M.Sc. (Computer Science) Semester–I (C.B.S.) Examination

DISCRETE MATHEMATICAL STRUCTURE

Compulsory Paper–1

Time : Three Hours] [Maximum Marks : 100

N.B. :— All questions are compulsory and carry equal marks.

EITHER

1. (A) Show that :

$$A \quad (B \odot C) = (A \odot B) \odot C$$

10

(B) Prove by Mathematical Induction that for $n \geq 1$

$$1.2 + 2.3 + \ldots + n.(n + 1) = \frac{n\,(n+1)\,(n+2)}{3}$$

10

OR

(C) Show that :

$((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee$

$(\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology. 10

(D) Obtain the principal disjunctive normal form of $(P \wedge Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$. 10

EITHER

2. (A) Let R be the relation whose matrix is

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

(i) Find the reflexive closure of R.

(ii) Find the symmetric closure of R. 10

(Contd.)

(B) If R and S are relations from A to B then show that :

(i) $R \subseteq S \Rightarrow R^{-1} \subseteq S^{-1}$

(ii) $(R \cup S)^{-1} = R^{-1} \cup S^{-1}$ 10

OR

(C) Let Z be the set of integers and let R be the relation called "congruence modulo 3" defined by

$R = \{ \langle x, y\rangle \mid x \in Z \wedge y \in Z \wedge (x-y) \text{ is divisible by } 3\}$ 10

Determine the equivalence classes generated by the elements of Z.

(D) Let $R = \{\langle 1, 2\rangle, \langle 3, 4\rangle, \langle 2, 2\rangle\}$ and

$S = \{\langle 4, 2\rangle, \langle 2, 5\rangle, \langle 3, 1\rangle, \langle 1, 3\rangle\}$. 10

Find $R \circ S$, $S \circ R$, $R \circ (S \circ R)$, $(R \circ S) \circ R$, $R \circ R$, $S \circ S$ and $R \circ R \circ R$.

EITHER

3. (A) Let $X = \{2, 3, 6, 12, 24, 36\}$ and the relation $\leq$ be such that $x \leq y$ if x divides y. Draw the Hasse diagram of $(X, \leq)$. 10

(B) Let $X = \{1, 2, 3\}$ and f, g, h and s be functions from X to X given by :

$f = \{\langle 1, 2\rangle, \langle 2, 3\rangle, \langle 3, 1\rangle\}$

$g = \{\langle 1, 2\rangle, \langle 2, 1\rangle, \langle 3, 3\rangle\}$

$h = \{\langle 1, 1\rangle, \langle 2, 2\rangle, \langle 3, 1\rangle\}$

$s = \{\langle 1, 1\rangle, \langle 2, 2\rangle, \langle 3, 3\rangle\}$

Find :

(i) $f \circ g$

(ii) $g \circ f$

(iii) $f \circ s$

(iv) $g \circ s$

(v) $s \circ s$. 10

OR

(C) Define lattice. Prove that if L is a bounded distributive lattice and if its complement exists, it is unique. 10

(D) On the set $A = \{a, b, c\}$, find all partial orders $\leq$ in which $a \leq b$. 10

EITHER

4. (A) Show that if G is an Abelian group, then every subgroup of G is a normal subgroup. 10

(B) Define :

(i) Finite-state Machine

(ii) Monoid

(iii) Submonoid

(iv) Ring

(v) Field. 10

OR

(C) Show that the language

$$L = \{a^K b^K \mid K \geq 1\}$$

is not a finite state language. 10

(D) Let $(\{a, b\}, *)$ be a semigroup where $a * a = b$.

Show that :

(i) $a * b = b * a$

(ii) $b * b = b$ 10

5. (A) Define :

(i) Duality Law

(ii) Well-formed formulas. 5

(B) Using characteristic function of a set show that

$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ 5

(C) Define :

(i) Recursion

(ii) Boolean Algebra. 5

(D) What is lattice homomorphism ? Explain in brief with example. 5

NIR/KW/18/2949

Master of Science (M.Sc.) Semester—I (CBCS) (Computer Science) Examination

DISCRETE MATHEMATICAL STRUCTURE

Paper—1

Paper—I

Time : Three Hours] [Maximum Marks : 80

Note :—(1) **All** questions are compulsory and carry equal marks.

(2) Draw neat and labelled diagram wherever necessary.

EITHER

1. (A) Give definition of characteristic function; using characteristic function prove that :

$$(A \oplus B) \oplus C = A \oplus (B \oplus C)$$ 8

(B) Obtain Principal Disjunctive Normal Form of :

$$P \rightarrow ((P \rightarrow Q) \wedge \neg(\neg Q \vee \neg P))$$ 8

OR

(C) What is Duality Law ? Write the duals of :

(i) $(P \vee Q) \wedge R$

(ii) $(P \wedge Q) \vee T$

(iii) $\neg(P \vee Q) \wedge (P \vee \neg(Q \wedge \neg S))$. 8

(D) Show that :

$$(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R.$$ 8

EITHER

2. (A) What is the value of ${}_nC_r$? Compute :

(i) ${}_{12}C_3$

(ii) ${}_{16}C_5$

(iii) ${}_{10}C_2$ 8

$nCr = \dfrac{n!}{(n-x)!\, x!}$

(B) Let A = {a, b, c, d, e} and R = {(a, a), (a, b), (b, c), (c, e), (c, d), (d, e)}. Compute :

(i) R^2

(ii) R^∞. 8

OR

(C) Obtain an explicit formula, for the recurrence relation $d_n = 2d_{n-1} - d_{n-2}$ with initial condition $d_1 = 1.5$ and $d_2 = 3$. 8

(D) Let A, B, C and D be sets, R a relation from A to B, S a relation from B to C, and T a relation from C to D. Then prove that $T \circ (S \circ R) = (T \circ S) \circ R$. 8

EITHER

3. (A) Define Euler path and circuit. Prove if a graph G has more than two vertices of odd degree, then there can be no Euler path in G. 8

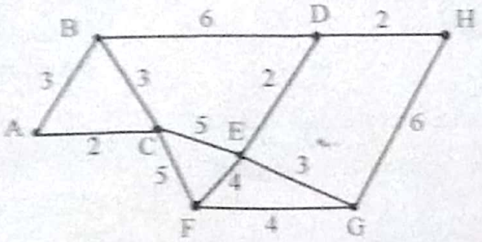
(B) Determine the Hasse diagram of the relation R.

Let A = {1, 2, 3, 4} and R = {(1, 1), (1, 2), (2, 2), (2, 4), (1, 3), (3, 3), (3, 4), (1, 4), (4, 4)}. 8

OR

(C) Prove – Let L be a bounded distributive lattice. If complement exist, it is unique. 8

(D) Determine a minimal spanning tree for the communication network shown below, using Prim's algorithm : 8



EITHER

4. (A) Let (S, $*$) and (T, $*'$) be monoids with Identities e and e' respectively. Let $f : S \to T$ be an isomorphism, then show that $f(e) = e'$. 8

(B) What is state transition table ? Draw the digraph of the machine, whose state transition table is shown below :

	0	1	2
s_0	s_1	s_0	s_2
s_1	s_0	s_0	s_1
s_2	s_2	s_0	s_2

8

OR

(C) Let T be the set of all even integers. Show that the semigroup (Z, +) and (T, +) are isomorphic. 8

(D) Let G be a set of all non-zero real numbers and let $a * b = \frac{ab}{2}$ then show that (G, $*$) is an abelian group. 8

5. (A) Let a, b and c be integers :

(i) If $a|b$ and $a|c$, then $a|b + c$

(ii) If $a|b$ or $a|c$, then $a|bc$. 4

(B) State and prove pigeonhole principle. 4

(C) Construct the tree for the following algebraic expression :

$$((3 * (1 - x)) \div ((4 + (7 - (y + 2))) * (7 + (x \div y))).$$ 4

(D) Explain :

(i) Semigroup

(ii) Monoid. 4

Master of Science (M.Sc.) Semester—I (C.B.C.S.) (Computer Science) Examination

DISCRETE MATHEMATICAL STRUCTURE

Paper—1

Paper—I

Time : Three Hours] [Maximum Marks : 80

N.B. :— ALL questions are compulsory and carry equal marks.

EITHER

1. (A) Define LCM and GCD.

Prove that if a and b are two positive integers, then GCD(a, b) · LCM(a, b) = a · b. 8

(B) Define elementary product and elementary sum.

Obtain disjunctive normal forms of :

(a) $P \wedge (P \to Q)$

(b) $\neg(P \vee Q) \rightleftarrows (P \wedge Q)$ 8

OR

(C) Prove the statement is true by using mathematical induction :

$$1 + 2^1 + 2^2 + \ldots\ldots + 2^n = 2^{n+1} - 1.$$ 8

(D) State the rules of generalization and specification.

Show that :

$$(x)\,(P(x) \to Q(x)) \wedge (x)\,(Q(x) \to R(x)) \Rightarrow (x)\,(P(x) \to R(x)).$$ 8

EITHER

2. (A) State and prove the pigeonhole principle. Compute :

(i) ${}_7C_4$

(ii) ${}_{16}C_5$ 8

(B) Let A = {1, 2, 3, 4, 5} and R = {(1, 1), (1, 2), (2, 3), (3, 5), (3, 4), (4, 5)}.

Draw digraph of R and compute R^2 and R^∞. 8

OR

(C) Define :

(i) Transposition

(ii) Even and odd permutation.

Determine the permutation :

$$P = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 4 & 5 & 7 & 6 & 3 & 1 \end{pmatrix}$$

is even or odd. 8

(D) Let A = {1, 2, 3, 4} and B = {a, b, c}. Let R = {(1, a), (1, b), (2, b), (2, c), (3, b), (4, a)} and S = {(1, b), (2, c), (3, b), (4, b)}. Compute : (a) $\overline{R}$, (b) $R \cap S$, (c) $R \cup S$ (d) R^{-1}. 8

(Contd.)

EITHER

3 (A) Define Hamiltonian path and circuit. Prove—Let the number of edges of G be m. Then G has a Hamiltonian circuit if $m \geq \frac{1}{2}(n^2 - 3n + 6)$, where n is no. of vertices. 8

(B) Let A = {1, 2, 3, 4, 12, 24}. Consider the partial order of divisibility on A, that is, if a and b ∈ A, a ≤ b if and only if a | b. Draw the Hasse diagram of the poset (A, ≤). 8

OR

(C) If $(L_1, \leq)$ and $(L_2, \leq)$ are lattices, then $(L, \leq)$ is a lattice. Where $L = L_1 \times L_2$ and the partial order ≤ of L is the product partial order. 8

(D) Define tree and spanning tree. Prove that—A tree with 'n' vertices has n – 1 edges. 8

EITHER

4. (A) Consider the binary operation * defined on the set A = {a, b, c, d} by the table :

*	a	b	c	d
a	a	c	b	d
b	d	a	b	c
c	c	d	a	a
d	d	b	a	c

Compute :

(i) c * d and d * c

(ii) b * d and d * b

(iii) a * (b * c) and (a * b) * c

(iv) Is commutative or associative ? 8

(B) What is state transition table ? Draw the digraph of the machine whose state transition table is shown below :

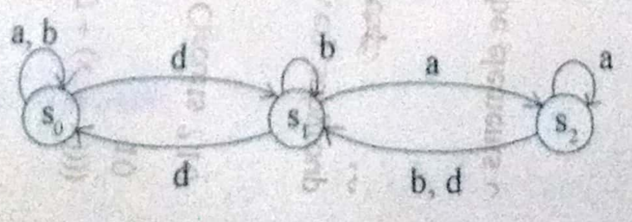
	0	1	2
s_0	s_1	s_0	s_2
s_1	s_0	s_0	s_1
s_2	s_2	s_0	s_2

8

OR

(C) Let G be the set of all non-zero real numbers and let $a * b = \frac{ab}{2}$. Show that (G, *) is an Abelian group. 8

(D) Let $S = \{s_0, s_1, s_2\}$ and I = {a, b, d}. Consider the finite-state machine M = (S, I, F) defined by the digraph as shown in figure below :



Compute the functions f_{bad}, f_{add} and f_{badadd} and verify that : 8

$f_{add} \circ f_{bad} = f_{badadd}$.

Master of Science (M.Sc.) (Computer Science) Semester—I (C.B.S.) Examination

DISCRETE MATHEMATICAL STRUCTURE

Compulsory Paper—1

Time : Three Hours] [Maximum Marks : 100

N.B. :— All questions are compulsory and carry equal marks.

EITHER

1. (a) Prove by mathematical induction

$$1^2 + 3^2 + 5^2 + \ldots\ldots (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3}$$ 10

(b) Obtain principle conjunctive normal form of

$(A \vee B) \wedge (B \vee C)(A \vee C)$ 10

OR

(c) Prove DeMorgan's theorem for set. 10

(d) Obtain principle disjunctive normal form of

$(P \wedge Q \vee R) \wedge (P \vee \dot{Q})$ 10

EITHER

2. (a) Let A = {a, b, c, d} and R be a relation on A that has matrix

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Show diagraph of it, also find indegree, outdegree for all vertices. 10

(b) Let R be a relation on set A. Then prove that R^∞ is transitive closure of R. 10

OR

(c) Let A = {1, 2, 3, 4, 5, 6}. Compute :

(i) (4.1 3 5) · (5 6 3)

(ii) (5 6 4) · (1 3 5) 10

(d) Explain properties of Relations. What is equivalence relation ? 10

EITHER

3. (a) Prove that :

Impt

Let L be a bounded distributive lattice. If a complement exists, it is unique. 10

(b) Define Hasse diagram. Draw Hasse diagram for the relation $A = \{1, 2, 3, 4\}$ whose matrix is

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

10

OR

(c) Prove that if G is a connected graph and every vertice has even degree, then there is an Euler circuit in G. 10

(d) Let the number of edges of G be m. Then prove that G has a Hamiltonian circuit if $m \geq \frac{1}{2}(n^2 - 3n + 6)$ (n is the number of vertices) 10

EITHER

4. (a) What are the properties of binary operation ? What is group ? 10

(b) What is Finite-state Machine ? 10

OR

(c) What is Semigroup ? Prove set P(s), where S is a set, together with the operation of union is a commutative semigroup. 10

(d) Let T be the set of all even integers. Show that the semigroup $(\mathbb{Z},+)$ and $(T, +)$ are isomorphic. 10

5. (a) What is Normal form ? 5

(b) What is pigeonhole principle ? 5

(c) What is Lattice ? 5

(d) What is isomorphism ? 5

NJR/KS/18/4107

Master of Science (M.Sc.) Semester—I (CBCS) (Computer Science) Examination

DISCRETE MATHEMATICAL STRUCTURE

Paper—1

Paper—I

Time : Three Hours]

[Maximum Marks : 80

N.B. :— All questions are compulsory and carry equal marks.

EITHER

1. (A) Obtain the principal disjunctive normal form of :

$$P \rightarrow ((P \rightarrow Q) \wedge \rceil(\rceil Q \vee \rceil P)).$$ 8

(B) Prove by mathematical induction that if $A_1, A_2, \ldots, A_n$ are any n sets, then :

$$\overline{\bigcap_{i=1}^{n} A_i} = \bigcup_{i=1}^{n} \overline{A_i}.$$ 8

OR

(C) If a and b are two positive integers, then GCD(a, b) · LCM(a, b) = ab. 8

(D) Prove that :

$$A - B = A \cap \overline{B}.$$ 8

EITHER

2. (A) Define transitive closure of relation R. Prove that R^{∞} is the transitive closure of relation R defined on set A. 8

(B) Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be invertible functions then prove that :

$$(g \circ f)^{-1} = f^{-1} \circ g^{-1}.$$ 8

OR

(C) Let R be a relation on set A = {1, 2, 3, 4, 5}

R = {(1, 2), (2, 1), (3, 4), (4, 3), (3, 5), (5, 3), (4, 5), (5, 4), (5, 5)}

Draw the graph and find the relational matrix. 8

(D) Find an explicit formula for sequence defined by $C_n = 3C_{n-1} - 2C_{n-2}$ with initial conditions $C_1 = 5$ and $C_2 = 3$. 8

EITHER

3. (A) Let L be a bounded distributive lattice. Show that if a complement exists, then it is unique. 8

(B) Prove that – If a graph G has more than two vertices of odd degree, then there can be no Euler path in G. 8

OR

(C) Show that if n is a positive integer and $p^2|n$, where p is prime number, then D_n is not Boolean Algebra. 8

(D) Let n be a positive integer and let D_n be the set of all positive divisors of n. Then D_n is a lattice under the relation of divisibility. If n = 20, then find D_n by using Hasse diagram. 8

EITHER

4. (A) Let G be the set of all non-zero real numbers and let :

$$a * b = \frac{ab}{2} \ \forall \ a, b \in G$$

Show that (G, $*$) is an abelian group. 8

(B) Let T be the set of all even integers. Show that the semigroup (z, +) and (T, +) are isomorphic. 8

OR

(C) Define group homomorphism and isomorphism. Prove that f is one to one if and only if $Ker(f) = \{e\}$. 8

(D) Let $(S, *)$ and $(T, *')$ be monoids with identities e and e′ respectively and let $f : S \to T$ be an isomorphism. Then show that $f(e) = e'$. 8

5. (A) Define :

(i) Power set

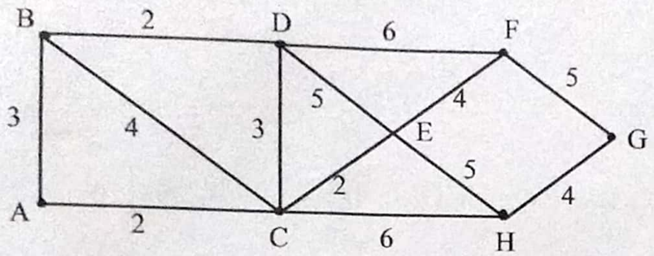
(ii) Symmetric difference. 4

(B) Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, determine whether the permutation is even or odd :

(i) $(6, 4, 2, 1, 5)$

(ii) $(4, 8) \circ (3, 5, 2, 1) \circ (2, 4, 7, 1)$. 4

(C) Find the Hamiltonian circuit for graph :



4

(D) Define :

(i) Semigroup

(ii) Monoid. 4

5. (A) Prove that :

$$\overline{A \cup B} = \overline{A} \cap \overline{B}.$$ 4

(B) How many different seven person committees can be formed each containing three women from an available set of 20 women and four men from an available set of 30 men ? 4

(C) Define :

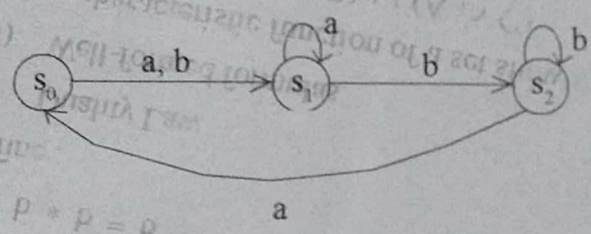
(i) Graph

(ii) Subgraph

(iii) Discrete graph

(iv) Complete graph. 4

(D) Construct the state transition table of the finite state machine of the digraph.



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Shri Shivaji Education Society Amravati's Science College, Congress Nagar, Nagpur.

Master of Science (Computer Science)Semester-I

CBCS

Test Examination -2019-20

Paper- I

(Discrete Mathematics)

Time:3 Hours] **[Max. Marks: 80**

N.B.: (1) All questions are compulsory.
(2) Draw a well labeled diagram wherever necessary.

1.EITHER

(A) Prove by Mathematical Induction that for $n\geq 1$ 8

$$1.2+2.3+\ldots\ldots+n(n+1)=\frac{n(n+1)(n+2)}{6}$$

(B)Show that :

$((P\vee Q)\wedge\neg(\neg P\wedge(\neg Q\vee\neg R)))\vee(\neg P\wedge\neg Q)\vee(\neg P\wedge\neg R)$ is a Tautology. 8

OR

(C) Obtain Principal Disjuctive Normal form of: 8

$P\Rightarrow((P\Rightarrow Q)\wedge\neg(\neg Q\vee\neg P))$

(D) Prove De'Morgan's theorem for set. 8

2. EITHER

(A) Obtain an explicit formula for the recurrence relation$d_n=2d_{n-1}-d_{n-2}$ with initial condition $d_1=1.5$ & $d_2=3$ 8

(B) Let R be relation from A to B & let A1 & A2 be subsets of Athen 8

prove that: R(A1UA2)=R(A1)U R(A2)

OR

(C) Explain properties of Relations. What is equivalence relation? 8

(D)LetA={a,b,c,d}& R={(a,a),(a,b),(a,c),(b,a),(b,b),(c,a),(c,b),(d,a)}show matrix relation, diagraph of it also find indegree, outdegreefor all vertices. 8

3. EITHER

(A)Let the number of edges of G be m.Then prove that G has a Hamiltonian circuit if 8

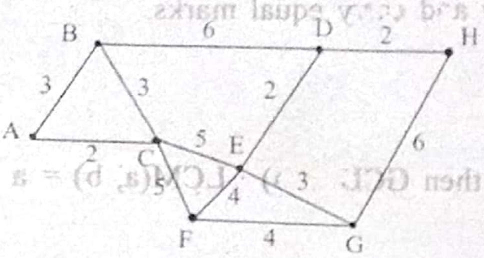
$m\geq\frac{1}{2}(n^2-3n+6)$ (n is the number of vertices)

(B)Determine the Hass diagram of the relation R let A={1,2,3,4}and R={(~~1,1),(1,2~~) 8
R={(1,1),(1,2),(2,2),(2,4),(1,3),(3,3),(3,4),(1,4),(4,4)}

OR

(C)Define Euler path and circuit.Prove if a graph G has more than two vertices of odd degree, there can be no Euler path in G. 8

(D)Determine a minimal spanning tree for the communication network shown below, using Prim's algorithm : 8



4. EITHER

(A)Let G be a group and let a and b be elements of G then prove that :

(i) (a–1)–1 = a

(ii) (ab)–1 = b–1.a–1

(B) Prove that if H and K are two normal subgroups of group G then H ∩ G is also a normal subgroup of G. 8

OR

(C)Let G be a group and let : H = {x : x ∈ G and xa = ax for all a ∈ G} then show that H is normal subgroup of G. 8

(D)Let R be a congruence relation on the group (G, ∗). Prove that the semigroup (G/R, ∗) is a group. 8

5. (A) What is Duality law? Write the duals of :

i)(PVQ)∧R ii)┐(PVQ)∧(PV┐(Q∧┐S))

(B) Let A={a,b,c,d,e} & R={(a,a),(a,b),(b,c),(c,c),(c,d),(d,e)} 4

Compute i)R^2 ii)R^∞

(C)Construct the tree for the following algebraic expression: 4

((3*(1-X))÷((4+(7-(Y÷2)))*(7+(X÷Y)))

(D) Explain: 4

(i)Semigroup

(ii)Monoid