(1) Chapter

Gradient

The gradient of scalar point function $\phi(x,y,z)$ denoted by $\nabla\phi$ (or) grad ϕ and defined by $\nabla\phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\phi$

$$\nabla\phi = \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial z}.$$

Note:-

→ The directional derivative of ϕ in the direction of $\bar{a}$ is

$$\nabla\phi \cdot \frac{\bar{a}}{|\bar{a}|}$$

→ The maximum directional derivative is $|\nabla\phi|$

(P) Formula:-

(1) The normal vector to the surface ϕ is $\nabla\phi$

(2) If $\bar{n}_1 = \nabla\phi$ is normal at $P(x_1, y_1, z_1)$

$\bar{n}_2 = \nabla\phi$ is normal at $P(x_2, y_2, z_2)$

then the ~~ideal~~ angle b/w two normals is given by

$$\cos\theta = \frac{\bar{n}_1 \cdot \bar{n}_2}{|\bar{n}_1||\bar{n}_2|}$$

(3) The unit normal vector is $\frac{\nabla\phi}{|\nabla\phi|}$

(1) Find the unit normal vector at the point $(1, -1, 2)$ to the surface $x^2y + y^2z + z^2x = 5$

Sol:- The unit normal vector is $\frac{\nabla\phi}{|\nabla\phi|}$

Let $\phi = x^2y + y^2z + z^2x - 5 = 0$

Now $\nabla\phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)$

$(x^2y + y^2z + z^2x - 5)$

$$= \hat{i}\frac{\partial}{\partial x}\{x^2y+y^2z+z^2x-5\}+\hat{j}\frac{\partial}{\partial y}\{x^2y+y^2z+z^2x-5\}$$

$$+\hat{k}\frac{\partial}{\partial z}\{x^2y+y^2z+z^2x-5\}$$

$$= \hat{i}(2xy+z^2)+\hat{j}(x^2+2yz)+$$

$$\hat{k}(y^2+2zx)$$

$$\boxed{\nabla\phi = (2xy+z^2)\hat{i}+(x^2+2yz)\hat{j}+(y^2+2zx)\hat{k}}$$

$$[\nabla\phi]_{(1,-1,2)} = (-2+4)\hat{i}+(1-4)\hat{j}+(1+4)\hat{k}.$$

$2(-1)(2)$

$2(-1)(2)$

$$= 2\hat{i}-3\hat{j}+5\hat{k}$$

-4

$$\boxed{(\nabla\phi)_{(1,-1,2)} = 2\hat{i}-3\hat{j}+5\hat{k}}$$

Now $|\nabla\phi| = \sqrt{4+9+25}$

$$= \sqrt{38}$$

unit normal vector $= \dfrac{2\hat{i}-3\hat{j}+5\hat{k}}{\sqrt{38}}$

Find the unit normal vector at $(2,-2,3)$ to the surface $x^2y+2xz=4$

Sol:- $\phi = x^2y+2xz-4=0$

$\nabla\phi = \hat{i}\frac{\partial}{\partial x}(x^2y+2xz-4) + \hat{k}\frac{\partial}{\partial y}(x^2y+2xz-4)\ \hat{k}\frac{\partial}{\partial z}(x^2y+2xz-4)$

$= \hat{i}(2xy+2z) + \hat{j}(x^2) + \hat{k}(2x)$

$= (2xy+2z)\hat{i} + x^2\hat{j} + 2x\hat{k}$

Now

$[\nabla\phi]_{(2,-2,3)} = (-8+6)\hat{i} + 4\hat{j} + 4\hat{k}$

$= 2\hat{i}+4\hat{j}+4\hat{k}$

$2(-2)(2)$. 6.

4 4 $|\nabla\phi| = \sqrt{4+16+16}$ 32 4 36

$= \sqrt{36} = 6$

unit normal vector $= \dfrac{2\hat{i}+4\hat{j}+4\hat{k}}{6}$

(3) Find ~~unit normal vector~~ directional derivative of $\phi = x^2yz + 4xz^2$ at pt $(1,-2,-1)$ in the direction $2\hat{i} - \hat{j} - 2\hat{k}$.

Sol:- Let $\phi = x^2yz + 4xz^2$

$$\nabla\phi = \hat{i}\frac{\partial}{\partial x}(x^2yz + 4xz^2) + \hat{j}\frac{\partial}{\partial y}(x^2yz + 4xz^2) + \hat{k}\frac{\partial}{\partial z}(x^2yz + 4xz^2)$$

$$\boxed{\nabla\phi = (2xyz + 4z^2)\hat{i} + (x^2z)\hat{j} + (x^2y + 8xz)\hat{k}}$$

$(\nabla\phi)_{(1,-2,-1)} = (8\hat{i}) + \cdot$

$$\boxed{(\nabla\phi)_{(1,-2,-1)} = 8\hat{i} - \hat{j} - 10\hat{k}}$$

$\bar{a} = 2\hat{i} - \hat{j} - 2\hat{k}$

$|\bar{a}| = \sqrt{4+1+4} = \sqrt{9} = 3$

Now directional derivative of ϕ in the direction of $\bar{a}$ is at $(1,-2,-1)$

$$\nabla\phi \cdot \frac{\bar{a}}{|\bar{a}|}$$

$$= \frac{(8\hat{i} - \hat{j} - 10\hat{k}) \cdot (2\hat{i} - \hat{j} - 2\hat{k})}{3}$$

$$= \frac{16 + 1 + 20}{3} = \frac{37}{3}$$

17
31
53
16

(?) Find the maximum directional derivative of surface $\phi = x^2 y z^3$ at the point $(1,-1,2)$

Sol:- Let $\phi = x^2 y z^3$

Now $\nabla\phi = \hat{i}\frac{\partial}{\partial x}(x^2 y z^3) + \hat{j}\frac{\partial}{\partial y}(x^2 y z^3) + \hat{k}\frac{\partial}{\partial z}(x^2 y z^3)$

$= (2xyz^3)\hat{i} + (x^2 z^3)\hat{j} + (3x^2 y z^2)\hat{k}$

$(\nabla\phi)_{(1,-1,2)} = -16\hat{i} + 8\hat{j} - 12\hat{k}$

MOD is

$|\nabla\phi| = \sqrt{256+64+144}$

$|\nabla\phi| = \sqrt{464}$

H.W

(5) Find the directional derivative of the surface $\phi = x^2+y^2+z^2$ at the point $(1,-1,1)$ in the direction of $\hat{i}-3\hat{j}+2\hat{k}$.

Sol:- $\nabla\phi = 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$

$1+9+4$

$(\nabla\phi)_{(1,-1,1)} = 2\hat{i} - 2\hat{j} + 2\hat{k}$

$\nabla\phi \cdot \bar{a} = (2\hat{i} - 2\hat{j} + 2\hat{k}) \frac{\hat{i}-3\hat{j}+2\hat{k}}{\sqrt{14}}$

$$= \frac{2+6+4}{\sqrt{14}} = \frac{12}{\sqrt{14}}$$

⑥ Find the directional derivative of xyz^2 at the point $(1, 0, 3)$ in the direction of $\hat{i} - \hat{j} + 2\hat{k}$.

Sol:- Let $\phi = xyz^2$

$$\nabla\phi = yz^2\hat{i} + xz^2\hat{j} + 2xyz$$

$$= 9\hat{j} \qquad 1+1+4$$

$$\frac{9\hat{j}\,(\hat{i} - \hat{j} + 2\hat{k})}{\sqrt{6}}$$

$$= \frac{-9}{\sqrt{6}}$$

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Angle problems:-

1) Find the angle b/w the normals to the surface $xy = z^2$ at $(1,4,2)$ and $(-3,-3,3)$

Sol:- Now let $\phi = xy - z^2 = 0$.

Now $\nabla\phi = \frac{\partial}{\partial x}(xy - z^2)\hat{i} + \frac{\partial}{\partial y}(xy - z^2)\hat{j} + \frac{\partial}{\partial z}(xy - z^2)\hat{k}$

$\nabla\phi = y\hat{i} + x\hat{j} - 2z\hat{k}$.

Now the normal at point $(1,4,2)$

$(\nabla\phi)_{(1,4,2)} = 4\hat{i} + \hat{j} - 4\hat{k}$.

the normal at point $(-3,-3,3)$

$(\nabla\phi)_{(-3,-3,3)} = -3\hat{i} - 3\hat{j} - 6\hat{k}$.

Now the angle b/w normals is

$$\cos\theta = \frac{\nabla\phi_1 \cdot \nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}$$

$$= \frac{(9\hat{i} + \hat{j} - 4\hat{k})(-3\hat{i} - 3\hat{j} - 6\hat{k})}{(\sqrt{16+1+16})(\sqrt{9+9+36})}$$

32

36
118

$$= \frac{-12 - 3 + 24}{\sqrt{33}\sqrt{54}} = \frac{9}{\sqrt{33}\sqrt{54}}$$

(2) Find the angle of the surface $\phi = 3x^2y - y^3z^2$ at the point $(1,-2,-1)$, $(1,2,0)$

Sol:

Let $\phi = 3x^2y - y^3z^2$

$\nabla\phi = \cancel{6xy\hat{i} - 3y^2z^2}$

$= 6xy\hat{i} + (3x^2 - 3y^2z^2)\hat{j} - 2y^3z\hat{k}$.

$\nabla\phi_{(1,-2,-1)} = -12\hat{i} - 9\hat{j} - 16\hat{k}$.

$\nabla\phi_{(1,2,0)} = 12\hat{i} + 3\hat{j}$.

$$\cos\theta = \frac{(\nabla\phi_1)\cdot\nabla\phi_2}{|\nabla\phi_1||\nabla\phi_2|}$$

$$= \frac{(-12\hat{i} - 9\hat{j} - 16\hat{k})(12\hat{i} + 3\hat{j})}{(\sqrt{144+81+256})(\sqrt{144+9})}$$

481

$$= \frac{-144-27}{\sqrt{481}\sqrt{153}} = \frac{-171}{\sqrt{481}\sqrt{153}}$$

$$= \frac{57}{\sqrt{481}\sqrt{17}}$$

Imp

1) Find the constants a, b such that the surface $3x^2-2y^2-3z^2+8=0$ is orthogonal to $ax^2+y^2=bz$ at the point $(-1,2,1)$

Sol: Let $\phi_1 = 3x^2-2y^2-3z^2+8=0$

$\phi_2 = ax^2+y^2-bz=0$

Now $\nabla\phi_1 = \hat{i}\frac{\partial}{\partial x}(3x^2-2y^2-3z^2+8)$

$+\hat{j}\frac{\partial}{\partial y}(3x^2-2y^2-3z^2+8)+\hat{k}\frac{\partial}{\partial z}(3x^2-2y^2-3z^2+8)$

$\nabla\phi_1 = 6x\hat{i}-4y\hat{j}-6z\hat{k}$

$\nabla\phi_2 = 2ax\hat{i}+2y\hat{j}-b\hat{k}$

Now $\nabla\phi_1$ is orthogonal to $\nabla\phi_2$ ~~then~~

$(-1,2,1)$ the $(\nabla\phi_1)(\nabla\phi_2)$

$(\nabla\phi_1)_{(-1,2,1)} = -6\hat{i}-8\hat{j}-6\hat{k}$

$(\nabla\phi_2)_{(-1,2,1)} = -2a\hat{i}+4\hat{j}-b\hat{k}$

Now $(\nabla\phi_1)(\nabla\phi_2) = 12a-32+6b$

$\Rightarrow 12a+6b-32=0$

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$(-1,2,1)$ lies on $\phi_2 = ax^2+y^2-bz=0$

$$a+4-b=0$$

$$\boxed{a-b=-4} \quad (2)$$

$(2)\times 6 = \boxed{6a-6b=-24} \quad (3)$

solve (1) & (3)

$$6a+3b=16$$
$$6a-6b=-24$$
$$- \quad + \quad +$$

$$9b=40$$

$$\boxed{b=\frac{40}{9}}$$

b in (2)

$\cancel{6a-6\left(\frac{40}{}\right.}$

$$a-\frac{40}{9}=-4$$

$$a=-4+\frac{40}{9}$$

$$=\frac{-36+40}{9}=\frac{4}{9}$$

$$\boxed{a=\frac{4}{9}}$$

Formula Involving Gradient :-

If $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$, $r = |\bar{r}|$

P.T i) $\nabla r = \frac{\bar{r}}{r}$

ii) $\nabla f(r) = \frac{f'(r)}{r} \bar{r}$

iii) $\nabla \left(\frac{1}{r}\right) = \frac{-\bar{r}}{r^3}$

proof :-

$\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$r = |\bar{r}| = \sqrt{x^2 + y^2 + z^2}$

$r^2 = x^2 + y^2 + z^2$

p.diff w.r.to 'x'

$2r \frac{\partial r}{\partial x} = 2x$

$\boxed{\frac{\partial r}{\partial x} = \frac{x}{r}}$

lly w.r.to 'y'

$2r \frac{\partial r}{\partial y} = 2y$

$\boxed{\frac{\partial r}{\partial y} = \frac{y}{r}}$

& $\boxed{\frac{\partial r}{\partial z} = \frac{z}{r}}$

i) $\nabla r = \frac{\bar{r}}{r}$

$$\nabla r = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) r$$

$$= \hat{i}\frac{\partial r}{\partial x} + \hat{j}\frac{\partial r}{\partial y} + \hat{k}\frac{\partial r}{\partial z}$$

$$= \hat{i}\frac{x}{r} + \hat{j}\frac{y}{r} + \hat{k}\frac{z}{r}$$

$$= \frac{1}{r}\left[x\hat{i} + y\hat{j} + z\hat{k}\right]$$

$$\therefore \boxed{\nabla r = \frac{\bar{r}}{r}}$$

ii) To prove $\nabla f(r) = \frac{f'(r)}{r}\bar{r}$

$$\nabla f(r) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) f(r)$$

$$= f'(r)\frac{\partial r}{\partial x}\hat{i} + f'(r)\frac{\partial r}{\partial y}\hat{j} + f'(r)\frac{\partial r}{\partial z}$$

$$\nabla f(r) = f'(r)\left[\frac{x}{r}\hat{i} + \frac{y}{r}\hat{j} + \frac{z}{r}\hat{k}\right]$$

$$= \frac{f'(r)}{r}\left[x\hat{i} + y\hat{j} + z\hat{k}\right]$$

$$\boxed{\nabla f(r) = \frac{f'(r)}{r}\bar{r}}$$

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iii) To prove $\nabla\left(\frac{1}{r}\right) = \frac{-\bar{r}}{r^3}$

$$\nabla\left(\frac{1}{r}\right) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(\frac{1}{r}\right)$$

$$= \hat{i}\frac{\partial}{\partial x}\left(\frac{1}{r}\right) + \hat{j}\frac{\partial}{\partial y}\left(\frac{1}{r}\right) + \hat{k}\frac{\partial}{\partial z}\left(\frac{1}{r}\right)$$

$$= \frac{-1}{r^2}\frac{\partial r}{\partial x}\hat{i} + \hat{j}\frac{-1}{r^2}\frac{\partial r}{\partial y} + \frac{-1}{r^2}\frac{\partial r}{\partial z}\hat{k}$$

$$= \frac{-1}{r^3}\left[x\hat{i} + y\hat{j} + z\hat{k}\right]$$

$$\boxed{\nabla\left(\frac{1}{r}\right) = \frac{-1}{r^3}\bar{r}} \Rightarrow \boxed{\nabla\left(\frac{1}{r}\right) = \frac{-\bar{r}}{r^3}}$$

(1) P.T $\nabla(r^n) = n \cdot r^{n-2}\bar{r}$

Sol:- Let $\nabla(r^n) = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) r^n$

$$= \hat{i}\, n r^{n-1}\frac{\partial r}{\partial x} + n r^{n-1}\frac{\partial r}{\partial y}\hat{j} + n r^{n-1}\frac{\partial r}{\partial z}\hat{k}$$

$$= n \cdot r^{n-1}\left[\frac{x}{r}\hat{i} + \frac{y}{r}\hat{j} + \frac{z}{r}\hat{k}\right]$$

$$\boxed{\nabla(r^n) = \frac{n r^{n-1}}{r}\bar{r}}$$

(2) P.T $\bar{\nabla}(\log r) = \frac{\bar{r}}{r^2}$

Proof:- L.H.S: $\nabla(\log r)$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\log r$$

$$= \frac{1}{r}\hat{i}\frac{\partial r}{\partial x} + \frac{1}{r}\frac{\partial r}{\partial y}\hat{j} + \frac{1}{r}\frac{\partial r}{\partial z}\hat{k}$$

$= \frac{1}{r^2}\left[x\hat{i} + y\hat{j} + z\hat{k}\right]$

$$\boxed{\nabla(\log r) = \frac{\bar{r}}{r^2}}$$

(3) P.T $\nabla(\bar{r}\cdot\bar{a}) = \bar{a}$

proof:- L.H.S $= \nabla(\bar{r}\cdot\bar{a})$

NO

$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(\bar{r}\cdot\bar{a})$

$\bar{r}, = x\hat{i} + y\hat{j} + z\hat{k}$

$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$\bar{r}\cdot\bar{a} = a_1 x + a_2 y + a_3 z$.

Now $\nabla(\bar{r}\cdot\bar{a}) = \hat{i}\frac{\partial}{\partial x}a_1 x + \hat{j}\frac{\partial}{\partial y}a_2 y + \hat{k}\frac{\partial}{\partial z}a_3 z$

$= a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$$\boxed{\nabla(\bar{r}\cdot\bar{a}) = \bar{a}}$$

(4) find $\nabla r^{n-1} = n-1\, r^{n-3}\,\bar{r}$

proof:- $\nabla r^{n-1} = \hat{i}\frac{\partial}{\partial x}r^{n-1} + \hat{j}\frac{\partial}{\partial y}r^{n-1} + \hat{k}\frac{\partial}{\partial z}r^{n-1}$

$= n-1\, r^{n-2}\frac{\partial r}{\partial x}\hat{i} + \hat{j}\, n-1\cdot r^{n-2}\frac{\partial r}{\partial y} + n-1\, r^{n-2}\frac{\partial r}{\partial z}\hat{k}$

$= n-1\, r^{n-2}\left[\frac{\partial r}{\partial x}\hat{i} + \frac{\partial r}{\partial y}\hat{j} + \frac{\partial r}{\partial z}\hat{k}\right]$

$= \frac{n-1\cdot r^{n-2}}{r}\left[x\hat{i} + y\hat{j} + z\hat{k}\right]$

$\nabla r^{n-1} = \frac{n-1\cdot r^{n-2}}{r}(\bar{r}) \Rightarrow n-1\, r^{n-3}\,\bar{r}$

$$\boxed{\nabla r^{n-1} = n-1\, r^{n-3}\,\bar{r}}$$

(7) If $\phi = x^2y^3z^4$ & $\psi = xy + yz + zx$ Evaluate $\nabla(\phi\psi)$.

Sol:- Let $\nabla(\phi\psi) = (\nabla\phi)\psi + \phi(\nabla\psi)$

$$\nabla\phi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(x^2y^3z^4)$$

$$= 2xy^3z^4\hat{i} + 3x^2y^2z^4\hat{j} + 4x^2y^3z^3\hat{k}$$

$$\nabla\psi = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)(xy + yz + zx)$$

$$\nabla\psi = (y+z)\hat{i} + (x+z)\hat{j} + (y+x)\hat{k}$$

Now $(\nabla\phi)(\psi) + \phi(\nabla\psi) = \nabla(\phi\psi)$

$$\left(2xy^3z^4\hat{i} + 3x^2y^2z^4\hat{j} + 4x^2y^3z^3\hat{k}\right)(xy + yz + zx)$$

$$+ x^2y^3z^4\left[(y+z)\hat{i} + (x+z)\hat{j} + (x+y)\hat{k}\right]$$

(8) Show that $(\bar{a}\nabla)\phi = \bar{a}\cdot\nabla\phi$

Proof:- Let us consider

$$\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$$

$$\bar{a}\nabla = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k})\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)$$

$$\boxed{\bar{a}\nabla = a_1\frac{\partial}{\partial x} + a_2\frac{\partial}{\partial y} + a_3\frac{\partial}{\partial z}}$$

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$$\Rightarrow (\bar{a}\cdot\nabla)\phi = \left(a_1\frac{\partial}{\partial x} + a_2\frac{\partial}{\partial y} + a_3\frac{\partial}{\partial z}\right)\phi$$

$$\boxed{(\bar{a}\cdot\nabla)\phi = a_1\frac{\partial\phi}{\partial x} + a_2\frac{\partial\phi}{\partial y} + a_3\frac{\partial\phi}{\partial z}}$$

$$\cancel{(\bar{a}\cdot\nabla)\phi = 0}$$

$$= (a_1\hat{i} + a_2\hat{j} + a_3\hat{k})\left(\frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}\right)$$

$$= \bar{a}\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\phi$$

$$\boxed{(\bar{a}\cdot\nabla)\phi = \bar{a}\,\nabla\phi}$$

(2) S.T $(\bar{a}\cdot\nabla)\bar{r} = \bar{a}$

Proof:- Let $\bar{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$$(\bar{a}\cdot\nabla) = a_1\frac{\partial}{\partial x} + a_2\frac{\partial}{\partial x} + a_3\frac{\partial}{\partial y}$$

$$\text{Now } (\bar{a}\cdot\nabla)\bar{r} = \left(a_1\frac{\partial}{\partial x} + a_2\frac{\partial}{\partial y} + a_3\frac{\partial}{\partial z}\right)$$

$$(x\hat{i} + y\hat{j} + z\hat{k})$$

$$= a_1\frac{\partial\bar{r}}{\partial x} + a_2\frac{\partial\bar{r}}{\partial y} + a_3\frac{\partial r}{\partial z}$$

$$\boxed{(\bar{a}\cdot\nabla)\bar{r} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}}$$

$$\boxed{(\bar{a}\cdot\nabla)\bar{r} = \bar{a}}$$