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[1] import numpy as np
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Seminar 4

Pressure measurements

1. Hydrodynamics I

Problem 1 - Reynold's number

Pipe flow with discharge $Q = 0.025 \text{ m}^3/\text{s}$ in a pipe of a diameter 0.1 m. Is the flow laminar?

Solution of Problem 1

The relevant equations can be found in slides L4 - slides 10-11

The main equation is:

$$Re = \frac{\rho \cdot v \cdot D}{\mu}$$

For water at 20°, we use $\rho = 1000 \text{ kg/m}^3$ and $\mu = 1.005 \cdot 10^{-3} \text{ kg/m-s}$

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[2] #Given are:
D1 = 0.1 # m, diameter of the pipe
Q1 = 0.025 # m^3/s, discharge in pipe
rho1 = 1000 # kg/m^3, density of water
mu1 = 1.005* 10**-3

# interim calculation
A1 = (np.pi/4) * D1**2 # m^2 area of the pipe
v1 = Q1/A1 # m/s, velocity of pipe flow

#Calculations
Re = rho1*v1*D1/mu1

# output
print("The Reynold's number is {0:1.3f}".format(Re), "\n")

if Re < 2000:
    print("It is laminar flow")
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elif Re == 2000 and Re <= 4000:
    print("It is Transitional flow")
elif Re > 4000:
    print("It is Turbulent flow")

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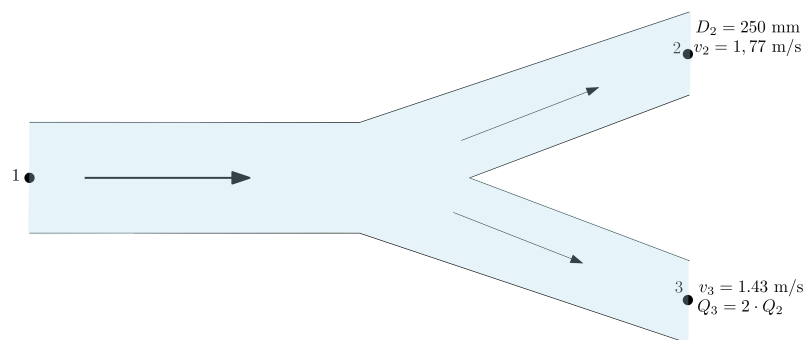
The Reynold's number is 316726.255

It is Turbulent flow

Problem 2 - Conservation of mass

Water flows through a branching pipeline as shown in the diagram. If the diameter, D_2 , is 250 mm, $v_2 = 1.77$ m/s and $V_3 = 1.43$ m/s:

1. what diameter, D_3 , is required for $Q_3 = 2Q_2$?
2. what is the total discharge at section 1?



Solution of Problem 2

The relevant equations are (check L4 slides 30)

The main equation is:

$$v_1 A_1 = v_2 A_2$$

with $A = \frac{\pi}{4} D^2$

$$\frac{\pi}{4} D_1^2 \cdot v_1 = \frac{\pi}{4} D_2^2 \cdot v_2$$

and

$$D_1 = \sqrt{\frac{v_2}{v_1} \cdot D_2^2}$$

Information provided in the problem are:

$$Q_3 = 2 \times Q_2$$

$$Q_1 = Q_2 + Q_3$$

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[3] # Given are:

v2_2 = 1.77 # m/s velocity section 2
v2_3 = 1.43 # m/s velocity in section 3
D2_2 = 250 # mm diameter of section 2

# interim calculation
D2_2m = 250/1000 # m, changing unit of D2_2 to m
Q2_2 = np.pi/4*D2_2m**2 *v2_2 # m^3/s, discharge Q2

# Solution problem
D2_3 = np.sqrt(2* (v2_2/v2_3) * D2_2m**2) # m, diameter section
3. Multiplied by 2 as Q3 = 2* Q2
Q2_3 = np.pi/4*D2_3**2 *v2_3 # m^3/s, discharge at Q2_3
Q2_1 = Q2_2 + Q2_3

# output
print("The pressure difference is {0:1.2f}".format(D2_3),"m",
"\n")
print("The Discharge at section 1  is
{0:1.2f}".format(Q2_1),"m\u00b2")
```

The pressure difference is 0.39 m

The Discharge at section 1 is 0.26 m²

Problem 3 - Conservation of momentum

A pipeline with a constant diameter of 0.3 m turns through an angle of 60°. The centreline of the pipe does not change elevation. The discharge through the pipeline is 0.1 m³/s of water, and the pressure at the bend is 30 m of water. Calculate the magnitude and direction of the resultant force on the pipe.

Solution of Problem 3

The relevant information and equations can be found in slides L3: 35-36

Equations to use are:

$$A_1 v_1 = A_2 v_s$$

$$P_1 \cdot A_1 - P_2 \cdot \cos \theta \cdot A_2 - F_{rx} =$$

$$= \rho \cdot Q \cdot (v_2 \cdot \cos \theta - v_1)$$

$$F_{r,y} - P_2 \cdot \cos \theta \cdot A_2 = \rho \cdot Q \cdot (v_2 \cdot \sin \theta)$$

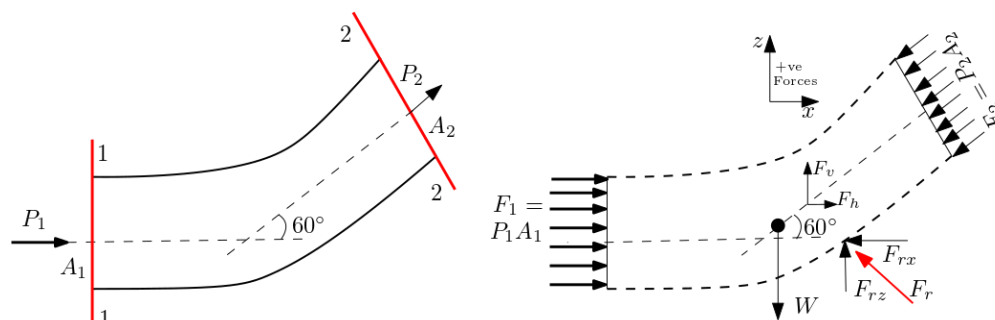
$$F_r = \sqrt{F_{r,x}^2 + F_{r,y}^2}$$

and

$$\theta = \arctan \left(\frac{F_{r,y}}{F_{r,x}} \right)$$

Solution of Problem 3

Draw the free body diagram of the problem



The given information are:

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[4] # Given are:

ae = 60 # degree, pipe bending
D3 = 0.3 # m, pipe diameter
Q3 = 0.1 # m^3/s, discharge through pipe
Ph = 30 # m , water height at the bend
rho3 = 1000 # kg/m^3, density of water
g3 = 9.81 # m/s^2, gravity

# interim calculation - #Calculation of A, v and P
ae_r = ae*np.pi/180
A3_1 = A3_2 = np.pi/4*D3**2 # pipe area at section 1 and 2.
Constant dia.
v3_1 = v3_2 = Q3/A3_1 #m^3/s, Q= AV and since A is equal and Q is
conserved
P3_1 = P3_2 = rho3*g3*Ph # N/m^2, since A and V are equal, P1 =
P2 = rho*g * h

# interim calculation - #Forces calculation in X-direction
F_1x = P3_1*A3_1 # N, Force at 1 in x, F= P.A
F_2x = P3_2*A3_2* np.cos(ae_r) # N, force at 2 in x
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F_rx = P3_1*A3_1 - P3_2*A3_2*np.cos(ae_r)-rho3*Q3*
(v3_2*np.cos(ae_r)-v3_1)
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[5] # interim calculation - #Forces calculation in y-direction
F_1y = 0
F_2y = P3_2*A3_2*np.sin(ae_r)
F_ry = rho3*Q3*(v3_2*np.sin(ae_r))+ P3_2*A3_2*np.sin(ae_r)

# calculation of part 1 and 2
F_r = np.sqrt(F_rx**2+ F_ry**2) # N, resultant force
Dir_r = np.arctan(F_ry/F_rx) # rad, direction of resultant in
radian
Dir_t = Dir_r*180/np.pi # Degree, direction of resultant in
degress

# Output
print("The resultant force is:{0:1.2f}".format(F_r), "N \n")
print("The resultant is acting at:
{0:1.2f}".format(Dir_t),"\u00b0 ")
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The resultant force is:20944.31 N

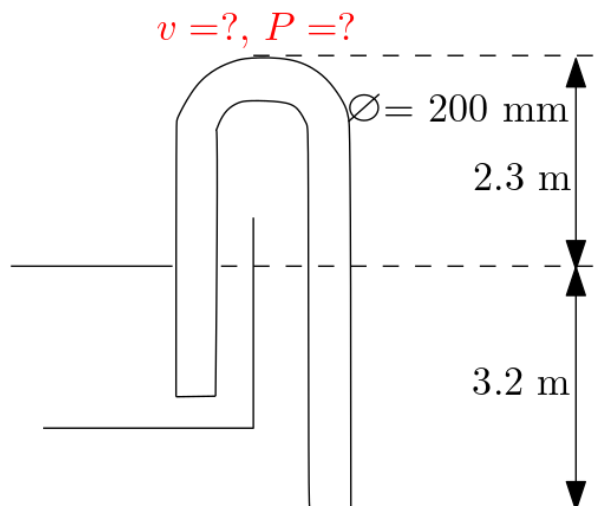
The resultant is acting at: 60.00 °

Problem 4- Conservation of Energy

From a large reservoir water is released via a syphon. The pipe diameter is 200 mm. The end of the syphon pipe is 3.2 m below the water level of the reservoir. The highest part of the syphon is 2.3 m above the water level of the reservoir. Water level remains constant, no losses. Calculate:

Calculate the discharge.

Calculate the pressure head at the highest point of the syphon.



Solution of Problem 4

The lecture contents on the problem can be found in slides L4- 45-48.

Basic equations are:

Bernoulli Equation:

$$z_1 + \frac{1}{2} \frac{v_1^2}{g} + \frac{P_1}{\rho \cdot g} = z_2 + \frac{1}{2} \frac{v_2^2}{g} + \frac{P_2}{\rho \cdot g}$$

and

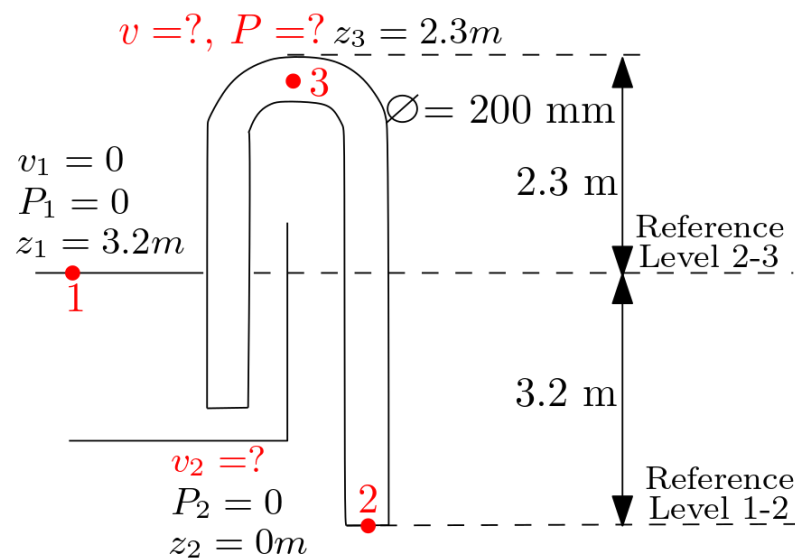
Continuity Equation:

$$A_1 v_1 = A_2 v_2$$

Steps to solve the problem:

Step 1: Drawing including the known and unknown variables v , P and z

Step 2: Decide which points and reference level to use for analysis and which calculation steps to take

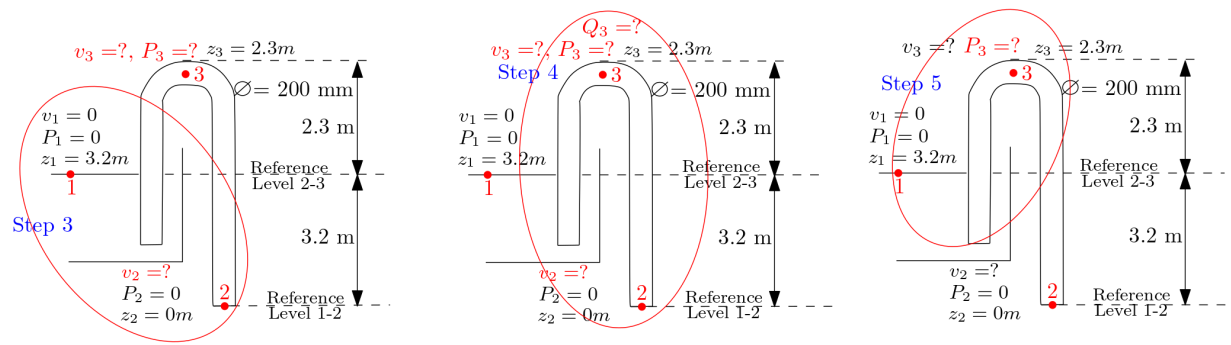


Solution of Problem 4

Step 3: Bernoulli equation for points 1-2 to calculate v_2 using reference level 1-2.

Step 4: Use Continuity equation for points 2-3 to calculate Q_2 .

Step 5: Bernoulli equation for points 1-3 to calculate P_3 using reference level 2-3.



```
[10] #Solution of Problem 4. Given are:
#Step 3 Bernoulli eq. 1-2 obtain v_2

z4_1 = 3.2 #m, elevation point 1 from ref 1-2
z4_2 = 0 # m, elevation point 2 from ref 1-2
v4_1 = 0 # m/s, velocity point 1
P4_1 = 0 # N/m^2, pressure point 1 (atmospheric)
P4_2 = 0 # N/m^2, pressure point 2 (atmospheric)
g4 = 9.81 # m/s^2, Gravity

#Using Bernoulli's equation
v4_2 = np.sqrt(2*g4*z4_1) #m/s- 3.2 + 0 + 0 = 0 + v^2/2g + 0

#output
print("Velocity at point 2 is: {0:1.3f}".format( v4_2), 'm/s')
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Velocity at point 2 is: 7.924 m/s

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[15] #Solution of Problem 4.
#Step 4- Continuity eq. 2-3, obtain Q_3

d4 = 0.2 # m, diameter of the pipe
v4_3 = v4_2 # m/s, Q and A are same so v is also same

#interim calculation
A4_2 = A4_3 = np.pi/4*d4**2 # m^2, area of cross-section. it is
same everywher

#calculation
Q4 = A4_3*v4_3 # m^3/s- discharge is conserved

#output
print("Discharge in the system is: {0:1.3f}".format( Q4),
      'm\u00b3/s')
```

Discharge in the system is: 0.249 m³/s

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[25] #Solution of Problem 4.
#Step 5- Bernoulli's eq. 1-3, obtain P3
#Avaialable are

z4_1u = 0 # m, elevation from ref line 2-3 to point 1
z4_3u = 2.3 # m, elevation from ref line 2-3 to point 3
v4_1 = 0 # m/s, velocity at point 1
P4_1 = 0 # N/m^2, atmospheric pressure
rho4 = 1000 # kg/m^3, fluid density
v4_3u = v4_3 # m/s velocity at point 3 from previous step

#calculation
#  $N/m^2 - 0+0+0 = z3 + v3^2/2.g + P3/rho*g$ 
P4_3 = (-z4_3u - v4_3u**2/(2*g4))*rho4*g4

#output
print("Pressure at point 3 in the system is:
{0:1.3f}".format(P4_3), 'N/m\u00b2')
```

Pressure at point 3 in the system is: -53955.000 N/m²

Assignment problem to be updated.