Que. Show that $GCD(a,b) \cdot LCM(a,b) = a \cdot b$. If $a=540, b=504$

$\longrightarrow$ $a = 540 = 2 \times 2 \times 3 \times 3 \times 3 \times 5$

$540 = 2^2 \times 3^3 \times 5^1 \times 7^0$

$a = 504 = 2 \times 2 \times 2 \times 3 \times 3 \times 7$

$= 2^3 \times 3^2 \times 7^1 \times 5^0$

$GCD(540,504) = 2^{min(2,3)} \times 3^{min(3,2)} \times 5^{min(1,0)} \times 7^{min(0,1)}$

$= 2^2 \times 3^2 \times 5^0 \times 7^0$

$= 4 \times 9$

$\boxed{GCD(540,504) = 36}$

$LCM(540,504) = 2^{max(2,3)} \times 3^{max(3,2)} \times 5^{max(1,0)} \times 7^{max(0,1)}$

$= 2^3 \times 3^3 \times 5^1 \times 7^1$

$= 8 \times 27 \times 5 \times 7$

$\boxed{LCM(540,504) = 7560}$

$GCD(540,504) \cdot LCM(540,504) = 36 \cdot 7560$

$= 272160$ ——— (I)

$a \cdot b = 540 \cdot 504 = 272160$ ——— (II)

from (I) & (II)

$\boxed{GCD(540,504) \cdot LCM(540,504) = a \cdot b}$

——— (Hence prove) ———

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Que. Show that $GCD(a,b) \cdot LCM(a,b) = a \cdot b$ If $a=100$, $b=80$

$a=70$, $b=150$

$a=190$, $b=34$

$a=125$, $b=650$

$\rightarrow a = 100 = 2 \times 2 \times 5 \times 5$

$= 2^2 \times 5^2$

2	100
2	50
5	25
5	5
	1

$b = 80 = 2 \times 2 \times 2 \times 2 \times 5$

$= 2^4 \times 5^1$

2	80
2	40
2	20
2	10
5	5
	1

$GCD(100,80) = 2^{\min(2,4)} \times 5^{\min(2,1)}$

$= 2^2 \times 5^1$

$= 4 \times 5$

$\boxed{GCD(100,80) = 20}$

$LCM(100,80) = 2^{\max(2,4)} \times 5^{\max(2,1)}$

$= 2^4 \times 5^2$

$= 16 \times 25$

$\boxed{LCM(100,80) = 400}$

$GCD(100,80) \cdot LCM(100,80) = 20 \cdot 400$

$= 8000$ ——— (I)

$a \cdot b = 100 \cdot 80 = 8000$ ——— (II)

from (I) & (II)

$\boxed{GCD(100,80) \cdot LCM(100,80) = a \cdot b}$

——— (Hence prove) ———

(2) $a=70,\ b=150$

2	70
5	35
7	7
	1

$\rightarrow\ a = 70 = 2\times5\times7$
$= 2^1\times5^1\times7^1\times3^0$

2	150
3	75
5	25
5	5
	1

$b = 150 = 2\times3\times5\times5$
$= 2^1\times3^1\times5^2\times7^0$

$GCD(70,150) = 2^{\min(1,1)}\times5^{\min(1,2)}\times7^{\min(1,0)}\times3^{\min(0,1)}$

$= 2^1\times5^1\times7^0\times3^0$

$= 2\times5\times1$

$\boxed{GCD(70,150) = 10}$

$LCM(70,150) = 2^{\max(1,1)}\times5^{\max(1,2)}\times7^{\max(1,0)}\times3^{\max(0,1)}$

$= 2^1\times5^2\times7^1\times3^1$

$= 2\times25\times21$

$= 50\times21$

$\boxed{LCM(70,150) = 1050}$

$GCD(70,150)\cdot LCM(70,150) = 10\cdot1050$
$= 10500$ ——— (I)

$a\cdot b = 70\cdot150 = 10500$ ——— (II)

from (I) & (II)

$\boxed{\therefore GCD(70,150)\cdot LCM(70,150) = a\cdot b}$

——— (Hence prove) ———

* Boolean Matrix *

Que. Define boolean matrix & state various operation perform on boolean matrix.

→ A matrix whose all elements either zero or one is known as boolean matrix or bit matrix.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

Various operation perform on boolean matrix is —

① Joint Operation ($\vee$) :→

Let A & B be any two boolean matrix then the joint operation is denoted by $A \vee B$ & is define as '

$$A \vee B = \begin{cases} 1 & \text{if } a_{ij} = 1 \text{ or } b_{ij} = 1 \\ 0 & \text{if } a_{ij} = 0 \text{ \& } b_{ij} = 0 \end{cases}$$

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{A \vee B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}}$$

(ii) Meet Operation :→

If A & B be any two boolean matrix then their meet operation is denoted by $A \cap B$ & it is defined as-

$$A \cap B = \begin{cases} 1 & \text{if } a_{ij}=1 \ \& \ b_{ij}=1 \\ 0 & \text{if } a_{ij}=0 \text{ or } b_{ij}=0 \end{cases}$$

$1 \cap 1 = 1$
0

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore \boxed{A \cap B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

(iii) Boolean Product ($\odot$) :→

If A & B are any two boolean matrix then boolean product is denoted by $A \odot B$ & is define as-

$$A \odot B = \begin{cases} 1 & \text{if } a_{ij}=1 \ \& \ b_{ij}=1 \\ 0 & \text{otherwise.} \end{cases}$$

ex→ (1) $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$$A \odot B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

$$= \begin{bmatrix} 1+1+0 & 1+0+1 & 0+1+1 \\ 0+1+0 & 0+0+1 & 0+1+1 \\ 1+0+0 & 1+0+0 & 0+0+0 \end{bmatrix}$$

$$\therefore \boxed{A \odot B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}}$$

Ex. 2 If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$.

find $A \cup B$, $A \cap B$, $A \odot B$

→ 1 $A \cup B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \cup \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$$\boxed{A \cup B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}}$$

2 $A \cap B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \cap \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$$\boxed{A \cap B = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}$$

③ $A \odot B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0+1+0 & 0+1+1 & 1+0+1 \\ 0+1+0 & 0+1+0 & 1+0+0 \\ 0+0+0 & 0+0+1 & 1+0+1 \end{bmatrix}$

$\boxed{A \odot B = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}}$

Ex. ③ If

$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$. $B = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

find. $A \cup B$, $A \cap B$, $A \odot B$

$\longrightarrow$ ① $A \cup B = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

$\boxed{A \cup B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}}$

② $$A \cap B = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \cap \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\boxed{A \cap B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}}$$

③ $$A \odot B = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+0+0 & 1+1+1+0 & 0+0+1+0 & 1+0+0+0 \\ 0+1+0+0 & 0+1+0+0 & 0+0+0+1 & 0+0+0+1 \\ 0+0+0+0 & 1+0+0+0 & 0+0+0+1 & 1+0+0+1 \\ 0+0+0+0 & 0+0+1+0 & 0+0+1+1 & 0+0+0+1 \end{bmatrix}$$

$$\boxed{A \odot B = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \end{bmatrix}}$$

***Que. If ~~ef~~ A, B & C are any three boolean matrix then show that-

(1) $A \cup (B \cup C) = (A \cup B) \cup C$

(2) $A \cap (B \cap C) = (A \cap B) \cap C$

(3) $A \odot (B \odot C) = (A \odot B) \odot C$

→ (1) Let,

A, B & C are any three boolean matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$A \cup (B \cup C) = (A \cup B) \cup C$$

$$L.H.S. = A \cup (B \cup C)$$

$$(B \cup C) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \vee \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$0+0=0$

$$(B \cup C) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$A \cup (B \cup C) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\boxed{A \cup (B \cup C) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}} \quad \text{——— (I)}$$

$R.H.S = (A \cup B) \cup C$

$$A \cup B = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$A \cup B = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$(A \cup B) \cup C = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \vee \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\boxed{(A \cup B) \cup C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}} \quad \text{——— II}$$

From I & II

L.H.S. = R.H.S.

$$\therefore \boxed{A \cup (B \cup C) = (A \cup B) \cup C}$$

2 $A \cap (B \cap C) = (A \cap B) \cap C$

→ L.H.S. = $A \cap (B \cap C)$

$$(B \cap C) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \wedge \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$(B \cap C) = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$1 + 1 = 1$

$$A \cap (B \cap C) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \wedge \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{A \cap (B \cap C) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}} \text{ --- } (I)$$

$$R.H.S. = (A \cap B) \cap C$$

$$(A \cap B) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \wedge \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$(A \cap B) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$(A \cap B) \cap C = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \wedge \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\boxed{(A \cap B) \cap C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}} \text{ --- } (II)$$

from (I) & (II)

L.H.S. = R.H.S.

$$\boxed{\therefore A \cap (B \cap C) = (A \cap B) \cap C}$$

(3) $A \odot (B \odot C) = (A \odot B) \odot C$

$\longrightarrow$ L.H.S. $= A \odot (B \odot C)$

$$(B \odot C) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+0 & 0+0+1 & 1+0+0 \\ 0+1+0 & 0+0+1 & 0+0+0 \\ 0+0+0 & 0+0+0 & 1+0+0 \end{bmatrix}$$

$$(B \odot C) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A \odot (B \odot C) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+1+0 & 1+1+0 & 1+0+0 \\ 0+1+0 & 0+1+0 & 0+0+1 \\ 1+0+0 & 1+0+0 & 1+0+1 \end{bmatrix}$$

$$\boxed{A \odot (B \odot C) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}} \text{ —— (I)}$$

R.H.S. $= (A \odot B) \odot C$

$$(A \odot B) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0 & 1+1+0 & 1+1+0 \\ 0+0+1 & 0+1+0 & 0+1+0 \\ 1+0+1 & 1+0+0 & 1+0+0 \end{bmatrix}$$

$$(A \odot B) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$$(A \odot B) \odot C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1+0 & 0+0+1 & 1+0+0 \\ 0+1+0 & 0+0+1 & 1+0+0 \\ 0+1+0 & 0+0+1 & 1+0+0 \end{bmatrix}$$

$$\boxed{(A \odot B) \odot C = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}} \text{ --- } (II)$$

From (I) & (II)

L.H.S. = R.H.S.

$$\therefore \boxed{A \odot (B \odot C) = (A \odot B) \odot C}$$

Que. ② If $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

S.T.

① $A \cup (B \cup C) = (A \cup B) \cup C$

② $A \cap (B \cap C) = (A \cap B) \cap C$

③ $A \odot (B \odot C) = (A \odot B) \odot C$

$\longrightarrow$ Act,

A, B & C are any three boolean matrix.

$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

① $A \cup (B \cup C) = (A \cup B) \cup C$

$\longrightarrow$ L.H.S. $= A \cup (B \cup C)$

$(B \cup C) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$(B \cup C) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$A \cup (B \cup C) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\boxed{A \cup (B \cup C) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}}$ ——— (I)

R.H.S. $= (A \cup B) \cup C$

$$(A \cup B) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$(A \cup B) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$(A \cup B) \cup C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \vee \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\boxed{(A \cup B) \cup C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}} \text{ ——— (II)}$$

$\therefore$ From (I) & (II)

L.H.S. = R.H.S.

$$\boxed{A \cup (B \cup C) = (A \cup B) \cup C}$$

(2) $A \cap (B \cap C) = (A \cap B) \cap C$

$\rightarrow$ L.H.S. $= A \cap (B \cap C)$

$$(B \cap C) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \wedge \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$(B \cap C) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$A \cap (B \cap C) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \wedge \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\boxed{A \cap (B \cap C) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}} \text{ ——— (I)}$$

Date 14/8/2012

R.H.S. = $(A \cap B) \cap C$

$(A \cap B) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \wedge \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$(A \cap B) = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$

$(A \cap B) \cap C = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \wedge \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\boxed{(A \cap B) \cap C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}}$ —— (II)

from (I) & (II)

L.H.S. = R.H.S.

$\boxed{\therefore \; A \cap (B \cap C) = (A \cap B) \cap C}$

(3) $A \odot (B \odot C) = (A \odot B) \odot C$

→ L.H.S. = $A \odot (B \odot C)$

$(B \odot C) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0+0 & 0+1 \\ 1+0 & 0+0 \end{bmatrix}$

$\boxed{(B \odot C) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}$

$$A \odot (B \odot C) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1 & 1+0 \\ 0+1 & 1+0 \end{bmatrix}$$

$$\boxed{A \odot (B \odot C) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}} \text{ — (I)}$$

R.H.S. $= (A \odot B) \odot C$

$$(A \odot B) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0+1 & 1+0 \\ 0+1 & 1+0 \end{bmatrix}$$

$$\boxed{(A \odot B) = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}}$$

$$(A \odot B) \odot C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+1 \\ 1+0 & 0+1 \end{bmatrix}$$

$$\boxed{(A \odot B) \odot C = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}} \text{ — (II)}$$

$\therefore$ From (I) & (II)

L.H.S. = R.H.S.

$$\boxed{\therefore A \odot (B \odot C) = (A \odot B) \odot C}$$

* Transpose of Matrix *

If A is any matrix then its transpose is denoted by A^T. It is the matrix obtain by interchanging rows into column & column into rows.

ex→ $A = \begin{bmatrix} 1 & 5 & -2 \\ 3 & 7 & 2 \\ 1 & 2 & 3 \end{bmatrix}$

$$\boxed{\therefore A^T = \begin{bmatrix} 1 & 3 & 1 \\ 5 & 7 & 2 \\ -2 & 2 & 3 \end{bmatrix}}$$

**

Que. ① If A & B are any two matrix then show that

① $(A^T)^T = A$

② $(A+B)^T = A^T + B^T$

③ $(AB)^T = A^T . B^T$

→ ① $(A^T)^T = A$

Consider,

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 2 \\ 3 & 0 & -3 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 3 & 3 \\ 2 & 1 & 0 \\ -1 & 2 & -3 \end{bmatrix}$$

denote
into

$$(A^T)^T = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 2 \\ 3 & 0 & -3 \end{bmatrix}$$

$$\boxed{(A^T)^T = A}$$

$$\boxed{\therefore \text{L.H.S.} = \text{R.H.S.}}$$

② $(A+B)^T = A^T + B^T$

→ Consider,

$\text{L.H.S.} = (A+B)^T$

$$(A+B) = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 2 \\ 3 & 0 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 5 \\ 5 & 3 & 1 \\ 1 & 7 & 1 \end{bmatrix}$$

$$(A+B) = \begin{bmatrix} 2 & 4 & 4 \\ 8 & 4 & 3 \\ 4 & 7 & -2 \end{bmatrix}$$

$$(A+B)^T = \begin{bmatrix} 2 & 8 & 4 \\ 4 & 4 & 7 \\ 4 & 3 & -2 \end{bmatrix} \quad \text{———} \quad \text{I}$$

$\text{R.H.S.} = A^T + B^T$

$$= \begin{bmatrix} 1 & 3 & 3 \\ 2 & 1 & 0 \\ -1 & 2 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 5 & 1 \\ 2 & 3 & 7 \\ 5 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 8 & 4 \\ 4 & 4 & 7 \\ 4 & 3 & -2 \end{bmatrix} \quad \text{———} \quad \text{II}$$

from I & II

$$\boxed{L.H.S. = R.H.S.}$$

3) $(AB)^T = B^T \cdot A^T$

$\longrightarrow$ $L.H.S. = (AB)^T$

$$(AB) = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 1 & 2 \\ 3 & 0 & -3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 5 \\ 5 & 3 & 1 \\ 1 & 7 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+10-1 & 2+6-7 & 5+2-1 \\ 3+5+2 & 6+3+14 & 15+1+2 \\ 3+0-3 & 6+0+21 & 15+0-3 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 1 & 6 \\ 10 & 23 & 18 \\ 0 & 27 & 12 \end{bmatrix}$$

$$(AB)^T = \begin{bmatrix} 10 & 10 & 0 \\ 1 & 23 & 27 \\ 6 & 18 & 12 \end{bmatrix} \text{ ——— } I$$

$R.H.S. = B^T \cdot A^T$

$$\begin{bmatrix} 1 & 5 & 1 \\ 2 & 3 & 7 \\ 5 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 & 3 \\ 2 & 1 & 0 \\ -1 & 2 & -3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+10-1 & 3+5+2 & 3+0+3 \\ 2+6-7 & 6+3+14 & 6+0+21 \\ 5+2-1 & 15+1+2 & 15+0+3 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 10 & 0 \\ 1 & 23 & 27 \\ 6 & 18 & 18 \end{bmatrix} \text{———} (II)$$

From (I) & (II)

$$\boxed{L.H.S. = R.H.S.}$$

→ let,

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 1 \\ 5 & 2 \end{bmatrix}$$

→ (1) $(A^T)^T = A$

Consider,

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$(A^T) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$(A^T)^T = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$$

$$(A^T)^T = A$$

$$\boxed{L.H.S. = R.H.S.}$$

② $(A+B)^T = A^T + B^T$

$\rightarrow$ L.H.S. $= (A+B)^T$

$A+B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 1 \\ 5 & 2 \end{bmatrix}$

$= \begin{bmatrix} 5 & 4 \\ 7 & 6 \end{bmatrix}$

$\boxed{(A+B)^T = \begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix}}$ —— I

R.H.S. $= A^T + B^T$

$= \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 5 & 7 \\ 4 & 6 \end{bmatrix}$ —— II

from I & II

$\boxed{\text{L.H.S.} = \text{R.H.S.}}$

③ $(AB)^T = B^T \cdot A^T$

$\rightarrow$ L.H.S. $= (AB)^T$

$$(AB) = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 \\ 5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 4+15 & 1+6 \\ 8+20 & 2+8 \end{bmatrix}$$

$$= \begin{bmatrix} 19 & 7 \\ 28 & 10 \end{bmatrix}$$

$$\boxed{(AB)^T = \begin{bmatrix} 19 & 28 \\ 7 & 10 \end{bmatrix}} \text{ —— } \mathrm{I}$$

$$B^T \cdot A^T = \begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4+15 & 8+20 \\ 1+6 & 2+8 \end{bmatrix}$$

$$\boxed{B^T \cdot A^T = \begin{bmatrix} 19 & 28 \\ 7 & 10 \end{bmatrix}} \text{ —— } \mathrm{II}$$

From I & II

$\boxed{\text{L.H.S.} = \text{R.H.S.}}$