EE5609 – Matrix Theory 2023
Practice Set 1

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<u>*Solutions are not to be returned*</u>

Reading Exercise

1. *Geometry of some 2-D linear transformations.*

   Inspect and understand Tables 1 to 4 in Chapter 1 of David Lay's textbook.

2. *Terminology related to linear transformations.*

   From Section 1.8 of David Lay, learn about *domain*, *codomain*, *image* and *range*.

   From Section 1.9 of David Lay, learn about *onto* and *one-to-one* functions. An onto function is also called *surjective*, and a one-to-one function is also called *injective.* A function that is both onto and one-to-one is called a *one-to-one correspondence* or a *bijection* or *invertible.*

3. *Geometric transformations*

   Section 7.1 of Boyd & Vandenberghe.

4. *Selector matrices*

   Section 7.2 of Boyd & Vandenberghe.

5. *Discrete-time convolution as matrix-vector multiplication.*

   Section 7.4 of Boyd & Vandenberghe.

Practice Set

Notation: Bold lower case letters will denote column vectors, and bold upper case letters are matrices. The symbol $\mathbb{F}$ denotes either $\mathbb{R}$ or $\mathbb{C}$.

1. *Questions from Boyd & Vandenberge's textbook.*

   (a) *Chapter 6, Exercise Problems:* 6.6, 6.7, 6.11, 6.13.

   (b) *Chapter 7, Exercise Problems:* 7.1 to 7.4.

   (c) *Chapter 10, Exercise Problems:* 10.1 to 10.6.

2. Give examples of linear transformations that are:

   (a) one-to-one but not onto.

   (b) onto but not one-to-one.

   (c) both onto and one-to-one (i.e., invertible).

3. Suppose $f : \mathbb{R}^2 \to \mathbb{R}^2$ is a function that reflects a point about the following straight line passing through the origin:
$$x_2 = x_1 \tan(\theta).$$
   Using the fact that f is a linear function find the matrix corresponding to f.

4. Suppose $\boldsymbol{A} \in \mathbb{F}^{m\times n}$. Is the function that maps $\boldsymbol{x} \in \mathbb{F}^n$ to $\boldsymbol{A}\boldsymbol{x} \in \mathbb{F}^m$ linear?

5. Suppose $\boldsymbol{A} \in \mathbb{F}^{m\times n}$. For $\boldsymbol{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_m \end{bmatrix} \in \mathbb{F}^m$ let

$$\begin{bmatrix} y_1 & \cdots & y_n \end{bmatrix} = \begin{bmatrix} x_1 & \cdots & x_m \end{bmatrix} \boldsymbol{A}.$$

Define the function f as $f(\boldsymbol{x}) = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$.

Prove that f is linear and find the matrix corresponding to f.

6. Determine if the following functions are linear. Support your answer with mathematical reasoning.

   (a) $f : \mathbb{R}^2 \to \mathbb{R}$, where $f(\boldsymbol{x}) = \max\{x_1, x_2\}$.

   (b) $f : \mathbb{F} \to \mathbb{F}^2$, where $f(x) = \begin{bmatrix} x \\ a \end{bmatrix}$. For which choices of $a \in \mathbb{F}$ is f linear, and for which choices of a is f non-linear?

   (c) $f : \mathbb{C}^2 \to \mathbb{C}^2$, where $f(\boldsymbol{x}) = \begin{bmatrix} \text{Real}(x_1) \\ \text{Real}(x_2) \end{bmatrix}$. Use $\mathbb{F} = \mathbb{C}$.
