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* Explicite Formula *

Imp Que. Find explicite formula from the sequence define by $C_n = 3C_{n-1} - 2C_{n-2}$ with initial cond^n $C_1 = 5$ & $C_2 = 3$

$\longrightarrow$

$$C_n = 3C_{n-1} - 2C_{n-2}$$

It is homogeneous equation of 2^{nd} degree.

$C_n = x^2$

$C_{n-1} = x$

$C_{n-2} = x^0 = 1$

$$x^2 = 3x - 2$$
$$x^2 - 3x = -2$$
$$x^2 - 3x + 2 = 0$$
$$x^2 - 1x - 2x + 2 = 0$$
$$x(x-1) - 2(x-1) = 0$$
$$(x-1)(x-2) = 0$$
$$x - 1 = 0 \quad \& \quad x - 2 = 0$$
$$\boxed{x = 1} \qquad \boxed{x = 2}$$

eq^n has roots (1, 2)

$S_1 = 1, S_2 = 2$

The general explicit Formula -

$$C_n = US_1^n + VS_2^n$$

$\underline{n = 1}$

$$C_1 = US_1^1 + VS_2^1$$

But, $C_1 = 5$, $S_1 = 1$, $S_2 = 2$

$$5 = U(1) + V(2)$$
$$5 = U + 2V$$
$$\boxed{U + 2V = 5} \text{ ——— (A)}$$

similarly,

$$C_n = Us_1^n + Vs_2^n$$

$\underline{\underline{n=2}}$

$$C_2 = Us_1^2 + Vs_2^2$$

$$C_2 = 3,\ s_1 = 1,\ s_2 = 2$$

$$3 = U(1) + V(4)$$

$$3 = U + 4V$$

$$\boxed{U + 4V = 3} \text{ ——— } (B)$$

from eq^n (A) & (B)

$$\begin{array}{r} \cancel{U} + 2V = 5 \\ \cancel{U} + 4V = 3 \\ - \quad - \quad - \\ \hline -2V = 2 \end{array}$$

$$\boxed{V = -1}$$

putting these value in eq^n (A)

$$U + 2V = 5$$

$$U + 2(-1) = 5$$

$$U - 2 = 5$$

$$U = 5 + 2$$

$$\boxed{U = 7}$$

Required equation -

$$C_n = Us_1^n + Vs_2^n$$

$$U = 7,\ V = -1,\ s_1 = 1,\ s_2 = 2$$

$$C_n = 7(1) + (-1)2^n$$

$$\boxed{C_n = 7 - 2^n}$$

Here, req. formula -

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$\boxed{C_n = 7 - 2^n}$

Req. series.

5, 3, −1, −9.

e. Find an explicite formula for the sequence define by −

$a_n = 4a_{n-1} + 5a_{n-2}$

with initial condⁿ $a_1 = 2$, $a_2 = 6$

→ $a_n = 4a_{n-1} + 5a_{n-2}$

It is homogeneous eqⁿ of 2nd degree.

$x^2 = 4x + 5$

$x^2 - 4x - 5 = 0$

$x^2 - 5x + 1x - 5 = 0$

$x(x-5) + 1(x-5) = 0$

$(x-5)(x+1) = 0$

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→ $a_n = 4a_{n-1} + 5a_{n-2}$

It is homogeneous equation of 2^{nd} degree.

$a_n = x^2$
$a_{n-1} = x$
$a_{n-2} = x^0 = 1$

$\therefore x^2 = 4x + 5$

$x^2 - 4x - 5 = 0$

$x^2 - 5x + x - 5 = 0$

$x(x-5) + 1(x-5) = 0$

$(x-5)(x+1) = 0$

$x + 1 = 0$, $x + 5 = 0$ | $x - 5 = 0$

$\boxed{x = -1}$, $x = -5$ | $\boxed{x = 5}$

eq^n has two roots $(-1, 5)$

$s_1 = -1, \; s_2 = 5$

The general explicite formula –

$a_n = U s_1^n + V s_2^n$

$n = 1$

$a_1 = U s_1 + V s_2$

But, $a_1 = 2, s_1 = -1, s_2 = 5$

$2 = U(-1) + V(5)$

$2 = -U + 5V$

$-U + 5V = 2$

$\boxed{U - 5V = -2}$ ——— (A)

Similarly,

$a_n = U s_1^n + V s_2^n$

$n = 2$

$a_2 = U s_1^2 + V s_2^2$

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But, $a_2 = 6$, $s_1 = -1$, $s_2 = 5$

$$6 = U(-1)^2 + V(5)^2$$

$$6 = U + 25V$$

$$\boxed{U + 25V = 6} \text{ ——— } (B)$$

from eq^n (A) & (B) we get,

$$\cancel{U} - 5V = -2$$

$$\cancel{U} + 25V = 6$$

$$- \quad - \quad -$$

$$30V = 8$$

$$V = \frac{8}{30}$$

$$\boxed{V = \frac{4}{15}}$$

putting these value in eq^n (A) we get,

$$U - 5V = -2$$

$$U - \cancel{5} \times \frac{4}{\cancel{15}\,3} = -2$$

$$U - \frac{4}{3} = -2$$

$$U = -2 + \frac{4}{3}$$

$$U = \frac{-6+4}{3}$$

$$\boxed{U = \frac{-2}{3}}$$

Required eq^n –

$$a_n = U s_1^n + V s_2^n$$

$$U = \frac{-2}{3}, \ V = \frac{4}{15}, \ s_1 = -1, \ s_2 = 5$$

$$\boxed{a_n = \frac{-2}{3}(-1)^n + \frac{4}{15}(5)^n}$$

Req. series –

2, 6, 11.78,

$\therefore$ No. of distinguishable word $= \dfrac{7!}{1!\,2!\,1!\,1!\,1!\,1!\,1!}$

$= \dfrac{5040}{2}$

$\boxed{\therefore \text{ No. of distinguishable word} = 2520}$

Que. Define following terms.

① Product Set :-

If A & B are two non-empty set then product set is denoted be - **A·B** & is define as -

$$\boxed{A \times B = \{(a,b) / a \in A \;\&\; b \in B\}}$$

② Relation :-

Let A & B are two non-empty set then relation R is a subset of AXB i.e. $R \subseteq A \times B$ we say that - a is related to b by R & we write it as $\boxed{a\,R\,b}$ Relation

V.V.imp

③ Matrix Relation :-

let A & B are finite set & R is a relation from A to B then matrix relation is denoted be - M_R & is define as -

$$M_R = \begin{cases} 1 & \text{if } (a_i b_i) \in R \\ 0 & \text{if } (a_i b_i) \notin R \end{cases}$$

(4) Directed Graph or Digraph of relation :-

If A is a finite set & R is a relation on A then we can also represent R in a pictorial form as follows :

Draw a small circle for each element of A this circle are called as vertices or nodes. Draw an arrow from vertices a_i to a_j from the relation the resultant pictorial relation is called directed graph or digraph.

(5) Indegree of Vertices :-

No. of arrows resting on a particular vertices is called indegree of vertices.

(6) Outdegree of Vertices :-

No. of arrows going away from a vertices are called outdegree of vertices.

*V.V.imp
Que. S-04

If $x = \{1, 2, 3, 4\}$ & $R = \{(x, y) / x > y\}$ Draw the graph of R & give its matrix.

→ $x = \{1, 2, 3, 4\}$

$y = \{1, 2, 3, 4\}$

$x \times y = \{ (1,1) (1,2) (1,3) (1,4)$

(2,1) (2,2) (2,3) (2,4)

(3,1) (3,2) (3,3) (3,4)

(4,1) (4,2) (4,3) (4,4)}

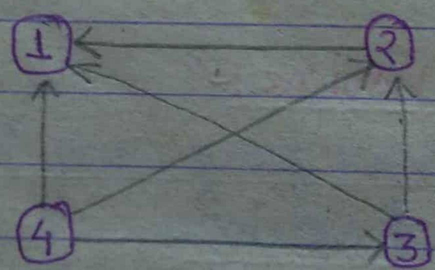
Relation – $R = \{(x,y) / x > y\}$

$R = \{(2,1)(3,1)(3,2)(4,1)(4,2)(4,3)\}$

Matrix Rel^n –

$$M_R = \begin{array}{c|c} x \backslash y & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

Digraph –



	1	2	3	4
I in	3	2	1	0
O out	0	1	2	3

Que. If $x = \{1,2,3,4\}$ & $R = \{(x,y) / x<y\}$ Draw the graph of R & give its matrix.

$\longrightarrow$ $x = \{1,2,3,4\}$

$y = \{1,2,3,4\}$

$x \times y = \{(1,1)(1,2)(1,3)(1,4)$
$(2,1)(2,2)(2,3)(2,4)$
$(3,1)(3,2)(3,3)(3,4)$
$(4,1)(4,2)(4,3)(4,4)\}$

Relation -

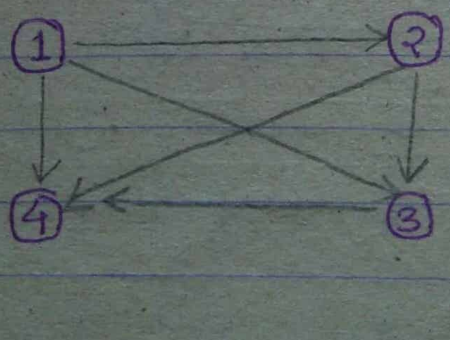
$R = \{(x,y) / x<y\}$

$R = \{(1,2)(1,3)(1,4)(2,3)(2,4)(3,4)\}$

Matrix Rel^n -

$$M_R = \begin{array}{c|c} x \backslash y & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Digraph -



	1	2	3	4
Iin	0	1	2	3
Oout	3	2	1	0

Que. 1 Find the relation R. define on $A = \{1,2,3,4\}$ if

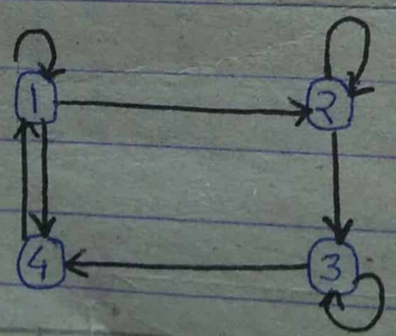
$$M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

→ $A = \{1,2,3,4\}$ $B = \{1,2,3,4\}$

$$M_R = \begin{array}{c|cccc} A \backslash B & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 1 \\ 2 & 0 & 1 & 1 & 0 \\ 3 & 0 & 0 & 1 & 1 \\ 4 & 1 & 0 & 0 & 0 \end{array}$$

Relation $R = \{(1,1)(1,2)(1,4)(2,2)(2,3)(3,3)(3,4)(4,1)\}$

Digraph -



	1	2	3	4
Iin	2	2	2	2
Oout	3	2	2	1

Que. ② If $A = \{1,2,3,4,5\}$ & $M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$.

find relation & digraph.

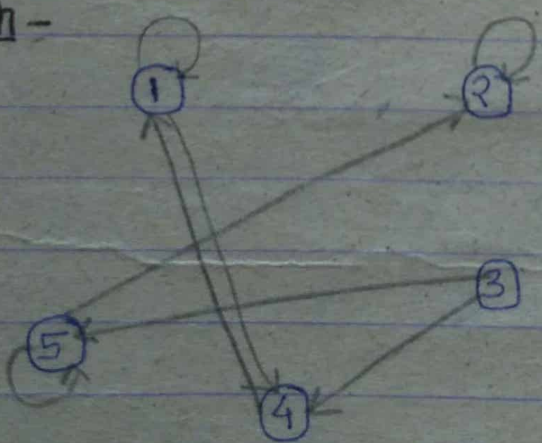
$\longrightarrow$ $A = \{1,2,3,4,5\}$ & $B = \{1,2,3,4,5\}$

$$M_R = \begin{array}{c|c} A \backslash B & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Relation-

$R = \{ (1,1)\ (1,4)\ (2,2)\ (3,4)\ (3,5)\ (4,1)\ (5,2)\ (5,5) \}$

Digraph -



	1	2	3	4	5
Iin	2	2	0	2	2
Oout	2	1	2	1	2

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**W-09

**Que ③ let $A = \{a, b, c, d\}$ & $M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

Find relⁿ & digraph.

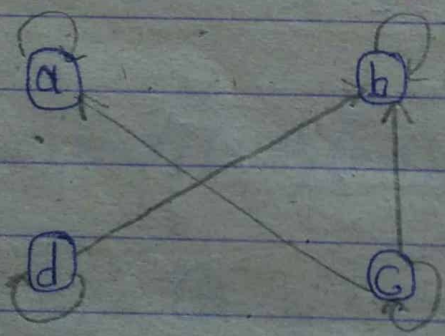
→ $A = \{a, b, c, d\}$ $B = \{a, b, c, d\}$

$$M_R = \begin{array}{c|cccc} A \backslash B & a & b & c & d \\ \hline a & 1 & 0 & 0 & 0 \\ b & 0 & 1 & 0 & 0 \\ c & 1 & 1 & 1 & 0 \\ d & 0 & 1 & 0 & 1 \end{array}$$

Relation -

$R = \{(a,a) (b,b) (c,a) (c,b) (c,c) (d,b) (d,d)\}$.

Digraph -



	a	b	c	d
Iin	2	3	1	1
Out	1	1	3	2

S-11 Que. 4 let $A = \{a, b, c, d\}$ & $M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

find rel^n & digraph

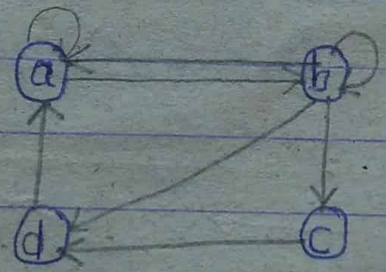
$\longrightarrow$ $A = \{a, b, c, d\}$ $B = \{a, b, c, d\}$

$$M_R = \begin{array}{c|c} A \backslash B & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Relation -

$R = \{ (a, a) (a, b) (b, a) (b, b) (b, c) (b, d) (c, d) (d, a) \}$

Digraph -



	a	b	c	d
Iin	3	2	1	2
Oout	2	4	1	1

Que ⑤ If $A = \{1, 2, 3, 4, 5\}$ & $M_R = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

find relation & digraph.

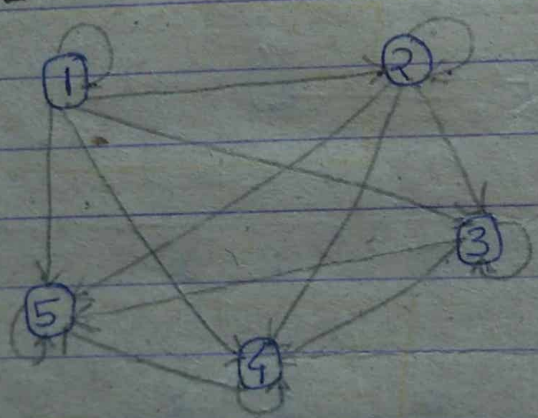
→ $A = \{1, 2, 3, 4, 5\}$ & $B = \{1, 2, 3, 4, 5\}$

$$M_R = \begin{array}{c|c} A \backslash B & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Relation -

$R = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (2,2)\ (2,3)\ (2,4)\ (2,5)$
$(3,3)\ (3,4)\ (3,5)\ (4,4)\ (4,5)\ (5,5)\}$

Digraph -



	1	2	3	4	5
I_{in}	1	2	3	4	5
O_{out}	5	4	3	2	1

Que. (1) To find R^2 & R^∞.

If $A = \{1,2,3,4,5\}$ & $M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$

$\longrightarrow$ If $A = \{1,2,3,4,5\}$ & $B = \{1,2,3,4,5\}$

$$M_R = \begin{array}{c|c} A \backslash B & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$R = \{ (1,1)\,(1,4)\,(2,2)\,(3,4)\,(3,5)\,(4,1)\,(5,2)\,(5,5) \}$

R^2 —

$1R_1$ & $1R_4 = 1R^2_4$

$1R_4$ & $4R_1 = 1R^2_1$

$3R_4$ & $4R_1 = 3R^2_1$

$3R_5$ & $5R_2 = 3R^2_2$

$3R_5$ & $5R_5 = 3R^2_5$

$4R_1$ & $1R_1 = 4R^2_1$

$4R_1$ & $1R_4 = 4R^2_4$

$5R_2$ & $2R_2 = 5R^2_2$

$5R_5$ & $5R_2 = 5R^2_2$

$\boxed{R^2 = \{ (1,4)\,(1,1)\,(3,1)\,(3,2)\,(3,5)\,(4,1)\,(4,4)\,(5,2) \}}$

$R^\infty = R^2 \cup$ Reflexsive Relation

$\boxed{R^\infty = \{ (1,4)\,(1,1)\,(3,1)\,(3,2)\,(3,5)\,(4,1)\,(4,4)\,(5,2)\,(2,2)\,(3,3)\,(5,5) \}}$

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*W-09 **Que ② let $A = \{a, b, c, d, e\}$ & $R = \{(a,a)(a,b)(b,c)(c,e)(c,d)(d,e)\}$
Compute R^2 & R^∞.

⟶ If $A = \{a, b, c, d, e\}$ & $B = \{a, b, c, d, e\}$

$R = \{(a,a)(a,b)(b,c)(c,e)(c,d)(d,e)\}$

R^2 -

aRa & $aRb = aR^2b$

aRb & $bRc = aR^2c$

bRc & $cRe = bR^2e$

bRc & $cRd = bR^2d$

cRd & $dRe = cR^2e$

$$\boxed{R^2 = \{(a,b)(a,c)(b,e)(b,d)(c,e)\}}$$

$R^\infty = R^2 \cup$ Reflexive Relation

$R^\infty = \{(a,b)(a,c)(b,e)(b,d)(c,e)(a,a)(b,b)(c,c)(d,d)$
$(e,e)\}$

Que. ③ If $A = \{1, 2, 3, 4, 5\}$ & $R = \{(1,1)(1,2)(2,3)(2,4)(3,4)$
$(4,1)(4,2)\}$

⟶ If $A = \{1, 2, 3, 4, 5\}$ & $B = \{1, 2, 3, 4, 5\}$

$R = \{ (1,1) (1,2) (2,3) (2,4) (3,4) (4,1) (4,2) \}$

R^2 -

$1R_1 \ \& \ 1R_2 = 1R^2_2$

$1R_2 \ \& \ 2R_3 = 1R^2_3$

$1R_2 \ \& \ 2R_4 = 1R^2_4$

$2R_3 \ \& \ 3R_4 = 2R^2_4$

$2R_4 \ \& \ 4R_1 = 2R^2_1$

$2R_4 \ \& \ 4R_2 = 2R^2_2$

$3R_4 \ \& \ 4R_1 = 3R^2_1$

$3R_4 \ \& \ 4R_2 = 3R^2_2$

$4R_1 \ \& \ 1R_1 = 4R^2_1$

$4R_1 \ \& \ 1R_2 = 4R^2_2$

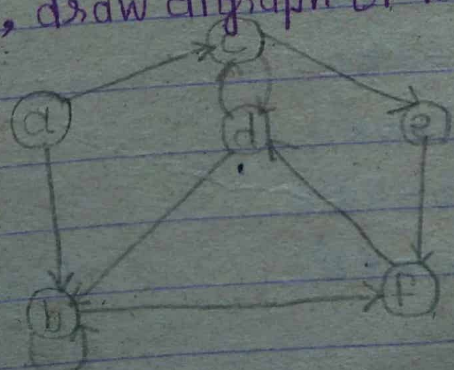
$4R_2 \ \& \ 2R_3 = 4R^2_3$

$4R_2 \ \& \ 2R_4 = 4R^2_4$

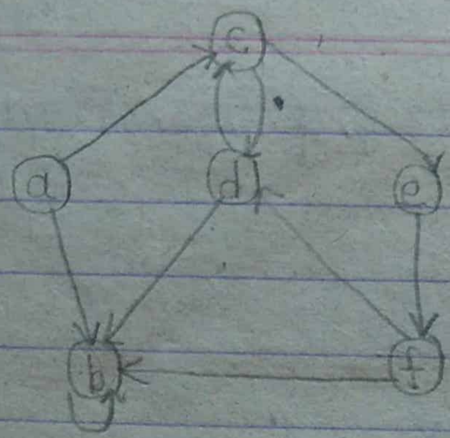
$R^2 = \{ (1,2) (1,3) (1,4) (2,4) (2,1) (2,2) (3,1) (3,2) (4,1) (4,2) (4,3) (4,4) \}$

$R^\infty = \{ (1,2) (1,3) (1,4) (2,4) (2,1) (2,2) (3,1) (3,2) (4,1) (4,2) (4,3) (4,4) (1,1) (3,3) (5,5) \}$

W-08 Que. 4 If $A = \{$
find the R whose digraph is given in fig. hense find R^2, draw digraph of R^2 & find R^∞.



→



$R = \{(a,b)(a,c)(b,b)(b,f)(c,d)(c,e)(d,b)(d,c)(e,f)(f,d)\}$

aRb & $bRb = aR^2b$

aRb & $bRf = aR^2f$

aRc & $cRd = aR^2d$

aRc & $cRe = aR^2e$

bRb & $bRf = bR^2f$

bRf & $fRd = bR^2d$

cRd & $dRb = cR^2b$

cRd & $dRc = cR^2c$

cRe & $eRf = cR^2f$

dRb & $bRb = dR^2b$

dRb & $bRf = dR^2f$

dRc & $cRd = dR^2d$

dRc & $cRe = dR^2e$

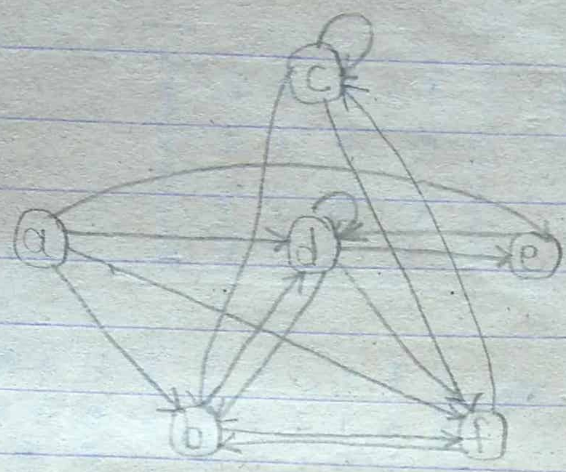
eRf & $fRd = eR^2d$

fRd & $dRb = fR^2b$

fRd & $dRc = fR^2c$

$R^2 = \{(a,b)(a,f)(a,d)(a,e)(b,f)(b,d)(c,b)(c,c)(c,f)(d,b)(d,f)(d,d)(d,e)(e,d)(f,b)(f,c)\}$

$R^\infty = \{(a,b)(a,f)(a,d)(a,e)(b,f)(b,d)(c,b)(c,c)(c,f)(d,b)(d,f)(d,d)(d,e)(e,d)(f,b)(f,c)(a,a)(b,b)(e,e)(f,f)\}$



* To find M_R^2 & M_R^∞ *

Que. If $A=\{1,2,3,4\}$ & $M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

find M_R^2 & M_R^∞.

→ $A=\{1,2,3,4\}$ & $B=\{1,2,3,4\}$

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$M_R^2 = M_R \cdot M_R$$

$$= \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0+1 & 1+1+0+0 & 0+1+0+0 & 1+0+0+0 \\ 0+0+0+0 & 0+1+0+0 & 0+1+1+0 & 0+0+1+0 \\ 0+0+0+1 & 0+0+0+0 & 0+0+1+0 & 0+0+1+0 \\ 1+0+0+0 & 1+0+0+0 & 0+0+0+0 & 1+0+0+0 \end{bmatrix}$$

$$\boxed{M_R^2 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix}}$$

$$M_R^3 = M_R^2 \cdot M_R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0+1 & 1+1+0+0 & 0+1+1+0 & 1+0+1+0 \\ 0+0+0+1 & 0+1+0+0 & 0+1+1+0 & 0+0+1+0 \\ 1+0+0+1 & 1+0+0+0 & 0+0+1+0 & 1+0+1+0 \\ 1+0+0+1 & 1+1+0+0 & 0+1+0+0 & 1+0+0+0 \end{bmatrix}$$

$$\boxed{M_R^3 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}}$$

$$M_R^4 = M_R^3 \cdot M_R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0+1 & 1+1+0+0 & 0+1+1+0 & 1+0+1+0 \\ 1+0+0+1 & 1+1+0+0 & 0+1+1+0 & 1+0+1+0 \\ 1+0+0+1 & 1+1+0+0 & 0+1+1+0 & 1+0+1+0 \\ 1+0+0+1 & 1+1+0+0 & 0+1+1+0 & 1+0+1+0 \end{bmatrix}$$

$$\boxed{M_R^4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}}$$

Since,

$$M_R^3 = M_R^4$$

$$\therefore M_R^\infty = M_R \cup M_R^2 \cup M_R^3$$

$$M_R^\infty = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \cup \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \cup \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\boxed{\therefore M_R^\infty = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}}$$

Que. If $A = \{a, b, c, d\}$ $M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

find M_R^2 & M_R^∞

→ $A = \{a, b, c, d\}$ & $B = \{a, b, c, d\}$

& $M_R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

$$M_R^2 = M_R \cdot M_R$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0+0 & 0+0+0+0 & 0+0+0+0 & 0+0+0+0 \\ 0+0+0+0 & 0+1+0+0 & 0+0+0+0 & 0+0+0+0 \\ 1+0+1+0 & 0+1+1+0 & 0+0+1+0 & 0+0+0+0 \\ 0+0+0+0 & 0+1+0+1 & 0+0+0+0 & 0+0+0+1 \end{bmatrix}$$

$$\boxed{M_R^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}}$$

since,

$$M_R = M_R^2$$

$$M_R^\infty = M_R$$

$$\boxed{M_R^\infty = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}}$$

Que. If $A = \{1, 2, 3, 4, 5\}$ & $M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$

find M_R^2 & M_R^∞

Date 27/11/12

$\longrightarrow$ $A = \{1, 2, 3, 4, 5\}$ & $B = \{1, 2, 3, 4, 5\}$

$$\& \ M_R = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$M_R^2 = M_R \cdot M_R$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+0+1+0 & 0+0+0+0+0 & 0+0+0+0+0 & 1+0+0+0+0 & 0 \\ 0+0+0+0+0 & 0+1+0+0+0 & 0+0+0+0+0 & 0+0+0+0+0 & 0 \\ 0+0+0+1+0 & 0+0+0+0+1 & 0+0+0+0+0 & 0+0+0+0+0 & 1 \\ 1+0+0+0+0 & 0+0+0+0+0 & 0+0+0+0+0 & 1+0+0+0+0 & 0 \\ 0+0+0+0+0 & 0+1+0+0+1 & 0+0+0+0+0 & 0+0+0+0+0 & 1 \end{bmatrix}$$

$$\boxed{M_R^2 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}}$$

$$M_R^3 = M_R^2 \cdot M_R$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{M_R^3 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}}$$

$$M_R^4 = M_R^3 \cdot M_R$$

$$= \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{M_R^4 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}}$$

Since,

$$M_R^3 = M_R^4$$

$$M_R^{\infty} = M_R^3$$

$$\boxed{M_R^{\infty} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}}$$

*** v.v.imp *** Transitive Closure *

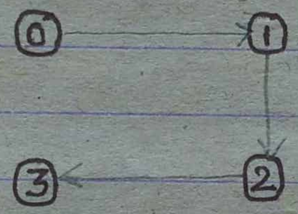
-12
-05
-04

Que. let $A=\{0,1,2,3\}$ & R be the relation on the set A
$R=\{(0,1)(1,2)(2,3)\}$ find transitive closure.

→ let, (I) **Diagraph method** –

$A=\{0,1,2,3\}$ & $B=\{0,1,2,3\}$

$R=\{(0,1)(1,2)(2,3)\}$



$$\boxed{T.C. \text{ of } R=\{(0,1)(0,2)(0,3)(1,2)(1,3)(2,3)\}}$$

(II) **Matrix Method** –

$A=\{0,1,2,3\}$ & $B=\{0,1,2,3\}$

$R=\{(0,1)(1,2)(2,3)\}$

$$\boxed{M_R=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&0&0\end{bmatrix}}$$

$M_R^2 = M_R \cdot M_R$

$$=\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&0&0\end{bmatrix}\cdot\begin{bmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&0&0\end{bmatrix}$$

$$\boxed{M_R^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}$$

$$M_R^3 = M_R^2 \cdot M_R$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{M_R^3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}$$

$$M_R^4 = M_R^3 \cdot M_R$$

$$= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{M_R^4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}$$

$$M_R^5 = M_R^4 \cdot M_R$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{M_R^5 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}$$

Since, $M_R^4 = M_R^5$

T.C. of $R = M_R \cup M_R^2 \cup M_R^3 \cup M_R^4$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cup \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cup \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cup \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{T.C. of } R = \begin{array}{c|c} A \backslash B & \begin{matrix} 0 & 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 0 \\ 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

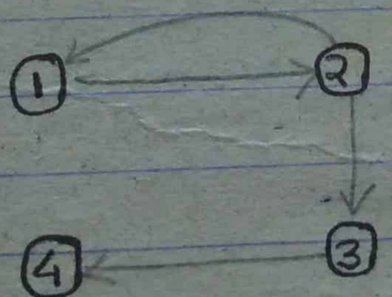
$\boxed{\therefore \text{ T.C. of } R = \{(0,1)(0,2)(0,3)(1,2)(1,3)(2,3)\}}$

2. let $A = \{1, 2, 3, 4\}$ $R = \{(1,2)(2,3)(3,4)(2,1)\}$ find transitive closure of R.

(I) Diagraph method –

→ let, $A = \{1,2,3,4\}$ & $B = \{1,2,3,4\}$

$R = \{(1,2)(2,3)(3,4)(2,1)\}$



$\boxed{R = \{(1,2)(1,1)(1,3)(1,4)(2,1)(2,2)(2,3)(2,4)(3,4)\}}$

(II) Matrix method –

$A = \{1, 2, 3, 4\}$, $B = \{1, 2, 3, 4\}$

$R = \{(1,2)\ (2,3)\ (3,4)\ (2,1)\}$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$M_R^2 = M_R \cdot M_R$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{M_R^2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}$$

$M_R^3 = M_R^2 \cdot M_R$

$$= \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{M_R^3 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}$$

$$M_R^4 = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{M_R^4 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}$$

since,

$$M_R^2 = M_R^4$$

$$\text{T.C. of } R = M_R \cup M_R^2 \cup M_R^3$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cup \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cup \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{T.C. of } R = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

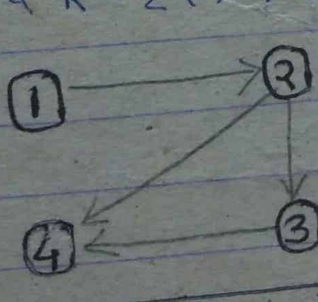
$$\boxed{\text{T.C. of } R = \{(1,1)(1,2)(1,3)(1,4)(2,1)(2,2)(2,3)(2,4)(3,4)\}}$$

15-10 ** Que. let $A = \{1,2,3,4\}$ & $R = \{(1,2)(2,3)(3,4)(2,4)\}$
find T.C. of R

→ (I) <u>Diagraph method</u> –

let $A = \{1,2,3,4\}$ & $B = \{1,2,3,4\}$

& $R = \{(1,2)\ (2,3)\ (3,4)\ (2,4)\}$



$$\boxed{R = \{(1,2)\ (1,3)\ (1,4)\ (2,3)(2,4)(3,4)\}}$$

29/11/12

(II) Matrix method -

let $A = \{1, 2, 3, 4\}$ & $B = \{1, 2, 3, 4\}$

& $R = \{(1,2)(2,3)(3,4)(2,4)\}$

$$\boxed{M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}}$$

$$M_R^2 = M_R \cdot M_R$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{M_R^2 = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}$$

$$M_R^3 = M_R^2 \cdot M_R$$

$$= \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{M_R^3 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}$$

$$M_R^4 = M_R^3 \cdot M_R$$

$$= \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{M_R^4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}$$

$$M_R^5 = M_R^4 \cdot M_R$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{M_R^5 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}$$

Since,

$$M_R^4 = M_R^5$$

$$\text{T.C. of } R = M_R \cup M_R^2 \cup M_R^3 \cup M_R^4$$

$$= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cup \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cup \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \cup \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\text{T.C. of } R = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{\text{T.C. of } R = \{(1,2)(1,3)(1,4)(2,3)(2,4)(3,4)\}}$$

* Cyclic Permutation *

Que. Define cyclic permutation with suitable example.

→ Consider, a finite set $A = \{x_1, x_2, \ldots x_n\}$ & let $t_1, t_2, \ldots t_k$ be k element of the set A & the permutation $p: A \to A$ is defined by $p(t_1) = t_2$ $p(t_2) = t_3$ $p(t_3) = t_4 \ldots\ldots p(t_{r-1}) = p(t_r)$. It is called cyclic permutation. It is denoted by $(t_1, t_2 \ldots\ldots t_r)$

W-06

Ex. ① let $A = \{1,2,3,4,5,6\}$ compute $(4,1,3,5)\cdot(5,6,3)$

$(5,6,3)\cdot(4,1,3,5)$

→ let $A = \{1,2,3,4,5,6\}$

① $(4,1,3,5)\cdot(5,6,3)$

② $(5,6,3)\cdot(4,1,3,5)$

$$(4,1,3,5) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 5 & 1 & 4 & 6 \end{pmatrix}$$

$$(5,6,3) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 5 & 4 & 6 & 3 \end{pmatrix}$$

$$(4,1,3,5)\cdot(5,6,3) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 5 & 1 & 4 & 6 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \boxed{1} & 2 & 5 & 4 & 6 & 3 \end{pmatrix}$$

$$(4,1,3,5)\cdot(5,6,3) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 4 & 1 & 6 & 5 \end{pmatrix}$$

$$(1,3)\,(3,4)\,(4,1)\,(5,6)\,(6,5)$$

$\therefore$ It is odd permutation.

$$(5,6,3)\cdot(4,1,3,5) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 2 & 5 & 4 & 6 & 3 \end{pmatrix}\cdot\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 5 & 1 & 4 & 6 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 6 & 1 & 4 & 3 \end{pmatrix}$$

$$(1,5)\,(3,6)\,(4,1)\,(5,4)\,(6,3)$$

It is odd permutation.

06

Ex. ② let $A = \{1,2,3,4,5,6,7,8\}$ Explain the following permutation as a product of transposition:

ⓐ $(2,1,4,5,8,6)$

ⓑ $(3,1,6)\cdot(4,8,2,5)$

$\rightarrow$ $A = \{1,2,3,4,5,6,7,8\}$

$(2,1,4,5,8,6)$

$(3,1,6)\cdot(4,8,2,5)$

$$(2,1,4,5,8,6) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 1 & 3 & 5 & 8 & 2 & 7 & 6 \end{pmatrix}$$

$$(1,4)(2,1)(4,5)(5,8)(6,2)(8,6)$$

$\therefore$ It is even permutations.

$$(3,1,6) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 2 & 1 & 4 & 5 & 3 & 7 & 8 \end{pmatrix}$$

$$(4,8,2,5) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 5 & 3 & 8 & 4 & 6 & 7 & 2 \end{pmatrix}$$

$$(3,1,6)\cdot(4,8,2,5) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 2 & 1 & 4 & 5 & 3 & 7 & 8 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 5 & 3 & 8 & 4 & 6 & 7 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 6 & 5 & 1 & 8 & 4 & 3 & 7 & 2 \end{pmatrix}$$

$$= (1,6)(2,5)(3,1)(4,8)(5,4)(6,3)(8,2)$$

$\therefore$ It is odd permutation.

W-07

Ex. 3 let $A = \{1,2,3,4,5,6,7,8\}$ Determine whether the permutation is even or odd.

(i) $(6,4,2,1,5)$

(ii) $(4,8)\cdot(3,5,2,1)\cdot(2,4,7,1)$

→ let,

$$A = \{1,2,3,4,5,6,7,8\}$$

① $(6,4,2,1,5) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 1 & 3 & 2 & 6 & 4 & 7 & 8 \end{pmatrix}$

$= (1,5)(2,1)(4,2)(5,6)(6,4)($

∴ It is odd permutation.

② $(4,8) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 8 & 5 & 6 & 7 & 4 \end{pmatrix}$

$(3,5,2,1) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 5 & 4 & 2 & 6 & 7 & 8 \end{pmatrix}$

$(2,4,7,1) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 4 & 3 & 7 & 5 & 6 & 1 & 8 \end{pmatrix}$

$(4,8)\cdot(3,5,2,1)\cdot(2,4,7,1) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 2 & 3 & 8 & 5 & 6 & 7 & 4 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 1 & 5 & 4 & 2 & 6 & 7 & 8 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 4 & 3 & 7 & 5 & 6 & 1 & 8 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 1 & 8 & 5 & 7 & 2 & 6 & 3 & 4 \end{pmatrix}$

$= (2,8)(3,5)(4,7)(5,2)(7,3)(8,4)$

∴ It is even permutation.

S-08 Ex. ④ let $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$ be the permutation on the set $A = (1,2,3,4,5,6)$

Compute ① p^2
② find whether p is an even or odd permutation.

→ $p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$

$A = (1,2,3,4,5,6)$

$p^2 = p \cdot p = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 3 & 5 & 1 & 2 & 6 \end{pmatrix}$

$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 1 & 5 & 2 & 4 & 3 & 6 \end{pmatrix}$

$\boxed{p^2 = (2,5)(3,2)(5,3)}$

∴ It is an odd permutation

S-09 Ex. ⑤ Encode the message 'WHERE ARE YOU' by apping the permutation $(1,7,3,5,11) \cdot (2,6,9) \cdot (4,8,10)$

→

W	H	E	R	E	A	R	E	Y	O	U
1	2	3	4	5	6	7	8	9	10	11

$A = \{1,2,3,4,5,6,7,8,9,10,11\}$

$$\boxed{(1,7,3,5,11) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 7 & 2 & 5 & 4 & 11 & 6 & 3 & 8 & 9 & 10 & 1 \end{pmatrix}}$$

$$\boxed{(2,6,9) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & 6 & 3 & 4 & 5 & 9 & 7 & 8 & 2 & 10 & 11 \end{pmatrix}}$$

$$\boxed{(4,8,10) = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & 2 & 3 & 8 & 5 & 6 & 7 & 10 & 9 & 4 & 11 \end{pmatrix}}$$

$$(1,7,3,5,11)\cdot(2,6,9)\cdot(4,8,10) =$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 7 & 2 & 5 & 4 & 11 & 6 & 3 & 8 & 9 & 10 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & 6 & 3 & 4 & 5 & 9 & 7 & 8 & 2 & 10 & 11 \end{pmatrix} \cdot$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & 2 & 3 & 8 & 5 & 6 & 7 & 10 & 9 & 4 & 11 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 7 & 6 & 5 & 8 & 11 & 9 & 3 & 10 & 2 & 4 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \text{WHERE} & \text{ARE} & \text{YOU} \\ \text{RAEEU} & \text{YED} & \text{HRW} \end{pmatrix}$$