SMS-732, Modeling and Simulation
Assignment - 1 (Modeling Deterministic Systems)

Aircraft Performance Model

To illustrate the development of a deterministic, mathematical model, consider the level flight (at constant altitude of an aircraft at subsonic speeds – speed below the speed of sound at constant altitude, i.e., below MACH 1). Our aim is to find an equation for the velocity of the aircraft as a function of time for

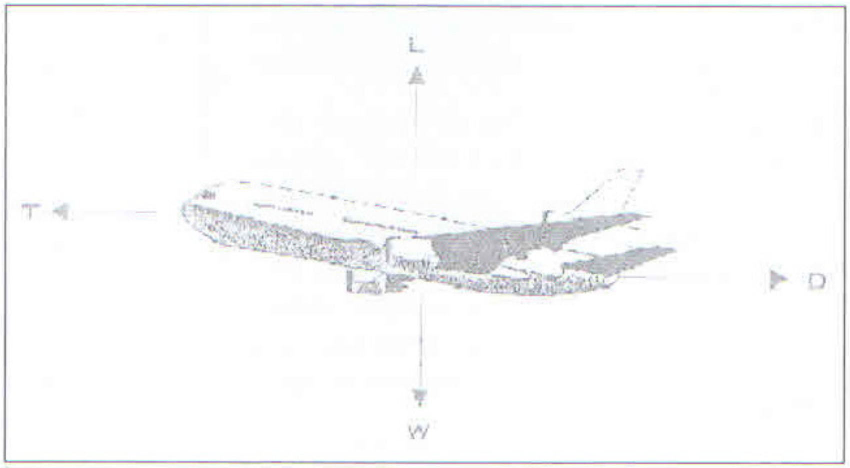

Figure 3. Forces on a cruise flight.

a given thrust level and altitude. We can first analyse the forces acting on the aircraft (*Figure* 3). The lift force L is balanced by the weight of the aircraft W.

$$L = W.$$

The thrust developed by the engine T is equal to drag force D.

$$T = D.$$

[We neglect the small angle between the thrust vector due to engine position and the direction of the drag force. We also neglect the moment caused by the separation of aerodynamic centre, the point at which the lift force acts and the centre of gravity through which the weight acts.]

Now the expressions for lift and drag forces are as follows:

$$L = \left(\frac{1}{2}\rho V^2\right) C_L * S$$

$$D = \left(\frac{1}{2}\rho V^2\right) C_D * S\,,$$

where ρ is the air density at that altitude, V is the velocity and $Q=1/2\rho V^2$, the dynamic pressure, C_L and C_D are lift coefficient

Both lift and drag increase with the square of the velocity.

and drag coefficient respectively and S is the lift generating area, essentially the wing area.

Note that both lift and drag increase with the square of the velocity. Since for level flight $D = T$, we replace D by the 'thrust available', i.e., T. (This again is not a constant but depends on the rpm of the engine and the altitude). We equate L with weight since weight is the most important factor for an aircraft. Note that the weight keeps decreasing while the aircraft is in flight due to fuel consumption. Let the rate of decrease in weight be constant.

$$\frac{dW}{dt} = -k$$

[k depends on the thrust level of the engine]

We employ an empirical equation called 'drag polar' relating C_L with C_D.

$$C_D = C_{D_0} + KC_L^2$$

Where C_{D_0} and K are constants. We shall use later the following 'drag polar' equation:

$$C_D = 0.025 + 0.035\,C_L^2.$$

With this collection of equations, we can write the model equations:

$$W = L = \left(\frac{1}{2}\rho V^2\right) C_L S$$

or

$$C_L = \frac{\frac{W}{S}}{\left(\frac{1}{2}\rho V^2\right)} \qquad (1)$$

$$T = \left(\frac{1}{2}\rho V^2\right) S\, C_D$$

$$= \left(\frac{1}{2}\rho V^2\right) S\left(C_{D_0} + KC_L^2\right) \quad . \qquad (2)$$

Substituting for C_L in (2) from (1), and solving for V, we can get the velocity V for a given weight. Therefore we can find velocity as a function of time, as weight varies with time. Note that at a constant thrust, the aircraft will accelerate in level flight due to decrease in weight.

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The air density ratio (where ρ/ρ_0 is the air density at altitude (meters) and ρ_0 is air density at sea level) is given by the approximate relation:

$$\sigma = \frac{\rho}{\rho_0} = \exp\left(-\frac{h}{\beta}\right).$$

where $\beta = 9296$ m (for altitudes less than 11 km). For example. for an altitude of 8000 m.

$$\sigma = e^{-\frac{8000}{9296}} = 0.4229 \cdot$$

Let us consider a numerical example: to calculate the velocity of a trainer aircraft with initial (all-up) weight of 10,000 kg and wing area 6m^2 at altitude of 8000 m. The engine thrust is taken as 1450 N.

The air density at 8 km $= 0.429 \times 1.225$ kg/m^2
$= 0.5255$ kg/m^2.

Substituting the numbers and solving for velocity V, we get $V =$ 80 m/s or 288 Km/hour.

We can write a computer program to calculate the velocity as a function of time as the fuel is consumed. By integrating the velocity with time we can obtain the range of distance travelled for a given fuel weight. We can also find the optimal height for specified conditions. Such models are routinely used by the airline industry.

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Prepare a simulation model on your computer in Lab; choose your preferred language such as MATLAB, C++ or any other, in which you feel comfortable.

(Ref: Resonance, March 2001)