Unit 2 :-

Counting :-

The function as the basic application which are implemeilate on discrete data or object that application involves principle of counting following are some important type including in counting.

1. Permutation
2. combination
3. Pigeon hole principle
4. recurence relation
5. mathematical induction

1. Permutation :-

A permutation is an arrangement of number of object in sum definate order taken at some or at all type. The total number of permutation of disting object i.e n taken are 'r' at a time is denoted by

→ $P(n,r)$

where $1 \leq r \leq n$

The number of permutation of n object taken 'r' at a time is determine by following formula.

$$P(n,r) = \frac{n!}{(n-r)!} \quad \text{OR} \quad \boxed{{}^nP_r = \frac{n!}{(n-r)!}}$$

Example :-

Determine the value of following

(a) $4P_2 = \frac{4!}{(4-2)!} = \frac{4 \times 3 \times \cancel{2 \times 1}}{\cancel{2 \times 1}}$

$= 12$

(b) $9P_3 = \frac{9!}{(9-3)!} = \frac{9 \times 8 \times 7 \times \cancel{6 \times 5 \times 4 \times 3 \times 2 \times 1}}{\cancel{6 \times 5 \times 4 \times 3 \times 2 \times 1}}$

$= 504$

(c) $P(15,3) = \frac{15!}{(15-3)!} = \frac{15 \times 14 \times 13 \times \cancel{12}}{\cancel{12 \times 11 \times 10}}$

$= 2730$

(d) $20P_3 = \frac{20!}{(20-3)} = \frac{20}{17} = \frac{20 \times 19 \times 18 \times \cancel{17 \times 16}}{\cancel{17 \times 16 \times 15}}$

$= 6840$

(e) $52P_4 = \frac{52!}{(52-4)} = \frac{52}{48} = \frac{52 \times 51 \times 50 \times 49 \times \cancel{48 \times 47}}{\cancel{48 \times 47}}$

$= 6497400$

(f) $5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5 \times 4 \times 3 \times \cancel{2 \times 1}}{\cancel{2 \times 1}}$

$= 60$

word statement:- How many variable name of 8 letters can be formed form the letter a, b, c, d, e, f, g, h, i if no letter is reepted?

$n = 9 \quad r = 8.$

$^nP_r = \frac{n!}{(n-r)!} = \frac{9!}{(9-8)!} = \frac{9}{1}.$

There are 9 letters given initialy that is $n = 9$ &
There are 8 letter are to be related that is $r = 8$.
∴ total number of variable name of 8 letters is P_{n-r}.

$$9! = \frac{9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{1}$$

$$= \underline{\underline{362880}}$$

② How many six digit number can be formed from the digits 0,1,2,3,4,5,6 7, If no digit is repeated.

$n = 8 \quad r = 6$

$$nP_r = \frac{n!}{(n-r)!} = \frac{8!}{(8-6)!} = \frac{8}{2}$$

$$= \frac{8 \times 7 \times 6 \times 5 \times 4 \times 3 \times \cancel{2 \times 1}}{\cancel{2 \times 1}}$$

$$nP_r = \underline{\underline{20160}}$$

imp.

Permutation with repetition

It is a permutation when all object are not different, some are repeated. hence the number of permutation of n object of which n_1 object are of one kind and n_2 object of another kind.

When all are taken at a time is called repeated permutation and it is determine by the formula.

$$K = \frac{n!}{n_1!\, n_2!}$$

Q. Determines the number of permutation that can be made out of letter of the word that is PROGRAMMING.

Soln.

There are 11 letter in the word programming out of which G, M, R are repeated.

therefore $n = 11$ $n_1 = 2$ for G.

$n_2 = 2$ for M

$n_3 = 2$ for R.

hence the permutation is.

$$K = \frac{n!}{n_1!\, n_2!\, n_3!} = \frac{11}{2!\, 2!\, 2!}$$

$$= \frac{11 \times 10 \times 9 \times 8 \times 7 \times \cancel{6}^{3} \times 5 \times \cancel{4}^{2} \times \cancel{3} \times \cancel{2} \times 1}{\cancel{2} \times 1 \times \cancel{2} \times 1 \times \cancel{2} \times 1}$$

$$= 4989600$$

Page No.:
Date: / /

There are 4 blue 3 red & 2 black pins in a box. this are drown. ~~d~~ /I determines all the different permutation.

→ $n = 9$ $n_1 = 4$ $n_2 = 3$ $n_1 = 2$.

$$k = \frac{n!}{n_1!\, n_2!\, n_3!}$$

$$= \frac{9}{4!\, 3!\, 2}$$

$$= \frac{\cancel{9} \times \cancel{8} \times 7 \times \cancel{6} \times 5 \times \cancel{4} \times \cancel{3 \times 2 \times 1}}{\cancel{4 \times 3 \times 2 \times 1}\; \cancel{3} \times \cancel{2} \times 1\; \cancel{2} \times 1}$$

$= 1260$

3) How many distinguishable permultation of the letter MISSISSIPPI.

4) How many different variable name can be form by using the letter A, A, A B, B, B, B C, C, C,

5) How many distigeistable permutation in the word BANANA.

Page No.:
Date: / /

Combination is selection of sum or all object from a set of given object where order of object does not matter.
The number of combination of 'n' object taken 'r' at a time is represented $C(n,r)$. Or ^nC_r.

The combination of n object taken r object is determine by the formula

$$^nC_r = \frac{n!}{r!(n-r)!}$$

(1) $10C6 =$

$n = 10 \quad r = 6$

$$= \frac{10}{6!(10-6)!}$$

$$= \frac{\cancel{10}^{5} \times \cancel{9}^{3} \times \cancel{8}^{2} \times \cancel{7} \times 6 \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1}{6 \times \cancel{7} \times \cancel{5} \times \cancel{4} \times \cancel{3} \times \cancel{2} \times 1 \, (\cancel{4} \times \cancel{3} \times \cancel{2} \times 1)}$$

$= 210$

(2) $52C_4$

$= n = 52 \quad r = 4.$

$$= \frac{52}{4!(52-4)!}$$

$= 270725$

Pigeon hole principle :-

Summer 2017 - (4) or (5) mark
winter 2018 - (4) mark

Theorem :- S.T if 'n' pigeons are assigned to 'm' pigeons hole & m<n then, there is a atleast one
Statement :- pigeon that contain two or more pigeon.

Proof :- Let as able 'n' pigeon with the number 1 true n. & m pigeon. hole with the no 1 true m.

Now, starting pigeon 1 and pigeon hole 1, assign each pigeon in order to the pigeon hole with the same number.

So, we can assign as many pigeons as possible to disting pigeon holes.

But, as we know that the pigeon holes are less than pegeons that is m<n.

Thus there remains m-n pigeons that have not get

It been assinged to of pigeon hole. hence there is at least one pigeon hole that will be assign two or more pigeon.

example :-
Show that at least two must have their bday in the same month. if 13 people assembled.

In the room. we assign each person to the month of year on which he was born. Since are 12 month and there are 13 people

i.e in this case the pigeon are 13 people. & pigeon hole are month in the year.

Date: __/__/

So accordingly to the pigeon hole principle there must be atleast 2 people assigned to the same month.

<u>OR</u>

Proof :-

Statement - It state that if n pigeon are assigne 'n' pigeon hole & If the number of pigeon are very large then the number of pigeon hole must contain at least

$[(n-1)/m]+1$ pigeons

→ We can prove that the methods of contradiction.

Assume that each pigeon hole does not contain more that

$[(n-1)/m]+1]$ pigeons

then there will be atmost

$m[(n-1)/m] \leq m(n-1)/m$

$= n-1$ pigeon in all

But the number of pigeons are very large than the pigeons holes hence this in contradiction to are assumption.

hence the for given n pigeon hole of the pigeon hole must contain atleast.

Date: __ / __ / __

Recursion relation / recurence

A recursion relation or recurence is an equation that recursively defined a sequence, that is each term of sequence is defined as a function of the preciding term.

A recursive formula must a combine by information about the beining of the sequence, this information is called as initial condition of sequence.

Note - A recurrence reln also called difference equation.

example :- The recurence reln $a_n = a_{n-1} + 3$ ---- with initial conn $a_1 = 4$

it define the sequence 4, 7, 10, 3.

There are two ways to find out the recursive formula or explicit formula

① Backtracking
② theorem base method OR using characteristics method.

Theorem base method

Q. find and explicit formula for the sequence defined by the recursive relation.

$C_n = 3C_{n-1} - 2C_{n-2}$ with initial condition $C_1 = 5$ $C_2 = 3$

Soln

Given :- The recurence is.

$C_n = 3C_{n-1} - 2C_{n-2}$

The given recurence relation is a linear homogeneous relation & it is associated with $x^2 = 3x - 2$

Rewrite this equation as

$x^2 - 3x + 2 = 0$ which is linear eqn and we have to solve it by method of factorisation.

2x2

$= x^2 - 2x - x + 2$

$= (x-2)(x-1)$

$= x = 2 \quad x = 1$

∴ value of x are nothing but a two roots of given linear equation

$s_1 = 2, s_2 = 1$.

Now, by theorem base method the explicit formula can be given by

$$\boxed{C_n = U s_1^n + V s_2^n} \quad \text{— (A)}$$

Now,
1st we have to find out the value of u & v
hence by using eqn (A) we can put $n=1$, $s_1=1$ & $n=2$ $s_2=2$.

for $n=1$

$C_1 = U s_1^n + V s_2^n$

$= U(1)^1 + V(2)^1$

$= (2)+(1)$

$= \boxed{2U+V}$

$\boxed{C_1 = U+2V}$ --- (1)

for $n=2$

$C_2 = U s_1^n + V s_2^n$

$= (2)+(1)$

$= U(1)^2 + V(2)^2$

$= \boxed{4U+V}$

$= U+4V$

$\boxed{C_2 = U+4V}$ --(2)

By eqn (1) & (2) we get.
$C_1=5$ $C_2=3$.

$C_1 = U+2V$	$C_2 = U+4V$
$5 = U+2V = 5+2V$	$3 = U+4V$ $= 3-4V$
$U = 7$	$V = -1$

Now form eqn (A) put the value of U, V, s_1 & s_2

$U=7$ $V=-1$
$s_1=1$ $s_2=2$

$C_n = U s_1^n + V s_2^n$

$C_n = 7 + (-1)2^n$

$C_n = 7-2^n$

Date: __ /__ /__

Relation & diagram.

Let A and B are two non-empty set. a relation which is denoted by R from set A to B is a subset of 'A X B'

If $R \subseteq (A \times B)$ and $(a,b) \in R$ hence we say that a is related to be by relation R and it is written as aRb if a is not related to b by relation R then we write as 'a $\not R$ b'

example :-

let A = {1, 2, 3, 4, 5} define the following relation R(<) on A that is A : aRb if (a<b)

sol^n :-

given A = {1, 2, 3, 4, 5}

hence, the relation R is

$$R = \{(1,2)\ (1,3)\ (1,4)\ (1,5)\ (2,3)\ (2,4)\ (2,5)\ (3,4)\ (3,5)\ (4,5)\}$$

Graph of relation or directed graph or diagram.

Q.5 :→ imp A relation can also represented by drawing its graph which is called as directed graph.

Let R be relation on a set $x = \{x_1, x_2, x_3 \ldots x_n\}$

The element of x. are represented by point or circles. called as nodes.

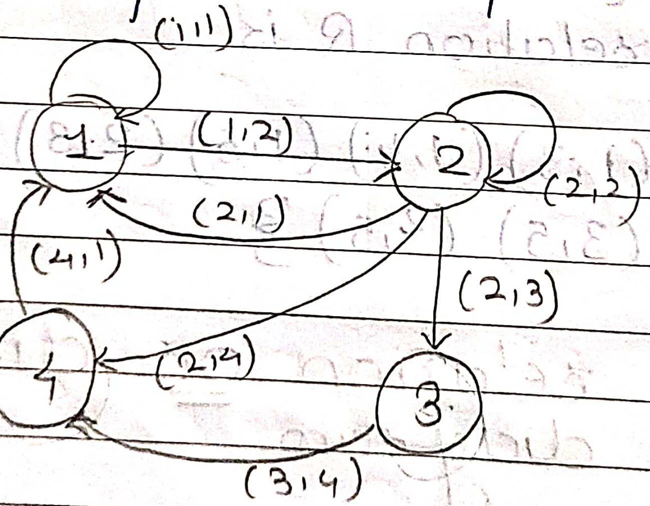
This nodes may also be called vertapes. we connects nodes by arc and put and arrow ($\rightarrow$) on arc in direction from x_i x_j when all nodes by arc with to proper arrow, we get relation arc.

examples:-

Let A = { 1,2,3,4 } and R = { (1,1) (1,2) (2,1) (2,2) (2,3) (2,4) (3,4) (4,1) }
Draw diagram of relation R.

Given

A = { 1, 2, 3, 4 }
R = { (1,1) (1,2) (2,1) (2,2) (2,3) (2,4) (3,4) (4,1) } then the diagram are shown below.



MR =

R \ C	1	2	3	4
1	1	1	0	0
2	1	1	1	1
3	0	0	0	1
4	1	0	0	0

Indegree Outdegree.

If R is a relation of set A
- and $a \in b$ then the indegree is number of $b \in a$ such that $(b, a) \in R$ the
- outdegree of a is the number of $b \in a$ such that $(a, b) \in R$

Example $A = \{a, b, c, d\}$ and R be the relation on A that has the matrix show below. $M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

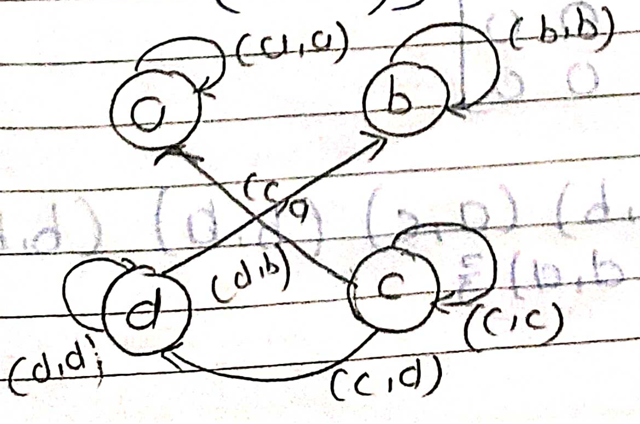
construct the diagram and find out indegree and outdegree of all verteeces.

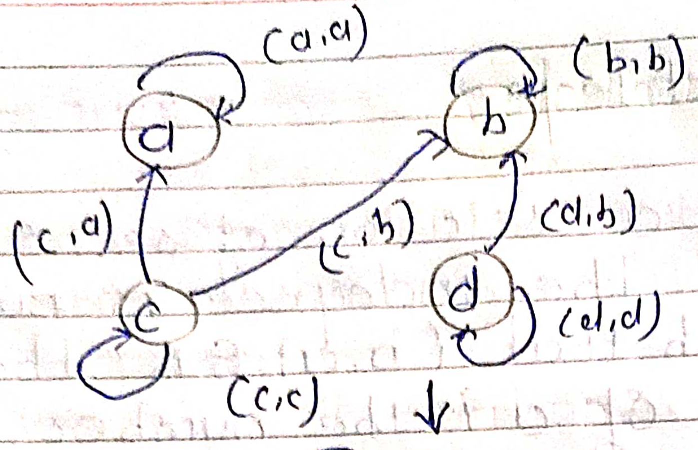
Soln:-

Given $A = \{a, b, c, d\}$

$$M_R = \begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \hline \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \end{array}$$

$R = \{(a, a)\ (b, b)\ (c, a)\ (c, d)\ (c, c)\ (d, b)\ (d, d)\}$





Nodes	In-degree	out-degree
a	2	1
b	3	1
c	1	3
d	1	2

Que- let $A = \{a, b, c, d\}$ & R be a relation on A that has matrix.

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

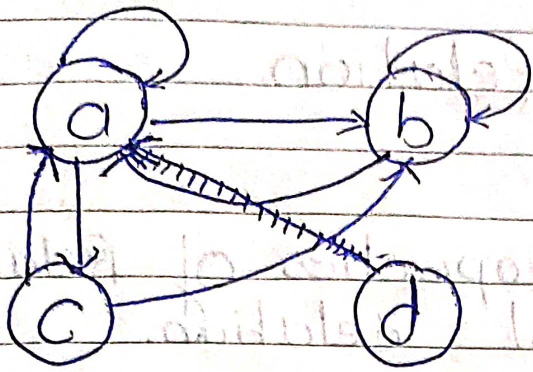
show diagraph of it also find indegree & outdegree of all verticle

→ Given $A = \{a, b, c, d\}$

$$M_R = \begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$R = \{(a,a)\ (a,b)\ (a,c)\ (b,b)\ (b,b)\ (c,a)$
$(c,b)\ (d,a)\}$

Page No.:______
Date:__ /__ /__



Nodes	Indegree	outdegree
a	2	3
b	3	2
c	1	2
d	0	1

Homework

1> let $X = \{1, 2, 3, 4\}$ & $R = \{(x, y) / x < y\}$
draw matrix indegree & outdegree

2> let $x = \{1, 2, 3, 4\}$ & $R = \{(1,1)\ (1,4)\ (4,1)$
$(4,4)\ (2,2)\ (2,3)\ (3,2)\ (3,3)$ find MR
& matrix of indegree & outdegree

Imp

Equivalence Relation

properties

Ques: Explain the properties of Relation which is a equivalent relation.

Properties of relation:-

① Reflective relation :-

A relation are on set A is reflexive if $(a, a) \in R$ for all $a \in A$, that is if $aRa \ \forall \ a \in A$

② Irreflexive relation :-

A relation are on set A is Irreflexive if $(a, a) \notin R$ for all $a \notin A$ that is if $a \not R a \ \forall \ a \in A$

example :-

ⓐ Let $A = \{(a, a) \mid a \in A\}$ so that, A is the relation of equality on the set A than A is reflexible. $(a, a) \in A$ for $\forall a \in A$

ⓑ let $A = \{1, 2, 3\}$ & $R = \{(1, 1)(1, 2),$ Then R is not reflexive. $(2, 2) \notin R$ & $(3, 3) \in R$ also R is not Inflexisive $(1, 1) \in R$.

Equivalence Relation :

Note :-

1. R is reflexive if ⇒ aRa or $(a,a) \in R$.
2. R is symmetric if ⇒ aRb then bRa.
3. R is transitive if ⇒ aRb & bRc ⇒ aRc.

Symmetric :- { (1,2) (2,1) }
transitive :- { (1,2,4) (1,4) }

ex :-

A = { 1, 2, 3, 4, 5 } & R = { (1,1) (1,2) (1,3) (2,1) (2,2) (3,1) (2,3) (3,3) (4,4) (3,2) (5,5) }
Determine R is equivalence Reln or not.

Given :- A = { 1, 2, 3, 4, 5 }
R = { (1,1) (1,2) (1,3) (2,1) (2,2) (3,1) (2,3) (3,3) (4,4) (3,2) (5,5) }

Here we have to determine R is equival Relatn or not

Now, we have to prove
R is reflexive R is symmetric R is transitive

1) Here Relative :- we observed that reflexivity is satisfied it means

R is reflexive as (1,1) (2,2) (3,3) (4,4) (5,5) belongs to R hence R reflective

(2) Symmetric →

aRb & bRa.

• Here we have to proof that R symmetric it means if A related to b that mean ARb & bRa.

Here;

1) (1,2) ∈ R (2,1)
(a,b) ∈ R (b,a) ∈ R

2) (1,3) ∈ R (3,1) ∈ R
3) (2,3) ∈ R (3,2) ∈ R

Hence from above eqn. R is symmetric

(3) transative →

here we have to prove R transative, it means

aRb, bRc ⇒ aRc.

In the given relation we observed that

• (1,2) ∈ R & (2,3) ∈ R ⇒ (1,3) ∈ R.

also

• (3,1) ∈ R & (1,3) ∈ R ⇒ (3,3) ∈ R

Hence from the above relation R is transative.

Since R is reflexive, transative & symmetric

Q Let R be the relation whose matrix is.

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

① find the reflexive closure of R

② find the symmetric closure of R.

Given

$$R = \begin{array}{c|ccccc} & a & b & c & d & e \\ a & 1 & 0 & 0 & 1 & 1 \\ b & 0 & 0 & 1 & 0 & 1 \\ c & 1 & 1 & 1 & 0 & 0 \\ d & 0 & 1 & 1 & 0 & 0 \\ e & 0 & 0 & 1 & 0 & 1 \end{array}$$

R = { (a,a) (a,d) (a,e) (b,c) (b,e) (c,a) (c,b) (c,c) (d,b) (d,c) (e,c) (e,e) }

① Reflexive:-
{ (a,a) (c,c) (e,e) } belongs to R hence R is reflexive

② Symmetric ⇒
aRb & bRa.

- Here we have to proof that R symmetric
It means aRb & bRa.

Here,
(b,c) ∈ R (c,b)

Hence from above eqn R is symmetric.

Home work

Page No.
Date:

Q. Let $A = \{a, b, c, d\}$ & $R = \{(a,a) (b,a) (b$
$(c,c) (d,d) (d,c)\}$
Determine R is equivalence relation or not

Q. let $A = \{1, 2, 3, 4\}$ & $R = \{(1,1) (1,3) (2,2)$
$(2,4) (3,1) (3,3) (4,2) (4,4)\}$
Determine R is equivalence relation or not

R & S are relations from A to B then
show that,

1. $\cancel{R \subseteq S \Rightarrow R^{-1} \subseteq S^{-1}}$
2. $\cancel{(R \cup S)^{-1} \Rightarrow R^{-1} \cup S^{-1}}$

1. $(R \cup S)^{-1} = R^{-1} \cap S^{-1}$
2. $(R \cap S)^{-1} = R^{-1} \cup S^{-1}$

1. $(R \cup S)^{-1} = R^{-1} \cap S^{-1}$

Let.

$(b,a) \in R^{-1} \cap S^{-1}$

$b \in R^{-1}$ and $a \in S^{-1}$ $\forall x \in R$.

$(a,b) \in (R \cup S)^{-1}$

similarly.

$(R \cup S)^{-1} \Rightarrow R^{-1} \cap S^{-1}$

$(a,b) \in (R \cup S)$

$(b,a) \in (R \cup S)^{-1}$

$(b,a) \in R^{-1}$ or $(b,a) \in S^{-1}$

$(a,b) \in R^{-1}$ or $(a,b) \in S^{-1}$

① $(R \cup S)^{-1} = R^{-1} \cap S^{-1}$

soln:-

let $(a,b) \in (R \cup S)^{-1}$

so we have, $(b,a) \in (R \cup S)$

Now,

$(b,a) \in R$ or $(b,a) \in S$

This means that,

$(a,b) \in R^{-1}$ or $(a,b) \in S^{-1}$

hence

$(a,b) \in R^{-1} \cap S^{-1}$ ---- ①

convensly,

hence $(a,b) \in R^{-1} \cap S^{-1}$

so we have. $(a,b) \in R^{-1}$ or $(b,a) \in S^{-1}$

This means that,

$(b,a) \in R$ and $(b,a) \in S$

so $(b,a) \in R \cup S$

hence

$(a,b) \in (R \cup S^{-1})$ ---- ②

③ $(R \cap S)^{-1} = R^{-1} \cup S^{-1}$

soln:- let $(a,b) \in (R \cap S)^{-1}$

so we have $(b,a) \in (R \cap S)$

Now,

$(b,a) \in R$ and $(b,a) \in S$

This means that

$(a,b) \in R^{-1}$ and $(a,b) \in S^{-1}$

hence

$(a,b) \in R^{-1} \cup S^{-1}$ ---- ①

convesly,

hence $(a,b) \in R^{-1} \cap S^{-1}$

so we have $(a,b) \in R^{-1}$ or $(b,a) \in S^{-1}$

$(b,a) \in R$ or $(b,a) \in S$
so $(b,a) \in R \cap S$.
hence
$(a,b) \in (R \cap S)^{-1}$; --- (11)
from (1) & (2)
$(R \cap S)^{-1} = R^{-1} \cup S^{-1}$

Q. let R be relation from A to B & let A_1 & A_2 be subset of A then prove that,
$R(A_1 \cup A_2) = R(A_1) \cup R(A_2)$. --- (10) mark.

Theorem:-

Let R be a relation from A to B and let A_1 & A_2 be subsets of A then.
1) If $A_1 \subseteq A_2$, then $R(A_1) \subseteq R(A_2)$.
2) If $(A_1) \cup (A_2) = R(A_1) \cup R(A_2)$
3) $R(A_1 \cap A_2) \subseteq R(A_1) \cap R(A_2)$.

Proof:- (a) If $A_1 \subseteq A_2$, then $R(A_1) \subseteq R(A_2)$
Let,
R be a relation from A to B and A_1 & A_2 be subset of A.
(a) If $y \in R(A_1)$ then xRy, $\forall x \in A$,
$\therefore A_1 \subseteq A_2$, $x \in A_2$
Thus,
$y \in R(A_2)$

$\boxed{R(A_1) = R(A_2)}$

similarly. - 11ly.

(b) If $y \in R(A_1 \cup A_2)$ by defn

xRy $\forall x$ in $A_1 \cup A_2$

If x is in A_1 then xRy

we must have, $y \in R(A_1)$

By the same argument if x is in A_2 then $y \in R(A_2)$

If either case.

Thus, we have show that.

$R(A_1 \cup A_2) \subseteq R(A_1) \cap R(A_2)$

conversely,

since,

$A_1 \subseteq (A_1 \cup A_2)$

we know that.

IIIly

$R(A_2) \leq R(A_1 \cup A_2)$

Thus, $R(A_1) \cup R(A_2) \subseteq R(A_1 \cup A_2)$

$\boxed{\therefore R(A_1) \cup R(A_2) = R(A_1) \cup R(A_2)}$

(3) If $y \in R(A_1 \cap A_2)$ then, $\forall x$ in $A_1 \cap A_2$, xRy

$\therefore x$ is in both A_1 & A_2

it follows that, y is in both $R(A_1)$ and $R(A_2)$

$\therefore y \in R(A_1) \cap R(A_2)$.

$\boxed{\therefore R(A_1 \cap A_2) \subseteq R(A_1) \cap R(A_2)}$

hence proved.

Q. Let R be a relation on set A then prove that R^∞ is a transitive closer of R.

Warshall's algorithum

If relation R for a set is not transitive we need to apply closer to make it transitive this closer is known as transitive closer, we used a warshall's method to find the transitive method.

e.g.

Let A = {a, b, c, d} and {(a,a), (a,b), (b,a), (b,c) (c,d)} find the transitive closure

Steps :-

Step No 1 :- We first write the matrix M_R of the relation R and denoted it by W_0.

Step No 2 : We write a blank matrix of order four denote it by W_1 and transfer all 1's from W_0.

continue until you get same matrix.

Step 1 :-

$$W_0 = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 0 \\ b & 1 & 0 & 1 & 0 \\ c & 0 & 0 & 0 & 1 \\ d & 0 & 0 & 0 & 0 \end{array}$$

column 1, 2, 3, 4 (a, b, c, d); row 1a, 2b, 3c, 4d

$$W_1 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$C \downarrow p_i \downarrow 2$ $\quad$ $R \downarrow q_j \downarrow 2$ $\quad$ (2,2)

$$W_2 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$C \downarrow p_i \downarrow 1, 2$ $\quad$ $R \downarrow q_j \downarrow 1, 2, 3$

(1,1) (1,2) (1,3) (2,1) (2,2) (2,3)

$$W_3 = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \left[\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$C \downarrow p_i \downarrow 1, 2$ $\quad$ $R \downarrow q_j \downarrow 4$ $\quad$ (1,4) (2,4).

$W_4 =$ (rows a, b, c, d; columns a, b, c, d)

$C \downarrow p_i \downarrow 1, 2, 3$ $\quad$ $R \downarrow q_j \downarrow \phi$ (Null).

(No relation found.)

R = {(a,a) (a,b) (a,c) (a,d) (b,a) (b,b) (b,c) (b,d) (c,d)}.

eg:- let $A = \{1, 2, 3, 4\}$ and $R = \{(1,2)(2,1)(2,3)(3,4)\}$ find the transitive closure

eg :- let $A = \{4, 6, 8, 10\}$ & $R = \{(4,14)(4,10), (6,6)(6,8), (6,8), (8,10)\}$ is a relation on set A determine closure of R

Q. let $A = \{1, 2, 3, 4\}$ and $R = \{(1,2)(2,1)(2,3)(3,4)\}$

column

row

step 1 = W_0

$$\begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$$W_1 = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

c ↓ p_i ↓ 2   r ↓ q_j ↓ 2

$$W_2 = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{cccc} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

p_i ↓ 1, 2 q_j ↓ 2, 3

$$W_3 = \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \left[\begin{array}{cccc} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

$\{(1,1)(1,2), (1,3)(2,2)(2,\ldots(2,1)$.

p_i ↓ 1, 2 q_j ↓ 4 $\rightarrow (1,4)(2,4)$

Page No.: ____
Date: __ / __ / __

$W_4 \rightarrow$ No relation found.

$R = \{(1,1)(1,2)(1,3)(1,4)(2,1)(2,2)(2,3)$
$(2,4)\ (3,4)\}.$

$A = \{4,6,8,10\}$ & $R = \{(4,4)\ (4,10)$
$(6,6)\ (6,8)(8,6)\ (8,10)\}.$

$$W_0 = \begin{array}{c} \\ 4 \\ 6 \\ 8 \\ 10 \end{array} \begin{array}{c} \begin{array}{cccc} 4 & 6 & 8 & 10 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$W_1 = \begin{array}{c} \\ 4 \\ 6 \\ 8 \\ 10 \end{array} \begin{array}{c} \begin{array}{cccc} 4 & 6 & 8 & 10 \end{array} \\ \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} \end{array}$$

$p_i \quad q_j$
$\downarrow \quad \downarrow$
1
4

Permutation function.

A permutation on set A is bijection from (A X A) A into A that is itself. Here the set A is a finite if set A contain an element. then there n factorial different permutation on A.

The matrix for desencling the function on a finite set is to least / to domain across the top row & image of each element exactly below it.

e.g. Consider A = {a, b}, determine all the permutation of A.

Soln :-

The set A has two element hence, it has 2! = 2 then.

$$P_1 = \begin{pmatrix} a & b \\ a & b \end{pmatrix}, \quad P_2 = \begin{pmatrix} a & b \\ b & a \end{pmatrix}$$

* Inverse of Permutation :-

e.g Consider the permutation.

$$P = \begin{pmatrix} a & b & c & d \\ d & c & a & b \end{pmatrix}$$

Determine inverse P^{-1} :- To find P^{-1}

$$P^{-1} = \begin{pmatrix} d & c & a & b \\ a & b & c & d \end{pmatrix} \quad P^{-1} = \begin{pmatrix} d & a & b & c \\ a & c & d & b \end{pmatrix}$$

clockwise.
cyclic

Date: / /

* composition of two permutation

e.g. consider a finite set $A = \{5,6,7\}$

let $p_1 = \begin{pmatrix} 5 & 6 & 7 \\ 5 & 7 & 6 \end{pmatrix}$ & $p_2 \begin{pmatrix} 5 & 6 & 7 \\ 6 & 5 & 7 \end{pmatrix}$

be two permutation of A. Determine $P_1 \circ P_2$ & $P_2 \cdot P_1$

$$P_1 \cdot \circ P_2 = \begin{pmatrix} 5 & 6 & 7 \\ 5 & 7 & 6 \end{pmatrix} \cdot \begin{pmatrix} 5 & 6 & 7 \\ 6 & 5 & 7 \end{pmatrix} = \begin{pmatrix} 5 & 6 & 7 \\ 6 & 7 & 5 \end{pmatrix}$$

$$P_2 \cdot P_1 = \begin{pmatrix} 5 & 7 & 6 \\ 6 & 5 & 7 \end{pmatrix} \cdot \begin{pmatrix} 5 & 6 & 7 \\ 5 & 7 & 6 \end{pmatrix} = \begin{pmatrix} 5 & 6 & 7 \\ 7 & 5 & 6 \end{pmatrix}$$

* cyclic permutation :-

consider the permutation $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 4 & 6 & 1 & 5 & 3 & 2 \end{pmatrix}$

soln :-

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 7 & 4 & 6 & 1 & 5 & 3 & 2 \end{pmatrix}$$

① $= (1\ 7\ 2\ 4)$ ② $= (3, 6)$.

e.g. consider a set $A = \{a, b, c, d, e, f, g, h\}$. Now find the product $\{e, f, c, d, a\}$ $\{a, b, h, g\}$

$$\{e, f, c, d, a\} = \begin{pmatrix} a & b & c & d & e & f & g & h \\ e & b & d & a & f & c & g & h \end{pmatrix}$$

$a = e = e$

$c = d = d$

$$\{a, b, h, g\} = \begin{pmatrix} a & b & c & d & e & f & g & h \\ b & h & c & d & e & f & a & g \end{pmatrix}$$

$\{e, f, c, d, a\} \circ (a, b, h, g)$.

$$\begin{pmatrix} a & b & c & d & e & f & g & h \\ e & h & d & b & f & c & a & g \end{pmatrix}$$

① let $A = \{1, 2, 3, 4, 5, 6\}$ compute

① $(4\,1\,3\,5) \cdot (5\,6\,3)$

② $(5\,6\,4) \cdot (1\,3\,5)$

①

$$(4135) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 5 & 1 & 4 & 6 \end{pmatrix}$$

$$(563) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 5 & 4 & 6 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 6 & 1 & 4 & 3 \end{pmatrix}$$

②

$$(564) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 3 & 5 & 6 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 5 & 1 & 6 & 4 \end{pmatrix}$$

$$(135) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 5 & 4 & 1 & 6 \end{pmatrix}$$

Page No.
Date: / /

Show that where composition of two disjoint cycles is comulative consider the set $A = \{a, b, c, d, e, f, g\}$
Now find a the product of:

$(a\,d\,g)\cdot(b\,c\,f)$ & $(b\,c\,f)\cdot(a\,d\,g)$

① $(a\,d\,g) = \begin{pmatrix} a & b & c & d & e & f & g \\ d & b & c & g & e & f & a \end{pmatrix}$

$(b\,c\,f) = \begin{pmatrix} a & b & c & d & e & f & g \\ a & c & f & d & e & b & g \end{pmatrix}$

$= \begin{pmatrix} a & b & c & d & e & f & g \\ d & c & f & g & e & b & a \end{pmatrix}$

② $(b\,c\,f)\cdot(a\,d\,g)$.

$(b\,c\,f) = \begin{pmatrix} a & b & c & d & e & f & g \\ a & c & f & d & e & b & g \end{pmatrix}$

$(a\,d\,g) = \begin{pmatrix} a & b & c & d & e & f & g \\ d & b & c & g & e & f & a \end{pmatrix}$

$= \begin{pmatrix} a & b & c & d & e & f & g \\ d & c & f & g & e & b & a \end{pmatrix}$

Date: __/__/__

from ① & ②

composition of two disjoined cycle is commitative hence proved.

Imp.

Path & relation & diagram

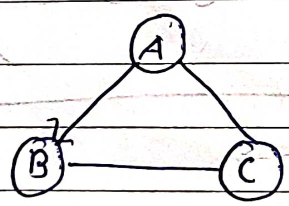
consider a relation R on a set A a path of length L in R from A to B is a finite sequence $\pi = a, x_1, a_2 x_2 \ldots a_n x_{n-1} b$. beinging with A & ending with B such that,

$aRx_1, x_1 R x_2, \ldots, x_{n-1} R b$.

Note that,

length n envolves. $n+1$ element of A.

Cycle :-



A path that beigns & ends at the same vertex is called cycle.

e.g. consider.

Let $A = \{a, b, c, d, e\}$ & $R = \{(a,a), (a,b) (b,c) (c,e) (c,d) (d,e)\}$

compute. i) R^2 ii) R^∞

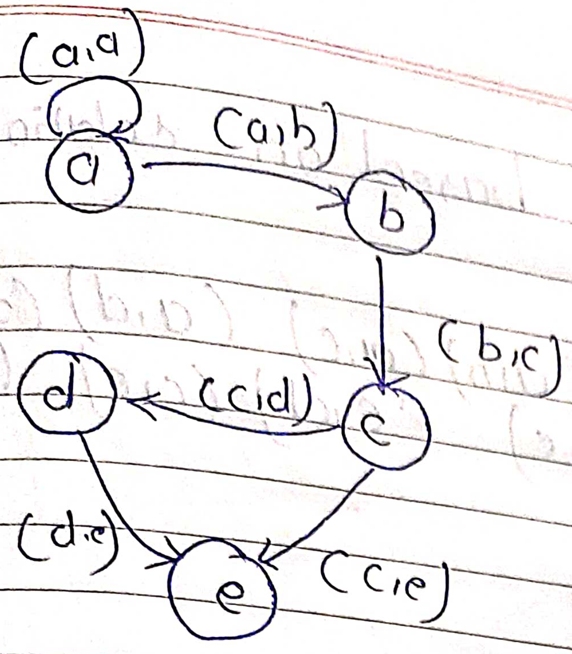
Soln :-

Given,

$A = \{a, b, c, d, e\}$

$R = \{(a,a) (a,b) (b,c) (c,e) (c,d) (d,e)\}$

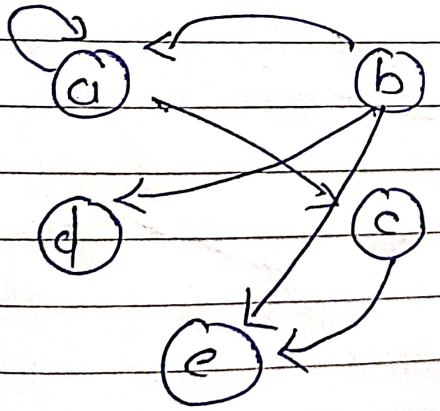
So the diagraph of R is shown below.



a) To find R^2 from the digraph following are the paths of length $n=2$.

$a R^2 a$ since aRa and aRa.
$a R^2 b$ since aRa and aRb.
$a R^2 c$ since aRa and aRc.
$b R^2 e$ since bRb and bRe.
$b R^2 d$ since bRb and bRd.
$c R^2 e$ since cRc and cRe.

$R^2 = \{(a,a)\ (a,b)\ (a,c)\ (b,e)\ (b,d)\ (c,e)\}$



(b) To find R^∞ we need all ordered pair of vertive for which there is path of any length from the first vertex to the second hence from the previous