② $|A|$ = no. of element in set A

$$\boxed{|A| = 4}$$

③ $|P(A)|$ = no of element in powe set A

$$\boxed{|P(A)| = 16}$$

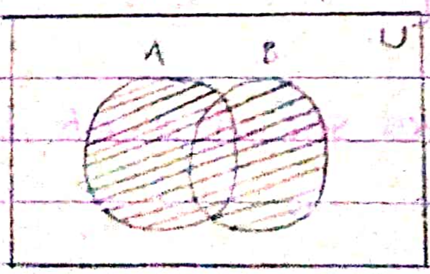
* Operations of set *

① Union :- If A & B are any two set then union is denoted be $A \cup B$. It is the set consisting of all elements of set A or set B. It is define as ~~$A \cup B$~~

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

$$A \cup B = \{x \mid x \in A \cup x \in B\}$$

Venn diagramatically it is represented as -



▨ $A \cup B$

ex→ $A = \{1,2,3,5,7\}$

$B = \{1,2,4,6,8,9\}$

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

$\boxed{A \cup B = \{1,2,3,4,5,6,7,8,9\}}$

* Properties of Union *

① If A and B are any two set then show that $A \cup B = B \cup A$

→ L.H.S. $= A \cup B$

$= \{x \mid x \in A \text{ or } x \in B\}$

$= \{x \mid x \in A \cup x \in B\}$

$= \{x \mid x \in B \cup x \in A\}$

$= \{x \mid x \in B \cup A\}$

$= B \cup A$

$=$ R.H.S.

$\boxed{\therefore \text{L.H.S.} = \text{R.H.S.}}$

② If A, B & C are any three set then show that $A \cup (B \cup C) = (A \cup B) \cup C$

→ L.H.S. $= A \cup (B \cup C)$

$= \{x \mid x \in A \text{ or } x \in (B \cup C)\}$

$= \{x \mid x \in A \cup x \in (B \cup C)\}$

$= \{x \mid x \in A \cup (x \in B \cup x \in C)\}$

$= \{x \mid (x \in A \cup x \in B) \cup x \in C\}$

$= \{x \mid (x \in A \cup B) \cup x \in C\}$

$= \{x \mid x \in (A \cup B) \cup C\}$

$= (A \cup B) \cup C$

$=$ R.H.S.

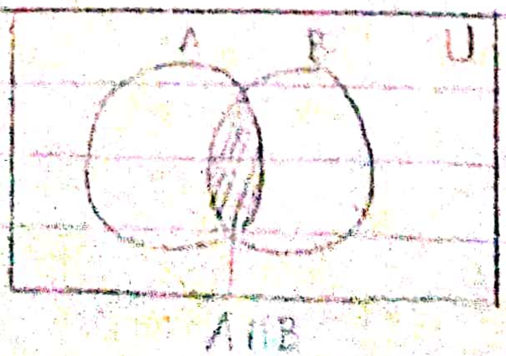
$\therefore$ $\boxed{\text{L.H.S.} = \text{R.H.S.}}$

(2) Intersection :- If A & B are any two set then their intersection is denoted by $A \cap B$. It is the set consisting of all elements of set A and set B. Symbolically it is defined as

$$A \cap B = \{x \mid x \in A \ \& \ x \in B\}$$

$$A \cap B = \{x \mid x \in A \cap x \in B\}$$

Venn diagrammatically it is represented as -



▨ → $A \cap B$

ex→ $A = \{1, 2, 3, 5, 7\}$

$B = \{2, 3, 6, 7, 8\}$

$A \cap B = \{x \mid x \in A \ \& \ x \in B\}$

$\boxed{A \cap B = \{2, 3, 7\}}$

* Properties of Intersection *

(1) If A, B & C are any three set then –

(1) $A \cap B = B \cap A$

(2) $A \cap (B \cap C) = (A \cap B) \cap C$

(1) L.H.S. $= A \cap B$

$= \{x \mid x \in A \ \cancel{\text{and}} \ x \in B\}$

$= \{x \mid x \in A \cap x \in B\}$

$= \{x \mid x \in B \cap x \in A\}$

$= \{x \mid x \in B \cap A\}$

$= B \cap A$

$=$ R.H.S.

$\boxed{\text{L.H.S.} = \text{R.H.S.}}$

(2) $L.H.S. = A \cap (B \cap C)$

$= \{x \mid x \in A \text{ and } x \in (B \cap C)\}$

$= \{x \mid x \in A \cap x \in (B \cap C)\}$

$= \{x \mid x \in A \cap (x \in B \cap x \in C)\}$

$= \{x \mid (x \in A \cap x \in B) \cap x \in C\}$

$= \{x \mid (x \in A \cap B) \cap x \in C\}$

$= \{x \mid x \in (A \cap B) \cap C\}$

$= (A \cap B) \cap C$

$= R.H.S.$

$\boxed{\therefore L.H.S. = R.H.S.}$

Que. If $A = \{1, 2, 3, 5, 7\}$ $B = \{2, 3, 4, 6, 7, 8\}$

$C = \{1, 5, 3, 8\}$

S.T. (1) $A \cap B = B \cap A$

(2) $A \cap (B \cap C) = (A \cap B) \cap C$

(3) $A \cup B = B \cup A$

(4) $A \cup (B \cup C) = (A \cup B) \cup C$

$\longrightarrow$ (1) $A \cap B = B \cap A$

L.H.S. $A \cap B = \{x | x \in A \ \& \ x \in B\}$
$= \{2, 3, 7\}$

R.H.S. $B \cap A = \{x | x \in B \ \& \ x \in A\}$
$= \{2, 3, 7\}$

$\therefore \boxed{A \cap B = B \cap A}$

---

(3) $A \cap (B \cap C) = (A \cap B) \cap C$

→ **L.H.S** $A \cap (B \cap C)$

$\because (B \cap C) = \{x | x \in B \ \& \ x \in C\}$
$= \{3, 8\}$

$A \cap (B \cap C) = \{x | x \in A \ \& \ x \in (B \cap C)\}$
$= \{3\}$

R.H.S. $= (A \cap B) \cap C$

$\therefore (A \cap B) = \{x | x \in A \ \& \ x \in B\}$
$= \{2, 3, 7\}$

$(A \cap B) \cap C = \{x | x \in (A \cap B) \ \& \ x \in C\}$
$= \{3\}$

$\boxed{\therefore A \cap (B \cap C) = (A \cap B) \cap C}$

---

(3) $A \cup B = B \cup A$

→ **L.H.S** $= A \cup B$
$= \{x | x \in A \text{ or } x \in B\}$
$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

R.H.S $= B \cup A$

$= \{x \mid x \in B \text{ or } x \in A\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

$\therefore$ $\boxed{\text{L.H.S.} = \text{R.H.S.}}$

④ $A \cup (B \cup C) = (A \cup B) \cup C$

$\rightarrow$ L.H.S. $= A \cup (B \cup C)$

$\because B \cup C = \{x \mid x \in B \text{ or } x \in C\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

$A \cup (B \cup C) = \{x \mid x \in A \text{ or } x \in (B \cup C)\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

R.H.S. $= (A \cup B) \cup C$

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

$(A \cup B) \cup C = \{x \mid x \in (A \cup B) \text{ or } x \in C\}$

$= \{1, 2, 3, 4, 5, 6, 7, 8\}$

$\boxed{\therefore \text{L.H.S.} = \text{R.H.S}}$

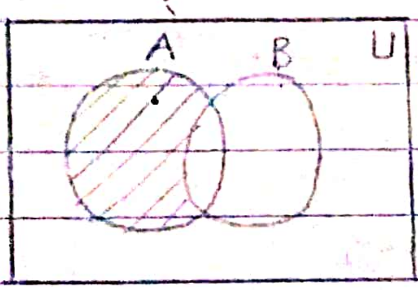
4/8/12

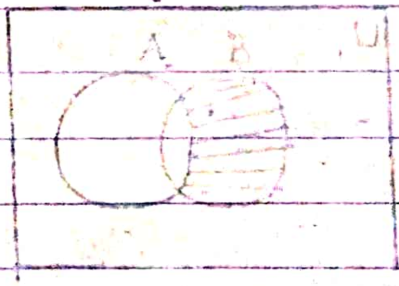
* Symmetric difference of a set *

If A & B are any two set then symmetric diff. of set A with respect to B is denoted by $A-B$. It is the set consisting of all element of set A which are not in set B. Symbollically it is define as

$$A-B=\{x \mid x\in A \ \& \ x\notin B\}$$
$$=\{x \mid x\in A \cap x\notin B\}$$

Venn diagrammatically represented as -





▨ ⇒ A−B   ▨ ⇒ B−A

ex → $A=\{1,2,3,5,7\}$

$B=\{1,2,4,6,7,8,9\}$

$A-B=\{x/x\in A \cap x\notin B\}$

$\boxed{A-B=\{3,5\}}$

$B-A=\{x/x\in B \cap x\notin A\}$

$\boxed{B-A=\{4,6,8,9\}}$

4/8/12

v.v.imp

***Que. If A & B are any two set then show that

1. $A-(A\cap B) = A-B$
2. $A-B = A\cap \overline{B}$
3. $A-B = A\cap \overline{B}^{c}$

→ 1. $A-(A\cap B) = A-B$

$$
\begin{aligned}
L.H.S. &= A-(A\cap B) \\
&= \{x \mid x\in A \ \&\ x\notin (A\cap B)\} \\
&= \{x \mid x\in A \cap x\notin (A\cap B)\} \\
&= \{x \mid x\in A \cap (x\notin A \cup x\notin B)\} \\
&= \{x \mid (x\in A\cap x\notin A) \cup (x\in A \cap x\notin B)\} \\
&= \{x \mid (x\in A\cap x\in \overline{A}) \cup (x\in A \cap x\notin B)\} \\
&= \{x \mid (x\in A\cap \overline{A}) \cup (x\in A-B)\} \\
&= \{x \mid (x\in \phi) \cup (x\in A-B)\} \\
&= \{x \mid x\in A-B\} \\
&= A-B \\
&= R.H.S.
\end{aligned}
$$

$\boxed{\therefore L.H.S. = R.H.S.}$

4/8/12

② $A-B = A \cap \overline{B}$

$\longrightarrow$ $L.H.S. = A-B$

$= \{x \mid x \in A \ \& \ x \notin B\}$

$= \{x \mid x \in A \cap x \notin B\}$

$= \{x \mid x \in A \cap x \in \overline{B}\}$

$= \{x \mid x \in (A \cap \overline{B}\}$

$= A \cap \overline{B}$

$= R.H.S.$

$\boxed{\therefore L.H.S. = R.H.S.}$

③ $A-B = A \cap B^c$

$\longrightarrow$ $L.H.S. = A-B$

$= \{x \mid x \in A \ \& \ x \notin B\}$

$= \{x \mid x \in A \cap x \notin B\}$

$= \{x \mid x \in A \cap x \in \overline{B}\}$

$= \{x \mid x \in A \cap x \in B^c\}$

$= \{x \mid x \in A \cap B^c\}$

$= A \cap B^c$

$= R.H.S.$

$\boxed{\therefore L.H.S. = R.H.S.}$

4/8/2021

Que. If $A = \{1, 3, 5, 7, 9\}$

$B = \{1, 2, 3, 4, 8, 9, 10\}$

$C = \{3, 4, 5, 6, 8, 9, 10\}$

Find $A-B$, $B-C$, $A-C$, $C-B$, $B-A$, $C-A$.

① $A-B$

$A-B = \{x \mid x \in A \ \& \ x \notin B\}$

$\therefore \boxed{A-B = \{5, 7\}}$

② $B-C = \{x \mid x \in B \ \& \ x \notin C\}$

$\therefore \boxed{B-C = \{1, 2\}}$

③ $A-C = \{x \mid x \in A \ \& \ x \notin C\}$

$\boxed{\therefore A-C = \{1, 7\}}$

④ $C-B = \{x \mid x \in C \ \& \ x \notin B\}$

$\boxed{\therefore C-B = \{5, 6\}}$

⑤ $B-A = \{x \mid x \in B \ \& \ x \notin A\}$

$\boxed{\therefore B-A = \{2, 4, 8, 10\}}$

⑥ $C-A = \{x \mid x \in C \ \& \ x \notin A\}$

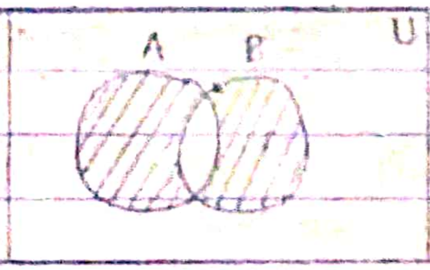
$\boxed{\therefore C-A = \{4, 6, 8, 10\}}$

* Symmetric Difference of boolean sum *

If A and B are any two set then symmetric difference of boolean sum is denoted by $A+B$ or $A \oplus B$. & is define as -

$$A \oplus B = A+B = \{x \mid x \in A-B \text{ or } x \in B-A\}$$
$$= \{x \mid x \in A-B \cup x \in B-A\}$$

Venn Diagrammatically it is represented as -



$A \oplus B$

for ex → $A = \{1,2,3,5,7\}$

$B = \{1,2,3,4,5,6,8,9\}$

$A \oplus B = \{x \mid x \in A-B \cup x \in B-A\}$

$A-B = \{7\}$

$B-A = \{4,6,8,9\}$

$$\boxed{A \oplus B = \{4,6,7,8,9\}}$$

Que. If A is any set then show that -

① $A + A = \phi$

→

$L.H.S = A + A$

$= \{x \mid x \in A - A \text{ or } x \in A - A\}$

$= \{x \mid x \in (A - A) \cup x \in (A - A)\}$

$= \{x \mid (x \in A \cap x \notin A) \cup (x \in A \cap x \notin A)\}$

$= \{x \mid (x \in A \cap x \in \bar{A}) \cup (x \in A \cap x \in \bar{A})\}$

$= \{x \mid (x \in A \cap \bar{A}) \cup (x \in A \cap \bar{A})\}$

$= \{x \mid x \in \phi \cup x \in \phi\}$

$= \{x \mid x \in \phi \cup \phi\}$

$= \{x \mid x \in \phi\}$

$= \phi$

$= R.H.S.$

$\boxed{\therefore L.H.S. = R.H.S}$

6/8/12

Que. If ~~then~~ $U = \{a, b, c, d, e, f, g, h, k\}$

$A = \{a, b, c, g\}$, $B = \{d, e, f, g\}$, $C = \{a, c, f\}$ $D = \{f, h, k\}$

~~then~~ compute $\bar{A}$, $A \oplus B$, $C \oplus D$, $C \cup D$

$\longrightarrow$ ① $\bar{A} = \{x \mid x \in U \cap x \notin A\}$

$$\boxed{\bar{A} = \{d, e, f, h, k\}}$$

② $A \oplus B = \{x \mid x \in A - B \cup x \in B - A\}$

$A - B = \{x \mid x \in A \cap x \notin B\}$

$= \{a, b, c\}$

$B - A = \{x \mid x \in B \cap x \notin A\}$

$= \{d, e, f\}$

$$\boxed{A \oplus B = \{a, b, c, d, e, f\}}$$

③ $C \oplus D = \{x \mid x \in C - D \cup x \in D - C\}$

$C - D = \{x \mid x \in C \cap x \notin D\}$

$= \{a, c\}$

$D - C = \{x \mid x \in D \cap x \notin C\}$

$= \{h, k\}$

$$\boxed{C \oplus D = \{a, c, h, k\}}$$

④ $C \cup D = \{x \mid x \in C \cup x \in D\}$

$= \{a, c, f, h, k\}$

$\overline{C \cup D} = \{x \mid x \in U \cap x \notin C \cup D\}$

$\boxed{\overline{C \cup D} = \{b, d, e, g\}}$

Que. let,

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{1, 2, 4, 6, 8\}$

$B = \{2, 4, 5, 9\}$

$C = \{x \mid x \text{ is a +ve integer and } x^2 < 16\}$

$D = \{7, 8\}$

$\overline{A}$, $\overline{C}$, $A - B$, $C - D$, $A \oplus B$, $C \oplus D$, $\overline{C \cup D}$, $\overline{C \cap D}$

→ Let,

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$A = \{1, 2, 4, 6, 8\}$

$B = \{2, 4, 5, 9\}$

$C = \{1, 2, 3\}$

$D = \{7, 8\}$

6/8/12

(1) $\overline{A} = \{x \mid x \in U \cap x \notin A\}$

$$\boxed{\overline{A} = \{3, 5, 7, 9\}}$$

(2) $\overline{C} = \{x \mid x \in U \cap x \notin C\}$

$$\boxed{\overline{C} = \{4, 5, 6, 7, 8, 9\}}$$

(3) $A - B = \{x \mid x \in A \cap x \notin B\}$

$$\boxed{A - B = \{1, 6, 8\}}$$

(4) $C - D = \{x \mid x \in C \cap x \notin D\}$

$$\boxed{C - D = \{1, 2, 3\}}$$

(5) $A \oplus B = \{x \mid x \in A - B \cup x \in B - A\}$

$A - B = \{1, 6, 8\}$

$B - A = \{5, 9\}$

$$\boxed{A \oplus B = \{1, 5, 6, 8, 9\}}$$

(6) $C \oplus D = \{x \mid x \in C - D \cup x \in D - C\}$

$C - D = \{1, 2, 3\}$

$D - C = \{7, 8\}$

$$\boxed{C \oplus D = \{1, 2, 3, 7, 8\}}$$

⑦ $C \cup D = \{x \mid x \in C \cup x \in D\}$

$= \{1, 2, 3, 7, 8\}$

$\overline{C \cup D} = \{x \mid x \in U \cap x \notin C \cup D\}$

$\boxed{\overline{C \cup D} = \{4, 5, 6, 9\}}$

⑧ $C \cap D = \{x \mid x \in C \cap x \in D\}$

$= \{\ \}$

$\overline{C \cap D} = \{x \mid x \in U \cap x \notin C \cap D\}$

$\boxed{\overline{C \cap D} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}}$

Que. $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}$

$A = \{1, 7, 3, 5, 11\}$

$B = \{2, 6, 9\}$

$C = \{4, 8, 10\}$

$D = \{7, 8, 9, 10\}$

→ ① $\overline{A} = \{x \mid x \in U \cap x \notin A\}$

$\boxed{\overline{A} = \{2, 3, 4, 5, 6, 8, 9, 10\}}$

6/8/12

(2) $\bar{C} = \{x \mid x \in U \cap x \notin C\}$

$\boxed{\bar{C} = \{1, 2, 3, 5, 6, 7, 9, 11\}}$

(3) $A - B = \{x \mid x \in A \cap x \notin B\}$

$\boxed{A - B = \{1, 7, 3, 5, 11\}}$

(4) $C - D = \{x \mid x \in C \cap x \notin D\}$

$\boxed{C - D = \{4\}}$

(5) $A \oplus B = \{x \mid x \in A - B \cup x \in B - A\}$

$A - B = \{1, 7, 3, 5, 11\}$

$B - A = \{2, 6, 9\}$

$\boxed{A \oplus B = \{1, 2, 3, 5, 6, 7, 9, 11\}}$

(6) $C \oplus D = \{x \mid x \in C - D \cup x \in D - C\}$

$C - D = \{4\}$

$D - C = \{7, 9\}$

$\boxed{C \oplus D = \{4, 7, 9\}}$

⑦ $C \cup D = \{x \mid x \in C \cup x \in D\}$

$= \{4, 7, 8, 9, 10\}$

$\boxed{\overline{C \cup D} = \{1, 2, 3, 5, 6, 11\}}$

⑧ $C \cap D = \{x \mid x \in C \cap x \in D\}$

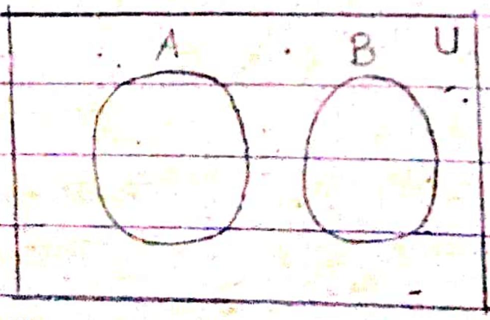
$= \{8, 10\}$

$\boxed{\overline{C \cap D} = \{1, 2, 3, 4, 5, 6, 7, 9, 11\}}$

* **Disjoint set :-**

If A and B are any two set and if $A \cap B = \phi$ then A and B is said to be disjoint set.

Diagrammatically it is represented as –



$A \cap B = \phi$

A and B are disjoint set

6/2/12

ex → $A = \{a, b, c\}$

$B = \{1, 2, 3\}$

$A \cap B = \{x / x \in A \cap x \in B\}$

$= \{\phi\}$

A & B are disjoint set.

* Induction Method *

Que. show by mathematical induction that for all $n \geq 1$

$$1 + 2 + 3 + \cdots\cdots n = \frac{n(n+1)}{2}$$

→ $p(n) = 1 + 2 + 3 + \cdots\cdots n = \frac{n(n+1)}{2}$

(I) B.S.

We have to show that $p(n)$ is true for $n = 1$

$$p(n) = \frac{n(n+1)}{2}$$

$\text{L.H.S.} = p(n) = p(1) = 1$

$\text{R.H.S.} = \frac{n(n+1)}{2} = \frac{1(1+1)}{2} = \frac{2}{2} = 1$

$$\boxed{\therefore \text{L.H.S.} = \text{R.H.S.}}$$

$\therefore p(n)$ is true for $n = 1$

(II) I.S.

(a) We have to s.t. $p(n)$ is true for $n=k$

$$p(n) = 1+2+3+\cdots\cdots+n = \frac{n(n+1)}{2}$$

$\underline{n=k}$

$$\boxed{p(k) = 1+2+3+\cdots\cdots+k = \frac{k(k+1)}{2}} \quad \text{—— (I)}$$

$p(n)$ is true for $n=k$

(b) Now, we have to s.t. $p(n)$ is true for $n=k+1$

$$p(n) = 1+2+3+\cdots\cdots+n = \frac{n(n+1)}{2}$$

$\underline{n=k+1}$

$$p(k+1) = 1+2+3+\cdots\cdots+(k+1) = \frac{k+1(k+1+1)}{2}$$

$$p(k+1) = 1+2+3+\cdots\cdots k+(k+1) = \frac{(k+1)(k+2)}{2}$$

$$\text{L.H.S.} = 1+2+3+\cdots\cdots k+(k+1)$$

$$= \frac{k(k+1)}{2} + \frac{(k+1)}{1} \quad \text{—— from (I)}$$

$$= \frac{k(k+1)+2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

$$= \text{R.H.S.}$$

$$\boxed{\text{L.H.S.} = \text{R.H.S.}}$$

$\therefore$ $p(n)$ is true for $n=k+1$

7/8/12

$p(n)$ is true for $n=1$, $n=k$ & $n=k+1$
Hense,
it is true for $n \geq 1$.

S-06
Que. Show that $p(n) = 1+5+9+\ldots\ldots+(4n-3) = n(2n-1)$

$\rightarrow$ $p(n) = 1+5+9+\ldots\ldots+(4n-3) = n(2n-1)$

① B.S.

We have to s.t. $p(n)$ is true for $n=1$

$p(n) = n(2n-1)$

L.H.S. $= p(n) = p(1) = 1$

R.H.S. $= n(2n-1) = 1(2\times1-1) = 1\times1 = 1$

$\boxed{\text{L.H.S.} = \text{R.H.S.}}$

$p(n)$ is true for $n=1$

② I.S.

ⓐ We have to s.t. $p(n)$ is true for $n=k$

$p(n) = 1+5+9+\ldots\ldots+(4n-3) = n(2n-1)$

$n=k$

$\boxed{p(k) = 1+5+9+\ldots\ldots+(4k-3) = k(2k-1)}$ — ①

$p(n)$ is true for $n=k$

7/8/12

(b) Now, we have to s.t. $p(n)$ is true for $n=k+1$

$$p(n) = 1+5+9+\cdots\cdots+(4n-3) = n(2n-1)$$

$$\underline{n=k+1}$$

$$p(k+1) = 1+5+9+\cdots\cdots+(4(k+1)-3) = k+1(2(k+1)-1)$$

$$p(k+1) = 1+5+9+\cdots\cdots+(4k+1) = k+1(2k+1)$$

$$p(k+1) = 1+5+9+\cdots\cdots+(4k-3)+(4k+1) = k+1(2k+1)$$

$$L.H.S. = 1+5+9+\cdots\cdots+(4k-3)+(4k+1)$$

$$= k(2k-1)+4k+1$$

$$= 2k^2-k+4k+1$$

$$= 2k^2+3k+1$$

$$= 2k^2+2k+k+1$$

$$= 2k(k+1)+1(k+1)$$

$$= (k+1)(2k+1)$$

$$= R.H.S.$$

$$\boxed{L.H.S. = R.H.S.}$$

$p(n)$ is true for $n=k+1$

Hense, $p(n)$ is true for $n=k$, $n=k$ & $n=k+1$

Hense, it is true for $n \geq 1$

7/8/12

6-08 Que. Show by mathematical induction

$p(n) = 2+4+6+ \dots 2n = n(n+1)$

$\longrightarrow$

$p(n) = 2+4+6+ \dots 2n = n(n+1)$

① B.S.

We have to S.T. $p(n)$ is true for $n=1$

$p(n) = n(n+1)$

$L.H.S. = p(n) = p(1) = 2$

$R.H.S. = n(n+1) = 1(1+1) = 1 \times 2 = 2$

$\boxed{L.H.S. = R.H.S.}$

$p(n)$ is true for $n=1$

② I.S.

ⓐ We have to S.T. $p(n)$ is true for $n=k$

$p(n) = 2+4+6+ \dots +2n = n(n+1)$

$n=k$

$\boxed{p(k) = 2+4+6+ \dots +2k = k(k+1)}$ —— ①

$p(n)$ is true for $n=k$

(b) We have to S.T. $p(n)$ is true for $n=k+1$

$$p(n) = 2+4+6+\cdots\cdots +2n = n(n+1)$$

$$\underline{n=k+1}$$

$$p(k+1) = 2+4+6+\cdots\cdots +2(k+1) = k+1(k+2)$$

$$p(k+1) = 2+4+6+\cdots\cdots +2k+2 = (k+1)(k+2)$$

$$p(k+1) = 2+4+6+\cdots\cdots +2k+(2k+2) = (k+1)(k+2)$$

$$\begin{aligned}
L.H.S. &= 2+4+6+\cdots\cdots +2k+(2k+2) \\
&= k(k+1)+2k+2 \\
&= k^2+k+2k+2 \\
&= k^2+3k+2 \\
&= k^2+2k+k+2 \\
&= k(k+2)+1(k+2) \\
&= (k+1)(k+2) \\
&= R.H.S.
\end{aligned}$$

$$\boxed{L.H.S. = R.H.S}$$

$\underline{p(n) \text{ is true for } n=k+1}$

W-08
Que. $p(n) = 1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$

→ $p(n) = 1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$

(1) B.S.

We have to S.T. $p(n)$ is true for $n=1$

$\therefore p(n) = \frac{n^2(n+1)^2}{4}$

L.H.S. $= p(n) = p(1) = 1$

R.H.S. $= \frac{n^2(n+1)^2}{4} = \frac{1^2(1+1)^2}{4} = \frac{1 \cdot 4}{4} = 1$

$\boxed{\therefore \text{L.H.S.} = \text{R.H.S.}}$

$p(n)$ is true for $n=1$

(2) I.S.

(a) We have to S.T. $p(n)$ is true for $n=k$

$\therefore p(n) = 1^3 + 2^3 + 3^3 + \dots + n^3 = \frac{n^2(n+1)^2}{4}$

$n=k$

$$\boxed{p(k) = 1^3 + 2^3 + 3^3 + \dots + k^3 = \frac{k^2(k+1)^2}{4}} \quad \text{(I)}$$

$p(n)$ is true for $n=k$

(b) Now, we have to show that for $n=k+1$

$$P(n) = 1^3+2^3+3^3+\ldots\ldots+n^3 = \frac{n^2(n+1)^2}{4}$$

$$n=k+1$$

$$P(k+1) = 1^3+2^3+3^3+\ldots\ldots+(k+1)^3 = \frac{(k+1)^2(k+1+1)^2}{4}$$

$$P(k+1) = 1^3+2^2+3^3+\ldots\ldots+k^3+(k+1)^3 = \frac{(k+1)^2(k+2)^2}{4}$$

$$L.H.S. = 1^3+2^2+3^3+\ldots\ldots+k^3+(k+1)^3$$

$$= \frac{k^2(k+1)^2}{4} + \frac{(k+1)^3}{1} \quad \text{from (I)}$$

$$= \frac{k^2(k+1)^2+4(k+1)^3}{4}$$

$$= \frac{k^2(k^2+2k+1)+4(k^3-3k^2+3k^2-1)}{4}$$

$$= \frac{k^4+2k^3+k^2+4k^3-12k^2+12k^2-4}{4}$$

$$= \frac{k^4+6k^3+k^2+4}{4} \quad \frac{k^4+2k^3+k^2+4k^3-12k^2+12k-4}{4}$$

$$= k^2(k^2+6) \quad \frac{k^4+6k^3-11k^2+12k-4}{4}$$

$$= \frac{(k+1)^2\left[k^2+4(k+1)\right]}{4}$$

$$= \frac{(k+1)^2\left[k^2+4k+4\right]}{4}$$

$$= \frac{(k+1)^2 [k^2+2k+2k+4]}{4}$$

$$= \frac{(k+1)^2 [k(k+2)+2(k+2)]}{4}$$

$$= \frac{(k+1)^2 [(k+2)(k+2)]}{4}$$

$$= \frac{(k+1)^2 (k+2)^2}{4}$$

$= R.H.S.$

$\boxed{L.H.S. = R.H.S.}$

p(n) is true for n=k+1

Hense,

p(n) is true for n=1, n=k & n=k+1

Hense,

it is true for $n \geq 1$

Que. $p(n) = 1^2+3^2+5^2+ \dots (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3}$

$\rightarrow$ $p(n) = 1^2+3^2+5^2+ \dots (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3}$

① B.S.

We have to S.T. $p(n)$ is true for $n=1$

$\therefore p(n) = \frac{n(2n+1)(2n-1)}{3}$

$L.H.S. = p(n)$

$= p(1)$

$= 1$

$R.H.S. = \frac{n(2n+1)(2n-1)}{3}$

$= \frac{1(2+1)(2-1)}{3}$

$= \frac{1 \cdot \cancel{3} \cdot 1}{\cancel{3}}$

$= 1$

$\boxed{L.H.S. = R.H.S.}$

$\therefore p(n)$ is true for $n=1$

I.S.

ⓐ We have to S.T. $p(n)$ is true for $n=k$

$p(n) = 1^2 + 3^2 + 5^2 + \ldots\ldots (2n-1)^2 = \frac{n(2n+1)(2n-1)}{3}$

$n=k$

$\boxed{p(k) = 1^2 + 3^2 + 5^2 + \ldots\ldots (2k-1)^2 = \frac{k(2k+1)(2k-1)}{3}}$ — ①

$p(n)$ is true for $n=k$

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(b) We have to s.T. p(n) is true for n=k+1

$$p(n) = 1^2+3^2+5^2+\cdots\cdots(2n-1)^2 = \frac{n(2n+1)(2n-1)}{3}$$

$$n=k+1$$

$$p(k+1) = 1^2+3^2+5^2+\cdots\cdots(2(k+1)-1)^2 = \frac{(k+1)(2(k+1)+1)(2(k+1)-1)}{3}$$

$$p(k+1) = 1^2+3^2+5^2+\cdots\cdots(2k+2-1)^2 = \frac{(k+1)(2k+2+1)(2k+2-1)}{3}$$

$$p(k+1) = 1^2+3^2+5^2+\cdots\cdots(2k+1)^2 = \frac{(k+1)(2k+3)(2k+1)}{3}$$

$$p(k+1) = 1^2+3^2+5^2+\cdots\cdots+(2k-1)^2+(2k+1)^2$$

$$= \frac{k(2k+1)(2k-1)}{3} + \frac{(2k+1)^2}{1}$$

$$= \frac{k(2k+1)(2k-1)+3(2k+1)^2}{3}$$

$$= \frac{(2k+1)[k(2k-1)+3(2k+1)]}{3}$$

$$= \frac{(2k+1)[2k^2-k+6k+3]}{3}$$

$$= \frac{(2k+1)[2k^2+5k+3]}{3}$$

$$= \frac{(2k+1)[2k^2+3k+2k+3]}{3}$$

$$= \frac{(2k+1)[k(2k+3)+1(2k+3)]}{3}$$

$$= \frac{(2k+1)(k+1)(2k+3)}{3}$$

$$= R.H.S.$$

$$\boxed{L.H.S. = R.H.S.}$$

P(n) is true for n=k+1

Hense,

P(n) is true for n=1, n=k, n=k+1

Hence,

it is true for $n \geq 1$

Que. $P(n) = 1 + a + a^2 + \ldots\ldots a^{n-1} = \frac{a^n - 1}{a-1}$

$\longrightarrow$ $p(n) = 1 + a + a^2 + \ldots\ldots a^{n-1} = \frac{a^n - 1}{a-1}$

① B.S.

we have to s.T. for p(n) is true for n=1

$$\therefore P(n) = \frac{a^n - 1}{a-1}$$

$$L.H.S. = p(n) = P(1) = 1$$

$$R.H.S. = \frac{a^n - 1}{a-1} = \frac{a^1 - 1}{a-1} = \frac{\cancel{a-1}}{\cancel{a-1}} = 1$$

$$\boxed{L.H.S. = R.H.S.}$$

P(n) is true for n=1

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③ I.S.

ⓐ We have to s.T. p(n) is true for n=k

$$p(n) = 1+a+a^2+ \dots a^{n-1} = \frac{a^n-1}{a-1}$$

$\underline{n=k}$

$$\boxed{p(k) = 1+a+a^2+ \dots +a^{k-1} = \frac{a^k-1}{a-1}} \quad \text{—— ①}$$

ⓑ We have to s.T. p(n) is true for n=k+1

$$p(n) = 1+a+a^2+ \dots +a^{n-1} = \frac{a^n-1}{a-1}$$

$\underline{n=k+1}$

$$p(k+1) = 1+a+a^2+ \dots +a^{k+\cancel{1}-\cancel{1}} = \frac{a^{k+1}-1}{a-1}$$

$$p(k+1) = 1+a+a^2+ \dots +a^k = \frac{a^{k+1}-1}{a-1}$$

$$p(k+1) = \underline{1+a+a^2+ \dots +a^{k-1}} + a^k = \frac{a^{k+1}-1}{a-1}$$

$$L.H.S. = 1+a+a^2 + \dots + a^{k-1} + a^k$$

$$= \frac{a^k-1}{a-1} + \frac{a^k}{1}$$

$$= \frac{a^k-1+a^k(a-1)}{a-1}$$

$$= \frac{\cancel{a^k}-1+a^{k+1}-\cancel{a^k}}{a-1}$$

$$= \frac{a^{k+1}-1}{a-1}$$

$$= R.H.S$$

$\boxed{L.H.S. = R.H.S.}$

p(n) is true for n=k+1

Hense,

p(n) is true for n=1, n=k & n=k+1.

Hense,

it is also true for $n \geq 1$

Que. $p(n) = a + ar + ar^2 + \ldots\ldots\ ar^{n-1} = \frac{a(1-r^n)}{(1-r)}$

$\longrightarrow$ $p(n) = a + ar + ar^2 + \ldots\ldots\ ar^{n-1} = \frac{a(1-r^n)}{(1-r)}$

① B.S.

We have to S.T. p(n) is true for n=1

$\therefore p(n) = \frac{a(1-r^n)}{(1-r)}$

$L.H.S. = p(n)$

$= p(1)$

$= a$

$R.H.S. = \frac{a(1-r^n)}{(1-r)}$

$= \frac{a\cancel{(1-r^1)}}{\cancel{(1-r)}}$

$= a$

$\boxed{L.H.S. = R.H.S}$

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p(n) is true for n = 1

② I.S.

ⓐ We have to s.T. p(n) is true for n=k

$$p(n) = a + ar + ar^2 + \dots \dots \; ar^{n-1} = \frac{a(1-r^n)}{(1-r)}$$

$n = k$

$$\boxed{p(k) = a + ar + ar^2 + \dots \dots \; ar^{k-1} = \frac{a(1-r^k)}{(1-r)}} \quad \text{— (I)}$$

ⓑ We have to s.T. p(n) is true for n=k+1

$$p(n) = a + ar + ar^2 + \dots \dots \; ar^{n-1} = \frac{a(1-r^n)}{(1-r)}$$

$n = k+1$

$$p(k+1) = a + ar + ar^2 + \dots \dots \; ar^{k+1-1} = \frac{a(1-r^{k+1})}{(1-r)}$$

$$p(k+1) = a + ar + ar^2 + \dots \dots \; ar^{k} = \frac{a(1-r^{k+1})}{(1-r)}$$

$$p(k+1) = a + ar + ar^2 + \dots \dots + ar^{k-1} + ar^{k} = \frac{a(1-r^{k+1})}{(1-r)}$$

$$L.H.S. = a + ar + ar^2 + \dots \dots + ar^{k-1} + ar^{k}$$

$$= \frac{a(1-r^k)}{(1-r)} + \cancel{a(1-r^{k+1})} \frac{ar^k}{1}$$