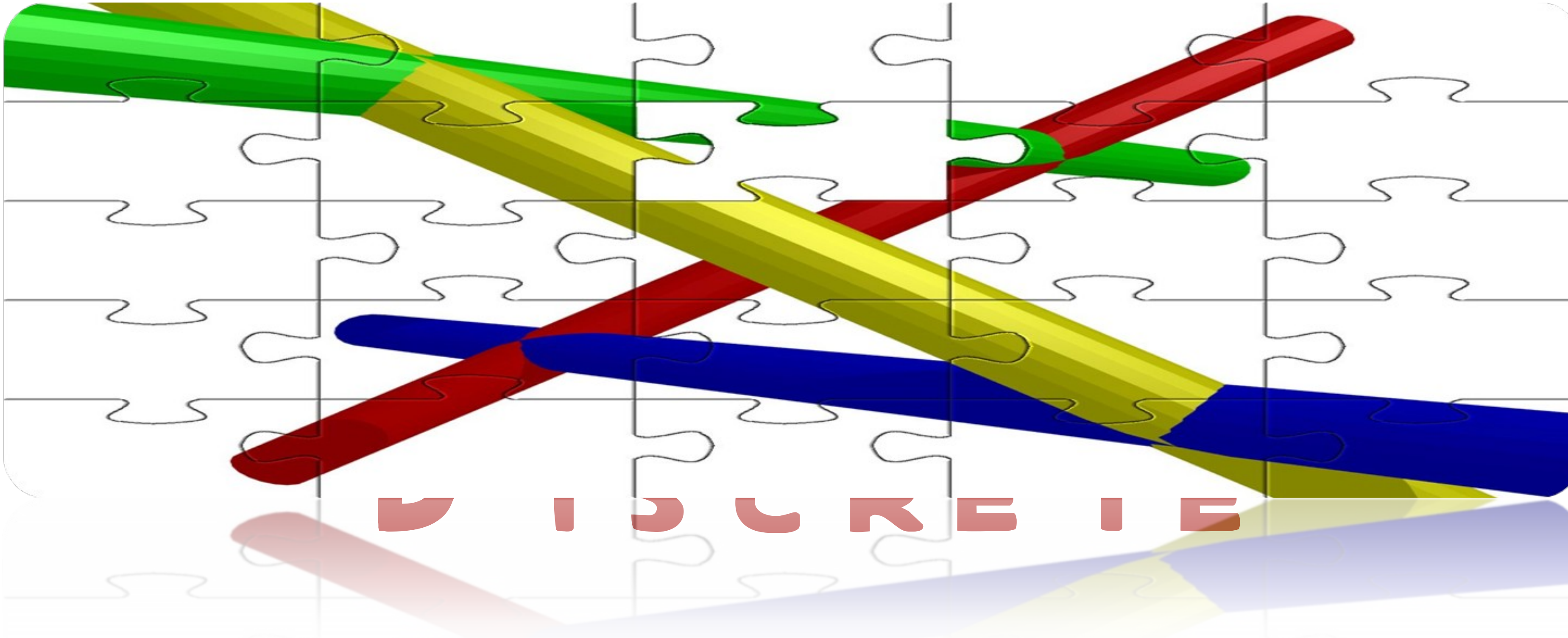
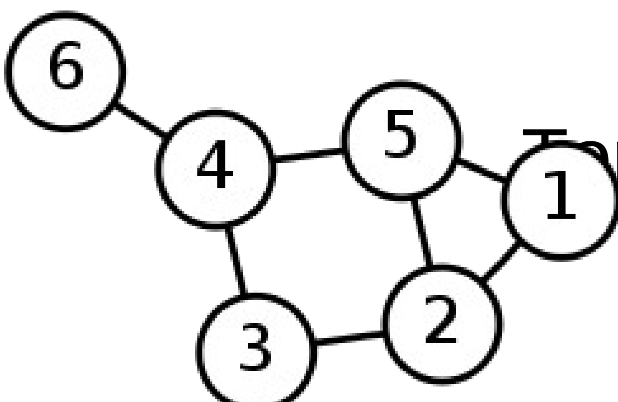
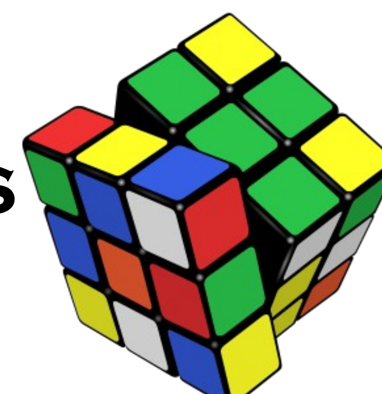


Structures

(Discrete Mathematics)

Topic: **Set Operations**





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Set operations: Union

- Formal definition for the union of two sets:
  A U B = { x | $x \in$ A **or** $x \in$ B } or
  A U B = { $x \in$ U| $x \in$ A **or** $x \in$ B }
- Further examples
  - $\{1, 2, 3\} \cup \{3, 4, 5\} = \{1, 2, 3, 4, 5\}$
  - $\{a, b\} \cup \{3, 4\} = \{a, b, 3, 4\}$
  - $\{1, 2\} \cup \emptyset = \{1, 2\}$
- Properties of the union operation
  - $A \cup \emptyset = A$ Identity law
  - $A \cup U = U$ Domination law
  - $A \cup A = A$ Idempotent law
  - $A \cup B = B \cup A$ Commutative law
  - $A \cup (B \cup C) = (A \cup B) \cup C$ Associative law

- Exercise:
- Write a formula to determine the cardinality of union of two sets A and B

- Set presentation:
  A set of natural numbers less than 5 can be presented by any of the following two ways:
  1. $\{x | x \in \mathrm{N} \wedge x < 5\}$
  2. $\{x \in \mathrm{N} | x < 5\}$

Set operations: Intersection

- Formal definition for the intersection of two sets:
  $A \cap B = \{ x \mid x \in A$ **and** $x \in B \}$
- Examples
  - $\{1, 2, 3\} \cap \{3, 4, 5\} = \{3\}$
  - $\{a, b\} \cap \{3, 4\} = \varnothing$
  - $\{1, 2\} \cap \varnothing = \varnothing$
- Properties of the intersection operation
  - $A \cap U = A$ Identity law
  - $A \cap \varnothing = \varnothing$ Domination law
  - $A \cap A = A$ Idempotent law
  - $A \cap B = B \cap A$ Commutative law
  - $A \cap (B \cap C) = (A \cap B) \cap C$ Associative law

Exercise-intersection

- $\{n \in \mathrm{N} | \mathrm{n} > 5\} \cap \{n \in \mathrm{N} | n < 10\} =?$
- $\{n \in \mathrm{N} | \mathrm{n} > 5\} \cap \{n \in \mathrm{N} | n < 10\} =?$
- $\{x \in \mathrm{z} | x \geq 0\} \cap \{x \in \mathrm{z} | x \leq 0\} =?$
- $\mathrm{z} \cap \mathrm{N} =?$
- $\{n \in \mathrm{z} | n\ is\ even\} \cap \{n \in \mathrm{z} | n\ is\ odd\}$

Exercise-union

- $\{n \in \mathrm{N} | \mathrm{n} > 5\} \cup \{n \in \mathrm{N} | n < 10\} =?$
- $\{n \in \mathrm{N} | \mathrm{n} > 5\} \cup \{n \in \mathrm{N} | n < 10\} =?$
- $\{x \in \mathrm{z} | x \geq 0\} \cup \{x \in \mathrm{z} | x \leq 0\} =?$
- $\mathrm{z} \cup \mathrm{N} =?$
- $\{n \in \mathrm{z} | n\ is\ even\} \cup \{n \in \mathrm{z} | n\ is\ odd\}$

Disjoint sets

- Formal definition for disjoint sets:

  two sets are disjoint if their intersection is the empty set

- Further examples
  - {1, 2, 3} and {3, 4, 5} are not disjoint
  - {a, b} and {3, 4} are disjoint
  - {1, 2} and ∅ are disjoint
    - Their intersection is the empty set
  - ∅ and ∅ are disjoint!
    - Because their intersection is the empty set

Set operations: Difference

- Formal definition for the difference of two sets:

  $A - B = \{ x \mid x \in A$ **and** $x \notin B \}$

- Further examples
  - $\{1, 2, 3\} - \{3, 4, 5\} = \{1, 2\}$
  - $\{a, b\} - \{3, 4\} = \{a, b\}$
  - $\{1, 2\} - \varnothing = \{1, 2\}$
    - The difference of any set S with the empty set will be the set S

Complement sets

- Formal definition for the complement of a set:
  $\overline{A} = \{ x \mid x \notin A \} = A^c$
  - Or U – A, where U is the universal set
- Further examples (assuming U = Z)
  - $\{1, 2, 3\}^c = \{ \ldots, -2, -1, 0, 4, 5, 6, \ldots \}$
  - $\{a, b\}^c = Z$
- Properties of complement sets
  - $(A^c)^c = A$ Complementation law
  - $A \cup A^c = U$ Complement law
  - $A \cap A^c = \varnothing$ Complement law

Set identities

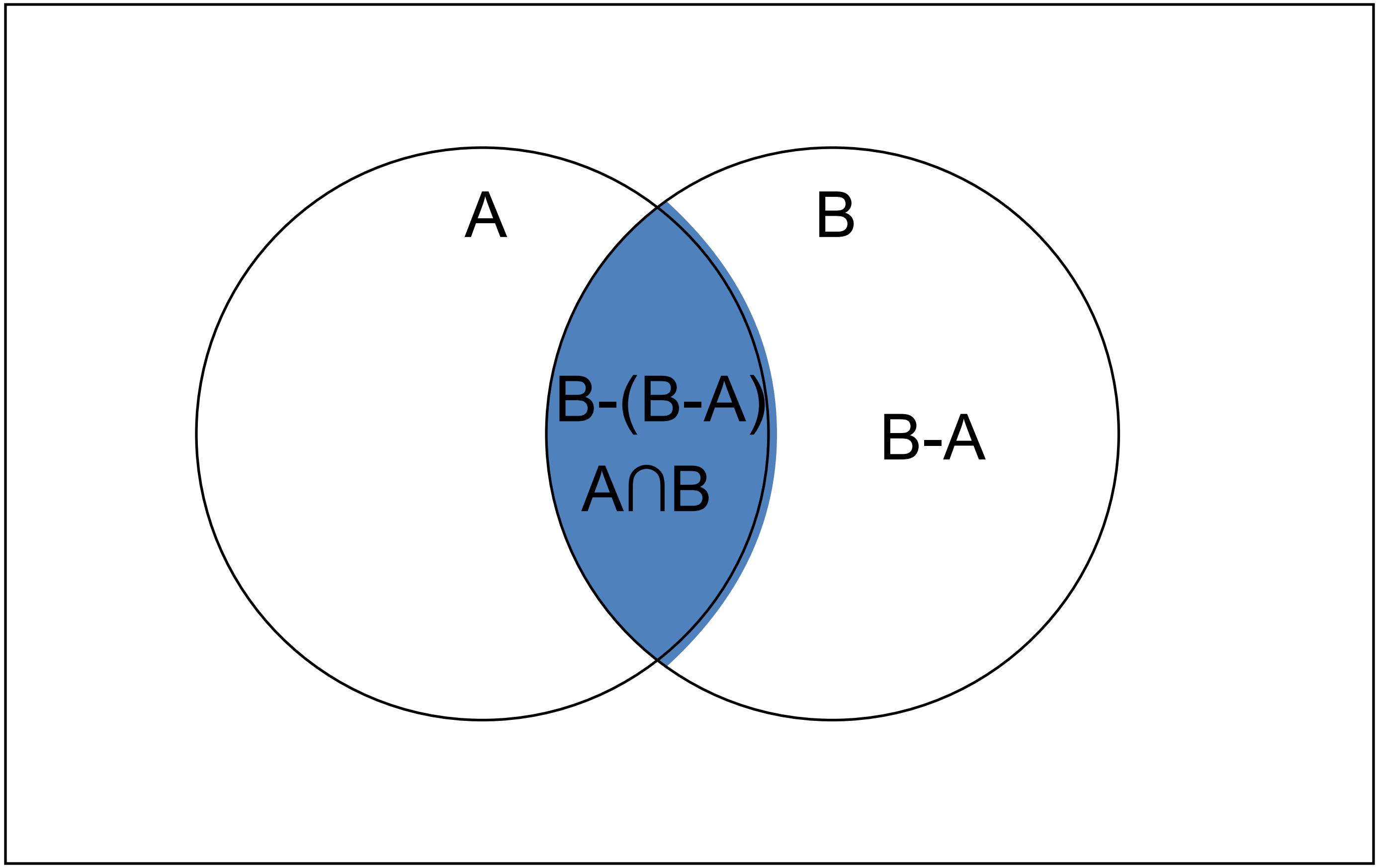
$A \cup \varnothing = A$ $A \cap U = A$	Identity Law	$A \cup U = U$ $A \cap \varnothing = \varnothing$	Domination law
$A \cup A = A$ $A \cap A = A$	Idempotent Law	$(A^c)^c = A$	Complementation Law
$A \cup B = B \cup A$ $A \cap B = B \cap A$	Commutative Law	$(A \cup B)^c = A^c \cap B^c$ $(A \cap B)^c = A^c \cup B^c$	De Morgan's Law
$A \cup (B \cup C)$ $= (A \cup B) \cup C$ $A \cap (B \cap C)$ $= (A \cap B) \cap C$	Associative Law	$A \cap (B \cup C) =$ $(A \cap B) \cup (A \cap C)$ $A \cup (B \cap C) =$ $(A \cup B) \cap (A \cup C)$	Distributive Law
$A \cup (A \cap B) =$ A $A \cap (A \cup B) =$ A	Absorption Law	$A \cup A^c = U$ $A \cap A^c = \varnothing$	Complement Law

How to prove a set identity

- For example: $A \cap B = B - (B - A)$
- Four methods:
  - Use the basic set identities
  - Use membership tables
  - Prove each set is a subset of each other
  - Use set builder notation and logical equivalences

What we are going to prove…

$$A \cap B = B-(B-A)$$



Proof by Set Identities

- $A \cap B = A - (A - B) = B - (B - A)$

Proof: $A - (A - B) = A - (A \cap B^c)$

$= A \cap (A \cap B^c)^c$

$= A \cap (A^c \cup B)$

$= (A \cap A^c) \cup (A \cap B)$

$= \emptyset \cup (A \cap B)$

$= A \cap B$

Showing each is a subset of the others

- $(A \cap B)^c = A^c \cup B^c$

Proof: Want to prove that

$(A \cap B)^c \subseteq A^c \cup B^c$ and $A^c \cup B^c \subseteq (A \cap B)^c$

(i) $x \in (A \cap B)^c$

$\Rightarrow x \notin (A \cap B)$

$\Rightarrow \neg (x \in A \cap B)$

$\Rightarrow \neg (x \in A \wedge x \in B)$

$\Rightarrow \neg (x \in A) \vee \neg (x \in B)$

$\Rightarrow x \notin A \vee x \notin B$

$\Rightarrow x \in A^c \vee x \in B^c$

$\Rightarrow x \in A^c \cup B^c$

(ii) Similarly we show that $A^c \cup B^c \subseteq (A \cap B)^c$

Exercise

- Let A, B, and C be sets. Show that:

a) $A - B = A \cap \bar{B}$

b) $A \cup (B - A) = A \cup B$

c) $(A - C) \cap (C - B) = \emptyset$