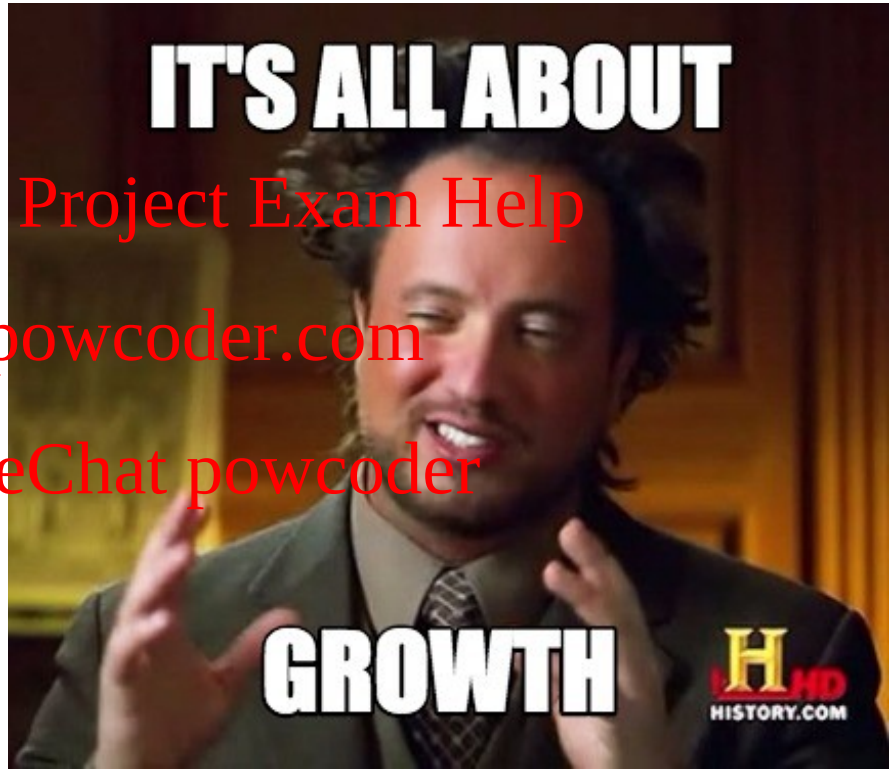


Biz:
a = 50
b = 5
c = 0.5 (rise / run) = 50/25
e = 2.71828



# Let’s practice

A_gompertzCurve.R



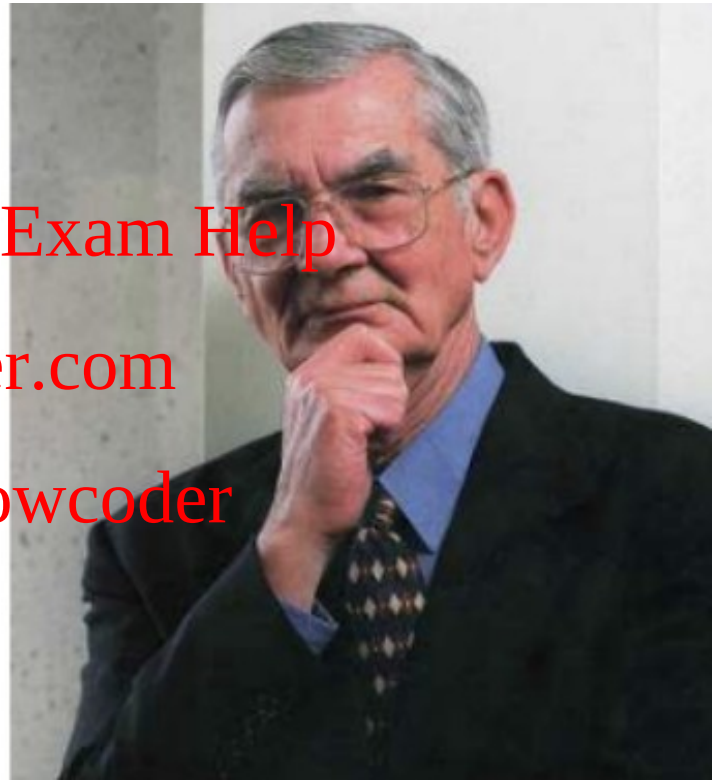
# Agenda

| Start | End | Item |
|---|---|---|
| | | Gompertz |
| | | Bass Models |
| | | SIR |
| | | |
| | | |
| | | |
| | | |

# Not a meme…really Frank Bass (1926-2006

A New Product Growth Model Consumer for Durables, Management Science, Vol. 15 (January 1969)





One of 10 papers in the selection Top 10 Most Influential Papers published in the 50-year history of Management Science (2004)

# Two input parameters

Rates of Innovation

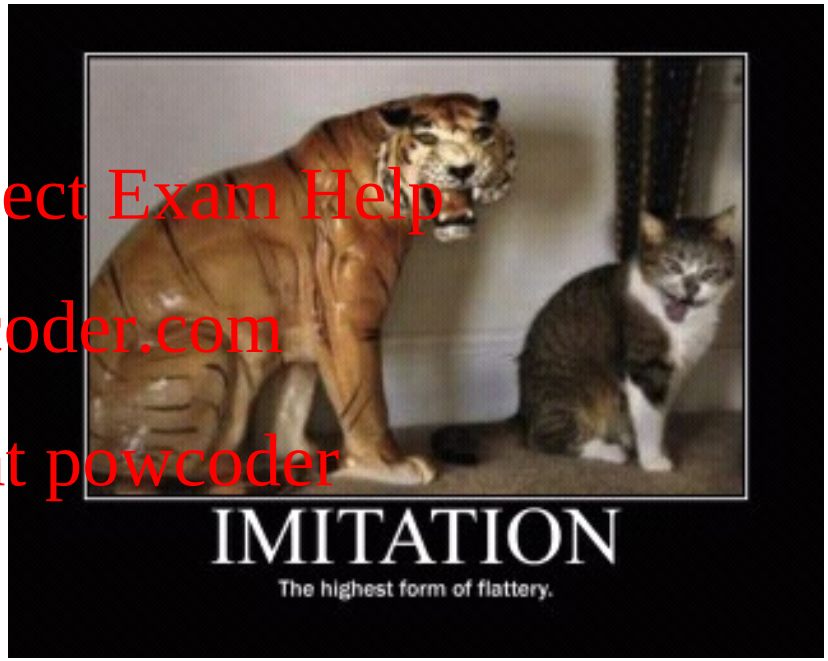
Some people buy the product because of product features, marketing etc.



# Two input parameters

## Rates of Imitation

Some people buy the product because they learn about others experience (word of mouth, ratings etc)



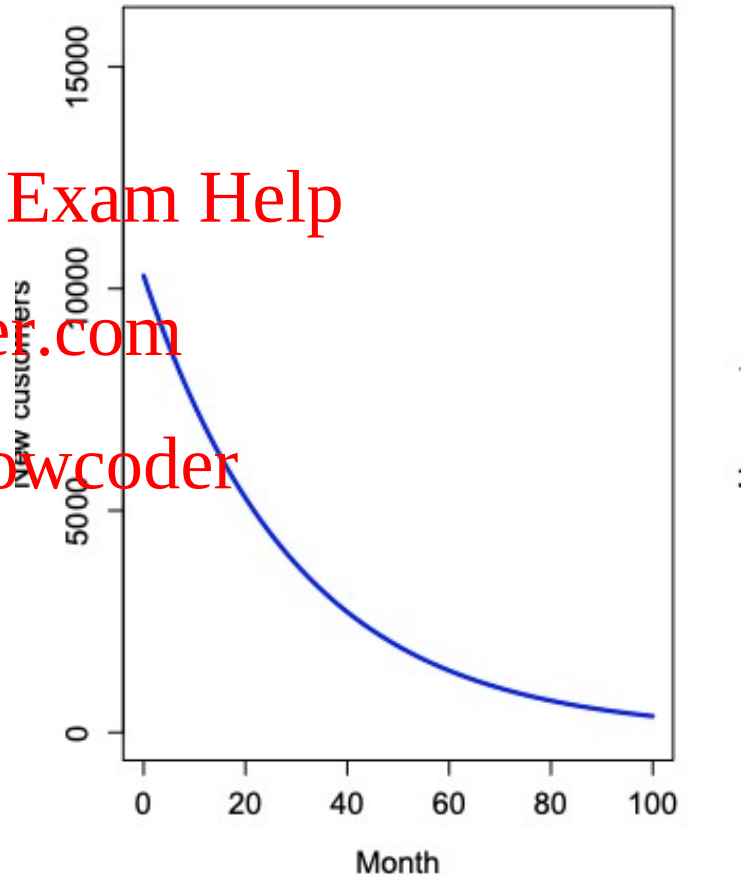
# Two input parameters (plus total market)

| | |
|---|---|
| Rates of Innovation | Some people buy the product because of product features, marketing etc. |
| Rates of Imitation | Some people buy the product because they learn about others experience (word of mouth, ratings etc) |

# Innovator Behavior

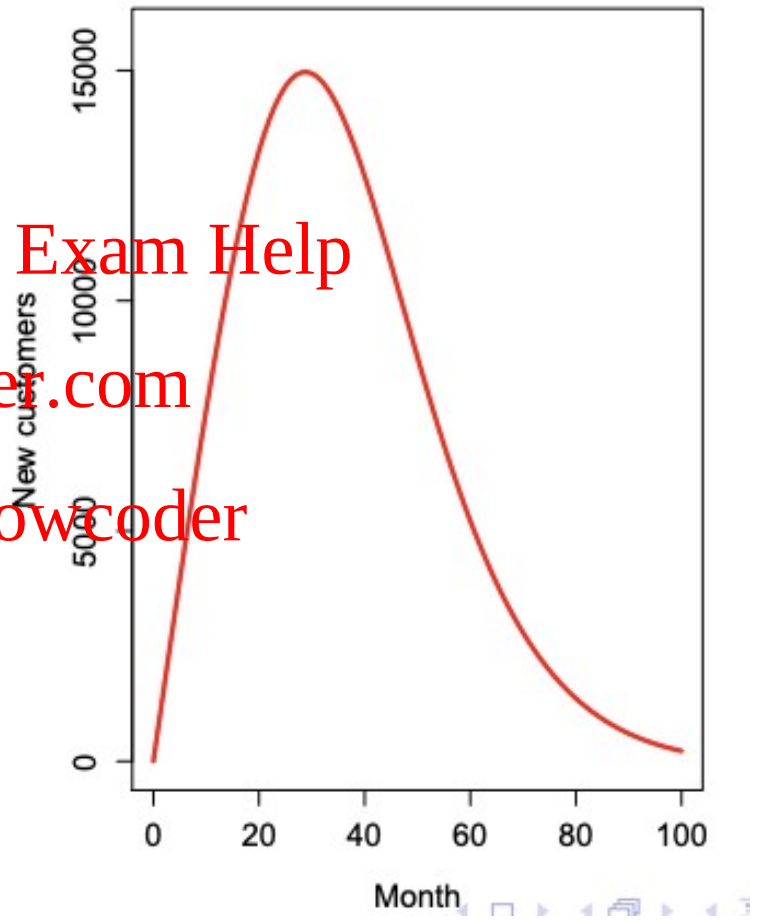


- Smaller total number
- Steep decline
- “People enticed & willing to take a chance”

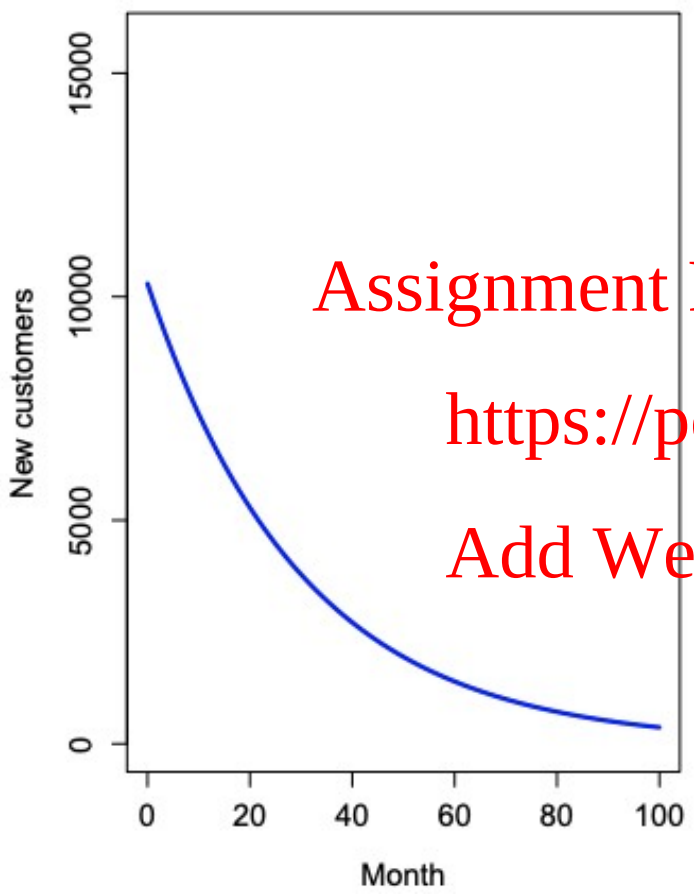


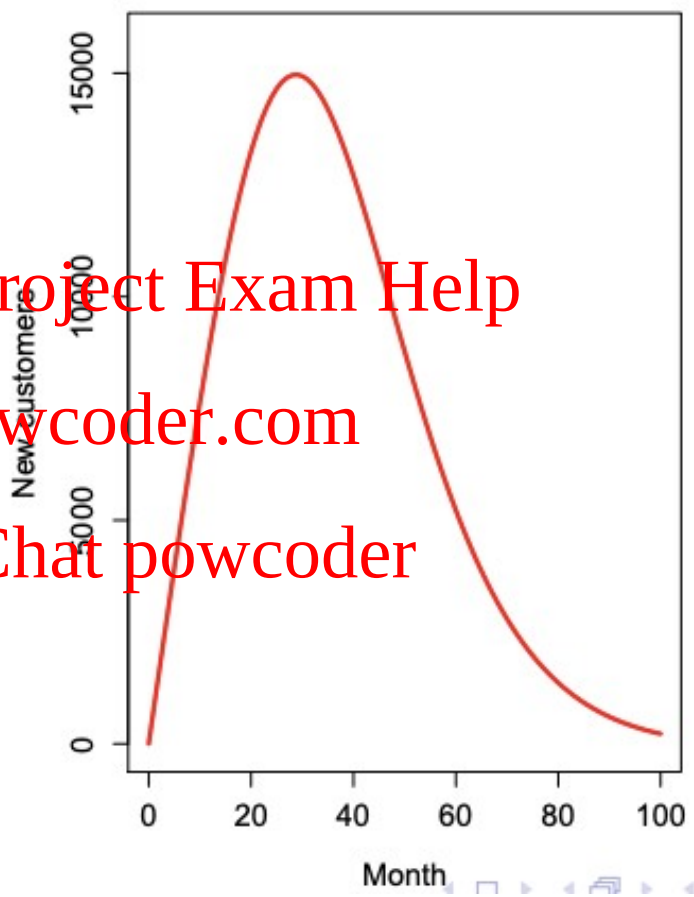
# Imitation Behavior

- Larger total number
- Steep incline as they learn from innovators
- “People eventually won over but total market is limited so a decline occurs eventually”



# Side by Side






Total new customers is the sum of these two parties

# BASS Models Find analog adoption parameters

## Parameters of the Bass Model in Several Product Categories

| Product/ Technology | Innovation parameter ($p$) | Imitation parameter ($q$) |
|---|---|---|
| B&W TV | 0.028 | 0.25 |
| Color TV | 0.005 | 0.84 |
| Air conditioners | 0.010 | 0.42 |
| Clothes dryers | 0.017 | 0.36 |
| Water softeners | 0.018 | 0.39 |
| Record players | 0.025 | 0.65 |
| Cellular telephones | 0.004 | 1.76 |
| Steam irons | 0.029 | 0.33 |
| Motels | 0.007 | 0.36 |
| McDonalds fast food | 0.018 | 0.54 |
| Hybrid corn | 0.039 | 1.01 |
| Electric blankets | 0.006 | 0.24 |

A study by Sultan, Farley, and Lehmann in 1990 suggests an average value of 0.03 for $p$ and an average value of 0.38 for $q$.



| | P | Q |
|---|---|---|
| Baseline case: US, consumer, durable, launch in '76 ... | 0.016 | 0.409 |
| For other cases, multiply by the following factors ... | | |
| Cellular telephone | 0.226 | 0.635 |
| Non durable product | 0.689 | 0.931 |
| Industrial | 1.058 | 1.149 |
| Non commercial innovation | 0.365 | 2.406 |
| Western Europe | 0.464 | 0.949 |
| Asia | 0.595 | 0.743 |
| Other regions | 0.796 | 0.699 |
| For each year after 1976, multiply by ... | 1.021 | 1.028 |

Christophe Van den Bulte: *Want to know how diffusion speed varies across countries and products? Try using a Bass model.* PDMA Visions 26(4) 2002, pp. 12-15

Takes some digging to get these for an analog product/service & there are differences!

# Let’s Practice

B_newProdForecasting.R

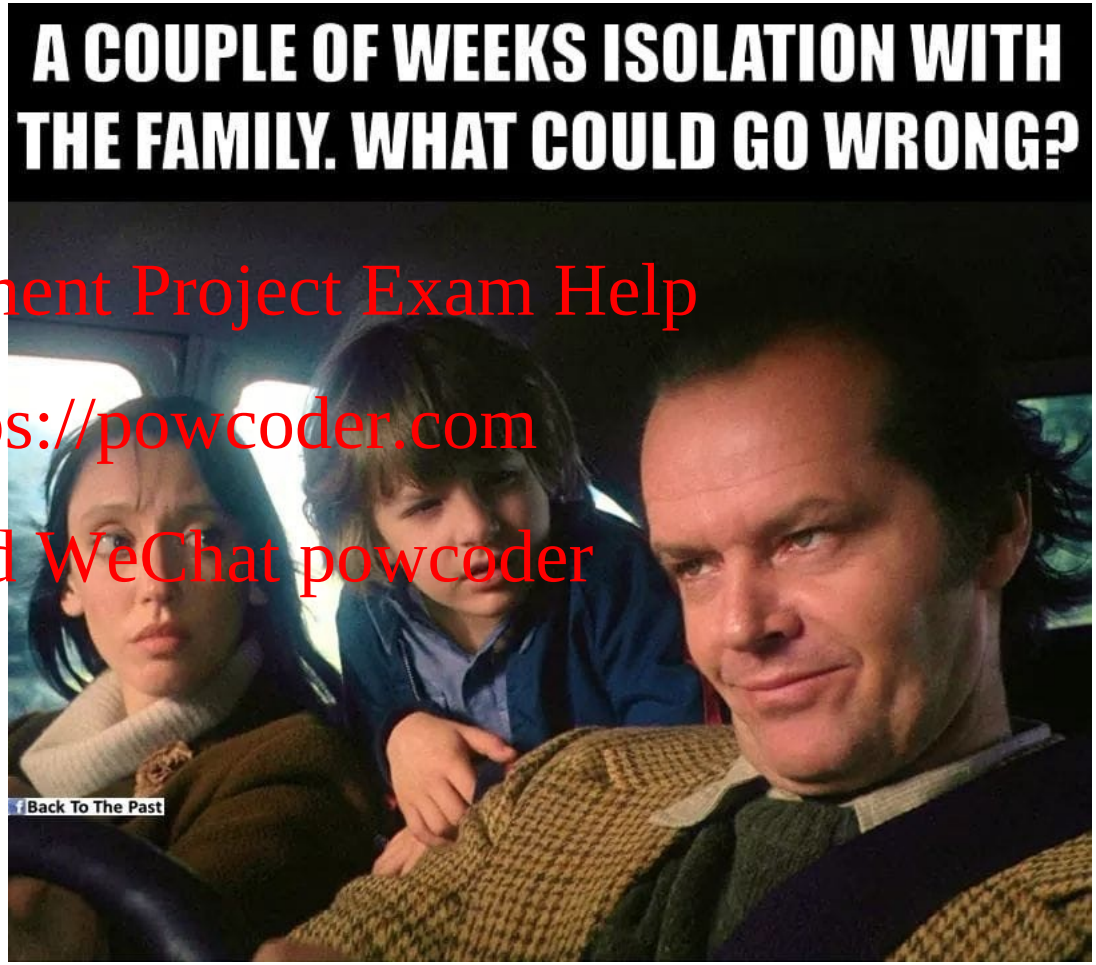
# Agenda

| Start | End | Item |
|---|---|---|
| | | Gompertz |
| | | Bass Models |
| | | SIR |
| | | |
| | | |
| | | |
| | | |

# SIR has 3 states of nature

Susceptible

People have no immunity.



# SIR has 3 states of nature




At some rate, people become infected

*For each infected person they may infect 1.2 people*

# SIR has 3 states of nature




At some rate, people become recovered or removed.
The longer someone is infected the longer the rate of infection has a chance for more infections.

# SIR has 3 states of nature



# SIR has 2 input parameters



# SIR has 3 states of nature








Beta – rate of infection for the disease

Gamma – rate of recovery

# SIR Assumptions




- No one joins the susceptible group, since we are ignoring births and immigration
- No on is re-infected (next slide)
- Everyone in population has the same probability of infection (for example does not account for elderly)
- Population is homogeneous (for example no social circles for immigrant or affluent communities…all the same

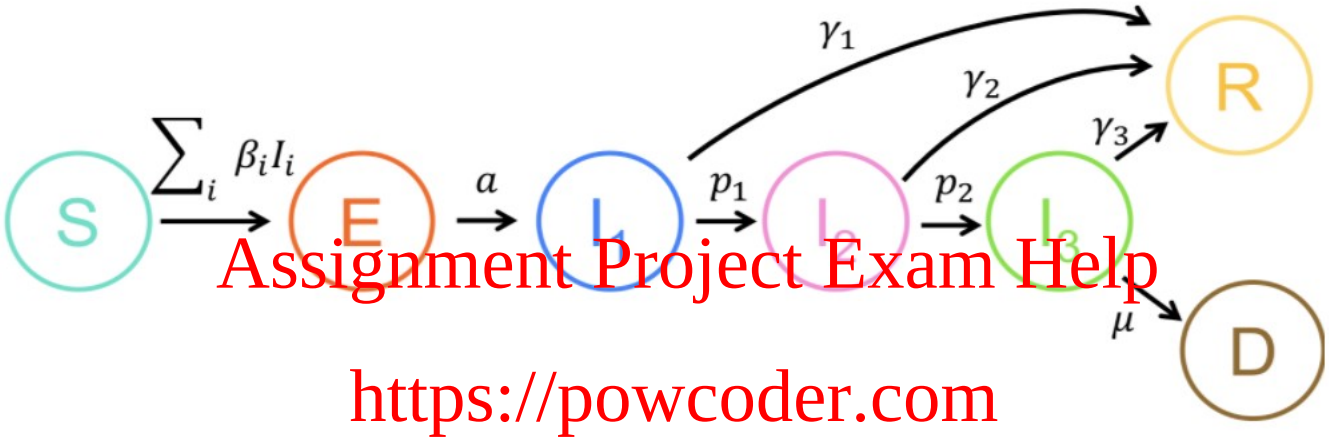
# SIR Assumptions




Other Similar Models:
No/Limited Immunity
SIRS (Susceptible - Infectious - Recovered - Susceptible)

Not complete infection
SEIR (Susceptible – Exposed – Infectious – Recovered)

# There can be more states of nature






- Susceptible
- Exposed
- Infected – not hospitalized
- Infected – hospitalized
- Infected – ICU
- Recovered
- Dead

# We will build the SIR model w/beta, gamma & total population




Beta – rate of infection for the disease

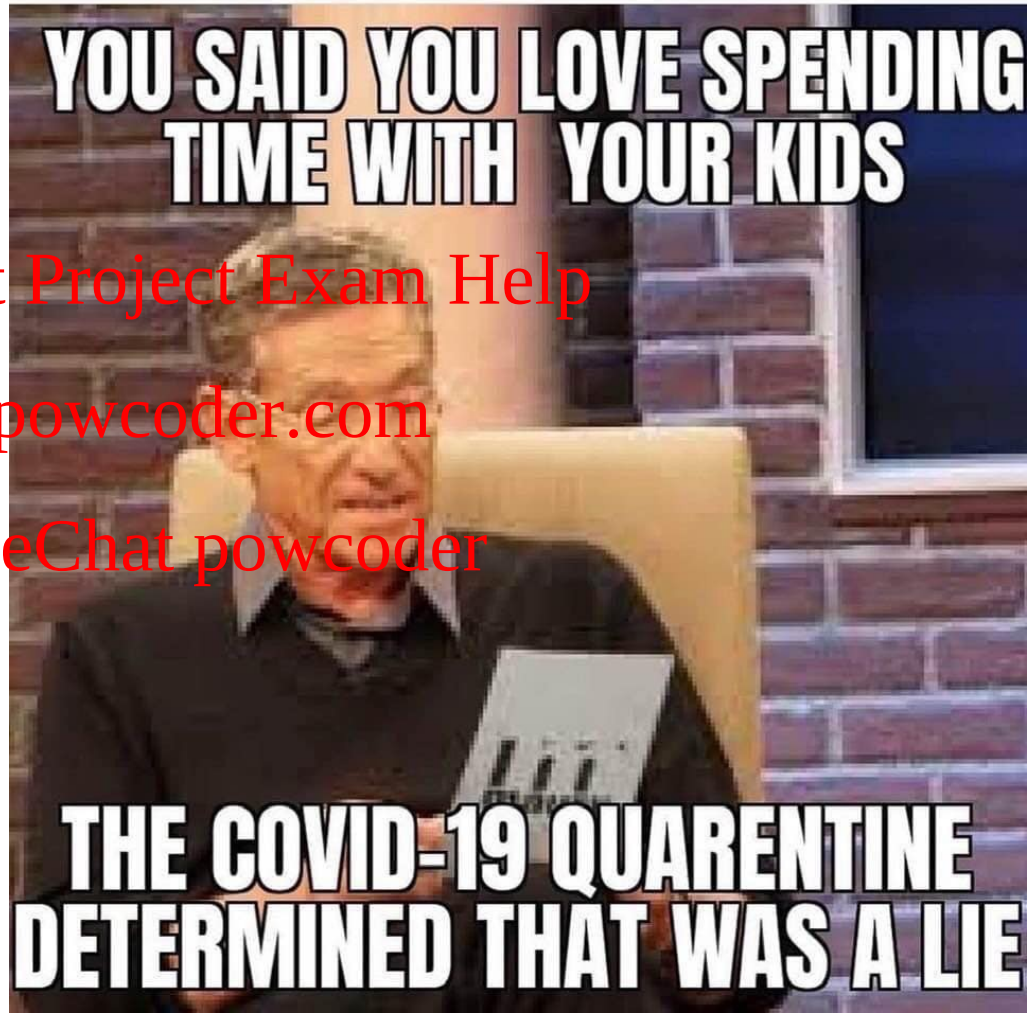


Gamma – rate of recovery

# Let’s Practice

C_SIR.R





# Thank you.




Every student in CSCI-96



HARVARD UNIVERSITY