UNIVERSITY OF MELBOURNE

SEMESTER 2 ASSESSMENT, NOVEMBER 2018

SCHOOL OF MATHEMATICS AND STATISTICS

MAST30028 NUMERICAL METHODS AND SCIENTIFIC COMPUTING

Exam duration — 3 hours

Reading time — 15 minutes

This paper is printed two-sided and consists of six (6) pages of examination.

Identical Examination Papers: There are none.

Authorized Materials: Pens, pencils, rubbers, rulers. Students may use any material from the subject website hosted on the Learning Management System (LMS), any MAST30028 files on the Lab Server and any part of the provided software system MATLAB. Students MAY NOT access their email accounts or any other material hosted on the LMS or any other websites.

Instructions to Invigilators: Each candidate should be issued with an examination booklet, and have access to a desktop computer equipped with MATLAB.

During reading time, students may read the paper, log in to a computer, delete all existing files from `C:\...\MATLAB` and move any relevant files from the server or the subject website to `C:\...\MATLAB`. They may start MATLAB but MAY NOT open any M-files.

Instructions to Students: This exam consists of (7) seven questions. Each question states the number of marks it is worth. The total number of marks available is 80.

At the end of the examination, zip up all M-files, figures and documents you wish to include into an archive (Right-click → Send to → Compressed (zipped) folder) labelled with your student number (e.g. `NMSCexam_350468.zip`) and upload via the exam page of the LMS (like you did with the assignments). The examination booklet will be collected by the invigilator.

This paper may be reproduced and lodged in the Baillieu Library.

Question 1 [8 marks]

Explain, in terms of concepts covered in this course, the output of the following MATLAB code:

a.
```
format short e
x = realmax*2; x/2
x = realmax+2; x-2
```

b.
```
format short e;
x=4;k=0;
while 4+x ~= 4
    x=x/2;k=k+1;
end
x
k
```

Question 2 [5 marks]

Explain, in terms of concepts covered in this course, the output of the following MATLAB code:


```
for n = [4 8 12 16]
   A = pascal(n);
   xTrue = ones(n,1)*10/3;
   b = A*xTrue;
   format long;
   x = A\b
   relerr = norm(x - xTrue)/norm(xTrue);
   fprintf('Relative error = %8.4e\n',relerr)
   pause
end
```

Feel free to modify the code to help you understand what is going on.

Question 3 [8 marks]

In lectures, the code `testEuler` was run to illustrate several features of Euler's Method. Explain **in detail** which three properties of Euler's method the code is demonstrating, and how it does this.

You may use any code used in the Labs.

Question 4 [14 marks]

The Simple Monte Carlo method can be used to approximate de nite integrals such as $I = \int_a^b f(x)\, dx$. By using points randomly placed in the interval $[a, b]$, the integral can be estimated by

$$\hat{I} = (b - a)f$$

where f is the sample mean of the function values at the random points

$$f = \frac{1}{N}\sum_{k=1}^{N} f(x_k)$$

a. Write a MATLAB function that returns the estimate $\hat{I}$ as described above, with de nition `function I = simpleMC(f,a,b,N)`

b. Since I haven't told you how to estimate the statistical error for this method from a single simulation, do the following instead. Write a driver `examMC.m` that

(i) repeats the simple MC simulation M = 1000 times, each one with N = 10,000 x values, to estimate the integral

$$I = \int_1^3 \exp(-x)\, dx$$

(ii) plots the estimates using a histogram plot

(iii) and hence computes a 95% con dence interval for $\hat{I}$, using the set of estimates for the simple MC method. You may use `mean` and `std`.

What properties of the histogram did you use to derive the 95% con - dence interval?

c. Compare the halfwidth of the con dence interval for the simple MC method with that obtained from hit-and-miss MC [you may adapt `hitmiss.m`].

d. How big would N have to be for the halfwidth to be $<$ 0.001, for each method? Comment.

Question 5 [16 marks]

The following data: t = 0.5, 1.0, 1.5 $\cdots$ 5.0, y =

7.1584e-02	8.5252e+00	1.0868e+01	2.1101e+01	3.1813e+01
4.9507e+01	9.9131e+01	1.6219e+02	2.8002e+02	4.8945e+02

is thought to show exponential behaviour: $y \sim c_1 c_2^t$

a. What form of plot would con rm exponential behaviour? Brie y explain why and con rm the behaviour with a suitable plot.

b. Inspired by your plot, transform the data to new variables in which the model appears linear in the parameters. Now t the transformed data to a line by constructing a suitable linear system and using \. Report the corresponding parameters c_1 and c_2.

c. Now t the original data directly to the exponential model, by using `lsqcurvefit`. Choose initial guesses for the parameters by using part (b). Compare the parameter values with those from part (b).

d. By plotting the data and both ts on the same plot, decide which method gives the best t for this data.

Question 6 [9 marks]

a. Use Newton's method, the method of bisection and `fzero` to nd the root of

$$f(x) = (\sin(x) - x)\exp(x)$$

For Newton's method, use $x_0 = 1$ and for the other methods use an initial interval of $[-1, 1]$.

Use tolerances of 10^{-10} and a maximum of 40 iterations, where possible.

b. Plot the residuals of each iterate for Newton's method and bisection on a suitable plot and describe what you see. For `fzero` use the `Display` option set to `iter`.

Comment on the performance of all three methods.

You may use any code used in the Labs.

Question 7 [20 marks]

The following ODEs describe a model for the spread of an infectious disease with lifelong immunity

$$\begin{aligned} \dot{s} &= -\beta s i \\ \dot{i} &= \beta s i - \gamma i \\ \dot{r} &= \gamma i \end{aligned}$$

where $s(t), i(t), r(t)$ are the fraction of the population susceptible to the disease, infected by the disease and recovered from the disease, respectively. There are 2 parameters β, γ, representing rates of transmission and recovery. Keep γ xed at $\gamma = 1/7$, corresponding to an infectious period of 7 days.

Suitable initial conditions are $(s_0, i_0, 0)$, where $s_0 + i_0 = 1$.

a. Write a MATLAB function `sir.m`, suitable for input into `ode23` etc., that describes the system given above, with the parameters set to $\beta = 3/7, \gamma = 1/7$.

b. Write a MATLAB driver that solves the system using `ode23`, with default tolerances. Test your driver by solving the problem with $i_0 = 0.01$, over the interval $[0, 100]$.

Use plot commands or edit the plot window so that your results look like Figure 1.

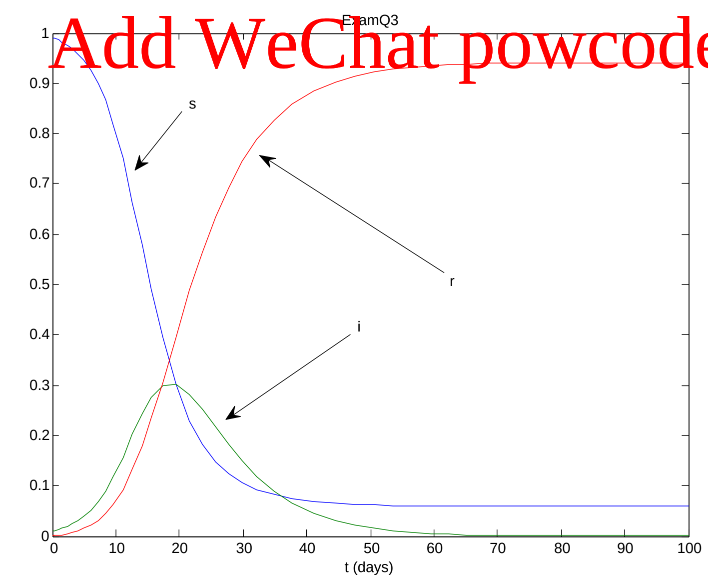

Figure 1: The epidemic model for $\beta = 3/7, \gamma = 1/7$.

c. Since s, i, r are the fractions of the population in each class, they must add to 1:

$$s(t) + i(t) + r(t) = 1$$

Check how accurately your numerical results satisfy this condition.

Since population fractions can't be negative, use an option to `odeset` that ensures that $s, i, r \geq 0$ and repeat your calculation.

d. Since r can always be found from s and i, we often only plot s (horizontal axis) versus i (vertical axis) (a *phase plot*). Plot your solution from (b) as a phase plot and use a Textarrow to indicate the initial condition.

The phase plot should look `chunky' i.e. not very smooth. By changing an option to `ode23`, produce a smoother phase plot.

e. If we add (equal) birth and death rates to this model, we get the equations:

$$\begin{aligned} \dot{s} &= -\beta s i + \mu - \mu s \\ \dot{i} &= \beta s i - \gamma i - \mu i \\ \dot{r} &= \gamma i - \mu r \end{aligned}$$

A realistic value for μ is $\mu = 1/(70 \times 365)$, corresponding to a human lifespan in days, but we will take $\mu = 1/(100 \times 7)$.

Write a new function M-file `sir_birth.m` describing this model, including the xed parameter μ.

Solve the new model for the case $\beta = 1.5/7, i_0 = 0.01$ over $[0, 6000]$ and describe what new features in the solution are produced by adding birth/death rates.

f. Using `tic/toc` to measure the time taken to solve the Initial Value Problem from part (e), compare the e ciency of `ode45,ode15s` for this problem. Do your results suggest this is a sti problem?

Reminder: You may use any code used in the Labs. Include your code and any relevant output and plots as evidence to support your answers. You may answer either in the booklet or in a document.

END OF THE EXAMINATION