

Q. ① let A_1, A_2, \dots, A_n be any n set show by mathematical induction -
$$\left(\bigcup_{i=1}^n A_i \right)^c = \bigcap_{i=1}^n A_i^c$$

→ B.S.] We have to s.t. $p(n)$ is true for $n=1$.

$$\left(\bigcup_{i=1}^n A_i \right)^c = \bigcap_{i=1}^n A_i^c$$

$$\text{L.H.S.} = \left(\bigcup_{i=1}^n A_i \right)^c = A_1^c$$

$$\text{R.H.S.} = \bigcap_{i=1}^n A_i^c = A_1^c$$

$$\boxed{\text{L.H.S.} = \text{R.H.S.}}$$

$p(n)$ is true for $n=1$.

I.S.]

② We have to s.t. $p(n)$ is true for $n=k$

$$\left(\bigcup_{i=1}^n A_i \right)^c = \bigcap_{i=1}^n A_i^c$$

$n=k$

$$\left(\bigcup_{i=1}^k A_i \right)^c = \bigcap_{i=1}^k A_i^c \quad \text{--- } \textcircled{I}$$

$p(n)$ is true for $n=k$

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(b) Now, we have to s.t. $p(n)$ is true for $n=k+1$

$$\left(\bigcup_{i=1}^n A_i \right) = \bigcap_{i=1}^n \overline{A_i}$$

$$\left(\bigcup_{i=1}^{k+1} A_i \right) = \bigcap_{i=1}^{k+1} \overline{A_i}$$

$$\text{L.H.S.} = \left(\bigcup_{i=1}^{k+1} A_i \right) = \overline{A_1 \cap A_2 \cap A_3 \dots \cap A_k \cap A_{k+1}}$$

$$\left(\bigcup_{i=1}^{k+1} A_i \right) = \overline{A_1 \cap A_2 \cap A_3 \dots \cap A_k} \cap \overline{A_{k+1}} \quad (\text{De Morgan's law})$$

$$\left(\bigcup_{i=1}^{k+1} A_i \right) = \left(\bigcup_{i=1}^k A_i \right) \cap \overline{A_{k+1}}$$

$$\left[\text{But, } \left(\bigcup_{i=1}^k A_i \right) = \bigcap_{i=1}^k \overline{A_i} \right]$$

$$\left(\bigcup_{i=1}^{k+1} A_i \right) = \bigcap_{i=1}^k \overline{A_i} \cap \overline{A_{k+1}}$$

$$\left(\bigcup_{i=1}^{k+1} A_i \right) = \bigcap_{i=1}^{k+1} \overline{A_i}$$

$$= \text{R.H.S.}$$

$$\boxed{\text{L.H.S.} = \text{R.H.S.}}$$

$$(2) \left(\overline{\bigcap_{i=1}^n A_i} \right) = \bigcup_{i=1}^n \overline{A_i}$$

→ B.S.] We have to s.T. $p(n)$ is true for $n=1$

$$\left(\overline{\bigcap_{i=1}^n A_i} \right) = \bigcup_{i=1}^n \overline{A_i}$$

$$\text{L.H.S.} = \left(\overline{\bigcap_{i=1}^n A_i} \right) = \overline{A_1}$$

$$\text{R.H.S.} = \bigcup_{i=1}^n \overline{A_i} = \overline{A_1}$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$\therefore p(n)$ is true for $n=1$.

I.S.]

(a) We have to s.T. $p(n)$ is true for $n=k$

$$\left(\overline{\bigcap_{i=1}^n A_i} \right) = \bigcup_{i=1}^n \overline{A_i}$$

$n=k$

$$\left(\overline{\bigcap_{i=1}^k A_i} \right) = \bigcup_{i=1}^k \overline{A_i} \quad \text{--- (I)}$$

$\therefore p(n)$ is true for $n=k$

(b) Now, we have to s.t. $p(n)$ is true for $n=k+1$

$$\left(\overline{\bigcap_{i=1}^n A_i} \right) = \bigcup_{i=1}^n \overline{A_i}$$

$$\left(\overline{\bigcap_{i=1}^{n=k+1} A_i} \right) = \bigcup_{i=1}^{k+1} \overline{A_i}$$

$$\text{L.H.S.} = \left(\overline{\bigcap_{i=1}^{k+1} A_i} \right) = \overline{A_1 \cap A_2 \cap A_3 \cap \dots \cap A_k \cap A_{k+1}}$$

$$\left(\overline{\bigcap_{i=1}^{k+1} A_i} \right) = \overline{A_1 \cap A_2 \cap A_3 \dots \cap A_k \cap A_{k+1}}$$

$$\left(\overline{\bigcap_{i=1}^{k+1} A_i} \right) = \left(\overline{\bigcap_{i=1}^k A_i} \right) \cup \overline{A_{k+1}}$$

$$\left[\text{But, } \overline{\bigcap_{i=1}^k A_i} = \bigcup_{i=1}^k \overline{A_i} \right]$$

$$\left(\overline{\bigcap_{i=1}^{k+1} A_i} \right) = \bigcup_{i=1}^k \overline{A_i} \cup \overline{A_{k+1}}$$

$$\left(\overline{\bigcap_{i=1}^{k+1} A_i} \right) = \bigcup_{i=1}^{k+1} \overline{A_i}$$

= R.H.S.

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$