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UNIT II

· COUNTING ·

The function has a basic application which are implemented on descrete data or objects, that application involves principal of counting. Following are some important type include in counting.

imp 1) Permutation
2) Combination
imp 3) Pigeon hole principal
4) Recurrence Relation
5) Mathematical Induction

1] Permutation :

A permutation is an arrangement of number of objects in some definate order taken at some or at all time. The total number of permutation of distinct object of i.e. 'n' taken 'r' at a particular time is denoted by $p(n,r)$

Where $1 \leq r \leq n$

The number of permutation of 'n' object taken 'r' at a time is determined by following formula:

$$p(n,r) = \frac{n!}{(n-r)!} \quad \text{or} \quad \boxed{{}^{n}p_{r} = \frac{n!}{(n-r)!}}$$

Ex Determines the value of following :

1) $^4P_2 = \frac{4!}{(4-2)!} = \frac{4!}{2!} = \frac{4\times3\times\cancel{2!}}{\cancel{2!}} = 12$

2) $^9P_3 = \frac{9!}{(9-3)!} = \frac{9!}{6!} = \frac{9\times8\times7\times\cancel{6!}}{\cancel{6!}} = 504$

3) $P(15,3) = \frac{15!}{(15-3)!} = \frac{15!}{12!} = \frac{15\times14\times13\times\cancel{12!}}{\cancel{12!}} = 2730$

4) $^{20}P_3 = \frac{20!}{(20-3)!} = \frac{20!}{17!} = \frac{20\times19\times18\times\cancel{17!}}{\cancel{17!}} = 6840$

5) $^{52}P_4 = \frac{52!}{(52-4)!} = \frac{52!}{48!} = \frac{52\times51\times50\times49\times\cancel{48!}}{\cancel{48!}} = 6497400$

6) $^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5\times4\times3\times\cancel{2!}}{\cancel{2!}} = 60$

* Word Problem

Ex: How many variables name of 8 letters can be form from the letter a, b, c, d, e, f, g, h, i, if no letter is repeated.

$\Rightarrow$ $^9P_8 = \frac{n!}{(n-r)!} = \frac{9!}{(9-8)!} = 9! = 362880$

$\because$ There are 9 letters given initially i.e. $n = 9$ & $r = 8$ 8 letters are to be related i.e. $r = 8$

$\therefore$ Total number of variables name of 8 letter is $(P(n,r)$

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Ex: How many 6 digit number can be form from the digit 0, 1, 2, 3, 4, 5, 6, 7. If there is no digit is repeated.

⇒ $n = 8, \; r = 6$

$${}^{8}P_{6} = \frac{8!}{(8-6)!} = \frac{8!}{2!} = \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times \cancel{2} \times 1}{\cancel{2}!}$$

$$= 20,160$$

* Permutation With Repeatation

It is a permutation when all object are not different. Some are repeated. Hence the number of permutation of n object of which n_1 objects are of 1 kind and n_2 object of another kind, when all are taken at a time is called repeated permutation & it is determined by the formula

$$K = \frac{n!}{n_1! \, n_2!}$$

Ex: Determines the number of permutation that can may out of the letters of word i.e. a ~~pro~~ PROGRAMMING?

⇒ Here are 11 letters in the word PROGRAMMING out of which G, R, M are repeated.

$\therefore \; n = 11 \qquad n_1 = 2$ for G, $n_2 = 2$ for M

$n_3 = 2$ for R

∴ permutation is

$$K = \frac{11!}{2! \times 2! \times 2!} = \frac{11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{2 \times 1 \times 2 \times 1 \times 2!}$$

$$= 4989{,}600$$

Ex: There are 4 blue, 3 red & 2 black pen in a box. This are drawn 1 by 1. Determine all the different permutation.

⇒ $n = 9 \quad n_1 = 4 \quad n_2 = 3 \quad n_3 = 2$

$$K = \frac{9!}{4! \times 2! \times 3!} = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{4 \times 3 \times 2 \times 1 \times 2 \times 1 \times 3 \times 2 \times 1}$$

$$= 1260$$

Ex: How many distinable permutation of the letter of word MISSISSIPPI

Ex: How many different variable name can be form by using the letter a, a, a, b, b, b, b, c, c, c

Ex: How many distinviable permutation of the letter in the word Banana

→ MISSISSIPPI

$n = 11 \qquad I = n_1 = 4 \qquad S = n_2 = 4 \qquad P = n_3 = 2$

$$K = \frac{n!}{n_1!\,n_2!\,n_3!} = \frac{11!}{4! \times 4! \times 2!}$$

$$= \frac{11 \times \cancel{10}^{5} \times \cancel{9}^{3} \times \cancel{8}^{2} \times 7 \times \cancel{6}^{3} \times 5 \times \cancel{4}^{2} \times \cancel{3} \times \cancel{2} \times 1}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{2} \times 1}$$

$$= 11 \times 5 \times 3 \times 7 \times 5 \times 3 \times 2$$

$$= 34650$$

→ a, a, a, b, b, b, b, c, c, c

$n = 10 \qquad a = n_1 = 3 \qquad b = n_2 = 4 \qquad c = n_3 = 3$

$$k = \frac{n!}{n_1! \times n_2! \times n_3!} = \frac{10!}{3! \times 4! \times 3!}$$

$$= \frac{10 \times \cancel{9}^{3} \times \cancel{8}^{4} \times 7 \times \cancel{6}^{3} \times 5 \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}{\cancel{3} \times \cancel{2} \times 1 \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{3} \times \cancel{2} \times 1}$$

$$= 12600$$

→ BANANA

$n = 6 \qquad A = n_1 = 3 \qquad n_2 = N = 2$

$$k = \frac{n!}{n_1!\,n_2!} = \frac{6!}{3! \times 2!} = \frac{6 \times 5 \times \cancel{4}^{2} \times \cancel{3} \times \cancel{2} \times 1}{\cancel{3} \times \cancel{2} \times 1 \times \cancel{2} \times 1}$$

$$= 60$$

* Combination :

A combination is a selection of same of all object from a set of given object where ordered of the object does not

The number of combination of n object taken r at a time is represented by C(n,r)

The combination of n object taken r at a time is determined by the formula

$$^{n}C_{r} = \frac{n!}{(n-r)! \times r!}$$

Determine the following eqn

1) $^{10}C_{6} = \frac{10!}{(10-6)! \times 6!} = \frac{10!}{4! \times 6!} = \frac{\overset{5}{\cancel{10}} \times \overset{3}{\cancel{9}} \times \overset{2}{\cancel{8}} \times 7 \times \cancel{6!}}{\cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \times \cancel{6!}} = 210$

2) $^{52}C_{4} = \frac{52!}{(52-4)! \times 4!} = \frac{52!}{(48! \times 4!} = \frac{52 \times 51 \times 50 \times 49 \times 48!}{4 \times 3 \times 2 \times 1 \times 48!}$

$= 270725$

3) $^{50}C_{45} = \frac{50!}{45! \times (50-45)!} = \frac{50!}{45! \times 5!} = \frac{50 \times 49 \times 48 \times 47 \times 46 \times 45!}{5 \times 4 \times 3 \times 2 \times 1 \times 45!}$

$=$

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W-18 Q. What is the value of $^{n}C_{r}$ compute.

1) $^{12}C_3$

2) $^{16}C_5$

3) $^{10}C_2$

(4) * Pigeon hole Principal

S-17 5m W-18 4m

Theorem:

Show that if n pigeon are assign to m pigeon hole and $m < n$ then there is a at least a one pigeon hole that contain two or more pigeon.

Proof:

Let us label the 'n' pigeon with the number 1 through 'n' and 'm' pigeon hold with the number 1 through 'm'.

How, starting with piegeon 1 and pigeon hole 1, assign each pigeon in order to the pigeon hole with the same number.

So, we can assign as many pigeons as possible to distinct pigeon holes.

But as we know that the pigeon hole are less than pigeon i.e. $m < n$.

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Thus, here remains $n-m$ pigeons that have not yet it been assign to a pigeon hole.

Hence, there is at least one pigeon hole that will be assign two or more pigeons.

Ex: show that at least two people must have their birthdate in the same month, if 13 people are asame assembled in the room

⇒ We assign each person to the month of the year on which he was born.

Since there are 12^{th} month in the year and their are 13 people i.e. In this case the pigeon are 13^{th} people and the pigeon hole are 12 month in the year i.e. 12

So accordingly to the pigeon hole principal their must be at least two people assign to the same month.

<u>OR</u> Statement:

It states that if n pigeon are assign to m pigeon hole and if the numbers of pigeon are very large then the number of pigeon hole must contain at least $[(n-1)/m]+1$ pigeons.

proof:

We can prove this by contradiction method Assume that each pigeon hole does not contain

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more than $[(n-1)/m]+1$ pigeon then there will be at most

$$m[(n-1)/m] \leq m(n-1)/m$$
$$= n-1 \text{ pigeons in all}$$

but the number of pigeon are very large than the pigeon hole.

Hence this is incontradiction to are assumption.

Hence for given pigeon hole one of the Pigeon hole must contain at least $[(n-1)/m]+1$

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Recursion Relation (Recurrence)

A recursion relation or Recurence is an equation that recursively define a sequence that is each term of sequence is defined as a function of the preceding terms.

A Recursive formula must be a combine by information above the begening of the sequence. This information is called as initial condition for sequence.

NOTE: A Recursive Relation is also called as difference relation.

Ex: The Recurrence Relation i.e.

$a_n = a_{n-1} + 3$ with initial condition $a_1 = 4$

it defines the sequence 4, 7, 10, 13, ...

There are two ways to find out the Recursive formula or explicit formula.

1) Back tracking
2) Theorem base method OR Using characteristic equation

* Theorem base Method

Q.1) Find and explicit formula for the sequence defi[ned] by the Recursive Relation.

$C_n = 3C_{n-1} - 2C_{n-2}$ with initial Condition

$C_1 = 5$ and $C_2 = 3$

Soln: The given Recurrence relation is ;

$C_n = 3C_{n-1} - 2C_{n-2}$

The given recurrence relation is a linear homogeneous relation and it is associated with

$x^2 = 3x - 2$

Rewrite the equation as,

$x^2 - 3x + 2 = 0$

which is linear equation & we have to Solve from method of factorization.

$= x^2 - 2x - x + 2 \quad = x(x-2) - 1(x-2)$

$= (x-2)(x-1)$

$= x = 2, \; x = 1$

$\therefore$ Value of x are nothing but two root of

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given linear equation,

i.e. $s_1 = 2$, $s_2 = 1$

Now,

By theorem base method the explicit formula can be given by

$$\boxed{C_n = Us_1^n + Us_2^n} \quad \text{---} (A)$$

Now, first we have to find out the values of U, V.

Hence by using eqn (A) we can put n=1

$s_1 = 2$ and $s_2 = 1$

For n = 1

$C_1 = Us_1^n + Vs_2^n$

$= U(2)^1 + V(2)^1$

$$\boxed{C_1 = 2U + 2V} \quad \text{---} (1)$$

for n = 2

$C_2 = Us_1^n + Vs_2^n$

$= U(2)^2 + V(1)^2$

$$\boxed{C_2 = 4U + V} \quad \text{---} (2)$$

Now, solving eqn (1) and (2) we get

$C_1 = 2U \cancel{+} V$

$C_2 = 4U + V$

$C_1 - C_2 = 2V$

$V = \dfrac{C_1 - C_2}{2} = \dfrac{3-5}{2} = \dfrac{2}{2} = 1$

$C_1 = 2U + V$

$5 = 2U + V$

$U = 5 + 2$

$U = 7$

$C_2 = 4U + V$

$3 = 4U + V$

$V = 3 - 4$

$V = -1$

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Now, from eq^n (A) put the value U, V, S_1 and

$$C_n = U S_1^n + V S_2^n$$

$$C_n = 7(1)^n + (-1) 2^n$$

$$\boxed{C_n = 7 - 2^n}$$

Q. obtain and explicit formula for the sequence define by $a_n = 4a_{n-1} + 5a_{n-2}$ with initial condition $a_1 = 2$ & $a_2 = 6$

⇒ The given Recurrence Relation is

$$a_n = 4a_{n-1} + 5a_{n-2}$$

The given Recurrence relation in a linear homogeneous relation and its associated with

$$x^2 = 4x + 5$$

Rewrite this eq^n as

$$x^2 - 4x + 5 = 0$$

which is linear equation and behave to solve it by method of factorization.

$$x^2 - 4x + 5 = 0$$

$$x^2 - 5x + x + 5$$

$$x(x-5) + 1(x-5)$$

$$(x-5)(x+1)$$

$$x = 5 \quad x = -1$$

Value of x are nothing but a two roots of given linear eq^n.

$$S_1 = 5 \quad \& \quad S_2 = -1$$

Now,

By th^m base method the explicit formula can be given by:

$a_n = Us_1^n + Vs_2^n$ - - - - (A)

How, 1st we have to find out the values of U & V.

Hence,

By using eq^n (A) we can put $n=5$, $S_1 = 5$

$S_2 =$

for $n = 5$

$a_5 = U(5)^5 + V(-1)^5$

$a_5 = 25U - V$ - - - - (1)

for $n = -1$

$a_{(-1)} = U(5)^{-1} + V(-1)^{-1}$

$a_1 = 5U - V$ - - - (2)

How solving eq^n (1) & (2)

$a_5 = 25U - \cancel{V}$

$a_1 = 5U - \cancel{V}$

put in eq^n (1)

* Relation And Diagram

let A and B are two not empty sets. A relation which in denoted by R, from set A to B. in a subset of 'A×B'.

If $R \subseteq A \times B$ and $(a,b) \subset R$. Hence we Say that A in Related to B. By Relation R and it in return as aRb.

If a is not Related to b. By relation R then we write '$a \not{R} b$'.

* eg: let A = {1,2,3,4,5}, Define the following relation R (<) on A that is A ⇒ aRb iff (a<b).

→ Given : A : {1,2,3,4,5}
Hence the Relation R in

R = {(1,2), (1,3), (1,4), (1,5), (2,3), (2,4), (2,5), (3,4), (3,5), (4,5)}

Graph of Relation OR Directed Graph or diagram

*** A Relation can also Represented by drawing its graph. It is called an directed draw.

let R in a Relation on a set.
$X = \{x_1, x_2, x_3 ---- x_n\}$, the elements of

*** X are Represented by points are circle called as nodes. these nodes may also be called Vertices. We connects nodes by arc and put arrow ($\rightarrow$) and the R in the direction from x_i to x_j.

When all nodes connected by R; with proper arrow ($\rightarrow$) we get a graph on the Relation R.

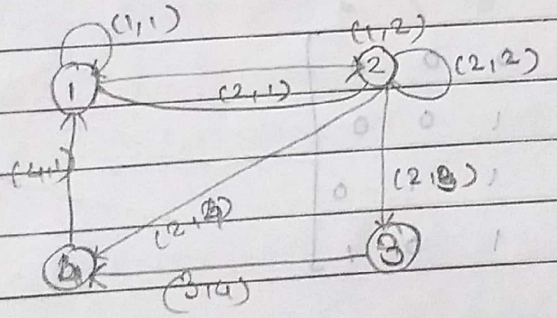
Que: let A = {1, 2, 3, 4} and R = {(1,1) (1,2) (2,1) (2,2) (2,3) (2,4) (3,4) (4,1)} Draw diagraph of relation 'R'.

⇒ Given A = {1, 2, 3, 4} and

R = {(1,1) (1,2) (2,1) (2,2) (2,3) (2,4) (3,4) (4,1)}

then the diagram

**
*



$$
MR = \begin{array}{c c} & \xrightarrow{\text{column}} \\ \begin{array}{c} 1 \\ 2 \\ \text{Row}\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}
$$

If R is a Relation on Set A and $a \in$ then in degree is number of $b \in a$ Such that $(b, a) \in R$

The outdegree a is a number of $b \in a$ Such that $(a, b \in R)$

Q. let $A = \{a, b, c, d\}$ and R be the Relation on a that has the matrix shown below.

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

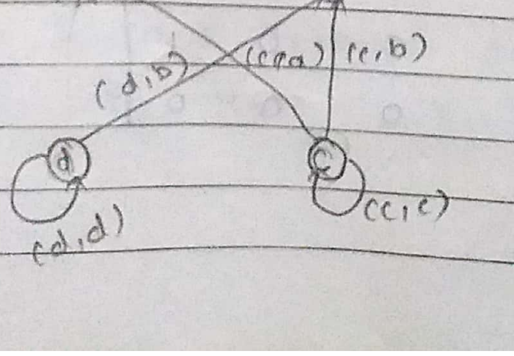
Construct the diagram.

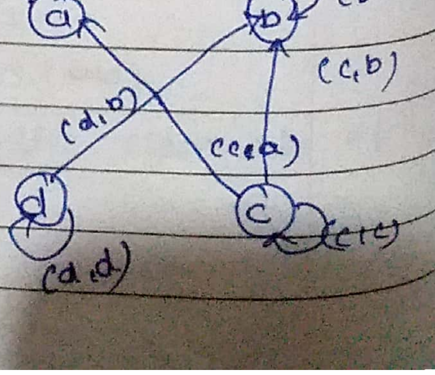
And find all in degree and outdegree or all Vertex.

→ Given : $A = \{a, b, c, d\}$

$$M_R = \begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

$R = \{(a, a), (b, b), (c, a), (c, b), (c, c), (d, b), (d, d)\}$





Node	Indegree	outdegree
a	2	1
b	3	1
c	1	3
d	1	2

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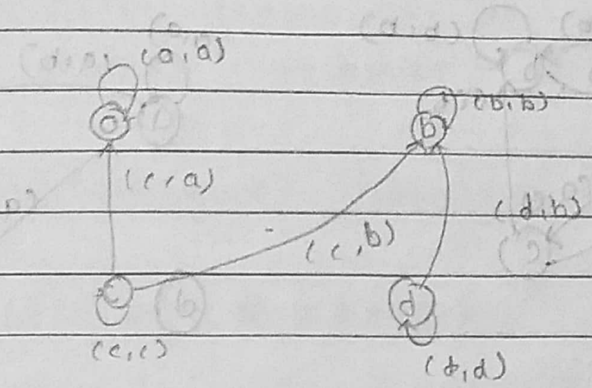
Q. From the above diagram following table gives the indegree and outdegree of vertices.



Nodes	Indegree	outdegree
a	2	1
b	3	1
c	1	3
d	1	2

Q. let $A = \{a, b, c, d\}$ & R be a relation on A that has matrix.

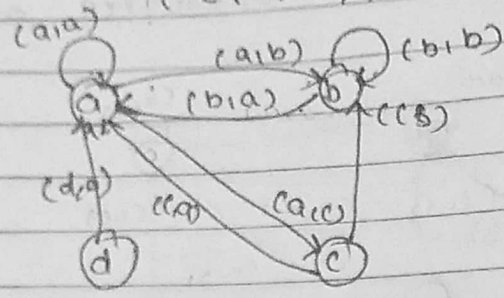
$$M_R = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

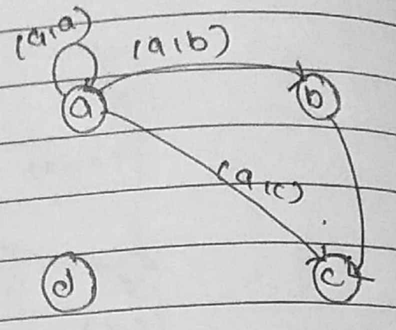
show: diagraph of it also find indegree & outdegree of all vertices.

⇒ Given : $A = \{a, b, c, d\}$

$$M_R = \begin{array}{c|cccc|} & a & b & c & d \\ a & 1 & 1 & 1 & 0 \\ b & 1 & 1 & 0 & 0 \\ c & 1 & 1 & 0 & 0 \\ d & 1 & 0 & 0 & 0 \end{array}$$

R = { (a,a) (a,b), (a,c), (b,a) (b,b) (c,a)
(c,b), (d,a) }





Nodes	Indegree	outdegree
a	4	3
b	3	2
c	1	2
d	-	1

H.W:

1) let x ± { 1,2,3,4 } & R = { (x,y) / x<y } draw digraph matrix indegree & outdegree.

2) let x = { 1,2,3,4 } & R = { (1,1) (1,4) (4,1) (4,4) (2,2) (2,3) (3,2) (3,3) find M_R & indegree & outdegree.

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* Equivalence Relation

Q. Explain properties of relation. What is equivalence relation? (reflexive)

* Properties of Relation

1) Reflexive Relation :

A relation R on set A is a reflexive if $(a, a) \in R$ for all $(\forall)$ $a \in A$ that is if aRa $\forall a \in A$

2) Irreflexive Relation :

A relation R on set A is irreflexive if $(a, a) \notin R$ for all $a \in A$ that is if $a \not R a$ $\forall a \in R$.

Ex: 1) Let $A = \{(a, a) / a \in A\}$ so that iA is the relation of equality on the set A than iA is reflexive. $(a, a) \in A$ for $\forall a \in A$.

2) Let $A = \{1, 2, 3\}$ & $R = \{(1,1) (1,2)$ $(a, a) \in A$ for $\forall a$ Then R is not reflexive $(2, 2) \in R$ & $(3, 3) \in R$

Also R is not irreflexive $(1, 1) \in R$.

Equivalence Relation

(b)

* Note :
1) R is reflexive if ⇒ a R a or (a, a) ∈ R.

2) R is symmetric if ⇒ a R b then b R a.

3) R is transitive if ⇒ a R b & b R c ⇒ a R c

Ex: let A = {1, 2, 3, 4, 5} & R = {(1,1) (1,2) (1,3) (2,1) (2,2) (3,1), (2,3) (3,3) (4,4) (3,2) (5,5)}
Determine R is equivalence relation or not.

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⇒ R is equivalence or not, it means
We have to prove that
R is reflexive, symmetric & transitive

(1) Reflexive :
Here in R we observe that reflexivity is satisfied. it means
R is reflexive as {(1,1) (2,2) (3,3) (4,4) (5,5)} ∈ R
∴ R is Reflexive.

2) Symmetric -
Here we have to prove to R is Symmetric a R b then b R a
If means a R b → b R a
Here,
(1,2) ∈ R & (2,1) ∈ R
(1,3) ∈ R & (3,1) ∈ R

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$(2,3) \in R$ & $(3,2) \in R$

$\therefore$ from the above elements R is symmetric.

3) Transitive :

Here we have to prove that R is transitive, it means

aRb & bRc $\Rightarrow$ aRc

in the given relation we observe that

aRb
bRc
aRc

$(1,2) \in R$ & $(2,3) \in R \Rightarrow (1,3) \in R$
$(3,1) \in R$ & $(1,3) \in R \Rightarrow (3,3) \in R$
$(1,3) \in R$ & $(3,1) \in R \Rightarrow (1,1) \in R$

$\therefore$ from the above element R is transitive

$\therefore$ R is equivalence Relation
since R is reflexive, transitive & symmetric.

Q. Let R be the relation whose matrix is

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

1) find the reflexive closure of R.
2) find the symmetric closure of R.

$\Rightarrow$ Given :

$$R = \begin{array}{c|ccccc|} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 0 & 1 & 0 & 1 \\ 3 & 1 & 1 & 1 & 0 & 0 \\ 4 & 0 & 1 & 1 & 0 & 0 \\ 5 & 0 & 0 & 1 & 0 & 1 \end{array}$$

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R = { (1,1) (1,4) (1,5) , (2,3) (2,5) , (3,1) (3,2) (3,3)
(4,2) (4,3) , (5,3) , (5,5) }

i] Reflexive :

Here in R we observe that reflexivity is satisfied

it means R is reflexive

{ (1,1) (3,3) (5,5) } ∈ R

∴ R is reflexive.

2) Symmetric :

Here we have to prove to R is Symmetric aRb then bRa

It means aRb → bRa

Here

(2,3) ∈ R ⇒ (3,2) ∈ R

∴ from the above elements R is symmetric.

Let R & S are relation from A to B then show that

i) $R \subseteq S \Rightarrow R^{-1} \subseteq S^{-1}$

ii) $(R \cup S)^{-1} \Rightarrow R^{-1} \cup S^{-1}$

1] $R \subseteq S \Rightarrow R^{-1} \subseteq S^{-1}$

$(a,b) \in (R \subseteq S)$

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1) let A = {a, b, c, d} & R = {(a, a) (b, a) (b, b) (c, c) (d, d) (d, c)} Determine R in equivalence relation or not.

⇒ Given relation A = {a, b, c, d} & R = {(a, a) (b, a) (b, b) (c, c) (d, d) (d, c)}

R is equivalence or not it means we have to prove that R is reflexive, symmetric & transitive

1] Reflexive :

Here in R we observe that reflexvity is satisfied.

It means R is reflexive

{(a, a) (b, b) (c, c) (d, d)} ∈ R

∴ R is reflexive

2] Symmetric :

Here we have to prove to R is symetric aRb then bRa

It means aRb → bRa

Here,

From the above elements R is not symmetric.

3] Transitive :

Here we have to prove that R is transitive.

It means aRb & bRc ⇒ aRc in the given relation we observe that

2] let $A = \{1, 2, 3, 4\}$ & $R = \{(1,1) (1,3) (2,2) (2,4) (3,1) (3,3) (4,4) (4,2)\}$ determine R is equivalence relation or not.

⇒ Given relation $A = \{1, 2, 3, 4\}$ & $R = \{(1,1) (1,3) (2,2) (2,4) (3,1) (3,3) (4,4) (4,2)\}$

R is equivalence or not it means we have to prove that

R is reflexive, symmetric & transitive.

1] Reflexive:

Here in R we observe that reflexvity is satisfied.

it means R is reflexvive.

$\{(1,1) (2,2) (3,3) (4,4)\}$

∴ R is reflexive

2] Symmetric:

Here we have to prove to R is symmetric aRb then bRa

it means aRb → bRa

Here,

$(2,4) \in R \rightarrow (4,2) \in R$

$(1,3) \in R \rightarrow (3,1) \in R$

∴ from the above elements R is summetric

3] Transitive:

Here we have to prove to R is transitive aRb & bRa then aRc

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It means aRb & $bRc \Rightarrow aRc$ in the given relation we observe that

$(1,3) \in R$ & $(3,1) \in R \Rightarrow (1,1) \in R$
$(2,4) \in R$ & $(4,2) \in R \Rightarrow (2,2) \in R$

$\therefore$ from the above elements R is transitive.

$\therefore$ R is equivalence Relation.
Since R is reflexive, symmetric & transitive.

—— x ——

Aishwarya

1/9/19

Q. Let R & S are relation from A to B then show that
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i] $(R \cup S)^{-1} = R^{-1} \cap S^{-1}$
ii] $(R \cap S)^{-1} = R^{-1} \cup S^{-1}$

$\Rightarrow$ i] $(R \cup S)^{-1} = R^{-1} \cap S^{-1}$

Let $(a,b) \in (R \cup S)^{-1}$
So, we have, $(b,a) \in (R \cup S)$

Now,
$(b,a) \in R$ OR $(b,a) \in S$

It means that
$(a,b) \in R^{-1}$ ~~or~~ and $(a,b) \in S^{-1}$

Hence
$\boxed{(a,b) \in R^{-1} \cap S^{-1}}$ --- ①

Conversly,
hence $(a,b) \in R^{-1} \cap S^{-1}$

So we have $(a,b) \in R^{-1}$ ~~and~~ or $(a,b) \in S^{-1}$

This means that,
$(b,a) \in R$ ~~and~~ or $(b,a) \in S$

So $(b, a) \in R \cup S$

hence $\boxed{(a, b) \in (R \cup S)^{-1}}$ — (ii)

from (i) & (ii)

$$\boxed{(R \cup S)^{-1} = R^{-1} \cap S^{-1}}$$

ii] $(R \cap S)^{-1} = R^{-1} \cup S^{-1}$

⇒

let $(a, b) \in (R \cap S)^{-1}$

so, we have

$(b, a) \in (R \cap S)$

Now,

$(b, a) \in R$ and $(b, a) \in S$

This means that

$(a, b) \in R^{-1}$ ~~and~~ or $(a, b) \in S^{-1}$

hence

$\boxed{(a, b) \in R^{-1} \cup S^{-1}}$ — (i)

Conversly,

hence $(a, b) \in R^{-1} \cup S^{-1}$

So, we have $(a, b) \in R^{-1}$ or $(a, b) \in S^{-1}$

This means that

$(b, a) \in R$ and $(b, a) \in S$

so $(b, a) \in R \cap S$

Hence $\boxed{(a, b) \in (R \cap S)^{-1}}$ — (ii)

from (i) & (ii)

$$(R \cap S)^{-1} = R^{-1} \cup S^{-1}$$

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Imp 8m

Q. Let R be relation from A to B & let A_1 & A_2 be subset of A then prove that, $R(A_1 \cup A_2) = R(A_1) \cup R(A_2)$

Sol^n: Theorem :

let R be a relation from A to B and let A_1 & A_2 be subset of A then

a) If $A_1 \subseteq A_2$, then $R(A_1) \subseteq R(A_2)$

b) $R(A_1 \cup A_2) = R(A_1) \cup R(A_2)$

c) $R(A_1 \cap A_2) \subseteq R(A_1) \cap R(A_2)$

Proof : a] If $A_1 \subseteq A_2$, then $R(A_1) \subseteq R(A_2)$

let R be a relation from A to B & A_1 & A_2 be subset of A.

ⓐ If $y \in R(A_1)$ then xRy, $\forall x \in A_1$

$\therefore A_1 \subseteq A_2$, $x \in A_2$

Thus,

$y \in R(A_2)$

$\therefore R(A_1) = R(A_2)$

$\therefore \boxed{R(A_1) \subseteq R(A_2)}$

ⓑ If $y \in R(A_1 \cup A_2)$ by def^n

xRy $\forall x$ in $A_1 \cup A_2$

If x is in A_1 then xRy

we must have, $y \in R(A_1)$

By the same argument if x is in A_2 then

$y \in R(A_2)$

In either case

Thus, we have show that

$R(A_1 \cup A_2) \subseteq R(A_1) \cap R(A_2)$

Conversly,

Since, $A_1 \subseteq (A_1 \cup A_2)$
we know that,
$R(A_1) \subseteq R(A_1 \cup A_2)$

$||^{ly}$,
$R(A_2) \subseteq R(A_1 \cup A_2)$

Thus,
$R(A_1) \cup R(A_2) \subseteq R(A_1 \cup A_2)$

$\therefore R(A_1) \cup R(A_2) = R(A_1 \cup A_2)$

$\boxed{c}$ If $y \in R(A_1 \cap A_2)$ then $\forall x$ in $A_1 \cap A_2$, xRy

$\therefore x$ is in both A_1 & A_2
it follows that, y is in both $R(A_1)$ and $R(A_2)$

$\therefore y \in R(A_1) \cap R(A_2)$

$\therefore \boxed{R(A_1 \cap A_2) \subseteq R(A_1) \cap R(A_2)}$

$\therefore$ Hence proved.

Q. Let R ... relation on set A then prove that R^{∞} is a transitive closure of R.

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WARSHANN'S ALGORITHM

If relation R for a set is not a transitive, we need to apply closer to make it transitive. This closure is known as transitive closure. We use the warshann's method to find transitive closure.

eg: let $A = \{a, b, c, d\}$ and $R = \{(a,b), (b,a) (b,c) (c,d)\}$ find the transitive closure.

⇒ Steps :

Step No 1 : We first write the matrix M_R of the Relation R and denote by W_0.

Step 2 :

We write a blank matrix of order 4, denote it by W_1 and transfer all 1's from W_0.

⋮

Continue until you get same matrix.

$$W_0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$W_1 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$1/1$

column	row
P_i	q_j
$\downarrow$,	
2	$2 \to (2,2)$

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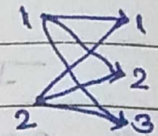
column row

$$W_2 = \begin{array}{cc|cccc} & & 1 & 2 & 3 & 4 \\ & & a & b & c & d \\ \hline 1 & a & 1 & 1 & 1 & 0 \\ 2 & b & 1 & 1 & 1 & 0 \\ 3 & c & 0 & 0 & 0 & 1 \\ 4 & d & 0 & 0 & 0 & 1 \end{array}$$

2/2 p_i q_j



(1,1)(1,2) (1,3)(2,1),(2,2) (2,3)

column row

$$W_3 = \begin{array}{cc|cccc} & & 1 & 2 & 3 & 4 \\ & & a & b & c & d \\ \hline 1 & a & 1 & 1 & 1 & 1 \\ 2 & b & 1 & 1 & 1 & 1 \\ 3 & c & 0 & 0 & 0 & 1 \\ 4 & d & 0 & 0 & 0 & 1 \end{array}$$

3/3 p_i q_j

1 ⟶ 4

2 ↗

(1,4) (2,4)

$$W_4 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & & & & \\ b & & & & \\ c & & & & \\ d & & & & \end{array}$$

: p_i q_j

1

2 ϕ

3 :

$W_4 = \{\phi\}$ (No relation found)

$R = \{(a,a)\ (a,b)\ (a,c)\ (a,d)\ (b,a)\ (b,b)\ (b,c)$
$(b,d)\ (c,d)\}$

(1)

* Permutation Function

A permutation on set A is ~~by~~ bijection from A into A (itself) Here is set A is a finite. If set A contains & elements then there are n! different permutation on A : The matrix form for descending the function on a finite set is to list domain across top row & across the image of each element exactly.

Eg: Consider $A = \{a, b\}$ determine all the permutation of A

⇒ The set A has two ~~dom~~ element. Hence, it has $2! = 2$ permutation are as follows :

$$P_1 = \begin{pmatrix} a & b \\ a & b \end{pmatrix} \qquad P_2 = \begin{pmatrix} a & b \\ b & a \end{pmatrix}$$

i] Inverse of a Permutation

e.g: Consider the permutation

$$P = \begin{pmatrix} a & b & c & d \\ d & c & a & b \end{pmatrix}$$

Determine inverse P^{-1} : To find P^{-1}.

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$$\Rightarrow \quad p^{-1} = \begin{pmatrix} d & c & a & b \\ a & b & c & d \end{pmatrix}$$

$$p^{-1} = \begin{pmatrix} d & a & b & c \\ a & c & d & b \end{pmatrix}$$

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* Composition of two permutation

e.g.: Consider a finite set $A = \{5, 6, 7\}$
let $P_1 = \begin{pmatrix} 5 & 6 & 7 \\ 5 & 7 & 6 \end{pmatrix}$ & $P_2 = \begin{pmatrix} 5 & 6 & 7 \\ 6 & 5 & 7 \end{pmatrix}$ be two permutation of A. Determine $P_1 \circ P_2$ and $P_2 \circ P_1$

$\Rightarrow$ $$P_1 \circ P_2 = \begin{pmatrix} 5 & 6 & 7 \\ 5 & 7 & 6 \end{pmatrix} \circ \begin{pmatrix} 5 & 6 & 7 \\ 6 & 5 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 6 & 7 \\ 6 & 7 & 5 \end{pmatrix}$$

$$P_2 \circ P_1 = \begin{pmatrix} 5 & 6 & 7 \\ 6 & 5 & 7 \end{pmatrix} \circ \begin{pmatrix} 5 & 6 & 7 \\ 5 & 7 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 6 & 7 \\ 7 & 5 & 6 \end{pmatrix}$$

* Cyclic Permutation

Ex: Consider the set $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ Determine the permutation P denote by the cycle.

Soln:

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Eg: Consider the permutation $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 4 & 6 & 1 & 5 & 3 & 2 \end{pmatrix}$

$\Rightarrow$ $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 4 & 6 & 1 & 5 & 3 & 2 \end{pmatrix}$

(1) $(1, 7, 2, 4)$ (2) $(3, 6)$

E.g: Consider a set $A = \{a, b, c, d, e, f, g, h\}$ Now find product $\{e, f, c, d, a\} \circ \{a, b, h, g\}$

$\Rightarrow$ $\{e, f, c, d, a\} = \begin{pmatrix} a & b & c & d & e & f & g & h \\ e & b & d & a & f & c & g & h \end{pmatrix}$

$\{a, b, h, g\} = \begin{pmatrix} a & b & c & d & e & f & g & h \\ b & h & c & d & e & f & a & g \end{pmatrix}$

$\{e, f, c, d, a\} \circ \{a, b, h, g\}$

$= \begin{pmatrix} a & b & c & d & e & f & g & h \\ e & h & d & b & f & c & a & g \end{pmatrix}$

$\therefore$ It is a cyclic permutation.

E.g: let $A = \{1, 2, 3, 4, 5, 6\}$ compute

(1) $(4, 1, 3, 5) \circ (5, 6, 3)$

(2) $(5, 6, 4) \circ (1, 3, 5)$

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1] $(4,1,3,5) \circ (5,6,3)$

$\Rightarrow (4,1,3,5) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 5 & 1 & 4 & 6 \end{pmatrix}$

$(5,6,3) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 5 & 4 & 6 & 3 \end{pmatrix}$

$(4,1,3,5) \circ (5,6,3) =$

$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 6 & 1 & 4 & 3 \end{pmatrix}$ ∴ It is cyclic permute

2] $(5,6,4) \circ (1,3,5)$

$\Rightarrow (5,6,4) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 5 & 6 & 4 \end{pmatrix}$

$(1,3,5) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 5 & 4 & 1 & 6 \end{pmatrix}$

$(5,6,4) \circ (1,3,5) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 5 & 1 & 6 & 4 \end{pmatrix}$

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Q. Show that composition of two disjoint cycles is commutative consider the set $A = \{a, b, c, d, e, f, g\}$ Now find the product of :
$(a\ d\ g) \circ (b\ c\ f)$ & $(b\ c\ f) \circ (a\ d\ g)$

→ $(a\ d\ g) = \begin{pmatrix} a & b & c & d & e & f & g \\ d & b & c & g & e & f & a \end{pmatrix}$

$(b, c, f) = \begin{pmatrix} a & b & c & d & e & f & g \\ a & c & f & d & e & b & g \end{pmatrix}$

$(a\ d\ g) \circ (b\ c\ f) = \begin{pmatrix} a & b & c & d & e & f & g \\ d & c & f & g & e & b & a \end{pmatrix}$ --- ①

$(b\ c\ f) = \begin{pmatrix} a & b & c & d & e & f & g \\ a & c & f & d & e & b & g \end{pmatrix}$

$(a\ d\ g) = \begin{pmatrix} a & b & c & d & e & f & g \\ d & b & c & g & e & f & a \end{pmatrix}$

$(b\ c\ f) \circ (a\ d\ g) = \begin{pmatrix} a & b & c & d & e & f & g \\ d & c & f & g & e & b & a \end{pmatrix}$ --- ②

From ① and ② composition of two disjoint cycle is commutative.

Hence proved

Imp. m

* Path in Relation & Digraph

Consider a relation R on set A. A path of length n in R from A to B is a finite sequence $\pi = a_1 x_1 \cdot a_2 x_2 \cdots a_n$ $\pi = a, x_1, x_2 \cdots x_{n-1}, b$ begining with A and ending with B such that $a R x_1, x_1 R x_2 \cdots x_{n-1} R b$

Note that, length n involves n+1 elements of A.

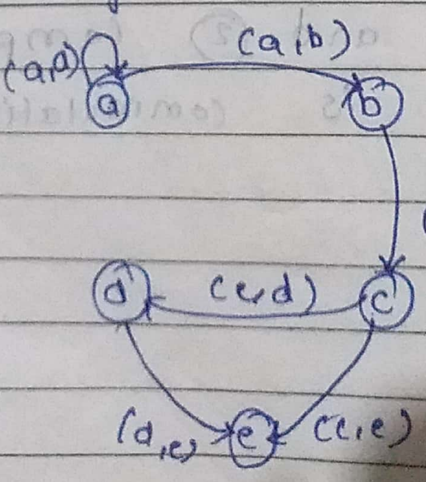
* Cycle :

A path that begins & ends at the same vertex is called cycle.

Q. let $A = \{a, b, c, d, e\}$ & $R = \{(a,a)\ (a,b)\ (b,c), (c,e)\ (c,d)\ (d,e)\}$ compute i) R^2. ii) R^{∞}

⇒ To find R^2 Given: $A = \{a, b, c, d, e\}$
$R = \{(a,a)\ (a,b)\ (b,c)\ (c,e)\ (c,d)\ (d,e)\}$

So the diagraph of R is shown below.



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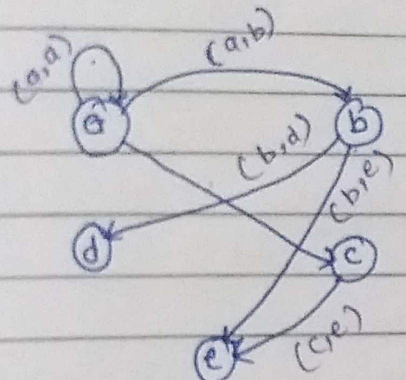
a] To find R^2 from the digraph following are the path, of length $n=2$

aR^2a since aRa and aRa
aR^2b since aRb and
aR^2c since
bR^2e since
bR^2d
cR^2e

$R^2 = \{(a,a)\ (a,b)\ (a,c)\ (b,c)\ (b,d)\ (c,e)\}$



b] To find R^∞ we need all ordered pair of vertices for which there is path of any length from the first vector to the second from the previous figure which is hence from the previous figure which is based on relation R we see that,

$R^\infty = \{(a,a)\ (a,b)\ (a,c)\ (a,d)\ (a,e)\ (b,c)$
$(b,d), (b,e)\ (c,d)\ (c,e)\ (d,e)$