In [11]:
```
import numpy as np
```

Seminar 2

1. **Fundamental of Hydrostatics**
2. **Forces on submerged surfaces**

Problem 1

Rectangular container of a width of 1 m and a length of 2 m is filled to a depth of 0.6 m with oil. Calculate the pressure at the bottom of the tank. What is the weight of the oil?

Solution of Problem 1

The liquid is OIL in the problem. The main equations are:

$$P = \rho \cdot g \cdot h \tag{1}$$

$$P_{abs} = P_{atm} + P_{gage} \tag{2}$$

$$\gamma = \rho \cdot g = \frac{M}{V} \cdot g = \frac{W}{V \cdot g} \cdot g = \frac{W}{V} \tag{3}$$

$$W = \gamma V \tag{4}$$

with γ = specific weight (N/m^3), and M is mass (Kg), W is weight (N) and V volume (m^3 .

other available information are:

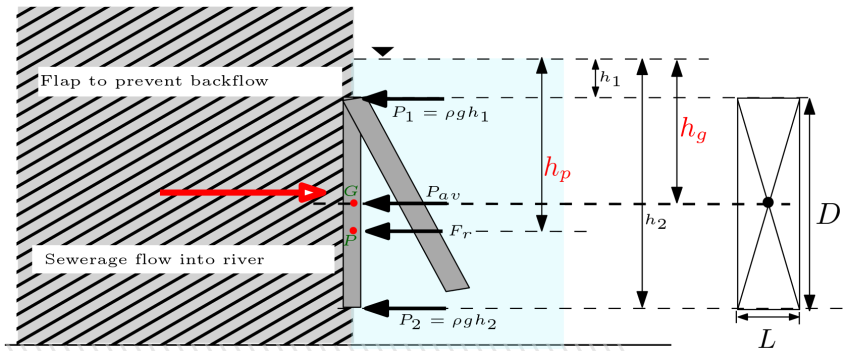
In [12]:

```
#Given,
dy_o = 800 # kg/m^3, density of oil
ga1 = 7850 # N/m^3, specific weight of oil
W1 = 2 # m, width of the tank
L1 = 2 # m, length of the tank
H1 = 0.6 # m, oil depth in the tank
g = 9.81 # m/s^2, earth's gravity
P1_atm = 101000 # N/m^2, Standard atmospheric pressure

# interim calculation
V1 = L1*W1*H1 # m^3, volume of the tank

#Calculations
P1_oi1 = dy_o*H1*g # N/m^2
P1_abs = P1_atm + P1_oi1
W1_oi1 = ga1* V1

# output
print("The pressure of the tank is {0:1.2f}".format(P1_abs),"N/m\u00b2", "\n")
print("The Weight of the oil in the tank is {0:1.2f}".format(W1_oi1),"N")
```

```
The pressure of the tank is 105708.80 N/m²

The Weight of the oil in the tank is 18840.00 N
```

Problem 2 - Vertically submerged tank

Given the following rectangular gate with $h_1 = 1$ m, $L = 2$ m and $D = 3$ m :



Determine for water:

1. Pressure at the bottom of the gate
2. resultant hydrostatic forces
3. depth at which the resultant force acts

Solution of Problem 2

The relevant equations are (check slides 18 - 20 of L2):

$$F_r = \rho \cdot g \cdot h_g \cdot A \tag{5}$$

$$h_g = h_1 + \frac{D}{2} \tag{6}$$

and

$$h_p = \left(\frac{I_g}{A \cdot h_g} \right) + h_g \tag{7}$$

with $I_g = \frac{L \cdot D^3}{12}$.

Information provided in the problem are:

In [13]:

```
# Given are

dy2 = 1000 # kg/m^3, water density
g2 = 9.81 # m/s^2
h2_1 = 1 # m, height from surface to gate top
D2 = 3 # m, depth of the tank
L2 = 2 # m, Length of tank

# interim calculation
A2 = L2*D2 # m^2, Area of tank
h2_g = h2_1 + D2/2 # m, height from top to centroid
I2_g = L2*D2**3/12 # m^4, second moment of area


# calculation
P2_bot = dy2 * g2* (h2_1 + D2) # N/m^2, P = rho.g.h, h = (h1+D) see fig.
F2_r = dy2 *g2*h2_g*A2 # N, Resultant force
h2_p = I2_g/(A2*h2_g)+ h2_g


# output
print("The pressure of tank bottom is {0:1.2f}".format(P2_bot),"N/m\u00b2", "\n")
print("The resultant force in the tank is {0:1.2f}".format(F2_r),"N",  "\n")
print("The location of resultant force is {0:1.2f}".format(h2_p), "m")
```

```
The pressure of tank bottom is 39240.00 N/m²

The resultant force in the tank is 147150.00 N

The location of resultant force is 2.80 m
```

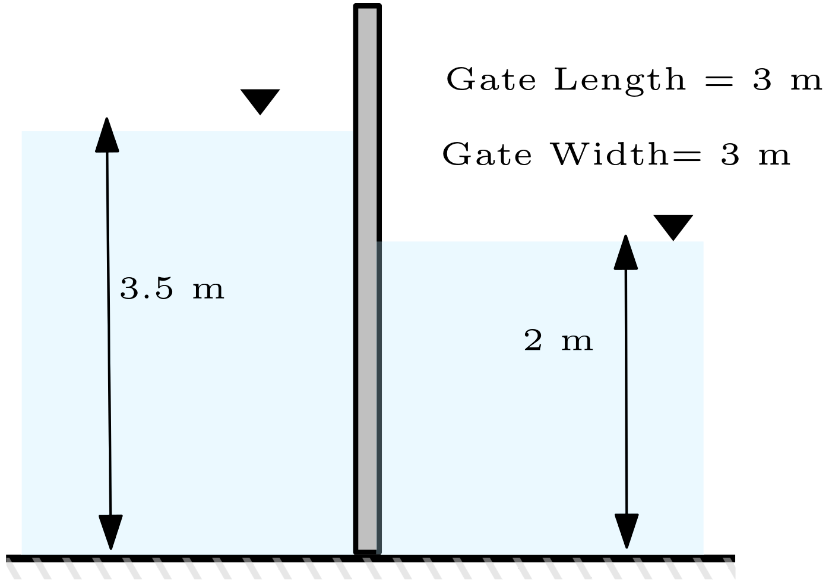
Problem 3 - Vertically submerged tank

A lock on a canal is sealed by a gate that is 3.0 m wide (see fig below). The gate is perpendicular to the sides of the lock. When the lock is used there is water on one side of the gate to a depth of 3.5 m, and 2.0 m on the other side.

(a) What is the hydrostatic force of the two sides of the gate?

(b) At what height from the bed do the two forces act?

(c) What is the magnitude of the overall resultant hydrostatic force on the gate and at what height does it act?

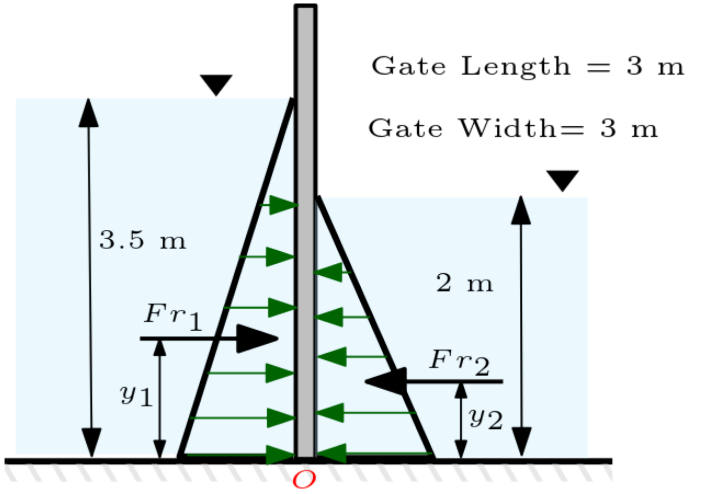


Solution of problem 3

The relevant equations are (check slides 18 - 20 of L2).

$$F_r = \rho \cdot g \cdot h_g \cdot A \tag{8}$$

Then, we draw the pressure diagram



Since both pressure diagram are triangular, F_r acts at 1/3 from the bases.

Other information are:

In [14]:

```
# Given
L3 = 3 # m, Gate length
W3 = 3 # m, Gate width
h_up = 3.5 # m, upstream water height
h_dn = 2 # m, downstream water height
dy3 = 1000 # kg/m^3, density of water
g3 = 9.81 # m/s^2, gravity

# interim calculation
hg_up = h_up/2 # m, centroid of surface upstream
hg_dn = h_dn/2 # m, centroid of surface downstream
A_up = L3*h_up # m^2, area upstream
A_dn = L3*h_dn # m^2 area downstream

# calculation (a) and (b)
F_up = dy3*g*hg_up*A_up  # N, resultant force up stream
F_dn = dy3*g*hg_dn * A_dn # N, resultant force up stream
y_up = 1/3*h_up # m, Upstream location of centre of pressure from bottom.
y_dn = 1/3*h_dn # m, downstream Location of centre of pressure from base.

# output
print("The resultant force upstream  is: {0:1.2E}".format(F_up),"N", "\n")
print("The resultant force downstream  is: {0:1.2E}".format(F_dn),"N", "\n")
print("The location of resultant force upstream from the bottom  is {0:1.2f}".format(y_up),"m",  "\n")
print("The location of resultant force downstream from the bottom  is {0:1.2f}".format(y_dn),"m", )
```

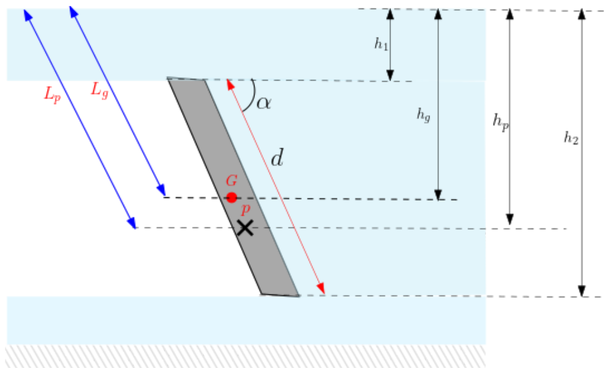
```
The resultant force upstream  is: 1.80E+05 N

The resultant force downstream  is: 5.89E+04 N

The location of resultant force upstream from the bottom  is 1.17 m

The location of resultant force downstream from the bottom  is 0.67 m
```

In [15]:
```
# Solution 3C

Fr_o = F_up - F_dn # N, +ve means the resultant force is upstream

# Moment F * y about O, at the base (see fig)

M_up = F_up * y_up # N-m, moment in upstream
M_dn = F_dn * y_dn # N-m, moment in downstream

# Location of Resultant force (Moment balance equation)
y_r = (M_up - M_dn)/Fr_o # m, moment in the system is conserved

#output
print("The overall resultant force  is: {0:1.2E}".format(Fr_o),"N", "\n")
print("The location of overall resultant force  is: {0:1.2f}".format(y_r),"m")
```

```
The overall resultant force  is: 1.21E+05 N

The location of overall resultant force  is: 1.41 m
```

Problem 4 - inclined submerged surface

The inclined circular gate is shown in figure below:



For $h_1 = 1$ m, $d = 0.6$ m, and $\alpha = 45^{\circ}$, determine:

a) Resultant force on the gate

b) Vertical depth to the centre of pressure h_p

Solution of Problem 4

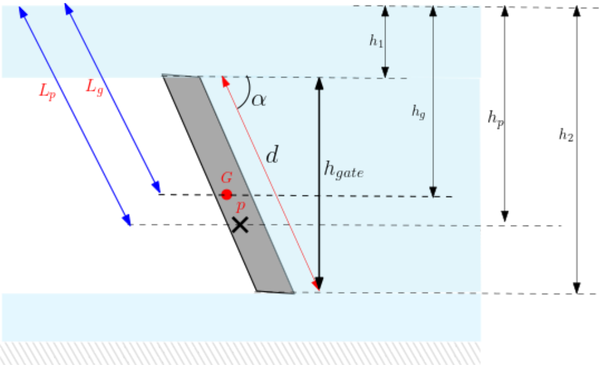
Reference lecture slides 25-27. Important equations are:

$$h_g = h_1 + \frac{h_{gate}}{2} \tag{9}$$

$$F_r = \rho \cdot g \cdot h_g \cdot A \tag{10}$$

$$L_p = \frac{I_g}{A \cdot L_g} \tag{11}$$

with $I_g = \pi R^4/4$ and $L_g = h_g/\sin\alpha$



In [16]:

```
# Given
h4_1 = 1 # m, free surface height
d4 = 0.6 # m, diameter circular gate
apa = 45 # degrees, inclined angle
dy4 = 1000 # kg/m^3, density water
g4 = 9.81 # m/s^2, gravity

# interim calculation
h_gate = d4*np.sin(apa*np.pi/180) # m, check np.sin and np. pi and why * pi/180
A4 = np.pi/4*(d4)**2 # m^2 area of gate
h4_g = h4_1 + h_gate/2 # m, location of surface centroid

# calculation (a)
F4_r = dy4*g*h4_g*A4

#output
print("The resultant force  is: {0:1.2f}".format(F4_r),"N")
```

```
The resultant force  is: 3362.11 N
```

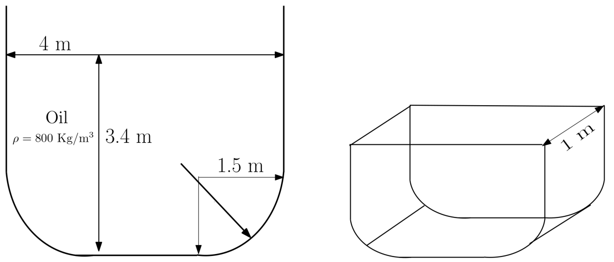
In [17]:

```
# solution 4(b) location h_p

# interim calculation
L4_g = h4_g/np.sin(45*np.pi/180) # m, inclined length from centroid
R4 = d4/2 # m , radius of the gate
I4_g = np.pi*R4**4/4 # m^4 second moment of area
L4_p = I4_g/(A4*L4_g) + L4_g # incline length from centre of force

#calculation
h4_p = L4_p*np.sin(45*np.pi/180) # vertical height to centre of force

#Output
print("The vertical depth to the centre of force  is: {0:1.2f}".format(h4_p),"m")
```

```
The vertical depth to the centre of force  is: 1.22 m
```

Problem 5 - curved submerged surface

The inclined circular gate is shown in figure below:



determine:

a) Resultant force on the curved surface.

b) Angle of the resultant force to the horizontal.

Solution of Problem 5

Reference lecture slides 30-31. Important equations are:

$$h_g = (D - R) + \frac{R}{2} \tag{12}$$

$$F_v = \rho \cdot g \cdot h_g \cdot A \tag{13}$$

$$F_h = \rho \cdot g \cdot V \tag{14}$$

$$F_r = \sqrt{F_h^2 + F_v^2} \tag{15}$$

and

$$\tan\phi = \frac{F_v}{F_h} \tag{16}$$

In [18]:

```
# Given
dy5_o = 800 # kg/m^3, density of oil
W5 = 4 # m, tank width
D5 = 3.4 # m, Depth of wetted surface
L5 = 1 # m, length of surface see fig left in question
R5_p = 1.5 # m, Curved section radius
g5 = 9.81 # m^2/s, gravity

# interim calculation
A5_p = R5_p*L5 # m^2, projected curved area
h5_g = (D5 - R5_p) + R5_p/2
V5 =  np.pi/4*R5_p**2*L5 + R5_p*L5*(D5-R5_p) # m^3, circular volume + rectangular volume
# Circular vol = pi/4 * R^2 * L and Rect. vol = R*L*(D-R)

# Calculations
F5_h = dy5_o*g5*h5_g*A5_p # N, Force horizontal
F5_v = dy5_o*g5*V5 # N, Force vertical
F5_r = np.sqrt(F5_h**2+F5_v**2) # N, Resultant force
phi_r = np.tanh(F5_v/F5_h) # rad, angle with horizontal surcface

#output
print("The horizontal force  is: {0:1.2f}".format(F5_h),"N", "\n")
print("The vertical force  is: {0:1.2f}".format(F5_v),"N", "\n")
print("The resultant force  is: {0:1.2f}".format(F5_r),"N", "\n")
print("The angle of resultant force to the horizontal : {0:1.2f}".format(phi_r),"rad", "\n")
print("The angle of resultant force to the horizontal : {0:1.2f}".format(phi_r*180/np.pi),"deg", "\n")
```

```
The horizontal force  is: 31195.80 N

The vertical force  is: 36235.36 N

The resultant force  is: 47814.01 N

The angle of resultant force to the horizontal : 0.82 rad

The angle of resultant force to the horizontal : 47.07 deg
```