```
[1] import numpy as np
```

Seminar 4

Fluids in Motion

1. **Hydrodynamics I**

Problem 1 - Reynold's number

Pipe flow with discharge $Q = 0.025$ m^3/s in a pipe of a diameter 0.1 m. Is the flow laminar?

Solution of Problem 1

The relevant equations can be found in slides L4 - 10-11

The main equation is:

$$Re = \frac{\rho \cdot v \cdot D}{\mu}$$

For water at 20°, we use $\rho = 1000$ kg/^{3} and $\mu = 1.005 \cdot 10^{-3}$ kg/m-s

```
[2] #Given are:
D1 = 0.1 # m, diameter of the pipe
Q1 = 0.025 # m^3/s, discharge in pipe
rho1 = 1000 # kg/m^3, density of water
mu1 = 1.005* 10**-3

# interim calculation
A1 = (np.pi/4) * D1**2 # m^2 area of the pipe
v1 = Q1/A1 # m/s, velocity of pipe flow

#Calculations
Re = rho1*v1*D1/mu1

# output
print("The Reynold's number is {0:1.3f}".format(Re), "\n")

if Re < 2000:
    print("It is laminar flow")
```

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    elif Re == 2000 and Re <= 4000:
        print("It is Transitional flow")
    elif Re > 4000:
        print("It is Turbulent flow")
```

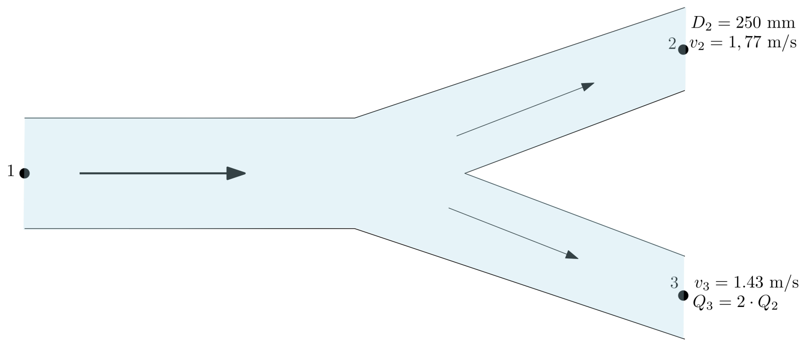
```
The Reynold's number is 316726.255

It is Turbulent flow
```

Problem 2 - Conservation of mass

Water flows through a branching pipeline as shown in the diagram. If the diameter, D_2, is 250 mm, $v_2 = 1.77$ m/s and $V_3 = 1.43$ m/s:

1. what diameter, D_3, is required for $Q_3 = 2Q_2$?
2. what is the total discharge at section 1?



Solution of Problem 2

The relevant equations are (check L4 slides 30)

The main equation is:

$$v_1 A_1 = v_2 A_2$$

with $A = \frac{\pi}{4} D^2$

$$\frac{\pi}{4} D_1^2 \cdot v_1 = \frac{\pi}{4} D_2^2 \cdot v_2$$

and

$$D_1 = \sqrt{\frac{v_2}{v_1} \cdot D_2^2}$$

Information provided in the problem are:

$Q_3 = 2 \times Q_2$

$$Q_1 = Q_2 + Q_3$$

```
[3]  # Given are:

v2_2 = 1.77 # m/s velocity section 2
v2_3 = 1.43 # m/s velocity in section 3
D2_2 = 250 # mm diameter of section 2

# interim calculation
D2_2m = 250/1000 # m, changing unit of D2_2 to m
Q2_2 = np.pi/4*D2_2m**2 *v2_2 # m^3/s, discharge Q2

# Solution problem
D2_3 = np.sqrt(2* (v2_2/v2_3) * D2_2m**2) # m, diameter section
3. Multiplied by 2 as Q3 = 2* Q2
Q2_3 = np.pi/4*D2_3**2 *v2_3 # m^3/s, discharge at Q2_3
Q2_1 = Q2_2 + Q2_3

# output
print("The diameter  is {0:1.2f}".format(D2_3),"m", "\n")
print("The Discharge at section 1  is
{0:1.2f}".format(Q2_1),"m\u00b2")
```

```
The diameter  is 0.39 m

The Discharge at section 1  is 0.26 m²
```

Problem 3 - Conservation of momentum

A pipeline with a constant diameter of 0.3 m turns through an angle of 60°. The centreline of the pipe does not change elevation. The discharge through the pipeline is 0.1 m^3/s of water, and the pressure at the bend is 30 m of water. Calculate the magnitude and direction of the resultant force on the pipe.

Solution of Problem 3

The relevant information and equations can be found in slides L3: 35-36

Equations to use are:

$$A_1 v_1 = A_2 v_s$$

$$P_1 \cdot A_1 - P_2 \cdot \cos\theta \cdot A_2 - F_{rx} =$$
$$= \rho \cdot Q \cdot (v_2 \cdot \cos\theta - v_1)$$

$$F_{r,y} - P_2 \cdot \sin\theta \cdot A_2 =$$
$$= \rho \cdot Q \cdot (v_2 \cdot \sin\theta)$$

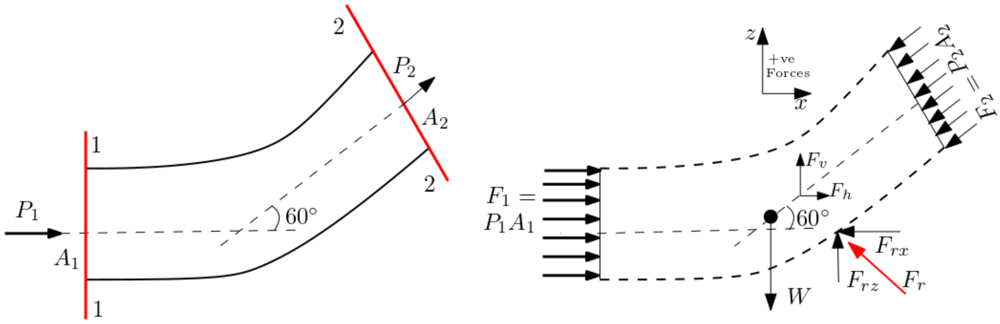
$$F_r = \sqrt{F_{r,x}^2 + F_{r,y}^2}$$

and

$$\theta = \arctan\left(\frac{F_{r,y}}{F_{r,x}}\right)$$

Solution of Problem 3

Draw the free body diagram of the problem



The given information are:

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[4]  # Given are:

ae = 60 # degree, pipe bending
D3 = 0.3 # m, pipe diameter
Q3 = 0.1 # m^3/s, discharge through pipe
Ph = 30 # m , water height at the bend
rho3 = 1000 # kg/m^3, density of water
g3 = 9.81 # m/s^2, gravity

# interim calculation -  #Calculation of A, v and P
ae_r = ae*np.pi/180
A3_1 = A3_2 = np.pi/4*D3**2 # pipe area at section 1 and 2. Constant dia.
v3_1 = v3_2 = Q3/A3_1 #m^3/s, Q= AV and since A is equal and Q is conserved
P3_1 = P3_2 = rho3*g3*Ph # N/m^2, since A and V are equal, P1 = P2 = rho*g * h

# interim calculation -  #Forces calculation in X-direction
F_1x = P3_1*A3_1 # N, Force at 1 in x,  F= P.A
F_2x = P3_2*A3_2* np.cos(ae_r) # N, force at 2 in x
F_rx = P3_1*A3_1 - P3_2*A3_2*np.cos(ae_r)-rho3*Q3*(v3_2*np.cos(ae_r)-v3_1)
```

[5]

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# interim calculation -  #Forces calculation in y-direction
F_1y = 0
F_2y = P3_2*A3_2*np.sin(ae_r)
F_ry = rho3*Q3*(v3_2*np.sin(ae_r))+ P3_2*A3_2*np.sin(ae_r)

# calculation of part 1 and 2
F_r = np.sqrt(F_rx**2+ F_ry**2) # N, resultant force
Dir_r = np.arctan(F_ry/F_rx) # rad, direction of resultant in radian
Dir_t = Dir_r*180/np.pi # Degree, direction of resultant in degress

# Output
print("The resultant force is:{0:1.2f}".format(F_r), "N \n")
print("The resultant is acting at: {0:1.2f}".format(Dir_t),"\u00b0 ")
```

```
The resultant force is:20944.31 N

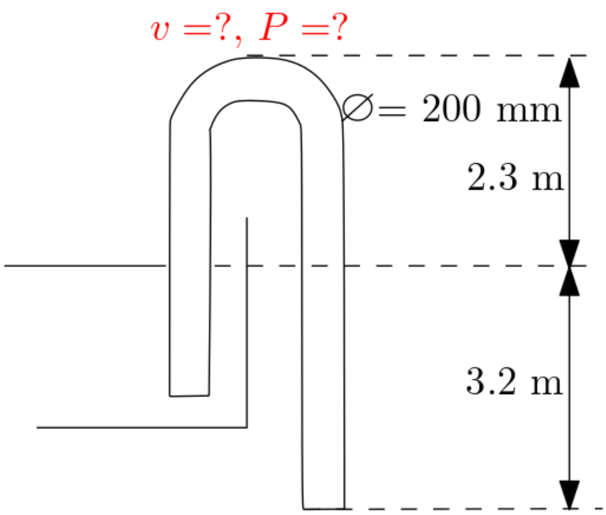
The resultant is acting at: 60.00 °
```

Problem 4- Conservation of Energy

From a large reservoir water is released via a syphon. The pipe diameter is 200 mm. The end of the syphon pipe is 3.2 m below the water level of the reservoir. The highest part of the syphon is 2.3 m above the water level of the reservoir. Water level remains constant, no losses.

Calculate the discharge.

Calculate the pressure head at the highest point of the syphon.



Solution of Problem 4

The lecture contents on the problem can be found in slided L4- 45-48.

Basic equations are:

Bernoulli Equation:

$$z_1 + \frac{1}{2}\frac{v_1^2}{g} + \frac{P_1}{\rho \cdot g} = z_2 + \frac{1}{2}\frac{v_2^2}{g} + \frac{P_2}{\rho \cdot g}$$

and

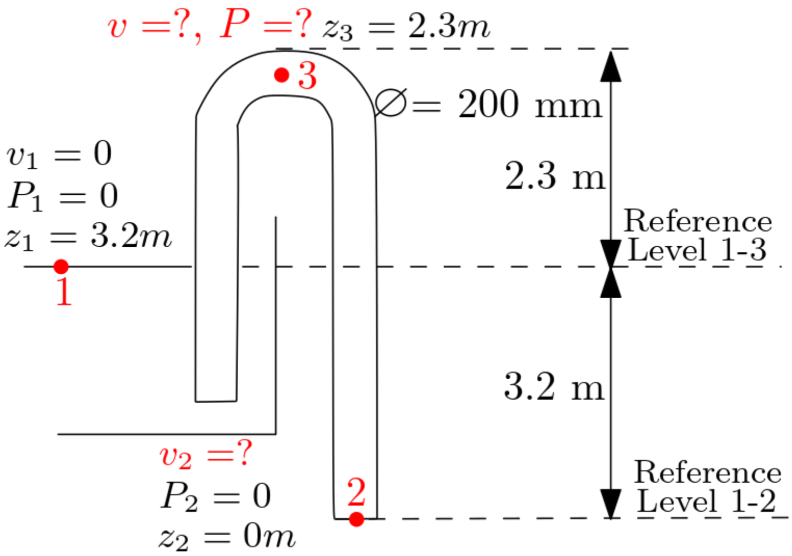
Continuity Equation:

$$A_1 v_1 = A_2 v_2$$

Steps to solve the problem:

Step 1: Drawing including the known and unknown variables v, P and z

Step 2: Decide which points and reference level to use for analysis and which calculation steps to take

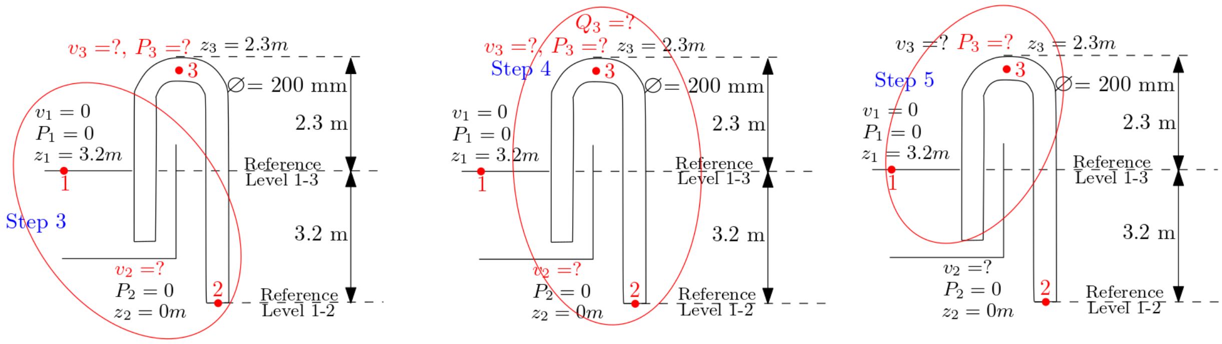


Solution of Problem 4

Step 3: Bernoulli equation for points 1-2 to calculate v_2 using reference level 1-2.

Step 4 Use Continuity equation for points 2-3 to calculate $Q_3 = Q$.

Step 5 Bernoulli equation for points 1-3 to calculate P_3 using reference level 2-3.



[9]

```
#Solution of Problem 4. Given are:
#Step 3 Bernoulli eq. 1-2 obtain v_2

z4_1 = 3.2 #m, elevation point 1 from ref 1-2
z4_2 = 0 # m, elevation point 2 from ref 1-2
v4_1 = 0 # m/s, velocity point 1
P4_1 = 0 # N/m^2, pressure point 1 (atmospheric)
P4_2 = 0 # N/m^2, pressure point 2 (atmospheric)
g4 = 9.81 # m/s^2, Gravity

#Using Bernouli's equation
v4_2 = np.sqrt(2*g4*z4_1) #m/s- 3.2 + 0 + 0 = 0 + v^2/2g + 0

#output
print("Velocity at point 2 is: {0:1.3f}".format( v4_2),  'm/s')
```

```
Velocity at point 2 is: 7.924 m/s
```

[7]

```
#Solution of Problem 4.
#Step 4- Continuity eq. 2-3, obtain Q_3

d4 = 0.2 # m, diameter of the pipe
v4_3 = v4_2 # m/s, Q and A are same so v is also same

#interim calculation
A4_2 = A4_3 = np.pi/4*d4**2  # m^2, area of cross-section. it is
same everywher

#calculation
Q4 = A4_3*v4_3 # m^3/s- discharge is conserved

#output
print("Discharge in the system is: {0:1.3f}".format( Q4),
 'm\u00b3/s')
```

```
Discharge in the system is: 0.249 m³/s
```

[8]

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#Solution of Problem 4.
#Step 5- Bernoulli's eq. 1-3, obtain P3
#Avaialable are

z4_1u = 0 # m, elevation from ref line 1-3 to point 1
z4_3u = 2.3 # m, elevation from ref line 1-3 to point 3
v4_1 = 0 # m/s, velocity at point 1
P4_1 = 0 # N/m^2, atmospheric pressure
rho4 = 1000 # kg/m^3, fluid density
v4_3u = v4_3 # m/s velocity at point 3 from previous step

#calculation
# N/m^2 - 0+0+0 = z3 + v3^2/2.g + P3/rho*g
P4_3 =(-z4_3u - v4_3u**2/(2*g4))*rho4*g4

#output
print("Pressure at point 3 in the system is:
{0:1.3f}".format(P4_3),  'N/m\u00b2')
```

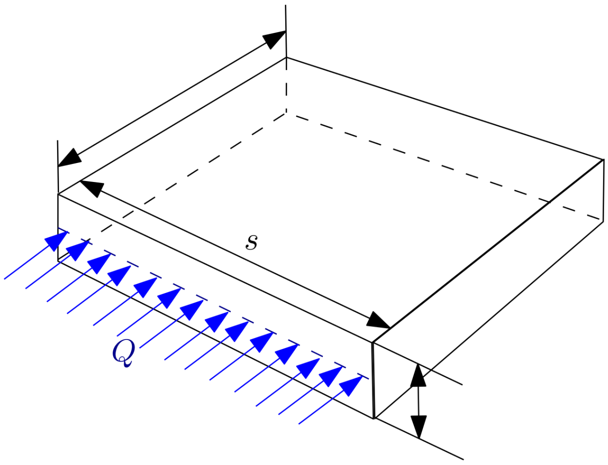
```
Pressure at point 3 in the system is: -53955.000 N/m²
```

Assignment problems

Assignment Problem 1 - Reynold's number

In a certain medical application, water at room temperature and pressure flows through a rectangular channel of length $L = 10$ cm, width $s = 1.0$ cm, and gap thickness $b = 0.30$ mm as in Fig. 1. The volume flow rate is sinusoidal with amplitude $\hat{Q} = 0.50$ mL/s and frequency $f = 20$ Hz, i.e., $Q = \hat{Q}\sin(2\pi ft)$.

Calculate the maximum Reynolds number based on maximum average velocity and gap thickness.



Assignment Problem 2 - Mass conservation equation

A hose attached with a nozzle is used to fill a 30 Litre bucket. The inner diameter of the hose is 2 cm, and it reduces to 0.8 cm at the nozzle exit If it takes 50 s to fill the bucket with water, determine

(a) the volume and mass flow rates of water through the hose, and

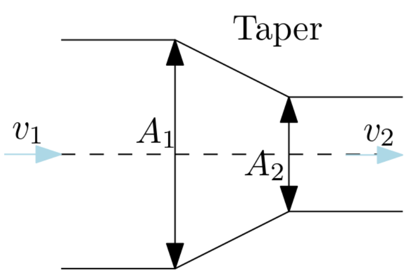
(b) the average velocity of water at the nozzle exit.

You can assume

- Flow through the hose is steady.
- There is no waste of water by splashing.

Assignment Problem 3 - Momentum conservation equation

A horizontal pipeline reduces in diameter using a standard, symmetrical taper section as shown below. Given the following information, calculate the force exerted by the water on the taper section: $Q = 0.42$ m^3/s, $D_1 = 0.60$ m, $D_2 = 0.30$ m, $P_1 = 25.30$ m of water, $P_2 = 23.61$ m of water, $\rho = 1000$ kg/m^3



[ ]

Assignment Problem 4 - Energy conservation equation

Water flows through a pipeline of constant diameter that is inclined upwards. On the centreline of the pipe, point 1 is 0.3 m below point 2. The pressure at point 1 is 9.3×10^3 N/m^2. What is the pressure at point 2 if there is no loss of energy?