

School of Mathematics and Statistics
MAST30028 Numerical Methods & Scientific Computing
Week 4

Drag and drop the folder Week4 from L: \MAST30028 to C:\...\MATLAB to :D\MATLAB and include it in the path. Now MATLAB knows how to find the files in Week4.

1 Confidence intervals

Exercise Set 1

This relates to material presented in Lecture 6 (Statistical errors).

- For your program in Exercise set 3a of Week 3, add code to compute a 95% confidence interval for the probability of winning the bet after performing n repetitions. Test your code.
- By modifying your driver from Exercise set 3b of Week 3 or otherwise, run the simulation 100 times. How many times does the confidence interval fail to contain the exact answer?
- * For the random experiment simulated in `dele1.m` (4 rolls of a die), write a program to compute a 95% confidence interval for the difference between the largest value of the 4 rolls and the smallest, after n repetitions.

2 Monte Carlo integration

This relates to material presented in Lecture 7 (Monte Carlo integration).

For a standard normal random variable,

$$Pr(Z \leq 1) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^1 e^{-x^2/2} dx = 0.5 + \frac{1}{\sqrt{2\pi}} \int_0^1 e^{-x^2/2} dx$$

Exercise Set 2

- (using numerical quadrature)

Use the MATLAB function `integral.m` to find the value of $Pr(Z \leq 1)$.

- (Crude Monte Carlo) Now write a MATLAB program to use crude Monte Carlo, as described below, to estimate $Pr(Z \leq 1)$.

“A second method uses the connection of the integral to the mean value of $f(x)$, not to be confused with the expected value of a random variable. The mean value of a function $f(x)$ over $[a,b]$ is just

$$\frac{1}{b-a} \int_a^b f(x) dx$$

In the Crude Monte Carlo method, we generate values of x from $U(a,b)$ and calculate the sample mean of the set of values $\{f(x_i)\}$. Our estimate of the integral is then just $b-a$ times the sample mean.”

How many function values do you need to get 2 decimal place accuracy?

- (directly)

generate a set of samples from a standard normal random variable using `randn`, and count how many are less than 1. A one-liner?

3 Floating point numbers

These relate to material in Lecture 7(Floating point numbers)

Exercise Set 3

- a. Run the M-file `floatgui` from the Week4 folder which illustrates a toy floating point system. Explain how the slider parameters `emin`, `emax`, `t` affect the set of machine numbers produced. What do you see switching to a log scale? Then back to linear scale?

- b. Explain the output of the following MATLAB code

```
• format short e;  
  x=1;k=0;  
  while x ~= 0  
    x=x/2;k=k+1;  
  end  
  x  
  k  
• format short e;  
  x=1;k=0;  
  while isfinite(x)  
    x=x*2;k=k+1;  
  end  
  x  
  k  
• format short e;  
  x=1;k=0;  
  while 1+x ~= 1  
    x=x/2;k=k+1;  
  end  
  x  
  k
```

- c. Predict and explain the result of the following commands (use `format long e`):

```
• x=realmax;x=x+1  
  x=realmax;x=2*x  
  x=x/2  
• x=realmin;x=x/2  
• x=1+eps  
  x=x-1  
  x=1+eps/2  
  x=x-1  
  x=8+eps  
  x=8+4*eps  
  x=8+5*eps
```