Set And Subset

A set is a well defined collection of object called elements or members of the sets.

① The collection of all wooden chair.
② Collection of real no. between 0 and 1.

* Well defined means it is possible to deside if a given object belongs to the collection or not it means the given set always represent a specific features.

* The way of describing a set that has finite number of element is by listing the element ab a set between the (bracket) {}.

* Thus the set of all positive integers that are less than 4 can be written as
  $A = \{1, 2, 3\}$

* We use the upper case letter such as A & C to denote sets and lower case letter such as a, b, c ---- x, y, z.
  To denote the member set

* We indicate the fact that x is an element of set $A = \{x\}$ by writting $x \in A$ and x is not in set A, we write $x \notin A$.

* Sometimes it is inconvinent to describe a by listing all of its element for that we a notation i.e. $P(x)$ to denote.

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p concerning the variables x.

The set defined $p(x)$ is written as $\{x \mid p(x)\}$ which is a collection of all object from which p is true and it is read as "The set of all x such as $p(x)$"

Ex: The set containing of all the letter in the word i.e. byte can be denoted by $\{b, y, t, e\}$ or can be written as $\{x / x$ is a letter in the "byte"$\}$

The set that has no element in its is called empty set and it is denoted by $\{\}$. ϕ

* We can't say two sets are said to be equal if they have some no. of element and some features of element and its is denoted by $A = B$.

* SUBSET :

If every element of set A is also an element of set B i.e. when x belongs to A then we can say that A is contained in B and it is denoted by $A \subseteq B$. If A is not a subset of B then we write $A \not\subseteq B$.

Ex: Let $A = \{1, 2, 3, 4, 5, 6\}$
$B = \{2, 4, 5\}$
$C = \{1, 2, 3, 4, 5\}$

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then $B \subseteq A$, $B \subseteq C$ & $C \subseteq A$

If A is any set then $A \subseteq A$ i.e every set a subset of itself also $A = B$ if $A \subseteq B$ hence $B \subseteq A$. The set A is called universe if it under discussion i.e. it contain all object and denoted by 'U' i.e. Universe set.

* A set A is called finite if it has n distinct element where $n \in N$ in this n is called quardility of set A and is denoted by $|A|$. A set that is not finite is called infinite.

* <u>Power Set</u> :

If A is a set than that of all subset of A is called a power set of A and B is denoted by $P(A)$.

Ex: Let $A = \{1, 2, 3\}$ then $P(A)$ consist of following subset of A as follows.

$P(A) : \{ \{\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2,$
$\{2, 1\}$
$\{1, 2, 3\}$

A pictorial representation between subset of universal set and their other operation defined by Venn diagram.

* A Venn diagram is symmetric representation.

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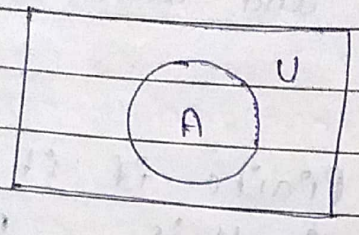
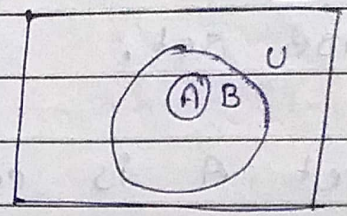
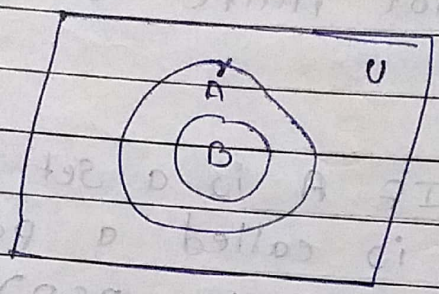
* of set by a point. The Universal set (U) is represented by the set of points in a rectangle and a set or subset is represented by circle following in the Venn diagram which shows universal set U and its subset is denoted by circle.



$A \subseteq B$



The Venn diagram of $A \not\subseteq B$



* Operation on Set

1] Union Set
2] Intersection set
3] Disjoint set
4] Complement set
5] Symmetric Difference

1) Union Set:

In A & B are two set then their union as the set conversion all the elements that belongs

and it is denoted by 'A∪B' ie the element belongs to either A or B.

Thus

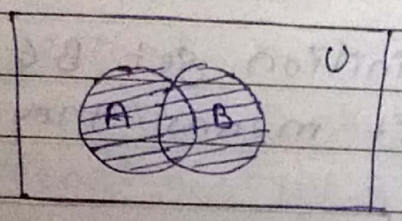
$$A \cup B = \{x / x \in A \text{ or } x \in B\}$$

Ex: Let $A = \{a, b, c, e, f, d, r, s\}$

find $A \cup B$

$A \cup B = \{a, b, c, e, f, d, r, s\}$

Venn diagram for $A \cup B$ is



2] <u>Intersection of sets</u> :

If A and B are two set their intersection is a set which common of all element that belongs both A and B i.e. common to both the sets and its is denoted $A \cap B$.

Thus $A \cap B = \{x / x \in A \text{ and } x \in B\}$

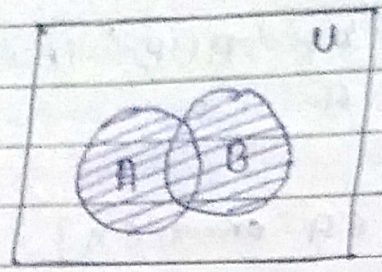
let $A = \{a, b, c, e, f\}$

$B = \{b, e, f, r, s\}$ and $C = \{a, f, u\}$

find $A \cap B$, $A \cap C$ and $B \cap C$

$A \cap B = \{b, e, f\}$

$A \cap C = \{a\}$ & $B \cap C = \{\ \}$

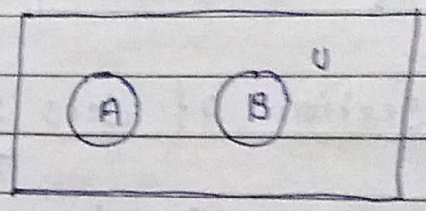


$A \cap B$

3] Disjoint Set:

Two set that have no common element are called disjoint set.

Ex: In previous definition set B & C have no common element it means they followed disjoint set are



4) Complement of Set:

If A & B two sets we defined the set of all element that belongs to A be not to B and we denoted by as follows,

i.e. $A \cdot B = \{x / x \in A \text{ and } x \in B\}$

Ex: Let $A = \{a, b, c\}$, $B = \{b, c, d, e\}$

$A - B = \{a\}$, $B - A = \{d, e\}$

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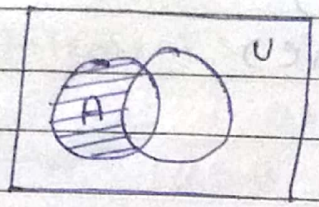
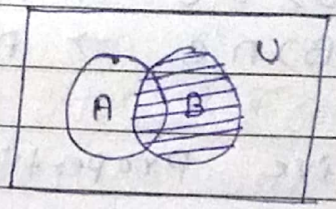
Venn Diagram for complement of set are



A - B



B - A

* Symmetric Difference *

If A and B are two we defined their symmetric diff. that belongs to A or B but to both A & B.
We denoted by $A \oplus B$

Thus,

$$A \oplus B = \{x / x \in A \text{ and } x \notin B\}$$

or

$$(x \in B \text{ and } x \notin A)$$

Ex: Let $A = \{a, b, c, d\}$ & $B = \{a, c, e, f, g\}$
then

$$A \oplus B = \{b, d, e, f, g\}$$

* Algebra on Sets:

Following are some algebric properties of set operation.

1) Commutative property

① $A \cup B \rightarrow B \cup A$

$A \cap B \rightarrow B \cap A$

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2) Associative property

① $(A \cup B) \cup C \rightarrow A \cup (B \cup C)$

$(A \cap B) \cap C \rightarrow A \cap (B \cap C)$

3) Distributive Properties

① $A \cap (B \cup C) \rightarrow (A \cap B) \cup (A \cap C)$

$A \cup (B \cap C) \rightarrow (A \cup B) \cap (A \cup C)$

4) Identity Property

① $A \cup A = A$

$A \cap A = A$

5) Complement property

① $(\bar{A}) = A$

$A \cup \bar{A} = U$

$A \cap \bar{A} = \phi$

$\phi = U$

$\bar{U} = \phi$

$\left.\begin{array}{l} \overline{A \cup B} = \bar{A} \cap \bar{B} \\ \overline{A \cap B} = \bar{A} \cup \bar{B} \end{array}\right\}$ (De-Morgan's Law)

6) Universal Set

① $A \cup U = A$

$A \cap U = A$

7) Empty set

① $A \cup \phi = A$ or $A \cup \{\} = A$

② $A \cap \phi = \phi$ or $A \cap \{\} = \{\}$

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Imp **Theorem 1]** : Prove that $\overline{A \cup B} = \overline{A} \cap \overline{B}$

Statement: Let x be an element hence by the defn of complement

$x \in \overline{A \cup B} \Leftrightarrow x \notin A \cup B$

Here,

$LHS = \overline{A \cup B}$ and $RHS = \overline{A} \cap \overline{B}$

$LHS = \overline{A \cup B}$

$= x \in \overline{A \cup B}$

$\Leftrightarrow x \notin A \cup B$

$\Leftrightarrow x \notin A$ or $x \notin B$

$\Leftrightarrow x \in \overline{A}$ or $x \in \overline{B}$

$\Leftrightarrow x \in \overline{A} \cap \overline{B}$

$LHS \Leftrightarrow \overline{A} \cap \overline{B}$

$\therefore LHS = RHS$

$\overline{A \cup B} = \overline{A} \cap \overline{B}$

Theorem 2] : Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$

Statement: Let x be an element hence by the defination of complement of set

$x \in \overline{A \cap B} \Leftrightarrow x \notin A \cap B$

Here,

$LHS = \overline{A \cap B}$ and $RHS = \overline{A} \cup \overline{B}$

$LHS = \overline{A \cap B}$

$= x \in \overline{A \cap B}$

$\Leftrightarrow x \notin A \cap B$

$\Leftrightarrow x \notin A$ or $x \notin B$

$\Leftrightarrow x \in \overline{A}$ and $x \in \overline{B}$

$\Leftrightarrow x \in \overline{A} \cup \overline{B}$

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LHS = RHS

$\overline{A \cap B} = \overline{A} \cup \overline{B}$

Theorem 3]: Let A & B are finite sets then

$|A \cup B| = |A| + |B| - |A \cap B|$

Proof: Let $A = \{a, b, c, d, e\}$

and $B = \{c, e, f, h, k, m\}$

Statement: Now, we have to find $A \cup B$

$A \cup B = \{a, b, c, d, e, f, h, k, m\}$

$A \cup B = 9$

$A \cap B = \{c, e\}$

$A \cap B = 2$

$|A| = \{a, b, c, d, e\}$

$= \{5\}$

$|B| = \{c, e, f, h, k, m\}$

$= \{6\}$

We have $|A \cup B| = 9$ $|A \cap B| = 2$

$|A \cap B| = 2$

$|A| = 5$

$|B| = 6$

Now, consider RHS of given expression

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$|A| + |B| - |A-B| = 5 + 6 - 2$

$= 11 - 2$

$= 9$

$= |A \cup B|$

$\therefore |A| + |B| - |A \cap B| = |A \cup B|$

$\therefore$ LHS = RHS

$|A \cup B| = |A| + |B| - |A \cap B|$

Hence proved

Theorem 4] Let A, B, C are three finite sets then $|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

Proof: Let $A = \{a, b, c, d, e\}$

$B = \{a, b, e, g, h\}$

$C = \{b, d, e, g, h, k, m, n\}$

$|A \cap B| = \{a, b, c\}$
$= \{3\}$

$|A| = \{a, b, c, d, e\}$
$= \{5\}$

$|B \cap C| = \{e, g, h\}$
$= \{3\}$

$|B| = \{a, b, e, g, h\}$
$= \{5\}$

$|A \cap C| = \{b, d, e\}$
$= \{3\}$

$|C| = \{b, d, e, g, h, k, m, n\}$
$= \{8\}$

$|A \cap B \cap C| = \{b, c\}$
$= \{2\}$

$|A \cup B \cup C| = \{a, b, c, d, e, g, h, k, m, n\}$
$= \{10\}$

Now consider RHS of given expression

$= |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

$= 5 + 5 + 8 - 3 - 3 - 3 + 2$

$= 10$

$= |A \cup B \cup C|$

$\therefore |A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$

* Cartesian product

It is a product of 1st element of first set to the first element of second set an so on it is called as cartesian product.

Let A & B be any two sets then sets of all ordered pairs such that first element of ordered pair is that element of A and second element is the element of B is called cartesian product of A & B & it is denoted by $A \times B$

$A \times B = \{(x, y) \mid x \in A \wedge y \in B)\}$

Ex: $A = \{1, 2\}$, $B = \{a, b\}$

$A \times B = \{(1, a)\ (1, b)\ (2, a)\ (2, b)\}$

→ Let $A = \{e, f, g, h\}$ & $B = \{1, 2, 3, 4\}$

$A \times B = \{(e, 1)\ (e, 2)\ (e, 3)\ (e, 4)\ (f, 1)\ (f, 2)\ (f, 3)\ (f, 4)\ (g, 1)\ (g, 2)\ (g, 3)\ (g, 4)\ (h, 1)\ (h, 2)\ (h, 3)\ (h, 4)\}$

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Show that

i) $(A \times (B \cup C) = (A \times B) \cup (A \times C)$

→ Let x be any element
consider $x \in A \times (B \cup C)$

$A \times (B \cup C) = \{(x, y) / \in A$ and $y \in (B \cup C)\}$

$= \{(x, y) / \in A$ and $(y \in B$ or $y \in C)\}$

$\{(x, y) / (x \in A)$ and $y \in B)$ or
$(x \in A$ and $y \in C)\}$ or by distributive law

$= \{(x, y) / x \in A$ and $y \in B\} \cup \{x, y) /$
$x \in A$ and $y \in C\}$

$= (A \times B) \cup (A \times C)$

$\therefore A \times (B \cup C) = (A \times B) \cup (A \times C)$

ii) $(A \times (B \cap C) = (A \times B) \cap (A \times C)$

→ Let x be any element
consider $x \in A \times (B \cap C)$

$A \times (B \cap C) = \{(x, y) / \in A$ or $y \in (B \cap C)\}$

$= \{(x, y) / \in A$ or $(y \in B \cdot y \in C)\}$

$= \{(x, y) / (x \in A$ or $y \in B)$ and
$(x \in A$ or $y \in C)\}$

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$= \{(x,y) / x \in A \text{ and } y \in B\} \cap \{(x,y) / x \in A$
$\text{or } y \in C\}$

$= (A \times B) \cap (A \times C)$

$\therefore \; A \times (B \cap C) = (A \times B) \cap (A \times C)$

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* Sequence :

Sequence is a each of object arranged in particular order it is mainly two types i.e.

1] finite sequence

2] Infinite sequence - 2 3 is an infinite

* Recursive formula :

Recursive formula it can be recursive form 1st term and the find out the next term.

Here the starting pt is given below

Ex: $C_1 = 5$, $C_n = 2C_{n-1}$, $2 \leq n \leq 6$

Now, $C_1 = 5$, $C_2 = 2C_{2-1} = 2C_1 = 2 \times 5 = 10$

$C_3 = 2$, $C_2 = 2C_{2-1} = 2C_1 = 2 \times 10 = 20$

$C_4 = 2$ $C_3 =$

$C_5 = 2$ $C_4 =$

$C_6 = 2$ $C_5 =$

Sequence = 5, 10, 20, 40, 80, 100

this are the sequence

Ex: Sequence = 3, 7, 11, 15, 19, 25

$C_1 = 3$, $C_n = 5 + 4 = 7$

$C_n = C_{n-1} + 4$

* Explicit formula :

Depending the position we find out the number of term.

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$a_n = n^2 \quad 1 \leq n \leq \infty$

ex: $S_n = (-4)^n, \; 1 \leq n$

if $n = 1$ & $S_1 = (-4) = -4$

$n = 2, \; S_2 = (-4)^2 = 16$

$n = 3 \quad S_3 = (-4)^3 = -64$

Sequence = $-4, 16, -64, \ldots$

Note: Alphabate → finite set of symbol

String → grouping this finite set of alphabet is known as string.

* Set Corresponding of Sequence:

The set of all distinct element in a sequence is called set corresponding to a sequence.

For ex: i] Sequence = a, b, a, b, a, b, b, b ---

→ set corresponding to a sequence is $\{(a, b)\}$

2] Sequence = 1, 3, 5, 7, 11

→ set corresponding to a sequence is $\{1, 3, 5, 7, 11\}$

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3] Write an explicit formula for sequence
2, 5, 8, 11, 14, 17

→ Explicit formula

$$a_n = 2 + 3(n-1)$$

* **Division in the Integer**

Imp

→ If m is an integer and n is a positive integer. We can plate the non negative integer multiples of n on a line and locate n.

If m is a multiple of n.

Say $n = gn$ we can write

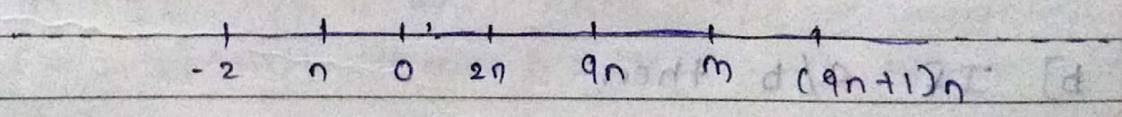
$\boxed{m = gn + r}$ Where $r = 0$ on the other line.

If m is not a multiple of n. we get g and be a first multiple of m and n line to the left of m therefore.

$$\boxed{r = m - qn}$$

then r is a distance from qn to m. So nearly $0 < r < n$ and again we have

$$\boxed{m = qn + r}$$



Theorem:

let a, b and c be integer

a) If $a|b$ and $a|c$, then $a|(b+c)$.
b) If $a|b$ and $a|c$, where $b > c$ then $a|(b-c)$
c) If $a|b$ and or $a|c$, then $a|bc$.
d) If $a|b$ and $b|c$, then $a|c$.

Proof: a] If $a|b$ then

$b = k_1 a$ where $k_1 \in I$

If $a|c$ then

$c = k_2 a$ where $k_2 \in I$

$b = k_1 a$ — ①
$c = k_2 a$ — ②

Adding ① & ②

$b + c = k_1 a + k_2 a$
$b + c = (k_1 + k_2) a$
$(b+c) = a$
$\Rightarrow \frac{a}{(b+c)}$

b] If $a|b$ then

$b = k_1 a$ where $k_1 \in I$

If $a|c$ then

$c = k_2 a$ where $k_2 \in I$. (1)

$b = k_1 a$ — ①
$c = k_2 a$ — ②

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Subtracting ① & ②

$$b - c = k_1 a - k_2 a$$
$$b - c = (k_1 - k_2) a$$
$$b - c = a$$
$$\Rightarrow \frac{a}{(b-c)}$$

c] If a/b then

$b = k_1 a$ — ① where $k_1 \in I$

multiply eqn ① by c

$cb = k_1 c a$ — ② $k_1 c \in I$

$= \frac{a}{bc}$

Similarly ;

If a/c then

$c = k_2 a$ — ② where $k_1 \in I$

multiply eqn ③ by 'b'

$bc = k_2 b a$ — ①, where $k_2 b \in I$

a/bc

$\therefore$ If a/b or a/c, then a/bc

d] If a/b & b/c then a/c

If a/b then $b = k_1 a$ — ① where $k_1 \in I$

If b/c then $c = k_2 b$ — ② where $k_2 \in I$

by putting value of b in eqn ②

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eqn (1) $\Rightarrow c = k_2 b$

$\Rightarrow c = k_2 (k_1 a)$

$\Rightarrow c = (k_2 k_1) a$

$\Rightarrow a/c$

* Prime Number :

A number i.e. $p > 1$ in Z^+ is called prime if and only iff positive integer that device p and 1.

ex: A number 2, 3, 5, 7, 11, 13 are the prime number.

4, 10, 16, 21, --- 50 on is are not a prime number.

for the given integer m and n we write m as $qn + r$ $1 \leq r \leq n$.

1) $m = 20$, $n = 3$
2) $m = 64$, $n = 87$
3) $m = 3$, $n = 22$
4) $m = 38$, $n = 12$

1] If $m = 20$ $n = 3$

$m = qn + r$ $q = 1, 2, 3, \ldots n$

$20 = 6 \times 3 + 2$

$20 = 18 + 2$

$20 = 20$

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* GCD (Greatest Common Divisor)

If a, b & k are in Z^+ and k/a, k/b we say that k is a common divisor of a and b. If d is the largest such that k, d is called the greatest common divisor OR GCD of a and b

$d = GCD(a, b)$

* Theorem : List Common Multiple (LCM)

If A, B & K are in Z^+ (i.e. any positive integer) and a/k, b/k we say that k is a common multiple of a & b. The smallest element such that k called it's 'c'.

∴ C is called ~~list~~ least common multiple OR LCM of (a, b)

i.e. $C = LCM(a, b)$

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** Imp * **Theorem :**

8m **Statement :**

If a & b are positive integer then

$GCD(a,b) \cdot LCM(a,b) = a \cdot b$

Proof :

Let $p_1, p_2 ----- p_k$ be a prime factor of either a or b.

then we write

$$a = p_1^{a_1} \cdot p_2^{a_2} ------ p_k^{a_k}$$
$$b = p_1^{b_1} \cdot p_2^{b_2} ------ p_k^{b_k}$$

Where sum of the a_i & b_i may be zero

If follows that,

$$GCD(a,b) = p_1^{\min(a_1, b_1)} \cdot p_2^{\min(a_2, b_2)} ------ p_k^{\min(a_k, b_k)}$$

and

$$LCM(a,b) = p_1^{\max(a_1, b_1)} \cdot p_2^{\max(a_2, b_2)} ------ p_k^{\max(a_k, b_k)}$$

$$GCD(a,b) \cdot LCM(a,b) = p_1^{a_1+b_1} \cdot p_2^{a_1+b_2} --- p_k^{a_k+b_k}$$

$$= (p_1^{a_1} \cdot p_2^{a_2} --- p_k^{a_k})(p_1^{b_1} \cdot p_2^{b_2} --- p_k$$

$$= a \cdot b$$

$$\therefore \boxed{GCD(a,b) \cdot LCM(a,b) = a \cdot b}$$

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Ex: Let $a = 540$, $b = 504$ factories a & b into prime

$\Rightarrow$ $a = 540 = 2 \times 2 \times 5 \times 3 \times 3 \times 3 = 2^2 \cdot 3^3 \cdot 5^1$

$b = 504 = 2 \times 2 \times 2 \times 3 \times 3 \times 7 = 2^3 \cdot 3^2 \cdot 7^1$

Thus, all the prime number that are the factor of either a & b.

$P_1 = 2$, $P_2 = 3$ $P_3 = 5$ $P_4 = 7$

then;

$a = 2^2 \cdot 3^3 \cdot 5^1 \cdot 7^0$

$b = 2^3 \cdot 3^2 \cdot 5^0 \cdot 7^1$

Now

$GCD(540, 504) =$

$= 2^{\min(2,3)} \cdot 3^{\min(3,2)} \cdot 5^{\min(1,0)} \cdot 7^{\min(1,0)}$

$= 2^2 \cdot 3^2 \cdot 5^0 \cdot 7^0$

$= 4 \cdot 9 \cdot 1 \cdot 1 = 36$

$LCM(540, 504) =$

$= 2^{\max(2,3)} \cdot 3^{\max(3,2)} \cdot 5^{\max(1,0)} \cdot 7^{\max(1,0)}$

$= 2^3 \cdot 3^3 \cdot 5^1 \cdot 7^1$

$= 8 \cdot 27 \cdot 5 \cdot 7$

$= 7560$

$GCD(540, 504) \cdot LCM(540 \cdot 504) = 36 \times 7560$

$= 272160$

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MATRIX

A matrix is a rectangular a
arranged in a M i.e. m Horizontal row
and N is a horizontal columns.

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{2n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

The ith row of A is $[a_{i1}, a_{i2} \cdots a_{in}]$
$1 < i \leq m$

and j^{th} column of $\begin{bmatrix} a_{j1} \\ a_{j2} \\ \vdots \\ a_{jn} \end{bmatrix}$ $1 < j \leq n$

We say that A is m/n and written as 'M × N'.

If $m = n$

we say that A is called a square matrix of order n and that the number $[a_{11}, a_{22} \cdots a_{nm}]$ for may diagonal of A.

Ex: 1) $B = \begin{bmatrix} 2 & 1 \\ 0 & 6 \end{bmatrix}$

$\Rightarrow$ where $m = 2$, $n = 2$

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$$A = \begin{bmatrix} 2 & 3 & 5 \\ 0 & -1 & 2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 3 \\ 4 & 6 \end{bmatrix} \quad C = \begin{bmatrix} 1, -1, 3, 4 \end{bmatrix}$$

$$D = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 0 & -1 \\ -1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$$

Find a_{12}, a_{23}, b_{21}, b_{12} d_{21}, e_{21} ?

$\Rightarrow$ $\therefore a_{12} = 3$ $\therefore a_{23} = 2$ $b_{21} = 4$ $b_{12} = 3$

$d_{21} = 2$ $e_{21} = -1$

* Diagonal of Matrix *

A square matrix $A = [a_{ij}]$ for which every entry of the main diagonal is zero i.e. $[a_{ij}] = 0$ for $i \neq j$ is called diagonal of matrix.

Ex: $A = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$ is a diagonal matrix.

* Equal Matrix *

Two matrix $m \times n$

$A = [a_{ij}]$ & $B = [b_{ij}]$ are set to be equal if $a_{ij} = b_{ij}$

where condition i is ranging from $1 < i \leq m$

and j is ranging from $1 < j \leq n$

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i.e. its corresponding element are also same

Ex: $A = \begin{bmatrix} 2 & -3 & -1 \\ 0 & 5 & 2 \\ 4 & -4 & 6 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & x & -1 \\ y & 5 & 2 \\ 4 & -9 & z \end{bmatrix}$

if $A = B$ then find x, y, z

$\Rightarrow \therefore x = -3 \quad y = 0 \quad z = 6$

* Zero Matrix *

A matrix all A of all entries are zero then that matrix is called as a zero matrix and its is denoted by zero

i.e. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

* Matrix Addition *

If $A = [a_{ij}]$ and $B = [b_{ij}]$ are $m \times n$ matrix then sum of A and B is the matrix $[C_{ij}]$ defined by $C_{ij} = a_{ij} + b_{ij}$

Where $i <$

C is obtained by adding corresponding elements of A and B and it is denoted by $C = A + B$.

Ex: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}$

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$$A+B = \begin{bmatrix} 1+5 & 2+6 \\ 3+7 & 4+8 \end{bmatrix} = C = \begin{bmatrix} 6 & 8 \\ 10 & 12 \end{bmatrix}$$

- Note: Sum of matrix A and B is defined only when A and B have the same number of rows and same number of columns.

* Matrix Multiplication *

If $A = [a_{ij}]$ is an $m \times p$ matrix $B = [b_{ij}]$ is $p \times n$ matrix then product of A and B denoted by AB is the $m \times n$ matrix.

Then $C = [c_{ij}]$ and it is denoted by c_{ij} =

$$c_{ij} = a_{ij} \cdot b_{1j} + a_{i2} \cdot b_{2j} + \cdots a_{ip} \cdot b_{pj}$$

Where $1 < i \leq m$

$1 < j \leq n$

Ex: $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ & $B = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

$$A \cdot B = \begin{bmatrix} 2+8 & 4+12 \\ 1+4 & 2+6 \end{bmatrix} = \begin{bmatrix} 10 & 16 \\ 5 & 8 \end{bmatrix}$$

* Identity Matrix

The matrix $m \times n$ is a diagonal matrix i.e. $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 1 \end{bmatrix}$ Where diagonal elements are I is called Identity matrix of order n.

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* Transpose of Matrix :

If $A = [a_{ij}]$ is $m \times n$ matrix then $n \times m$ matrix is equal to it

$A^T = (a_{ij}^T)$ where $a_{ij}^T = a_{ji}$

$1 \leq i \leq m$,

$1 \leq j \leq n$

is called transpose of matrix, and it is denoted by A^T.

Ex: Let matrix $A = \begin{bmatrix} 2 & -3 & 5 \\ 6 & 1 & 3 \end{bmatrix}$ $B = \begin{bmatrix} 3 & 4 & 5 \\ 2 & -1 & 0 \\ 1 & 6 & -2 \end{bmatrix}$

Find A^T & B^T

$\Rightarrow$ $A^T = \begin{bmatrix} 2 & 6 \\ -3 & 1 \\ 5 & 3 \end{bmatrix}$ $B^T = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -1 & 6 \\ 5 & 0 & -2 \end{bmatrix}$

* Symmetric Matrix of Transpose Matrix

A matrix $A = [a_{ij}]$ is called symmetric If transpose $A^T = A$

Ex: 1] $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & 5 \\ -3 & 5 & 6 \end{bmatrix}$ $A^T = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & 5 \\ -3 & 5 & 6 \end{bmatrix}$

Here $A^T = A$

$\therefore$ A is symmetrix matrix.

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2) $B = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & 5 \\ -3 & 5 & 6 \end{bmatrix}$ $B^T = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

Here $B^T \neq B$

$\therefore$ B is not symmetric matrix.

* Inverse of Matrix

If ~~A~~ and B are $n \times m$, then we say that B is an inverse of A.

if $AB = I_n$ and $BA = I_n$

Where I_n is an $n \times n$ identity matrix.

denoted by A^{-1}

Ex: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ & $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$ and we have

$ae + bg = 1$, $\rightarrow$ boolean matrix

$ce + dg = 0$,

and $af + bh = 0$, $cf + dh = 1$, Then find A & B are inverse matrix.

$\Rightarrow$ Given, $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$

$$\Rightarrow AB = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} e & f \\ g & h \end{bmatrix} \Rightarrow \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{--- (1)}$$

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$$BA = \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} ea + fc & eb + \\ ga + hc & gb + \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{--- (2)}$$

$\therefore$ By eq^n (1) & (2)

$$AB = BA = I$$

$\therefore$ So that A and B are inverse matrix.

Mathematical logic

* Statement & Notation:

Statements:

All declarative sentense one can assign one of the two possible truth value are called statement.

Truth Value:

Declarative sentense will be admitted in the object that have one and only one two possible values is called as truth value. This value is denoted by T (true) and F (false) and also denoted by symbol 1 and 0.

Ex: Canada is a country

$\Rightarrow$ Truth Value = T (true) and it is a statement.

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Ex: 100 + 1 = 102

⇒ It is not a statement
Truth value = F (false) = 0
It is a Equation.

Truth table
A truth table displace the relations betn truth values of proposition all statement.

* Connectives 14/08/19
* Propositions & logical operations

• Prepositions of statement:
A preposition is statement which is either true or false but not both.

Ex: Which of following of the statement.
1) Earth is round
It is a statement

2) Do you speak english
⇒ It is a not statement.
It is a question.

• Prepositions variables:
The lower case letter starting from p to onwards are used to represent preposition.

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Ex: p = India is an Asia
q = $2+2=4$

* Types of preposition of Statement.

1) Simple Statement:

A statement which does not contain any logical connectives is called simple statement.

2) Compound Statement:

A statement which contain one or more simple statement and some logical connectives is called compound statement.

Ex: p = The sun is shining today.
q = It is gold.
∴ Compound statement = $p + q$
= T
∴ C.S = The sun is shining today + It is g

* Logical Connectives

• Negation ($\sim$):

If p is a statement then negation of p 'NOT p' and it is denoted by '$\sim p$'. It follows that if p is true and $\sim p$ is false.

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- Truth table

p	~p
T	F
F	T

Ex: Give the example of following statement.

p : $2+3>1$
q : It is cold

⇒ p : $2+3>1$
p : 2+3 is greater than 1

q : It is cold
~q : It is hot.

* Conjunctions :

If p and q are the statement the conjunction of p and q is the compound statement of P and Q.

It is denoted by '$p \wedge q$'. The connectives '$\wedge$' is denoted by symbol '$\wedge$'. The compound statement p & q when both p and q are true otherwise false.

$\wedge$ + = F
$\vee$ = T

- Truth table

p	q	$p \wedge q$ (+)
T	T	T
T	F	F
F	T	F
F	F	F

Ex: from the conjunction of p & q for each of the following statement.

p : It is showing q : I am cold
p : $2 < 3$ q : $-5 > -8$

⇒ $q \wedge p$: It is showind and I am cold.
p : $2 < 3$ q : $-5 > -8$
$p \wedge q$: $2 < 3 \wedge -5 > -8$

* Disjunction :

If p and q are statement the disjunction of a p & q is a statement is 'p or q'. And it is denoted by '$p \vee q$'.

The Connective or denoted by symbol. The Compound stat $p \vee q$ is true. If at least one of them is true, it is false when p and q are false.

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$0 \cdot 0 = 0$ T $\wedge$
$0 \cdot 1 = 1$ F
$1 \cdot 0 = 1$ F
$1 \cdot 1 = 1$ F

V
T
T
T
F

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- Truth table (V) (•)

p	q	p V q
T	T	T
T	F	T
F	T	T
F	F	F

Make the truth table of the following statement.

1) $(p \wedge q) \vee (\sim p)$

p	q	$\sim p$	$p \wedge q$	$(p \wedge q) \vee (\sim p)$
T	T	F	T	T
T	F	F	F	F
F	T	T	F	T
F	F	T	F	T

2) $\sim p \wedge \sim q$

p	q	$\sim p$	$\sim q$
T	T	F	F
T	F	F	T
F	T	T	F
F	F	T	T

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3) $\sim(p \vee q) \wedge r$

p	q	r	$p \vee q$	$(p \vee q) \wedge r$	$\sim(p \vee q)$
T	T	T	T	T	F
T	T	F	T	F	T
T	F	T	T	T	F
T	F	F	T	F	T
F	T	T	T	T	F
F	T	F	T	F	T
F	F	T	F	F	T
F	F	F	F	F	T

* Conditional Statement :

If p & q are statement, the compound statement 'if p then q' is called conditional or indicational statement. The connective 'if ---- then' and it is denoted by symbol '$\rightarrow$'

• Truth table $\rightarrow$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

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Write the integration $p \to q$ for each of the following statement.

p : I am hungry
q : I will eat

If i am hungry then i will eat.

* Biconditional statement :

If p & q are statement then the compound statement p if and only iff q ($\leftrightarrow$) denoted by '$p \leftrightarrow q$'.

- Truth table

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	f	T

① $(p \to q) \leftrightarrow (\sim q \to \sim p)$
② $(p \to q) \leftrightarrow (p \vee q)$
③ $(q \to r) \leftrightarrow (q \wedge r)$

* $(p \to$

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1) $(p \to q) \leftrightarrow (\sim p \to \sim q)$

1	2	3	4	5	6	
p	q	$\sim p$	$\sim q$	$p \to q$	$\sim p \to \sim q$	$5 \leftrightarrow 6$
T	T	F	F	T	T	T
T	F	F	T	F	T	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

2) $(p \to q) \leftrightarrow (p \vee q)$

p	q	$p \to q$	$p \vee q$	$(p \to q) \leftrightarrow (p \vee q)$
T	T	T	T	T
T	F	F	T	F
F	T	T	T	T
F	F	T	F	F

3) $(p \to r) \leftrightarrow (q \wedge r)$

p	q	r	$p \to r$	$q \wedge r$	$(p \to r) \leftrightarrow (q \wedge r)$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	F	F
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	F	F	F
F	F	T	T	F	F
F	F	F	T	F	F

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★ Tautology:

A statement formula which is true regardless of two value of statement which is replace the variables in it is called universal valid formula or logical truth tautology

It means it contain only T in the final column of its truth table.

Ex: ~~Tra~~ at the following statement

$(p \to q) \Leftrightarrow (\sim q \to \sim p)$ is tautology

1	2	3	4	5	6	7
p	q	$p \to q$	$\sim p$	$\sim q$	$4 \to 5$	$3 \Leftrightarrow 6$
T	T	T	F	F	T	T
T	F	F	F	T	F	T
F	T	T	T	F	T	T
F	F	T	T	T	T	T

∴ Final column No 7 contains all T's so it is tautology.

★ Contradiction:

A statement that is always false called Contradiction.

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Show that the statement $P \wedge \sim P$ is a contradiction.

P	$\sim P$	$P \wedge \sim P$
T	F	F
T	F	F
F	T	F
F	T	F

$\therefore$ Final column contain all false.
so it is contradiction.

★ Contigency:

A statement that can be either true or false depending on the truth value of ~~the~~ its variable its called contigency.

Ex: Prove that the statement $(P \rightarrow Q) \rightarrow (P \wedge Q)$ is contiagency.

P	Q	$P \rightarrow Q$	$P \wedge Q$	$(P \rightarrow Q) \rightarrow (P \wedge Q)$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	F
F	F	T	F	F

As the value of the final column depending on the truth value of the variable

$\therefore$ It is a contigency.

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Q. 1: So that the following formulas which once are tautology, contradiction & contigency

a) $((P \to (\sim P)) \to \sim P$

b) $((P \wedge Q) \Leftrightarrow P)$

c) $((\sim Q \wedge P) \vee \sim Q)$

S-18 W-17 d) $((P \to (Q \to R) \to ((P \to Q) \to (P \to R)))$

⇒ 1) $((P \to (\sim P)) \to \sim P$

P	$\sim P$	$P \to (\sim P)$	$(P \to (\sim P)) \to \sim P$
T	F	F	T
T	F	F	T
F	T	T	T
F	T	T	T

∴ It is tautology

2) $((P \wedge Q) \Leftrightarrow P)$

P	Q	$P \wedge Q$	$(P \wedge Q) \Leftrightarrow P$
T	T	T	T
T	F	F	F
F	T	F	T
F	F	F	T

∴ It is contigency.

3) $(\sim Q \wedge P) \vee \sim Q)$

P	Q	$\sim Q$	$\sim Q \wedge P$	$(\sim Q \wedge P) \vee \sim Q)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	F
F	F	T	F	T

∴ It is contigency

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4) $((P \to (Q \to R) \to ((P \to Q) \to (P \to R)))$

1	2	3	4	5	6	7	8	9
P	Q	R	$Q \to R$	$P \to Q$	$P \to R$	$1 \to 4$	$5 \to 6$	$7 \to 8$
T	T	T	T	T	T	T	T	T
T	T	F	F	T	F	F	F	T
T	F	T	T	F	T	T	T	T
T	F	F	T	F	F	T	T	T
F	T	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	T	T	T

The final column T's all.

$\therefore$ It is tautology.

Q.2 From the given formula indicate which one are toutology & contradiction.

i) $(P \to (P \vee Q)$

P	Q	$P \vee Q$	$P \to (P \vee Q)$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

$\therefore$ It is tautology.

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2) $((\sim q \wedge p) \wedge q)$

p	q	$\sim q$	$(\sim q \wedge p)$	$(\sim q \wedge p) \wedge q$
T	T	F	F	F
T	F	T	T	F
F	T	F	F	F
F	F	T	F	F

$\therefore$ It is Contradiction.

* Equivalent OR Logical Equipment

If two prepositions are equal then we say that p & q are equivalent OR logical equivalent.

We denote that, '$p \cong q$'

Ex: To find equivalent statement

1) $p \to q \cong (\sim p) \vee q$

1	2	3	4	5
p	q	$\sim p$	$p \to q$	$(\sim p) \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

$\therefore$ eqn (4) & (5) are equivalent

2) $p \vee q \cong q \vee p$

p	q	$p \vee q$	$q \vee p$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	F

$\therefore$ eqn (3) & (4) are equivalent.

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3) $p \to q \cong (\sim p) \wedge q$

2)

p	q	$\sim p$	$p \to q$	$(\sim p) \wedge q$
T	T	F	T	F
T	F	F	F	F
F	T	T	T	T
F	F	T	T	F

$\therefore$ eq^n (4) & (5) values are not equal

$\therefore p \to q \ncong (\sim p) \wedge q$ are not equivalent.

4) show that prepositions $q \to p$ & $\sim p \to \sim q$ is not equivalent to $p \to q$

$\Rightarrow$

p	q	$q \to p$	$\sim p$	$\sim q$	$\sim p \to \sim q$	$p \to q$
T	T	T	F	F	T	T
T	F	T	F	T	T	F
F	T	F	T	F	F	T
F	F	T	T	T	T	T

$\therefore q \to p$ & $\sim p \to \sim q \ncong p \to q$

5) To show that $p \wedge q \cong q \wedge p$

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

$\therefore$ eq^n (3) & (4) values are equal.

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6) To show that $p \vee (q \vee r) \cong (p \vee q) \vee r$

7) De-morgan's law

1) $\sim(p \vee q) \cong \sim p \wedge \sim q$

2) $\sim(p \wedge q) \cong \sim p \vee \sim q$

* Derive Connectors

1) NAND GATE :

It means negation after AND of two statements. Assume p & q be a preposition then NAND of p & q be a preposition which is false when Both p & q are true, otherwise true. It is denoted by $p \uparrow q$

p	q	$p \uparrow q$
T	T	F
T	F	T
F	T	T
F	F	T

2) NOR Gate :

It means negation after OR of two statements. Assume p & q be a two prepositions. NOR of p & q be a preposition which is true when both p & q are false other wise false. It is denoted $p \downarrow q$

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P	q	$P \downarrow q$
T	T	F
T	F	F
F	T	F
F	F	T

$\downarrow$

TT = F
TF = F
FT = F
FF = T

3) X-OR Gate :

Assume P & q be two prepositions. XOR P & q are true or if q is true or not both and vice versa and it is denoted by $P \oplus q$

P	q	$P \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Both are true the value is true. And

$\oplus$

$\left.\begin{matrix} TT \\ FF \end{matrix}\right\} = F$ $\left.\begin{matrix} TF \\ FT \end{matrix}\right\} = T$

Ex: Construct the truth table for following.

1) $(A \oplus B) \oplus C$
2) $(A \uparrow B) \uparrow C$
3) show that $(P \oplus q) \vee (P \downarrow q)$ is equivalent $P \uparrow q$

$\Rightarrow$ 2) $(A \uparrow B) \uparrow C$

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- Construct the truth table following ~~Boxx~~ formula
$(\sim P \wedge (\sim Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R)$
- Give the truth table value P and q as T and R and s are ~~of~~. Find the truth table of the following statement.

a) $(P \wedge (Q \wedge R)) \vee \sim((P \vee Q) \wedge (R \vee S))$

$\Rightarrow$

$T \wedge (T \wedge F) \vee \sim((T \vee T) \wedge (F \vee F))$

$T \wedge F \vee \sim((T \vee T) \wedge (F \vee F))$

$T \wedge F \vee \sim(T \wedge (F \vee F))$

$T \wedge F \vee \sim(T \wedge F)$

$T \wedge F \vee \sim F$

$F \vee T$

$= T$

b) $(\sim(P \wedge Q) \vee \sim R) \vee (((\sim P \wedge Q) \vee \sim R) \vee S)$

$\Rightarrow (\sim(T \wedge T) \vee \sim F) \vee (((\sim T \wedge T) \vee \sim F) \vee F)$

$(\sim T \vee \sim F) \vee (((\sim T \wedge T) \vee \sim F) \vee F)$

$(\sim T \vee \sim F) \vee ((F \vee \sim F) \vee F)$

$(\sim T \vee \sim F) \vee (T \vee F)$

$T \vee (T \vee F)$

$T \vee T$

T

$\therefore (\sim(P \wedge Q) \vee \sim R) \vee (((\sim P \wedge Q) \vee \sim R) \vee S) = T$

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* Equivalence of Formula :

Let A and B are two statements formula and let P_1, P_2 ---- denotes all the variable occuring in both A & B. If truth value of A is equal to the value of B or for every one of the second possible sets of truth value.

If assign to P_1, P_2 ---- P_n then A & B are set to be equivalent. Here are some examples of formula which are equivalent. We should verify their equivalent by truth table.

1) $\sim\sim P \cong P$ (Negation of Negation P is equivalent to P.)

2) $P \vee P \cong P$ (P disjunction P is equivalent to P)

3) $(P \wedge \sim P) \vee Q$

A list of basic formula which is useful is given below in order to make the list complete we have to use a T, and F as a special variables in the sense that P can be replace by only a tautology & F by only contradiction.

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* Equivalent formula :

	'V'	'∧'	Law
1)	$P \vee P \Leftrightarrow P$	$P \wedge P \Leftrightarrow P$	
2)	$(P \vee (Q \vee R) \Leftrightarrow P \vee (Q \vee R)$	$(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R)$	Associative law
3)	$P \vee Q \Leftrightarrow Q \vee P$	$P \wedge Q \Leftrightarrow Q \wedge P$	Commutative law
4)	$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$	$P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R)$	Distributive law
5)	$P \vee F \Leftrightarrow P$	$P \wedge T \Leftrightarrow P$	–
6)	$P \vee T \Leftrightarrow T$	$P \wedge F \Leftrightarrow F$	–
7)	$P \vee \sim P \Leftrightarrow T$	$P \wedge F \Leftrightarrow F$	–
8)	$P \vee (P \wedge Q) \Leftrightarrow P$	$P \wedge (P \wedge Q) \Leftrightarrow P$	Absorption law
9)	$\sim(P \vee Q) \Leftrightarrow \sim P \wedge \sim Q$	$\sim(P \wedge Q) \Leftrightarrow \sim P \vee \sim Q$	De-Morgan's law

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$\neg$ — Negation sign

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Q. Show that $[\neg P \wedge (\neg Q \wedge R)] \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$

$\Rightarrow ([\neg P \wedge \neg Q] \wedge R) \vee (Q \vee P) \wedge R$

--- By distribution Law

$\rightarrow ([\neg P \wedge \neg Q] \vee [Q \vee P]) \wedge R$

--- By Associative law

$\rightarrow (\neg [P \vee Q] \vee [Q \vee P] \wedge R)$

$\because$ De' Morgan's law

$\rightarrow (\neg (P \vee Q) \vee [P \vee Q] \wedge R)$

--- By commutative law

Here in above expression

$[\sim [P \vee Q] \vee [P \vee Q]$ is a substitution instance for (i.e. $P \vee \neg P = T$)

$\Rightarrow T \wedge R$

$\Rightarrow R$ $\quad \because$ By law ($P \vee T = T$ & $P \wedge F = F$)

here $T \wedge R = R$

$\therefore \neg P \wedge (\neg Q \wedge R) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$ //

Q. Show that $[(P \vee Q) \wedge \neg (\neg P \wedge (\neg Q \vee \neg R))] \vee [(\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)]$ is tautology.

Imp

$\Rightarrow$ Using De-Morgan's law

$(\neg P \wedge \neg Q) = \neg (P \vee Q)$

$(\neg P \wedge \neg R) = \neg (P \vee R)$

$\therefore [(\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R) \Leftrightarrow (\neg (P \vee Q) \vee \neg (P \vee R)$

$\Leftrightarrow \neg [(P \vee Q) \wedge (P \vee R)]$ --- (1)

Also,

$\neg (\neg P \wedge (\neg Q \vee \neg R) \Leftrightarrow \neg [(\neg P \wedge \neg (Q \wedge R)]$

$\Leftrightarrow P \vee (Q \wedge R)$

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Hint Note: $P \to (Q \to R) \Rightarrow P \to (\neg Q \vee R)$

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$P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R)$ ..... (2)

... By distribution law

Thus,

$[(P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))] \vee [(\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R))]$

$\Rightarrow (P \vee Q) \wedge (P \vee Q) \wedge (P \vee R) \vee [\neg (P \vee Q) \wedge (P \vee R)]$

$\Rightarrow (P \vee Q) \wedge (P \vee R) \vee (\neg (P \vee Q) \wedge (P \vee R))$

$\therefore P \vee \neg P = T$

$\Rightarrow T$

$\therefore [(P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))] \vee [(\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)]$ is tautology.

Hence proved.

H.W.

1) Show that $P \to (Q \to R) \Leftrightarrow [P \to (\neg Q \vee R)] \Leftrightarrow (P \wedge Q) \to R$

2) Show that $P \to (Q \to R) \Leftrightarrow \neg P \to (P \to Q)$

3) Show that $P \to (Q \vee R) \Leftrightarrow (P \to Q) \vee (P \to R)$

4) Show that $(P \to Q) \wedge (R \to Q) \Leftrightarrow (P \vee R)$

5) Show that $\neg (P \to Q) \Leftrightarrow P \wedge \neg Q$

6) Show that $\neg (P \Leftrightarrow Q) \Leftrightarrow (P \wedge \neg Q) \vee (\neg P \vee Q)$

Hint: $\neg (P \Leftrightarrow Q) = \neg [(P \to Q) \wedge (Q \to P)]$

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* Duielity Law:

Q. What is a duielity law. Write the duals of following expression

i) $(P \vee Q) \wedge R$

ii) $(P \wedge Q) \vee T$

iii) $\sim(P \vee Q) \wedge (P \vee \sim(Q \wedge \sim S))$

Two ~~Fo~~ formulas A & A^* are ~~set~~ said to be duals of each other if either one can be obtained from the other by replacing conjunction ($\wedge$) by disjunction ($\vee$) and $\vee$ by $\wedge$

The connectives $\wedge$ & $\vee$ are also called duals of each other. If the formula A contains the special variable T or F, then A^* its duals is obtain by replacing T by F & F by T

Ex: i) $(P \vee Q) \wedge R$ ii) $(P \wedge Q) \vee T$

$\rightarrow (P \wedge Q) \vee R$ $\rightarrow (P \vee Q) \wedge F$

iii] $\sim(P \vee Q) \wedge (P \vee \sim(Q \wedge \sim S))$

$\rightarrow \neg(P \wedge Q) \vee (P \wedge \neg(Q \vee \neg S))$

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* Normal forms

A is set In mathematical logic A is set to be identically true and not tautology

If A has all the truth value as true similarily A is set be contradiction if all the truth value of A is false.

There is one more condition apart form contradiction & tautology. If A has truth value T for at least one combination of truth value assign then A is said to be satisfied condition

The problem of determining whether given formula is tautology or contradiction, or at least satisfied condition is known as decision problem. The construction of truth table may not be practical even with the aid of computer.

∴ We have to consider the other procedure known as reduction to normal form.

I] Disjunctive Normal form: (Sum or product)

It will be conveient to use the word in place of "conjunction" and sum place of "disjunction".

i] Elementry Product:

A product of variable & negation, a formula is called elementry product & etc

Elementry sum:

A sum of variable and then negation in a formula is called elementry sum.

★ Define disjunctive Normal form:

Let p & q are any two automatic variable, then p, ~~p & q~~, ~~q~~ & $\neg p \wedge q$, $\neg q \wedge p \wedge \neg p$, $p \wedge \neg p$, & $q \wedge \neg p$ are some examples of elementry product. on the other hand p, $\neg p \vee q$, $\neg q \vee p \vee \neg p$, $p \vee \neg p$, ~~q~~ $q \vee \neg p$ are the examples of elementry sum.

A formula which is equivalent to given formula at which consist of sum of elementry product is called as disjunctive Normal forms.

1] obtain the disjunctive Normal form the following formula

1) $p \wedge (p \rightarrow q)$

$\rightarrow p \wedge (\neg p \vee q)$ $\quad \because p \rightarrow q = \neg p \vee q$

$\rightarrow (p \wedge \neg p) \vee (p \wedge q)$ — by distributive law

2) $\neg(p \vee q) \Leftrightarrow (p \wedge q)$

$\rightarrow$

Consider $(p \Leftrightarrow q)$

we have to use equivalent formulas for above

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Statement formula which contains biconditional connectives so that, replace the given biconditional & conditional connectives.

$P \Leftrightarrow Q \Leftrightarrow (P \to Q) \wedge (Q \to P)$

$\Leftrightarrow (\neg P \vee Q) \wedge (\neg Q \vee P)$

$\Leftrightarrow (\neg P \wedge \neg Q) \vee (\neg P \vee P) \vee (Q \wedge \neg Q) \vee (Q \wedge P)$

--- By distributive law

$\Leftrightarrow (\neg P \wedge \neg Q) \vee F \vee F \vee (Q \wedge P)$

$\therefore P \vee F \Leftrightarrow P$ & $P \wedge T \Leftrightarrow P$

$\Leftrightarrow (\neg P \wedge \neg Q) \vee (Q \wedge P)$

$\Leftrightarrow (P \wedge Q) \vee (\neg P \wedge \neg Q)$

--- By commutative law

Hence, we get

$P \Leftrightarrow Q = (P \wedge Q) \vee (\neg P \wedge \neg Q)$

---- (A)

Now consider,

$\neg(P \vee Q) \Leftrightarrow (P \wedge Q)$

with the reference of eqn (A)

$\Rightarrow [\neg(P \vee Q) \wedge (P \wedge Q)] \vee [\neg(\neg(P \vee Q)) \wedge \neg(P \wedge Q)]$

---- By De-Morgan's law

$\Rightarrow [\neg P \wedge \neg Q \wedge P \wedge Q)] \vee [((P \vee Q) \wedge \neg P) \vee ((P \vee Q) \wedge \neg Q))]$

---- by distributive law

$\Rightarrow \neg P \wedge \neg Q \wedge P \wedge Q \vee (P \wedge \neg P) \vee (Q \wedge \neg P) \vee (P \wedge \neg Q) \vee (Q \wedge \neg P)$

---- By distributive law.

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$\Leftrightarrow$ = bicon

* Procedure to find out disjunctive Normal formas

1) If the connectives '$\rightarrow$' and "$\Leftrightarrow$" appear to a given formula then equivalent formula can be obtain by replacing this connectives with 'V' & '$\wedge$'

2) If the negation is applied to the formula & not to the variables then by using De-morgan's law and equivalent formula can be obtain apply negation to separate variable.

* Conjunctive Normal form

A formula which is equivalent to given formula and which consist of product of elementary sum is called a conjunctive normal form of a given formula. The method for obtaining a conjunctive normal form is similar to disjunctive normal form. The conjunctive normal form is not unique. Further more a given formula is identically true if every elementary sum in its conjunctive normal form in identically true.

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obtain the conjunctive normal form of the following ~~to~~ formula.

1) $P \wedge (P \rightarrow Q)$

$\rightarrow P \wedge (\neg P \vee Q)$

$\rightarrow (P \wedge \neg P) \wedge (P \wedge Q)$

$\rightarrow (P \wedge P) \wedge (P \vee Q) \wedge (\neg P \vee P) \wedge (\neg P \vee Q)$

$\rightarrow (P \vee P)$

* Principal Disjunctive Normal Form (PDNF):

$(\wedge) \vee (\wedge) \vee (\wedge) \rightarrow T$

$(P \vee Q) \rightarrow \neg R$

P	Q	R	$\neg R$	$P \vee Q$	$P \vee Q \rightarrow \neg R$	
T	T	T	F	T	F	
T	T	F	T	T	T	$(P \vee Q \vee \neg R)$
T	F	T	F	T	F	$\wedge$) $\vee$
T	F	F	T	T	T	$T \vee \neg Q \vee \neg R$
F	T	T	F	T	F	
F	T	F	T	T	T	$\neg P \vee Q \vee \neg R$
F	F	T	F	F	T	$\neg P \vee \neg Q \vee R$
F	F	F	T	F	T	$\neg P \vee \neg Q \vee \neg R$

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let p & q be a two statement variable. let us contruct all possible formula which consist of conjunction of p & its negation & conjunction of q & its negation. None of the formula to contains the both variables & its negation. for two variables p & q are 2^2. such formula given belows.

P

$P \wedge Q$

$P \wedge \neg Q$

$\neg P \wedge Q$

$\neg P \wedge \neg Q$

this formulas are called mean terms or boolean conjunction of P & Q. Following is the truth tables to shows that no two mean terms are equivalent. each mean term has the truth value for exactly one combination of the truth values of the variable P & q

P	Q	$\neg P$	$\neg Q$	$P \wedge \neg Q$	$\neg P \wedge Q$	$\neg P \wedge \neg Q$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	F	T	F
F	F	T	T	F	F	T

At least one true value & false value not be equivalent.

∴ There are two types of procedure used to find principal disjunctive Normal form.

1) Using truth table
2) Without Using truth table which is known as replacement method

Ex: find the PDNF for

$\neg P \vee Q = P \vee Q \wedge \neg P \vee Q \wedge \neg P \vee \neg Q$

P	Q	$\neg P$	$\neg P \vee Q$	
T	T	F	T	$P \vee Q$
T	F	F	F	
F	T	T	T	$\neg P \vee Q$
F	F	T	T	$\neg P \vee \neg Q$

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PDNF for

1) $P \to Q$ 2) $P \vee Q$ 3) $\neg(P \wedge Q)$

P	Q	$P \to Q$	$P \vee Q$	$P \wedge Q$	$\neg(P \wedge Q)$
T	T	T	T	T	T
T	F	F	T	F	T
F	T	T	T	F	T
F	F	T	F	F	F

1) $P \to Q \Rightarrow$

$P \vee Q \wedge \neg P \vee Q \wedge \neg P \vee \neg Q$

2) $P \vee Q) \Rightarrow P \vee Q \wedge P \vee \neg Q \wedge \neg P \vee Q$

3) $\neg(P \wedge Q) \Rightarrow P \vee Q \wedge P \vee \neg Q \wedge \neg P \vee Q$

Imp. Q. Obtain the principal disjunctive Normal form

$(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$

1	2	3	4	5	6	7	8	9
P	Q	R	$\neg P$	$P \wedge Q$	$\neg P \wedge R$	$Q \wedge R$	$5 \vee 6$	$8 \vee 7$
T	T	T	F	T	T	T	T	T
T	F	F	F	T	F	F	T	T
T	F	T	F	F	F	F	F	F
T	F	F	F	F	F	F	F	F
F	T	T	T	F	T	T	T	T
F	T	F	T	F	F	F	F	F
F	F	T	T	F	T	F	T	T
F	F	F	T	F	F	F	F	F

$(P \vee Q \vee R) \wedge (P \vee Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee \neg Q \vee R) \wedge$

$\cancel{(\neg P \vee \neg Q \vee \neg R)}$

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S-17 Ex: Obtain principal disjunctive normal form of
$(P \wedge Q \vee R) \wedge (P \vee Q)$

Ex: Obtain principal disjunctive normal form of
$P \rightarrow ((P \rightarrow Q) \wedge \neg (\neg Q \vee \neg P))$

S-17 ⇒

1	2	3	4	5	6	
P	Q	R	$P \wedge Q$	$P \vee Q$	$4 \vee R$	$6 \wedge 5$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	F	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	T
F	T	F	F	T	F	F
F	F	T	F	F	T	F
F	F	F	F	F	F	F

$\therefore (P \vee Q \vee R) \wedge (P \vee Q \vee \neg R) \wedge (P \vee \neg Q \vee R) \wedge (\neg P \vee Q \vee R$

1	2	3	4	5	6	7	8	
⇒ P	Q	$\neg P$	$\neg Q$	$P \rightarrow Q$	$\neg Q \vee \neg P$	$\neg(6)$	$5 \wedge 7$	$P \rightarrow 8$

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★ Principal Conjunctive Normal form

$(\vee) \wedge (\vee) \wedge (\vee)$ — F

Consider the formula which consist of disjunction in which each variable or its negation but not both appears only ones. following are some term which are called max term. The term are nothing but dual of min term.

$P \vee Q$, $P \vee \neg Q$, $\neg P \vee \neg Q$, $\neg P \vee Q$

Following to the truth table about for max term.

P	Q	¬P	¬Q	P∨Q	P∨¬Q	¬P∨¬Q	¬P∨Q
T	T	F	F	T	T	F	T
T	F	F	T	T	T	T	F
F	T	T	F	T	T	T	T
F	F	T	T	F	T	T	T

$P \vee \neg Q$

From the truth table it is clear that each of the max term has the truth values 'F' for exactly one combination of the truth values of the variable.

$(\vee) \wedge$

Obtain the PCNF following formula

1) $\neg(P \vee Q)$
2) $(\neg P \rightarrow R) \wedge (Q \Leftrightarrow P)$
3) $(P \rightarrow Q) \wedge (R \Leftrightarrow Q)$
4) Obtain PCNF of $(A \vee B) \wedge ((B \vee C)(A \vee C))$

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1) $\neg(P \vee Q)$

P	Q	$P \vee Q$	$\neg(P \vee Q)$	
T	T	T	F	$P \vee Q$
T	F	T	F	$P \vee \sim Q$
F	T	T	F	$\sim P \vee Q$
F	F	F	T	

$(P \vee Q) \wedge (P \vee \sim Q) \wedge (\sim P \vee Q)$

2) $(\neg P \to R) \wedge (Q \Leftrightarrow P)$

	P	Q	R	$\neg P$	$\neg P \to R$	$Q \Leftrightarrow P$	$(\neg P \to R) \wedge (Q \Leftrightarrow P)$
	T	T	T	F	T	T	T
	T	T	F	F	T	T	T
$P \vee \sim Q \vee R$	T	F	T	F	T	F	F
$P \vee \sim Q \vee \sim R$	T	F	F	F	T	F	F
$\sim P \vee Q \vee R$	F	T	T	T	T	F	F
$\sim P \vee Q \vee \sim R$	F	T	F	T	F	F	F
	F	F	T	T	T	T	T
$\sim P \vee \sim Q \vee \sim R$	F	F	F	T	F	T	F

3) $(P \to Q) \wedge (R \Leftrightarrow Q)$

P	Q	R	$P \to Q$	$R \Leftrightarrow Q$	$(P \to Q) \wedge (R \Leftrightarrow Q)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	T	T	T

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The process

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①

Imp * ***

Mathematical Induction *

Principal:

The process of establish the validity of general result involving natural numbers is the principal of mathematical induction.

Working:

let n_0 be a fixed integer. Suppose $P(n)$ is a statement involving the natural number n and we wish to prove that $p(n)$ is a truth for $n \geq n_0$.

1) **Basic of Induction:**

$P(n_0)$ is true i.e. $P(n)$ is a true for $n = n_0$.

2) **Induction step:** Assum that $P(k)$ is a true for $n = k$.

then $P(k+1)$ must also be true.

$P(n)$ is a true for all $n \geq n_0$.

Q.1 Prove that by mathematical Induction that for all $n \geq 1$, $1+2+3+\cdots+n = \frac{n(n+1)}{2}$

$\Rightarrow$ let $P(n)$ be the formula which represent the given series

$$P(n) = 1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

Now,

We have to use principal of mathematical induction i.e. we have to use basic step & induction step.

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Basic Step :

Here we have to prove the given expression for $n = 1$ i.e.

$$P(1) = \frac{n(n+1)}{2} = \frac{1(1+1)}{2} = 1$$

which is clearity true.

Induction Step :

Here we have to first assume that given expression is true for $n = k$ i.e. $P(k)$ is true.

Now,

We have to show that ~~each~~ if $P(k)$ is true then $P(k+1)$ is also true.

Hence,

$$P(k) = 1+2+3 \text{------} +k = \frac{k(k+1)}{2} \quad \text{---} (1)$$

& $$P(k+1) = 1+2+3 + \text{-----} + k + (k+1) = \frac{(k+1)(k+2)}{2} \quad \text{---} (2)$$

Which we have to prove that it is true.

Now consider,

R.H.S of $P(k+1)$ i.e.

$$1+2+3+\text{-----}+k+(k+1) = \frac{k(k+1)}{2} + (k+1)$$

$$= \frac{k(k+1) + 2(k+1)}{2}$$

$$= \frac{(k+1)\cdot(k+2)}{2} = P(k+1)$$

$$= R.H.S$$

$\therefore$ $P(k+1)$ is also true for $n = (k+1)$

$\therefore$ from eqn (1) & (2) be concluded that

$$\boxed{1+2+3+\text{-----}+n = \frac{n(n+1)}{2}}$$

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Q. Prove by mathematical induction by

$$5 + 10 + 15 + \cdots + 5n = \frac{5n(n+1)}{2}$$

⇒ let $P(n)$ be the formula which represent the given series.

$$P(n) = 5 + 10 + 15 + \cdots + 5n = \frac{5n(n+1)}{2}$$

Now,

We have to use principal of mathematical induction i.e. we have to use basic step and induction step.

Basic step :

Here we have to prove the given expression for $n = 5$ i.e.

$$P(5) = \frac{5n(5n+1)}{2} = \frac{5(5+5)}{2} = \frac{30}{2} = 15$$

which is clearity true.

Induction step :

Here we have to first assume that given expression is true for $n = k$ i.e. $P(k)$ is true.

Now, we have to show that $P(k)$ is true then $(P(k+1)$ is also, true.

Hence,

$$P(k) = 5 + 10 + 15 + \cdots 5k = \frac{5k(k+1)}{2} \quad \cdots (1)$$

& $P(k+1) = 5 + 10 + 15 + \cdots + 5k + (5k+1) = \cancel{(5k+1)}$

$$= \frac{(5k+1)\cancel{(5k+2)}}{2} \quad \cdots (2)$$

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$1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$ 26/08/19

Which we have to prove that it is true

Now consider

R.H.S of P(K+1) i.e.

$$5+10+15+\cdots+5K+(5K+1) = \frac{5K(K+1)}{2}+\frac{5K+2}{2}$$

$$= \frac{5K(K+1)+10(K+1)}{2}$$

$$= \frac{(K+1)(5K+10)}{2}$$

$$= \frac{(K+1)\cdot 5(K+2)}{2}$$

$$= \frac{5(K+1)(K+2)}{2}$$

$$= P(K+1)$$

$$= R.H.S.$$

$\therefore$ P(K+1) is also true for $n=(K+1)$

$\therefore$ from eqn ① & ② be concluded that

$$5+10+15+\cdots+5n = \frac{5n(n+1)}{2}$$

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W-16

5-1 Q: Show that $1^2+3^2+5^2+\cdots+(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

imp

→ let p(n) be the a formulas which represent to given series.

$$1^2+3^2+5^2+\cdots+(2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$$

Now,

We have to use the principal of mathematic

Q. prove by mathematical Induction that for $n \geq 1$

$1.2 + 2.3 + \cdots n(n+1) = \frac{n(n+1)(n+2)}{3}$ 27/08/19

induction i.e. we have to use basic step & Induction step.

Basic step: Here we have to prove the given expression for $n=1$ i.e.

$$P(1) = \frac{n(2n-1)(2n+1)}{3} = \frac{1(2\times1-1)(2\times1+1)}{3} = \frac{3}{3}$$

$$= 1$$

which is clearity true.

Induction step:

Here we have to first assume that given expression is true for $n=k$ i.e. $P(k)$ is true.

Now,

We have to show that $P(k)$ is true then $P(k+1)$ is also true.

Hence.

$$P(k) = 1^2 + 3^2 + 5^2 + \cdots + (2k-1) = \frac{k(2k+1)}{3} \quad \text{---- (1)}$$

& $P(k+1) = 1^2 + 3^2 + 5^2 + \cdots (2k-1)^2 + (2k+1) = (2k-1+1)$

Which we have to prove that it is true

Now consider,

R.H.S. of $P(k+1)$ i.e.

$$1^2 + 3^2 + 5^2 + \cdots (2k-1)^2 + (2k-1+1)^2 = k\frac{(2k-1)(2k+1)}{3} \cdot \frac{(2(k+1)-1)^2}{2}$$

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$$= \frac{k(2k-1)(2k+1)}{3} + \frac{(2k+1)^2}{1}$$

$$= \frac{k(2k-1)(2k+1) + 3(2k+1)^2}{3}$$

$$= \frac{(2k+1)[k(2k-1) + 3(2k+1)]}{3}$$

$$= \frac{(2k+1)[2k^2 - k + 6k + 3]}{3}$$

$$= \frac{(2k+1)(2k^2+5k+3)}{3}$$

$2k^2+5k+3$

$(2k+2)(2k+3)$

$2(k+1)(2k+3)$

$$= \frac{(2k+1)(k+1)(2k+3)}{3}$$

Q.

$\bar{\bar{A}} = A$, $\bar{A} \cdot B =$

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Q. Prove by mathematical induction that if $A_1, A_2 \ldots A_n$ are any n sets then

a) $\overline{\bigcup_{i=1}^{n} A_i} = \bigcap_{i=1}^{n} A_i$ b) $\overline{\bigcap_{i=1}^{n} A_i} = \bigcup_{i=1}^{n} \overline{A_i}$

c) $\left(\bigcap_{i=1}^{n} A_i\right) \cup B = \bigcap_{i=1}^{n} (A_i \cup B)$

Sol^n: a) The given example is an extended version of one of the De-morgan's law.

Here we have to prove this by principal of mathematical induction.

Basic step:

let $P(n) = \overline{\bigcup_{i=1}^{n} A_i} = \bigcap_{i=1}^{n} \overline{A_i}$

Now, we have prove that $P(1)$ is true

$P(1) = \overline{A_1}$ which is clearly true

$\overline{A_1} = A_1$

$P(n)$ is true for $n = 1$

Induction step:

Here we have to prove that if $P(k)$ is true then,

$P(k) = LHS \Rightarrow \overline{\bigcup_{i=1}^{k} A_i}$

$\overline{\bigcup_{i=1}^{k} A_i} = \overline{\overline{A_1} \cup \overline{A_2} \cup \overline{A_3} \cdots \cup \overline{A_k}}$

$= \overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \cdots \cap \overline{A_k} \cdots$

By De-morgans law

$= \bigcap_{i=1}^{k} A_i$

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$P(k) = RHS$

$P(k)$ is also true for $n = k$

$$P(k+1) = \overline{\bigcup_{i=1}^{k+1} A_i} = \overline{A_1 \cup A_2 \cup \cdots \cup A_k) \cup A_{k+1}}$$

$$= \overline{(A_1 \cup A_2 \cup \cdots \cup A_k) \cup (A_{k+1})}$$

--- By Associative law

$$= \overline{A_1 \cup A_2 \cup \cdots \cup A_k} \cap \overline{A_{k+1}}$$

-- By demorgan's law

$$P(k+1) = \overline{\bigcup_{i=1}^{k} A_i} \cap \overline{A_{k+1}} \quad \text{--- (1)}$$

$$= \bigcup_{i=1}^{k+1} \overline{A_i} = \bigcup_{i=1}^{k} A_i \cap \overline{A_{k+1}} \quad \text{--- (2)}$$

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2) The given example is an extended version of one of the De-Morgan's law.

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Here we have to prove this by principal of Mathematical induction.

Basic step : let $p(n)$

$$\overline{\bigcap_{i=1}^{n} A_i} = \bigcup_{i=1}^{n} \overline{A_i}$$

Now, we have to prove that $P(1)$ is true

$P(1) = \overline{A_1}$ which is clearly true.

$$\overline{A_1} = \overline{A_1}$$

$P(n)$ is true for $n = 1$

Induction step : Here we have to prove that if $P(k)$ is true, then

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$P(k) = LHS \quad \overline{\bigcap_{i=1}^{k} A_i}$

$\overline{\bigcap_{i=1}^{k} A_i} = \overline{\overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \cdots\cdots \cap A_k}$

$= \bigcup_{i=1}^{k} A_i$

$\therefore P(k) = R.H.S = \bigcup_{i=1}^{k} \overline{A_i}$

$P(k)$ is also true of $n = (k)$

$P(k+1) = \overline{\bigcap_{i=1}^{k+1} A_i} = \overline{(A_1 \cap A_2 \cap \cdots\cdots \cap A_k) \cap A_{k+1}}$

$= \overline{(A_1 \cap A_2 \cap \cdots\cdots \cap A_k)} \cup \overline{A_{k+1}} \cdots\cdots$ by Associative law

$= \overline{A_1} \cup \overline{A_2} \cup \cdots\cdots \cup \overline{A_k} \cap \overline{A_{k+1}}$

$P(k+1) = \bigcap_{i=1}^{k+1} \overline{A_i} \cap \overline{A_{k+1}}$ — ① By demorgan law

$= \bigcap_{i=1}^{k+1} \overline{A_i} = \bigcap_{i=1}^{k} A_i \cdot \cap \cap A_{k+1}$

8) To show $\left(\bigcap_{i=1}^{n} A_i\right) \cup B = \bigcap_{i=1}^{n} (A_i \cup B)$

⇒ By Mathematical induction

Basic Step : For $n = 1$

Let $P(n) = L.H.S = A_1 \cup B$

$P(1) = RHS = A_1 \cup B$

$\therefore LHS = RHS$

$\therefore A_1 \cup B = A_1 \cup B$

$\therefore P(1)$ is true for $n = 1$

Induction step :

for $n = k$

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$\therefore P(k) = LHS$

$$= \left(\bigcap_{i=1}^{k} A_i\right) \cup B = \bigcap_{i=1}^{n} (A_i \cup B)$$

$$P(k) = RHS = \bigcap_{i=1}^{k} (A_i \cup B)$$

$\therefore P(k)$ is also true for $n = k$,

simillarly,

for $n = k+1$

$$LHS = \left(\bigcap_{i=1}^{k+1} A_i\right) \cup B$$

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$$= (A_1 \cap A_2 \cap \cdots \cap A_k \cap A_{k+1}) \cup B$$

$$= \left(\left[\bigcap_{i=1}^{k} A_i\right] \cap A_{k+1}\right) \cup B$$

$$= \left(\left(\bigcap_{i=1}^{k} A_i\right) \cup B \cap (A_{k+1} \cup B)\right) \cdots$$

by distributive law

$$= \left(\bigcap_{i=1}^{k} (A_i \cup B) \cap (A_{k+1} \cup B)\right)$$

$$= \bigcap_{i=1}^{k+1} (A_i \cup B)$$

$$= RHS$$

$\therefore P(k+1)$ is true for $n = k+1$

$\therefore LHS = RHS$

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$$\therefore \left(\bigcap_{i=1}^{n} A_i\right) \cup B = \bigcap_{i=1}^{n} (A_i \cup B)$$

Hence proved.

Q. 5-17 10m

De-Morgan's theorem for set. Prove that
1) $(A \cup B)' = (A)' \cap (B)'$ 2) $(A \cap B)' = (A)' \cup (B)'$

Statement: De-Morgan's 1st law.

$$(A \cup B)' = (A)' \cap (B)'$$

The 1st law states that the complement of the union of two sets is the intersection of the complement.

Proof: $(A \cup B)' = (A)' \cap (B)'$

consider,

$x \in (A \cup B)'$

If $x \in (A \cup B)'$ then $x \notin A \cup B$ ---- By def^n of compliment

$x \in (A \cup B)'$ ---- Def^n $\notin$

$(x \in A \cap x \in B)$ ---- Def^n of '$\cup$'

$(x \in A)' \cap (x \notin B)$ ---- Def^n of $\notin$

$(x \notin A) \cap (x \notin B)$

$(x \in A') \cap (x \in B')$ --- Def^n of compliment

$x \in A' \cap B'$ ---- Def^n of '$\cap$'

$\therefore (A \cup B)' = (A)' \cap (B)'$

Hence proved.

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Statement :

De- Morgan's 2nd law

$(A \cap B)' = (A)' \cup (B)'$

The 2nd law states that the compliment of the intersection of two sets is the union of the compliments.

Proof: $(A \cap B)' = (A)' \cup (B)'$

consider,

$x \in (A \cap B)'$

If $x \in (A \cap B)'$ then $x \notin A \cap B$ ----

By def^n of compliment

$x \in (A \cap B)'$ ---- Def^n of $\notin$

$(x \in A \cup x \in B)'$ --- Def^n of '$\cap$'

$(x \in A)' \cup (x \notin B)$ --- Def^n of $\notin$

$(x \notin A) \cup (x \notin B)$ ---

$(x \in A)' \cup (x \in B)$ --- Def^n of Compliment

$x \in A' \cup B'$ --- Def^n of '$\cup$'

$\therefore$

$\therefore (A \cap B)' = (A)' \cup (B)'$

Hence proved.

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