Solution of Week 9 Lab (Prepared by Yuan Yin)

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1 Exercise 1:

1.1 Sensitivity of the Least Square Problem:

(Run 'ShowLSqV2.m' and try to understand the output.)

1.2 QR Factorization:

(Run and try to understand 'ShowQR' and 'qrExample. m'.)

1.3 Four Ways:

[1]:
```
%%file FourWays.m

function FourWays

clc

A = [1, 1, 1; 1, 1, 0; 0, 1, 1; 1, 0, 0; 0, 0, 1];
b = [89; 67; 53; 35; 20];
x_true = [35.125; 32.5; 20.625];

% Using Cholesky Factorization:
I = chol(A' * A);
x_chol = I \ (I' \ (A' * b));

% Using the QR Factorization:
[Q,R] = qr(A);
x_qr = R \ (Q' * b);

% Using \:
x_backslash = A \ b;

% Using Singular Value Decomposition:
[U,S,V] = svd(A);
x_svd = V * (S \ (U' * b));

% Calculating the error:
```

```
err_chol = norm(x_chol - x_true)
err_qr = norm(x_qr - x_true)
err_backslash = norm(x_backslash - x_true)
err_svd = norm(x_svd - x_true)
fprintf('As one can see, in this question, Cholesky factorization gives the smallest error.\n');

end
```

```
Created file '/Users/RebeccaYinYuan/MAST30028 Tutorial Answers Yuan Yin/WEEK 9/FourWays.m'.
```

[2]:
```
FourWays
```

```
err_chol =

   7.9441e-15


err_qr =

   2.5619e-14


err_backslash =

   1.2809e-14


err_svd =

   2.8643e-14

As one can see, in this question, Cholesky factorization gives the smallest error.
```


2 Exercise 2:

2.1 Steepest Descent:

Note that using Steepest Descent, we can still manage to find a local minimal of the 'banana' function, but it is a very slow and inefficient process which involves a lot of function evaluations. Also, if you run the script 'ShowBanana', you will see a characteristic zig-zag trajectory. This is because for the 'banana' function, if you follow its steepest descent direction, you will actually get a curve. However, our step is a straight-line segment.

2.2 Gauss-Newton Method:

For the undamped Gauss-Newton method, a poor initial guess will result in more iterations. Note that there are also some warning messages saying that the '$|residual|$' and the 'cost' will become infinity.

For the damped Gauss-Newton method, a poor initial guess will only result in more function evaluations.