27/8/12

① let $A_1, A_2 \ldots\ldots A_n$ be any n set show by mathematical induction-

$$\overline{\left(\bigcup_{i=1}^{n} A_i\right)} = \bigcap_{i=1}^{n} \overline{A_i}$$

→ B.S.] We have to s.T. p(n) is true for n=1

$$\overline{\left(\bigcup_{i=1}^{1} A_i\right)} = \bigcap_{i=1}^{n} \overline{A_i}$$

$$L.H.S. = \overline{\left(\bigcup_{i=1}^{n} A_i\right)} = \overline{A_1}$$

$$R.H.S. = \bigcap_{i=1}^{n} \overline{A_i} = \overline{A_1}$$

$$\boxed{L.H.S. = R.H.S.}$$

p(n) is true for n=1.

I.S]

① We have to s.T. p(n) is true for n=k

$$\overline{\left(\bigcup_{i=1}^{n} A_i\right)} = \bigcap_{i=1}^{n} \overline{A_i}$$

n=k

$$\overline{\left(\bigcup_{i=1}^{k} A_i\right)} = \bigcap_{i=1}^{k} \overline{A_i} \quad \text{——— (I)}$$

p(n) is true for n=k

(b) Now, we have to s.T. p(n) is true for n=k+1

$$\overline{\left(\bigcup_{i=1}^{n} A_i\right)} = \bigcap_{i=1}^{n} \overline{A_i}$$

n=k+1

$$\overline{\left(\bigcup_{i=1}^{k+1} A_i\right)} = \bigcap_{i=1}^{k+1} \overline{A_i}$$

$$\text{L.H.S.} = \overline{\left(\bigcup_{i=1}^{k+1} A_i\right)} = \overline{A_1 \cup A_2 \cup A_3 \ldots \cup A_k \cup A_{k+1}}$$

$$\overline{\left(\bigcup_{i=1}^{k+1} A_i\right)} = \overline{A_1 \cup A_2 \cup A_3 \ldots \cup A_k} \cap \overline{A_{k+1}}$$ (De-morgan's law)

$$\overline{\left(\bigcup_{i=1}^{k+1} A_i\right)} = \overline{\left(\bigcup_{i=1}^{k} A_i\right)} \cap \overline{A_{k+1}}$$

$$\left[\text{But, } \overline{\left(\bigcup_{i=1}^{k} A_i\right)} = \bigcap_{i=1}^{k} \overline{A_i}\right]$$

$$\overline{\left(\bigcup_{i=1}^{k+1} A_i\right)} = \bigcap_{i=1}^{k} \overline{A_i} \cap \overline{A_{k+1}}$$

$$\overline{\left(\bigcup_{i=1}^{k+1} A_i\right)} = \bigcap_{i=1}^{k+1} \overline{A_i}$$

= R.H.S.

$$\therefore \boxed{\text{L.H.S.} = \text{R.H.S.}}$$

② $$\overline{\left(\bigcap_{i=1}^{n} A_i\right)} = \bigcup_{i=1}^{n} \overline{A_i}$$

→ B.S.] We have to s.T. p(n) is true for n=1

$$\overline{\left(\bigcap_{i=1}^{n} A_i\right)} = \bigcup_{i=1}^{n} \overline{A_i}$$

$$L.H.S. = \overline{\left(\bigcap_{i=1}^{n} A_i\right)} = \overline{A_1}$$

$$R.H.S. = \bigcup_{i=1}^{n} \overline{A_i} = \overline{A_1}$$

$$\boxed{\therefore L.H.S. = R.H.S.}$$

$\therefore$ p(n) is true for n=1.

I.S.]

ⓐ We have to s.T. p(n) is true for n=k

$$\overline{\left(\bigcap_{i=1}^{n} A_i\right)} = \bigcup_{i=1}^{n} \overline{A_i}$$

$n=k$

$$\overline{\left(\bigcap_{i=1}^{k} A_i\right)} = \bigcup_{i=1}^{k} \overline{A_i} \quad \text{———} \; \text{(I)}$$

$\therefore$ p(n) is true for n=k

(b) Now, we have to s.T. p(n) is true for n=k+1

$$\overline{\left(\bigcap_{i=1}^{n} A_i\right)} = \bigcup_{i=1}^{n} \overline{A_i}$$

$$n=k+1$$

$$\overline{\left(\bigcap_{i=1}^{k+1} A_i\right)} = \bigcup_{i=1}^{k+1} \overline{A_i}$$

$$\text{L.H.S.} = \overline{\left(\bigcap_{i=1}^{k+1} A_i\right)} = \overline{A_1 \cap A_2 \cap A_3 \cap \cdots \cdots A_k \cap A_{k+1}}$$

$$\overline{\left(\bigcap_{i=1}^{k+1} A_i\right)} = \overline{A_1 \cap A_2 \cap A_3 \cdots\cdots \cap A_k \cup \overline{A_{k+1}}}$$

$$\overline{\left(\bigcap_{i=1}^{k+1} A_i\right)} = \overline{\left(\bigcap_{i=1}^{k} A_i\right)} \cup \overline{A_{k+1}}$$

$$\left[\text{But, } \overline{\bigcap_{i=1}^{k} A_i} = \bigcup_{i=1}^{k} \overline{A_i}\right]$$

$$\overline{\left(\bigcap_{i=1}^{k+1} A_i\right)} = \bigcup_{i=1}^{k} \overline{A_i} \cup \overline{A_{k+1}}$$

$$\overline{\left(\bigcap_{i=1}^{k+1} A_i\right)} = \bigcup_{i=1}^{k+1} \overline{A_i}$$

$$= \text{R.H.S.}$$

$$\boxed{\therefore \text{L.H.S.} = \text{R.H.S.}}$$