UNIT –I
MATRICES

1. Every square matrix can be expressed as the sum of a symmetric and sew-symmetric matrices in one and only way (uniquely).
2. Prove that $\frac{1}{2}\begin{pmatrix} 1+i & -1+i \\ 1+i & 1-i \end{pmatrix}$ is a unitary matrix.
3. Define the rank of the matrix and find the rank of the following matrix $\begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$
4. Find the value of '**k**' it the rank of matrix A is 2 where $A = \begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & k & 0 \end{bmatrix}$
5. Find the value of '**k**' it the rank of matrix A is 2 where $A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & k & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$
6. Find the rank of matrix $\begin{bmatrix} 1 & 4 & 3 & -2 & 1 \\ -2 & -3 & -1 & 4 & 3 \\ -1 & 6 & 7 & 2 & 9 \\ -3 & 3 & 6 & 6 & 12 \end{bmatrix}$
7. Reduce the matrix A to normal form and hence find its rank $A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$
8. Reduce the matrix A to normal form and hence find its rank $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$
9. Reduce the matrix A to normal form and hence find its rank $A = \begin{bmatrix} 1 & 2 & -1 & 3 \\ 4 & 1 & 2 & 1 \\ 3 & -1 & 1 & 2 \\ 1 & 2 & 0 & 1 \end{bmatrix}$
10. Reduce the matrix A to normal form and hence find its rank $A = \begin{bmatrix} 1 & 3 & 4 & 5 \\ 1 & 2 & 6 & 7 \\ 1 & 5 & 0 & 10 \end{bmatrix}$
11. Find the inverse of the matrix by elementary row operations (i.e. Gauss-Jordan method) $A = \begin{bmatrix} -1 & 3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 0 & 1 \end{bmatrix}$
12. Find the inverse of the matrix by elementary row operations(i.e. Gauss-Jordan method) $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

13. Find the inverse of the matrix by elementary row operations(i.e. Gauss-Jordan method) $A = \begin{bmatrix} 0 & 1 & 2 & 2 \\ 1 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 2 & 3 & 3 & 3 \end{bmatrix}$
14. Solve the system of linear equations by matrix method
$x + y + z = 6, 2x + 3y - 2z = 2, 5x + y + 2z = 13.$
15. Find whether the following systems of equations are consistent. If so solve them.
$x + 2y + 2z = 2,\ \ 3x - 2y - z = 5, 2x - 5y + 3z = -4,\ x + 4y + 6z = 0.$
16. Find whether the following systems of equations are consistent. If so solve them.
$3x + 3y + 2z = 1,\ \ x + 2y = 4, 10y + 3z = -2,\ 2x - 3y - z = 5.$
17. Find whether the following systems of equations are consistent. If so solve them.
$x + 2y + 3z = 1, 2x + 3y + 8z = 2, x + y + z = 3.$
18. Solve the system of equations $x + y + z = 6, x - y + 2z = 5, 3x + y + z = -8.$
19. Find whether the following systems of equations are consistent. If so solve them.
$x + 2y - z = 3,\ \ 3x - y + 2z = -1,\ 2x - 2y + 3z = 2,\ x - y + z = -1.$
20. Discuss for what values of **λ,μ** the simultaneous equations
$x + y + z = 6, x + 2y + 3z = 10, x + 2y + \lambda z = \mu$ Have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.
21. Discuss for what values of **λ,μ** the simultaneous equations
$2x + 3y + 5z = 9, 7x + 3y + 2z = 8,\ 2x + 3y + \lambda z = \mu$ Have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.
22. Discuss for what values of **λ,μ** the simultaneous equations
$x + y + z = 3,\ x + 2y + 2z = 6,\ x + 9y + \lambda z = \mu$ Have (i) no solution (ii) a unique solution (iii) an infinite number of solutions.
23. Show that the only real number **λ** for which the system $x + 2y + 3z = \lambda x, 3x + y + 2z = \lambda y,$
$2x + 3y + z = \lambda z$ has non-zero solution is 6 and solve them, when $\lambda = 6$.
24. Solve the system of equations
$x + y + w = 0,\ y + z = 0,\ x + y + z + w = 0,\ x + y + 2z = 0.$
25. Solve the system of equations $2x - y + 3z = 0,\ 3x + 2y + z = 0,\ x - 4y + 5z = 0.$
26. Show that the system of equations $2x - 2y + z = \boldsymbol{\lambda} x,\ 2x - 3y + 2z = \boldsymbol{\lambda} y, -x + 2y = \boldsymbol{\lambda} z$ can possess a non- trivial solution only if $\lambda = 1, \lambda = -3.$ obtain the general solution in each case.
27. Express the following system in matrix form and solve by Gauss Elimination method.
$2x + 2y + 2z + w = 6,\qquad 6x - 6y + 6z + 12w = 36,$
$4x + 3y + 3z - 3w = -1,\ \ 2x + 2y - z + w = 10.$
28. Use Gauss Seidel iteration method to solve the system
$10x + y + z = 12, 2x + 10y + z = 13, 2x + 2y + 10z = 14$
29. Use Gauss Seidel iteration method to solve the system
$20x + y - 2z = 17, 3x + 20y - z = -18, 2x - 3y + 20z = 25$
30. Use Gauss Seidel iteration method to solve the system
$10x - 2y - z - u = 3, -2x + 10y - z - u = 15,$
$-x - y + 10z - 2u = 27, -x - y - 2z + 10u = -9$

UNIT –II
Eigen Values and Eigen Vectors

1. Find the Eigen values and the corresponding Eigen vectors of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$
2. Find the Eigen values and the corresponding Eigen vectors of $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$
3. Find the Eigen values and the corresponding Eigen vectors of $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 3 \end{bmatrix}$
4. Prove that a square matrix **A** and its transpose $\mathbf{A}^{\mathbf{T}}$ have the same Eigen values.
5. If λ is an Eigen value of a non-singular matrix A, then prove $\frac{|A|}{\lambda}$ is an Eigen value of the matrix **adjA.**
6. Prove that the Eigen value of a unitary matrix have absolute value 1.
7. Prove that the Eigen values of a Hermitian matrix are all real.
8. Find a matrix **P** which transforms the matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$ to diagonal form. Hence calculate $\mathbf{A}^{\mathbf{4}}$. Find the Eigen values and Eigen vectors of **A**. Also determine the Eigen values of $\mathbf{A}^{\mathbf{-1}}$.
9. If $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 1 \\ -4 & 4 & 3 \end{bmatrix}$ find by Diagonalization (i) A^8 (ii) A^4
10. Find the diagonal matrix orthogonally similar to the following real symmetric matrix. Also obtain the transforming matrix. $A = \begin{bmatrix} 7 & 4 & -4 \\ 4 & -8 & -1 \\ -4 & -1 & -8 \end{bmatrix}$
11. State the Cayley-Hamilton theorem. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 8 & -8 & 2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$
12. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ Verify Cayley-Hamilton theorem. Find A^4 and $\mathbf{A}^{\mathbf{-1}}$ using Cayley-Hamilton theorem.
13. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & -1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$
14. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and hence find $A^{-1}$and find $B = A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$

Quadratic Forms

15. Find the nature of the quadratic form $\mathbf{2x^2+2y^2+2z^2+2yz}$.
16. Find the nature of the quadratic form, index and signature of $\mathbf{10x^2+2y^2+5z^2-4xy-10xz+6yz}$.
17. Reduce the quadratic form to the canonical form $\mathbf{x^2+y^2+2z^2-2xy+4zx+4yz}$.

18. Reduce the quadratic form to the canonical form **$3x^2-3y^2-5z^2-2xy-6yz-6zx$**.
19. Reduce the quadratic form **$3x^2+5y^2+3z^2-2yz+2xz-2xy$** to the canonical form by orthogonal reduction.
20. Find the Eigen vectors of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ and hence reduce **$6x^2+3y^2+3z^2-2yz+4zx-4xy$** to sum of squares.
21. Reduce the quadratic form **$3x_1^2+3x_2^2+3x_3^2+2x_1x_2+2x_1x_3-2x_2x_3$** into sum of squares from by an orthogonal transformation and give the matrix of transformation.
22. Reduce the quadratic form to the canonical form by an orthogonal reduction and state the nature of the quadratic form **$5x^2+26y^2+10z^2+4yz+14zx+6xy$.**

UNIT –III
Sequences and Series

1. Test for the convergence of the series $\sum_{n=1}^{\infty}\frac{2n-1}{n(n+1)(n+2)}$
2. Test for the convergence of the series $\sum_{n=1}^{\infty}\{\sqrt[3]{n^3+1}-n\}$
3. Test for the convergence of the series $\sum_{n=1}^{\infty}\frac{1}{2^n+3^n}$
4. Show that $\frac{2}{1^p}+\frac{3}{2^p}+\frac{4}{3^p}+\cdots$ is convergent for $p>2$ and divergent for $p\leq 2$
5. Test for the convergence of the series $\sum_{n=1}^{\infty}\sqrt{n^3+1}-\sqrt{n^3}$
6. Examine the convergence of the series $\sum_{n=1}^{\infty}\frac{(n^3-5n^2+7)}{(n^5+4n^4-n)}$
7. Examine the convergence of the series $\sum_{n=1}^{\infty}\frac{1}{n^{3/2}+n+1}$
8. Show that $1+\frac{2^p}{2!}+\frac{3^p}{3!}+\frac{4^p}{4!}+\cdots$ I convergent for all values of ‘p’.
9. Test for the convergence of the series $1+\frac{x}{2}+\frac{x^2}{5}+\frac{x^3}{10}+\cdots(x>0)$
10. Test for the convergence of the series $\sum_{n=1}^{\infty}\frac{n^2}{2^n}$
11. Test for the convergence of the series $\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}+\cdots\infty$
12. Test for the convergence of the series $\sum_{n=1}^{\infty}\frac{1.3.5.....(2n+1)}{2.5.8.....(3n+2)}$
13. Test for the convergence of the series $\frac{2}{1}+\frac{2.5.8}{1.5.9}+\frac{2.5.8.11}{1.5.9.13}+\cdots\infty$
14. Test for the convergence of the series $\sum_{n=1}^{\infty}\frac{[(n+1)!]^2x^{n-1}}{n},(x>0)$
15. Examine the convergence of the series $\frac{x}{1.2}+\frac{x^2}{2.3}+\frac{x^3}{13.40}+\cdots$
16. Examine the convergence or divergence of the series $\sum n^2x^{n+1},(x>0)$
17. Discuss the convergence of $\sum\frac{x^{2n}}{(n+2)\sqrt{(n+1)}},(x>0)$
18. Test for the convergence of the series $\sum_{n=1}^{\infty}\left(\frac{n^2}{2^n}+\frac{1}{n^2}\right)$
19. Test for the convergence of the series $\sum_{n=1}^{\infty}\frac{(\sqrt{5}-1)^n}{n^2+1}$

20. Test for the convergence of the series $\sum_{n=1}^{\infty} \frac{2n!}{n!n}$

21. Test for the convergence of the series $\frac{2}{1^2}x + \frac{3^2}{2^3}x^2 + \cdots + \frac{(n+1)^n}{n^{n+1}}x^n + \cdots (x > 0)$

22. Test for the convergence of the series $\frac{3x}{4} + \left(\frac{5}{6}\right)^2 x^2 + \left(\frac{7}{8}\right)^3 x^3 + \cdots$

23. Show that the harmonic series of order 'p', $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges for p>1 and diverges for p≤1.

24. Using integral test determine the convergence of the series $sin\pi + \frac{1}{4}sin\frac{\pi}{2} + \frac{1}{9}sin\frac{\pi}{3} + \cdots$

25. Test for the convergence of the series $\frac{x}{1} + \frac{x^3}{2.3} + \frac{1.3.x^5}{2.4.5} + \frac{1.3.5.x^7}{2.4.6.7} + \cdots$

26. Test for the convergence of the series $\sum \frac{(n!)^2}{(2n)!}x^{2n}$

27. Examine the convergence of $\frac{1}{3}x^2 + \frac{1.2}{3.5}x^3 + \frac{1.2.3}{3.5.7}x^4 + \cdots, (x > 0)$

28. Examine the convergence of $\sum \left[\frac{1.4.7\ldots.(3n-2)}{3.6.9\ldots3n}\right]^2$

29. Examine the convergence or divergence of $\sum \frac{1.3.5\ldots.(2n-1)}{2.4.6\ldots.2n(2n+2)}$

30. Test for the convergence of the series $\frac{3^2}{6^2} + \frac{3^2.5^2}{6^2.8^2} + \frac{3^2.5^2.7^2}{6^2.8^2.10^2} + \cdots$

31. Test for the convergence of the series $\frac{1}{3} + \frac{1.4}{3.6} + \frac{1.4.7}{3.6.9} + \frac{1.4.7.10}{3.6.9.12} + \cdots \infty$

32. Test for the convergence of the series $\frac{(a+x)}{1!} + \frac{(a+2x)^2}{2!} + \frac{(a+3x)^3}{3!} + \cdots$

33. Test for the convergence of the series $1 + \frac{x}{2} + \frac{2!}{3^2}x^2 + \frac{3!}{4^3}x^3 + \cdots$

34. Examine the convergence of the series $\frac{1}{1.3.5} - \frac{1}{3.5.7} + \frac{1}{5.7.9} - \frac{1}{7.9.11} + \cdots$

35. Examine the convergence of series $\frac{1}{1^2} - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \cdots$

36. Test for convergence, absolute convergence and conditional convergence of the series $\sum(-1)^{n-1}\left(\frac{1}{4n-3}\right)$

37. Test for convergence, absolute convergence and conditional convergence of the series $\frac{1}{5\sqrt{2}} - \frac{1}{5\sqrt{3}} + \frac{1}{5\sqrt{4}} - \cdots + (-1)^n\frac{1}{5\sqrt{n}} + \cdots$

38. Test for convergence, absolute convergence and conditional convergence of the series $\sum(-1)^n\frac{logn}{n^2}$

39. Test for convergence, absolute convergence and conditional convergence of the series $\sum(-1)^n\frac{sin\left(\frac{1}{\sqrt{n}}\right)}{n-1}$

40. Prove that the series $\frac{1}{2^3} - \frac{1}{3^3}(1+2) + \frac{1}{4^3}(1+2+3) - \frac{1}{5^3}(1+2+3+4) + \cdots \infty$ is conditionally convergent.

41. Test for convergence of logarithmic series $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} - \cdots$

42. Find the interval of convergence for the following series $\sum \frac{(n^2-1)}{n^2+1}x^n$

43. For what values of x the following series is convergent $x - \frac{x^2}{2^2} + \frac{x^3}{3^2} - \frac{x^4}{4^2} + \cdots$

UNIT –IV
Calculus

Mean Value theorems:

1. Write the statement of the Rolle's Theorem. Verify Rolle's theorem for $f(x) = e^x(sinx - cosx)\ in \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$.
2. Apply Rolle's theorem for $sinx\sqrt{cos2x}\ in \left[0, \frac{\pi}{4}\right]$ and find x such that $0<x < \frac{\pi}{4}$
3. Give an example for the function which one is not satisfies the Rolle's theorem with suitable explanation
4. State the Lagrange's mean value theorem. Verify the theorem for $f(x) = x(x-2)(x-3)\ in\ (0,4)$.
5. If a<b, prove that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$ using Lagrange's mean value theorem. Deduce the following: (i) $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$ (ii) $\frac{5\pi+4}{20} < \tan^{-1} 2 < \frac{\pi+2}{4}$
6. Prove that $\frac{\pi}{3} - \frac{1}{5\sqrt{3}} > \cos^{-1}\frac{3}{5} > \frac{\pi}{3} - \frac{1}{8}$ using Lagrange's mean value theorem.
7. Prove that $\frac{\pi}{6} + \frac{1}{5\sqrt{3}} < \sin^{-1}\frac{3}{5} < \frac{\pi}{6} + \frac{1}{8}$ using Lagrange's mean value theorem.
8. Using mean value theorem prove that $\tan x > x\ in\ 0 < x < \frac{\pi}{2}$
9. Prove using mean value theorem $|\sin u - \sin v| \leq |u - v|$.
10. State the Cauchy's mean value theorem. Find 'c' of Cauchy's mean value theorem on [a, b] for f(x) = e^x and g(x) = e^{-x} (a, b>0).
11. If f(x) =$\sqrt{x}$ and g(x) = $\frac{1}{\sqrt{x}}$ prove that 'C'of the Cauchy's generalized mean value theorem is the geometric mean of 'a' and 'b' for any a>0, b>0.

Applications of Definite Integral:

12. Find the volume of the reel shaped solid formed by the revolution about the y-axis the part of the parabola $y^2 = 4ax$ cut off by the latus-rectum.
13. Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (0<b<a) about the major axis.
14. Find the volume of the solid generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (0<b<a) about the minor axis.
15. Find the volume of spherical cap of height 'h' cut off from a sphere of radius 'a'.
16. Find the volume formed by the revolution of the loop of the curve $y^2(a+x) = x^2(3a-x)$ about the x-axis.
17. Find the surface area generated by the revolution of an arc of the catenary $y = ccosh\left(\frac{x}{c}\right)$ about the x-axis from $x = 0\ to\ x = c$.
18. Prove that the curved surface area of a sphere of radius r intercepted between two parallel planes at a distance 'a' and 'b' from the centre of the sphere is $2\pi r(b-a)$ when b>a and hence deduce the surface area of the sphere.

Gamma and beta functions:

19. Define beta function.
20. Prove that $B(m,n) = 2\int_0^{\frac{\pi}{2}} sin^{2m-1}\theta\ cos^{2n-1}\theta\ d\theta$

21. Prove that $B(m,n) = \int_0^\infty \frac{x^{m-1}}{(1+x)^{m+n}}dx = \int_0^\infty \frac{x^{n-1}}{(1+x)^{m+n}}dx$

22. Prove that $\frac{B(p,q+1)}{q} = \frac{B(p+1,q)}{p} = \frac{B(p,q)}{p+q}$ where P>0,q>0

23. Show that $\int_a^b (x-a)^m(b-x)^n dx = (b-a)^{m+n+1}B(m+1,n+1)$

24. Define Gamma function.

25. Write down the $B-\Gamma$ relation and prove it. (or) Prove that $B(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ where m>0, n>0.

26. Prove that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

27. Prove that $\int_0^\infty e^{-x^2}\,dx = \int_{-\infty}^0 e^{-x^2}\,dx = \frac{\sqrt{\pi}}{2}$

28. Prove that $\Gamma(n) = \int_0^1 (log\frac{1}{x})^{n-1}dx,\ n>0$

29. Evaluate $\int_0^{\frac{\pi}{2}} \sqrt{cot\theta}\ d\theta$

30. Evaluate $\int_0^1 x^4(log\frac{1}{x})^3 dx$

31. Prove that $\int_0^{\frac{\pi}{2}} sin^n\theta\ d\theta = \int_0^{\frac{\pi}{2}} cos^n\theta\ d\theta = \frac{\boldsymbol{\Gamma}\left(\frac{n+1}{2}\right)\sqrt{\pi}}{2\boldsymbol{\Gamma}(\frac{n+2}{2})}$

32. Evaluate $\int_0^\infty x^{-\frac{3}{2}}(1-e^{-x})dx$

33. Prove that $\int_0^\infty e^{-y^{\frac{1}{m}}}\,dy = m\,\Gamma(m)$

34. Prove that $\int_0^{\frac{\pi}{2}}[\sqrt{tan\theta}+\sqrt{sec\theta}\,]d\theta = \frac{1}{2}\Gamma\left(\frac{1}{4}\right)\left[\Gamma\left(\frac{3}{4}\right)+\frac{\sqrt{\pi}}{\Gamma\left(\frac{3}{4}\right)}\right]$

35. Evaluate $\int_0^\infty \frac{x}{(1+x^6)}dx$ using $B-\Gamma$ functions

36. Prove that $\int_0^1 x^m(logx)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$ where n is a positive integer and m>-1.

37. Evaluate $4\int_0^\infty \frac{x^2}{(1+x^4)}dx$ using $B-\Gamma$ functions

38. Prove that $\Gamma(m)\Gamma\left(m+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$

39. Evaluate $\int_0^1 \frac{x^2}{\sqrt{1-x^5}}dx$ in terms of Beta function.

40. Prove that $\int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{sin\theta}}\ d\theta \times \int_0^{\frac{\pi}{2}} \sqrt{sin\theta}\ d\theta = \pi$

UNIT –IV
Multivariable Calculus

1. State the Euler's theorem.
2. Verify Euler's theorem for $u = x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right)$ and also prove that $\frac{\partial^2 u}{\partial x\partial y} = \frac{x^2-y^2}{x^2+y^2}$
3. If $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$
4. If $u = \log\left(\frac{x^4+y^4}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$
5. If $u = \tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = sin2u$
6. Find $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$ if $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$

7. If $u = \sec^{-1}\left(\frac{x^3-y^3}{x+y}\right)$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\,cotu$, then evaluate $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\partial y} + y^2\frac{\partial^2 u}{\partial y^2}$
8. If $u^3+xv^2-uy=0$, $u^2+xyv+v^2=0$ find $\frac{\partial u}{\partial x}, \frac{\partial v}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial v}{\partial y}$
9. If $u = f(x-y, y-z, z-x)$ then prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$
10. Find the value of $\frac{\partial x}{\partial z}, \frac{\partial y}{\partial z}, \frac{\partial z}{\partial x}$ if $f(x,y,z) = 0$
11. If x+y+z=u,y+z=uv,z=uvw, then evaluate $\frac{\partial(x,y,z)}{\partial(u,v,w)}$
12. If $u = x^2 - 2y, v = x + y + z, w = x - 2y + 3z$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$
13. If $x = u(1-v), y = uv$ prove that **JJ'**=1.
14. If $u = 2xy, v = x^2 - y^2,\ x = r\cos\theta$ and $y = r\sin\theta$ find $\frac{\partial(u,v)}{\partial(r,\theta)}$
15. If $x = uv, y = \frac{u}{v}$ verify that $\frac{\partial(u,v)}{\partial(x,y)}\frac{\partial(x,y)}{\partial(u,v)} = 1$
16. If $x = \frac{u^2}{v}, y = \frac{v^2}{u}$ find $\frac{\partial(u,v)}{\partial(x,y)}$
17. Prove that $u = \frac{x^2-y^2}{x^2+y^2}, v = \frac{2xy}{x^2+y^2}$ are functionally dependent and find the relation between them.
18. If $x = u\sqrt{1-v^2} + v\sqrt{1-u^2}$ and $y = \sin^{-1}u + \sin^{-1}v$ then show that x and y are functionally related. Also find the relationship.
19. Prove that the functions **u=x+y+z, v=xy+yz+zx** and, **w=$x^2+y^2+z^2$** are functionally dependent and find the relation between them.
20. Find the maximum and minimum values of $f(x,y)=x^3+3xy^2-3x^2-3y^2+4$
21. Find the maximum and minimum values of $f(x,y)=3x^4-2x^3-6x^2+6x+1$
22. Find three positive numbers whose sum is 100 and whose product is maximum.
23. Find the rectangular parallelepiped of maximum volume that can be inscribed in a sphere.
24. Divide 24 into three parts such that the continued product of the first square of the second and cube of the third is maximum.
25. Find a point within a triangle such that the sum of the squares of its distances from the three vertices is minimum.
26. Find the minimum value of **$x^2+y^2+z^2$** given **x+y+z=3a**
27. Find the maximum and minimum values of $x + y + z$ subject to $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$
28. Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$
29. Find the maximum value of **$u=x^2y^3z^4$** if **2x+3y+4z=a**
30. Find the maximum and minimum distances of the point (3,4,12) from the sphere $x^2+y^2+z^2=1$