School of Mathematics and Statistics
MAST30028 Numerical Methods & Scientific Computing
Week 6

Drag and drop the folder `Week6` from `L: \MAST30028` to `C:\...\MATLAB` and include it in the path. Now MATLAB knows how to find the files in `Week6`. The folder includes a pdf file of Chapter 2 in Moler's textbook (`lu.pdf`).

1 Newton's method

This relates to material in Lecture 10.

Exercise Set 1

a. Show that Newton's method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

when applied to the function $f(x) = x^2 - 6$ gives an algorithm you have already coded.

b. A function to implement Newton's method is given is `Newton.m`. Read the code and make sure you understand how it works. What convergence criterion does it use? How does it differ from `Bisection.m`?

To see an example of usage, run `driverNewton`.

c. Investigate the roots of the following functions, with the specified initial values x_0 at least.

- $\cos(x) - x$ Use $x_0 = 1, 3, 6$
- $\log_e(x) - \exp(-x) = 0$ Use $x_0 = 2$
- $x^3 - 7x^2 + 14x - 8 = 0$ Use $x_0 = 1.1, 1.2 \ldots 1.9$

d. You can visualize Newton's method by a code such as `vizNewton.m`.

Use `vizNewton.m` to explain some of the behaviour seen in part c.

Explain the M-file (Challenging)

Open `MySqrt.m` and try to figure out exactly what every MATLAB command is doing.
Hint: what has this to do with Newton's method?
Run the script `ShowMySqrt.m` to see how it performs over, say, the interval [0.1, 5].

2 Secant method

This relates to material in Lecture 10.

Exercise Set 2

The secant method can be derived by replacing $f'(x_n)$ in Newton's method by the forward difference approximation

$$f'(x_n) \approx \frac{f(x_n) - f(x_{n-1})}{x_n - x_{n-1}}$$

Note that this results in a second-order recurrence relation i.e. two previous iterates are required to generate the next iterate. This means that 2 initial iterates are required, not necessarily bracketing.

a. By modifying `Newton.m` or otherwise, write your own function for the secant method.

b. Test your program by finding the root of the function $\cos x - x$.

c. Now see how the secant method performs in finding the root of $\log_e x - \exp(-x)$ using $x_0 = 1, x_1 = 2$. Compare with the performance of the other two methods.

3 Matrices in MATLAB

Work through or watch while I work through the M-file `lab6.m` introducing 2D arrays (matrices) in MATLAB. Then try the following:

Exercise set 3

a. How would you create the dot product of 2 vectors in MATLAB? How would you compute the Euclidean length of a vector?

Hint: don't use `length`!

b. How could you create a random (entries uniformly distributed) symmetric matrix? a random skew-symmetric matrix?

c. A magic square is a an $n \times n$ matrix in which each integer $1, 2, \ldots n^2$ appears once and for which all the row, column and diagonal sums are equal. MATLAB has a command `magic` that returns magic squares. Check its output at a few sizes and use MATLAB to verify the summation property.

Hint: `flipud` or similar commands may be useful.

d. For a 2D analogue of `linspace`, look up help for `meshgrid`. It creates matrices that are useful for creating surface plots of functions of 2 variables. Plot the function

$$f(x_1, x_2) = \min(1 - |x_1|, 1 - |x_2|)$$

on $[0, 1] \times [0, 1]$ using `mesh` or `surf`.

e. Try out

```
fprintf('%d\n', eye(4,2))
fprintf('%d %6.2f\n', eye(4,2))
```

to see how `fprintf` handles matrix input.

Exercise Set 4

Run the function `twodriver.m` to see the effect of not pivoting in a rather extreme 2×2 case.