Faculty of Civil Engineering
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# Hydraulics Modelling

## Sediment Transport

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# Contents - Sediment Transport

## Last Lecture contents

1. Navier-Stokes Equation and Saint Vennant Equation
2. Methods for solving Saint Vennant Equation
3. Model Calibration
4. Model Analysis - Sensitivity.

# Any Questions?

## Today's Lecture contents

1. Sediments and sediment transport
2. Factors affecting sediment transport
3. Sediment transport equations
4. Modeling sediment transport

# Sediment Transport

## Sediments and sediment transport

## Why sediment transport is important?

- **Too much sediment:** The most common issue. This normally leads to water quality problems. Also can alter waterways and create artificial banks.
- **Too little sediment:** Largely due to man-made structures (dams, canals etc). Less sediment leads to less nutrients for aquatic habitat.
- **Contaminated sediment:** They are mostly from point-source (effluent) but also diffuse (run-off). Long-term effect of water quality.
- **Scour:** Refers to removal (erosion) of sediment from streambed/banks leading to damage of hydraulic structures.

## Sediment and its properties

**Sediments** are small particles such as sand, gravel, clay, organic materials etc.

Sediments are distinguished between: **Cohesive** (e.g., clay, silt, small sizes) and **Non-Cohesive** (e.g., sand, gravel, larger sizes)

**Sediment Characterization**:

1. **Relative Density:** ($s$) also called *specific gravity*, which is

$$\boxed{s = \frac{\rho_s}{\rho}}$$

with $\rho_s \approx \mathbf{2650}$ kg/m$^3$ is a standard sediment density and $\rho$ is fluid density.

2. **Characteristic size:** Standard particle size tables (next slide) are set by governing bodies (e.g., DIN).

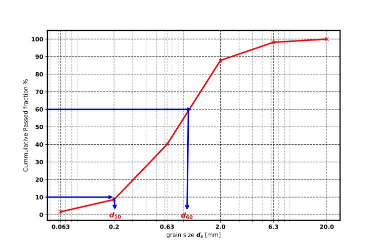
Particle size distribution plot is used to characterize the sediment

## Sediment and its properties

**Particle size** (from Chanson, 2004)

| Class name | Size range $d_s$ (mm) |
|---|---|
| Clay | $d_s < 0.002 - 0.004$ |
| Silt | $0.004 < d_s < 0.06$ |
| Sand | $0.06 < d_s < 2.0$ |
| Gravel | $2.0 < d_s < 64$ |
| Cobble | $64 < d_s < 256$ |
| Boulder | $256 < d_s$ |

**Particle size distribution (see Problem 1- Seminar-L08.ipynb)**




### Other factors

$d_{50}$ commonly used as a characteristics grain size.

**Uniformity Coefficient** ($S$): Commonly obtained from $S = \sqrt{\frac{d_{90}}{d_{10}}}$

Low $S < 2$ refers to well sorted sediment.

## Sediment and its properties

3. **Sediment mixtures densities**

   A. Dry sediment mixtures

   $$\boxed{(\rho_{sed})_d = (1-\eta)\rho_s}$$

   B. Wet sediment mixtures

   $$\boxed{(\rho_{sed})_w = \eta\rho + (1-\eta)\rho_s}$$

   with fluid density $\rho$, sediment density $\rho_s$ and porosity $\eta$. The value of $\eta$ is usually between 0.26 – 0.48

4 **Angle of repose** ($\phi$): Limiting slope angle at which grain begins to roll. Typically in river $\phi = 28^\circ$

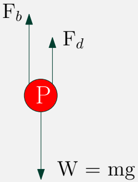
## Sediment and its properties

Three forces (Buoyancy $F_b$, Drag $F_d$ and Weight $W$) acts on the suspended particle. For moving water with velocity ($v$) in upwards direction, the particle will settle if $v > v_s$, with $v_s$ being the terminal velocity.

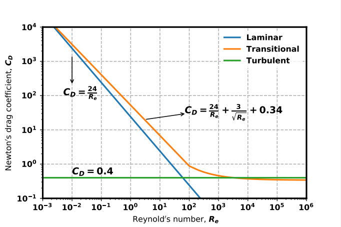
**Stokes' Law** presents $v_s$ as a function of densities of water ($\rho_w$) and particle ($\rho_w$), and particle diameter ($d$), given as

$$\boxed{v_s = \sqrt{\frac{4 \cdot g \cdot d}{3 \cdot C_D}\left(\frac{\rho_s - \rho_w}{\rho_w}\right)}}$$

with drag coefficient $C_D$ and acceleration due to gravity $g$




$C_D$ is a function of Reynolds number




**See Drag coefficient: (Drag Coff. see in HM-L08.ipynb)**

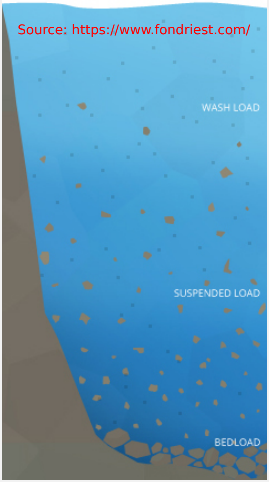
## Sediment Loads

**Sediment transport:** General term referring to transport of sediments in channels.

**Sediment load:** The transported materials, which can be:

- **Bedload**: Occurs for low velocity flows and/or large grain sizes. Controlled by bed shear stress (discussed later)
- **Suspended load**: Dominant at high velocity flows and/or small grain sizes.
- **Wash load**: Also a suspended load but for very small particle sizes ($<$ **0.00195**) mm that never settles at the bottom.

We will focus on **bedload**



# Sediment Transport

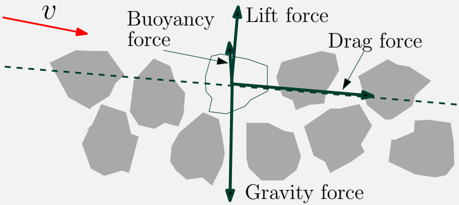
## Factors affecting sediment transport

## Forces of sediments

For a moving bed in a channel, **forces** acting on each sediment particle are:

1. Gravity force, $\boldsymbol{W = \rho_s g V_s}$
2. Buoyancy force, $\boldsymbol{F_b = \rho g V_s}$
3. Drag force, $\boldsymbol{F_D = C_D \rho A_s v^2/2}$
4. Lift force, $\boldsymbol{F_l = C_L \rho A_s v^2/2}$
5. Reaction forces of the surrounding grain

with volume of the particle $\boldsymbol{V_s}$, characteristic particle cross-section $\boldsymbol{A_s}$, characteristic velocity $\boldsymbol{v}$, drag and lift coefficients $\boldsymbol{C_D}$ and $\boldsymbol{C_L}$ and fluid density $\boldsymbol{\rho}$




$\boldsymbol{W}$ and $\boldsymbol{F_b}$ acts vertically while $\boldsymbol{F_D}$ acts along the flow direction.

The **reaction force** is dependent on grain deposition and packing

## Bed and critical stresses

**Threshold of sediment motion** $T_m$ defines the flow and boundary conditions for which the sediment transport just starts to occur.

$T_m$ is found to be related to:

- bed shear stress $\tau_0 = \rho g h S_0$
- particle density $\rho_s$
- fluid density $\rho$
- particle diameter $d$

These relates to three dimensionless quantities:

1. Froude number $= \frac{v_*}{\sqrt{g d_s}}$
2. Reynolds number $= \rho \frac{d v_*}{\mu}$
3. Specific gravity $= \frac{\rho_s}{\rho}$

with shear velocity $v_* = \sqrt{\tau_0/\rho}$, depth $h$ and bed slope $S_0$

**Sediment motion** begins when combination of $F_D, F_l, F_b$, called destablizing factor, becomes larger than $W$, an stabilizing factor $(\tau_*)$.

Introducing a threshold stress $\tau_c$, which is found to relate with $\tau_*$ as

$$\boxed{\tau_c = \tau_* g(\rho - \rho_w) d}$$

Bed load motion condition: $\tau_0 > \tau_c$

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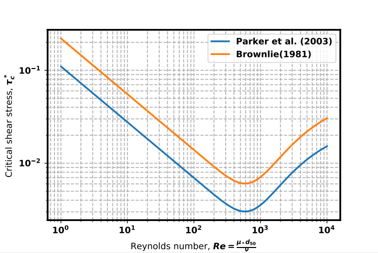
## Shields parameter for $T_m$: $\tau_0 > \tau_c$

All quantities except $\tau_*$ can be calculated. $\tau_*$, also called **Shields parameter**- a dimensionless factor, is obtained from Shields diagram:




**See Drag coefficient: (Shields Fig., see in HM-L08.ipynb)**

Shields figures shows:

- Laminar flow ($Re < 4-5$): $\tau_c < 0.035$
- Transition flow ($4-5 < Re < 75-100$): $0.035 < \tau_c < 0.04$
- Turbulent flow ($75-100 < Re$): $0.03 < \tau_c < 0.06$

**Example Problem** A channel with water depth 1.5 m and bed slope 1/500 is covered with stone of size 0.0034 m. Is the bed load in motion? **See Bed load motion: (Problem 2, see in Seminar-08.ipynb)**

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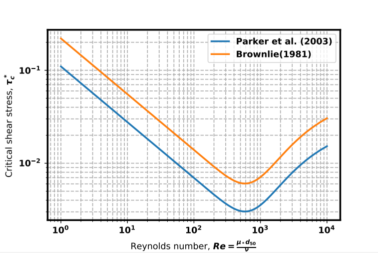


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Shields figures shows:

- Laminar flow ($Re < 4 - 5$): $\tau_c < 0.035$
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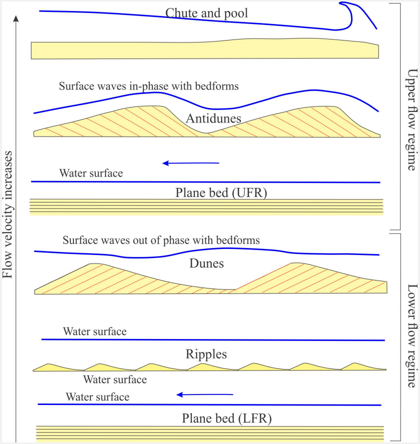
**Example Problem** A channel with water depth 1.5 m and bed slope 1/500 is covered with stone of size 0.0034 m. Is the bed load in motion?

**See Bed load motion: (Problem 2, see in Seminar-08.ipynb)**

## Bed formation due to sediment motion

The sediment transport results to:

- deformation of bed and change in bed-slope
- Characterized bed formations are: **Planes**, **Ripples**, **Dunes**, **Antidunes**
- Change in flow pattern (top and bottom)
- Several methods exist to predict bed forms. Water depth, bed shear stress, particle size etc. are required quantities.




Source: **https://www.geological-digressions.com/**

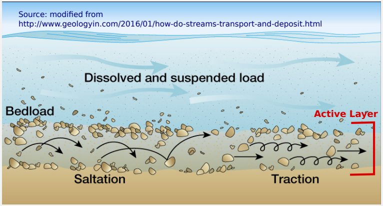
# Sediment Transport

## Sediment transport equations

## Sediment transport mechanisms

The sediment motion begins as $\tau_0 > \tau_c$. Transport mechanisms are:

- **Saltation:** bouncing of particularly small size particles.
- **Traction**: rolling of large size particles.
- **Dissolved and suspended load motion:** moving with flow




**Active layer** represents the bed portion where the transport takes place.

**Only Bed-load tranport item 1 and 2 is discussed.**

## Sediment transport empirical equations (1D)

- Several transport formulas are available in literature and they have been in development since *du Boys(1879)*[1].
- These formula have empirical constant in them. Therefore, they are only valid in suitable conditions, e.g., bedload, total load etc.
- These formulas mostly provide *volumetric sediment discharge per unit width* ($\boldsymbol{q_s}$) as a function of several factors (next slide).
- In general the available formulas can be divided into two groups: *Bed-load formula* and *total load formula*.
- The formulas can give a very different results for the same case. So, it is important to find the most suitable formula.

[1]du Boys, P. (1879), Le Rône et les rivires á lit affouiable: Étude du régime du Rhône et de l'action exercée par les eaux sur un lit à fond de graviers indéfiniment affouillable, Ann. Ponts Chaussees, 5(49), 141–195

## Sediment transport empirical equations (1D)

### Few bed-load formulas

| Reference | Formula |
|---|---|
| du Boys(1879) | $q_s = \lambda\tau(\tau - \tau_c)$ with $\lambda = \frac{0.54}{(\rho_s-\rho)g}$ |
| Einstein(1942) | $q_s = 2.15 \exp\left(-0.391\frac{\rho(s-1)gd}{\tau}\right)\sqrt{(s-1)gd^3}$ with $s = \frac{\rho_s}{\rho}$ |
| Meyer-Peter(1949) | $q_s = \sqrt{(s-1)gd^3}\left(\frac{4\tau}{\rho(s-1)gd} - 0.188\right)^{3/2}$ |
| Nielsen(1992) | $q_s = \sqrt{(s-1)gd^3}\left(\frac{12\tau}{\rho(s-1)gd} - 0.05\right)\sqrt{\frac{\tau}{\rho(s-1)gd}}$ |

*Check the validity range of formula before applying it.*

## Sediment transport empirical equations (1D)

### Few total sediment transport formulas

Total sediment transport ($q_{s(t)}$) refers to sum of bed-load ($q_{s(bl)}$) and suspended load transports ($q_{s(ss)}$).

| Reference | Formula |
|---|---|
| Einstein(1950) | $q_{s(t)} = q_{s(bl)}\left(1 + I_1 \ln\left(\frac{30d}{d_s}\right) + I_2\right)$ <br> $I_1$ and $I_2$ are integrals- obtained from chart. <br> $d_s$ particle size & $d$ is characteristic particle size |
| Engelund & Hansen(1967) | $q_{s(t)} = 0.4f\frac{\tau}{\rho}\sqrt{\frac{d_s}{(s-1)g}}$ <br> $f$ is friction factor |

Other formulas are provided by Graf(1971), van Rijn(1984) etc.

## Sediment transport empirical equations (1D)

Sediment transport formula are used in HEC-RAS for obtaining sediment load potential - which represents the transportable mass of a particular grain class in response to channel hydraulic properties.

**Example problem:**

The *bed-load transport* rate must be estimated for the Danube river at a particular cross-section. The known hydraulic data are flow rate of about 530 $m^3/s$, flow depth of 4.27 m and bed slope being about 0.0011. The channel bed is a sediment mixture with a median grain size of 0.012 m and the channel width is about 34 m.

Predict the sediment-load rate using the Meyer-Peter(1949) and the Einstein (1950) formula. Compare the obtained results.

**See Solution: (Problem 3- see in Seminar-08.ipynb)**

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# Sediment Transport

## Modeling sediment transport

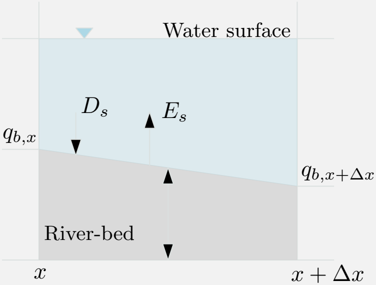
## Exner Equation

For larger-scale and complexities, conservative models are required to be developed and modelled.

Exner(1920, 1925) provides such conservation equation, commonly known as Exner equation, of sediment mass at the river bed.

The river-system used in Exner equation consists of:

- Volume transport rate per unit width $q_s$ varying in co-ordinate space (e.g. along $x$).
- Deposition and erosion rates $D_s$ and $E_s$ in the unit of volume per unit area
- Liquid property expressed by density $\rho$ and bed properties expressed by porosity $(\lambda_p)$



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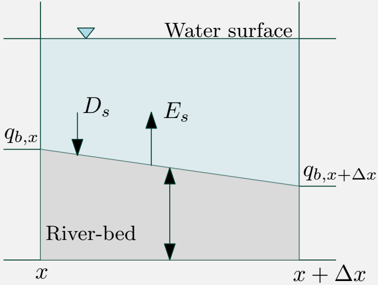

## Exner Equations

The Exner equation states that the difference between sediment entering and leaving a control volume must be stored or removed from storage, and is given as:

$$\frac{\partial}{\partial t}[\rho_s(1-\lambda_p)\eta]\Delta x = \rho_s[(q_{b,x} - q_{b,x+\Delta x} + (D_s - E_s)\Delta x]$$

Assuming $\lambda_p$ independent of time $t$, and letting $\Delta x \to 0$ leads to

$$\boxed{(1-\lambda_p)\frac{\partial \eta}{\partial t} = -\frac{\partial q_b}{\partial x} + D_s - E_s}$$

**This equation is solved in HEC-RAS**.

The 1D Exner model can be generalized to higher dimension as

$$\boxed{(1-\lambda_p)\frac{\partial \eta}{\partial t} = -\nabla \cdot \vec{\mathrm{q_b}} + D_s - E_s}$$

## Modelling with Exner equation

The HEC-RAS uses the following steps for balancing the Exner (continuity) equation:

1. Computing the **sediment transport capacity** as a function of grain class (i.e. computing the right side of the Exner equation)-
2. Computing the **sediment transport potential** for each grain class from sediment transport equations (Engelund-Hansen, Meyer-Peter etc.)
3. Computes **total transport capacity** as a sum of transport capacity for each grain class
4. Finally, the HEC-RAS compares computed capacity to supply for each grain class and calculates a sediment surplus or deficit by grain class.

# Thank You

Happy Modelling

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