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Search for Initial Conditions for Sustained Hopping of Passive Springy-Leg Offset-Mass Hopping Robot

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A springy-leg, offset-mass or SLOM hopper configuration is considered here, wherein the center of gravity of the body is offset from the line of action of spring force. It is observed that this feature tends to extend the passive hopping motion. A heuristics-based search strategy and a genetic algorithm-based search strategy are implemented for finding the initial conditions that result in extended hopping motion. [DOI: 10.1115/1.2745860]

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1 Introduction

Monopeds are the simplest dynamically stabilizable legged robots. A monoped can move only by hopping, which consists of alternating *flight* and *stance* phases. Monoped passive and active dynamic stability has been extensively studied. The term, passive dynamics, is used to refer to unpowered motion of a dynamical system from some initial state. Of course, sustained passive hopping motion is strictly not possible in real systems as there is always some loss of energy due to friction and impact affects.

Thompson and Raibert [1] studied passive, under-actuated dynamics of hopping machines in which the axis of the springy leg passed through the center of gravity (CG) of the body. A leg configuration for hopping is analyzed here, which can be termed the springy-leg, offset-mass (SLOM) configuration of monopod hopper. In this configuration, the CG of the body is offset from the line of action of the spring force. It is observed that this feature assists in achieving a somewhat sustained hopping motion. We have used the term SLOMPiSH to stand for *springy-leg offset-mass passive sustained hopping* and to mean execution of nearly periodic multiple hops before the consistency of the hops is lost.

The planar SLOM configuration appears to have been studied independently by two research groups [2,3]. Kuswadi et al. [3] studied this configuration with a single constant-force linear actuator between the leg and the body. The current and the previous value of this angular velocity, at the instant when the CG was directly above the point of contact of the foot, were used to compute the time duration of the actuation for an "on-off" controller. Shanmuganathan [2], on the other hand, used a reaction wheel actuator and demonstrated sustained hopping in simulations. His work considered both passive and under-actuated cases. In this paper, two methods of identifying initial conditions that result in SLOMPiSH motion are reported; namely, heuristics-based and genetic algorithm (GA)-based.

2 Mathematical Model of the Slom Hopper

The proposed SLOM hopper is schematically shown in Fig. 1 in stance and flight phases. The SLOM hopper consists of a body (mass m; moment of inertia I about the CG) and a massless leg (BS) connected to the body through a spring SP of stiffness k. The relative motion between the body and the leg is modeled as a prismatic joint.

The leg spring is offset by a distance d from the CG, and hence, the spring force does not act along a line passing through the CG. The leg is assumed to have a pointed foot at B. The ground is assumed rigid and the foot is assumed not to slip during the stance phase.

2.1 Equations of Motion. The dynamic equations of motion of the SLOM hopper are formulated for passive monopedal hopping in the vertical plane. The flight phase configuration of the SLOM hopper can be specified by the Cartesian position (x,y) of the CG and the leg orientation angle θ. On the other hand, the length l and the orientation θ, of the springy leg suffice to specify

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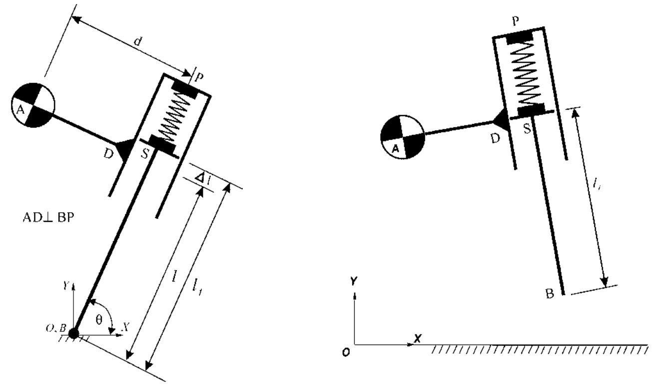

(*a*) SLOM Hopper during stance (*b*) SLOM Hopper during flight

Fig. 1 The Springy Leg Offset Mass (SLOM) Hopper

the configuration during the stance phase. The necessary equations for each phase of motion and conditions for take-off/landing are given below.

Stance Phase.

$$m\ddot{l} - md\ddot{\theta} - ml\dot{\theta}^2 + mg\sin\theta + k(l - l_f) = 0 \tag{1}$$

$$-md\ddot{l} + \{I + ml^2 + d^2\}\ddot{\theta} + 2ml\dot{l}\dot{\theta} + mg(l\cos\theta - d\sin\theta) = 0 \tag{2}$$

Flight Phase.

$$m\ddot{x} = 0 \tag{3}$$

$$m\ddot{y} = -mg \tag{4}$$

$$I\ddot{\theta} = 0 \tag{5}$$

Landing. Landing is assumed to occur at the instant when the foot touches the ground with a downward velocity (i.e., $y=0$; $\dot{y} < 0$). The impact of landing is associated with impulsive forces which cause jump in velocities. The initial conditions for stance phase can be obtained as follows [2]:

$$\dot{\theta}^+ = \frac{I\dot{\theta}^- + ml_f(-\dot{x}^-\sin\theta + \dot{y}^-\cos\theta)}{I + ml_f^2} \tag{6}$$

$$\dot{l}^+ = \dot{x}^+\cos\theta + \dot{y}^+\sin\theta + \dot{\theta}^+ d \tag{7}$$

where

$$\dot{x}^+ = \dot{x}^- - \sin\theta(l_f\dot{\theta}^+ + \dot{x}^-\sin\theta - \dot{y}^-\cos\theta) \tag{8}$$

$$\dot{y}^+ = \dot{y}^- + \cos\theta(l_f\dot{\theta}^+ + \dot{x}^-\sin\theta - \dot{y}^-\cos\theta) \tag{9}$$

and the superscripts "−" and "+" refer to time instants immediately before and after landing, respectively.

Take-off. Since the ground reaction, $R_y = m(\ddot{y} + g)$, the conditions for take-off are $\ddot{y} = -g$ and $\dddot{y} < 0$.

3 Search for Sustained Hopping Motion

3.1 Heuristic Search Algorithm. The SLOMPiSH motion is studied by simulating the hopping motion with a set of trial initial conditions in the flight phase (or the stance phase). By observing the resulting pattern of motion, the initial conditions are modified so as to obtain greater number of hops.

The dimension of search space for initial conditions is six during flight $(x, y, \theta, \dot{x}, \dot{y}, \dot{\theta})$ and four during stance $(l, \theta, \dot{\ell}, \dot{\theta})$. The dynamic behavior is independent of x and we assume zero vertical and angular velocities to restrict the search space to $F_R = \{y, \theta, \dot{x}\}$. This way, the determination of a SLOMPiSH trajectory reduces to the determination of the forward velocity for assumed values of (y, θ) at the instant of release. Results are presented for $y_0 = 0.65$ m and $\theta_0 = 90$ deg. The hopper parameters used in the simulation are: $m = 1$ kg; $k = 10$ kN/m; $I = 0.01$ kg m^2; $l_f = 0.5$ m; and $d = 0.1$ m.

Hopping patterns obtained from simulations with two different release velocities (2 m/s and 1 m/s) are shown in Figs. 2 and 3, respectively.

The hopper falls down on opposite sides in the two cases shown in Figs. 2 and 3. The heuristic assumption is that at some intermediate initial forward velocity in this range, the forward- and backward-falling tendencies will balance each other and the SLOM hopper can move forward with greater number of hops.

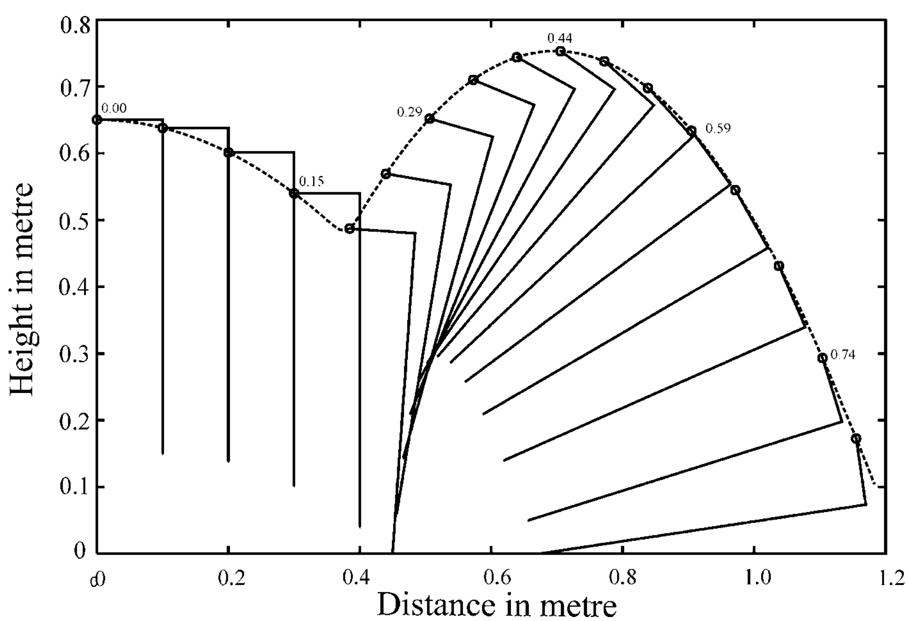

Fig. 2 Snapshots of SLOM hopper for release velocity of 2 m/s

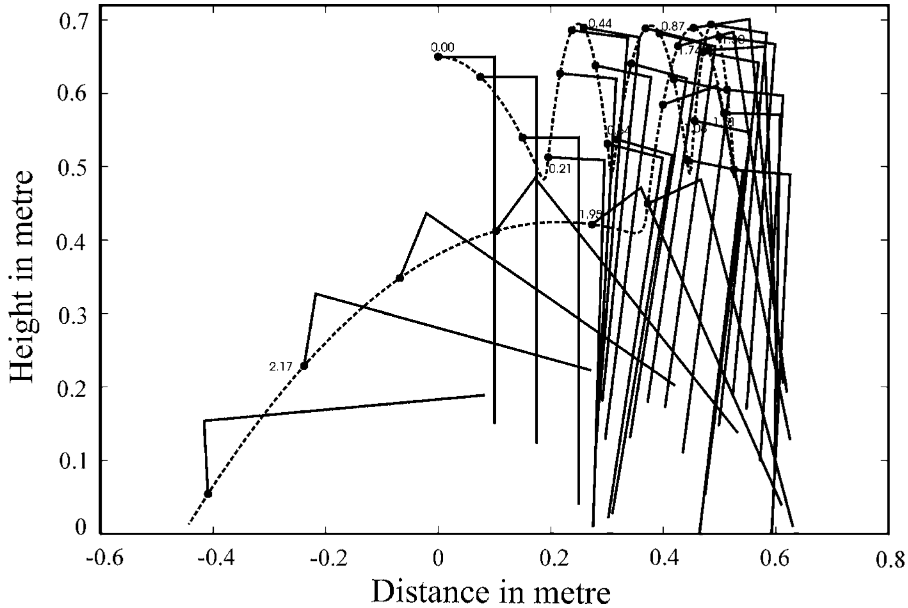

Fig. 3 Snapshots of SLOM hopper for release velocity of 1 m/s

Further, it is observed that graphs of the forward body and foot positions with respect to time begin to diverge just before the hopper falls.

Using these observations, the search procedure is coded in an algorithm that automatically searches for the initial conditions that yield sustained hopping. Initial bounds for release velocity are obtained by simulation for which the hopper falls in opposite directions for the upper and lower bounds. The appropriate release velocity within these bounds is then found using a bisection, which is terminated when *either* the number of hops prior to falling is more than a predefined limit *or* the width of the release velocity bound is smaller than some predefined tolerance.

For the system studied here, such a programmed search converged to a release velocity of 1.009743 m/s. The hopping motion is shown in Fig. 4. The numbers printed on each leg configuration indicate the time period between successive landing instants, which show near uniform hopping motion. For the parameters used, the stance time is typically about 10% of the cycle time.

3.2 Genetic Algorithm Based Search Strategy. The heuristic search method discussed above faces the problem of being trapped in local optima as it iteratively refines a single variable in a multi-dimensional state-space. Hence an evolutionary strategy in the form of a genetic algorithm (GA) has been implemented which operates on entire populations of candidate solutions in parallel. While this method does not get caught in local optima, it does not guarantee finding the global optima either. Reader interested in greater details of the method is referred to Yoshida et al. [4].

GA Problem Formulation. The aim of the GA search is to identify the initial conditions that result in SLOMPiSH motion. Number of consistent hops has been used as a measure to define the fitness function for a GA search. Consistency of hops can be found out by comparing the energy loss, time period, horizontal distance or maximum height between successive hops. We have used the maximum height, which is allowed to differ only by a small predefined fraction as compared to the value in the previous hop.

State Variables to be Optimized. The choice of the state variables to be optimized depends upon from where hopping motion is initiated. For hopping initiated from stance phase, the state variables chosen for optimization are the initial orientation of the leg θ_0 and the initial leg length l_0. The initial velocities are chosen to be zero (i.e., $\dot{l}_0=\dot{\theta}_0=0$). For the optimal θ_0 and l_0 another GA search is made to study the number of hops with nonzero initial velocities.

For hopping initiated from the flight phase, we assume zero vertical velocity, in order to restrict the search space to $F_R = [\theta\ \dot{\theta}\ \dot{x}\ y]$. Initially, a two-dimensional search is performed by fixing θ_0 to constant value and $\dot{\theta}_0$ to zero and optimizing $\dot{x}_0$ and y_0 in specified ranges. For these optimal values of $\dot{x}_0$ and y_0, a subsequent GA search is made to study the effect of variation of initial orientation and angular velocity on the number of hops.

GA Search Results. Genetic algorithms have been developed with different sets of operators. The version used here is based on

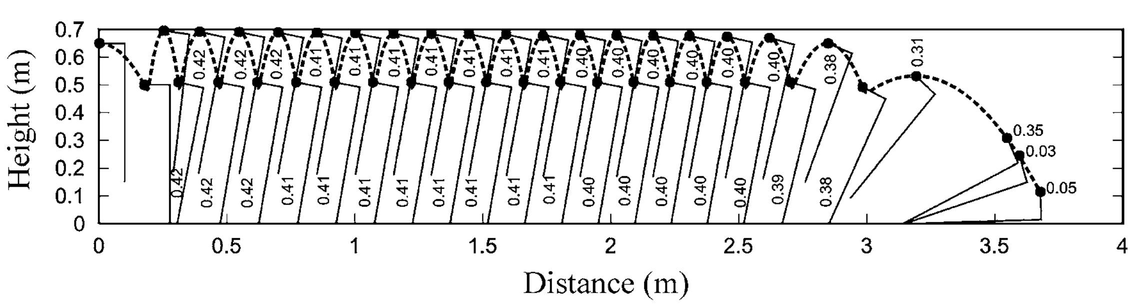

Fig. 4 Snapshots of the SLOM Hopper motion (with release velocity 1.009743 m/s)

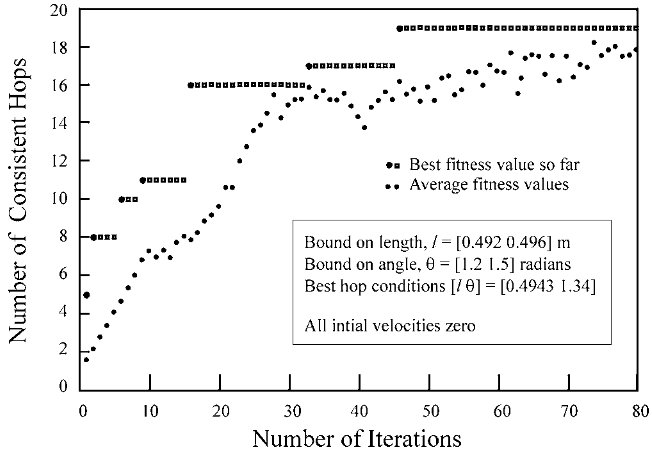

Fig. 5 Progress of GA-based search for suitable initial conditions

Shing and Parker [5].

Two types of crossovers, one "within individual" (crossover 1) and another "between a pair" (crossover 2) are used. Different values of mutation and crossover probabilities have been tried. Mutation probability of 0.05 and the two crossover probabilities of 0.4 and 0.2, respectively, have been found to be effective. Different population sizes ranging from 10 to 500 are tried. Typically convergence is observed within 50 to 200 generations. A population of 100 is found to be effective for most of the ranges of search variables. The state variables to be searched for are encoded in 12 to 16 bits binary strings. The GA-based search worked successfully in smaller as well as larger ranges, although, with a smaller range the convergence rate is found to be higher. The typical progress of a GA search for suitable conditions initiated from stance phase is shown in Fig. 5.

The upper points represent the best fitness value of iterations until that iteration, while a lower point represents the average fitness value until that iteration. In this case, after 50 iterations, no progress is seen in the fitness value.

A genetic algorithm search initiated from flight phase is carried out with the same range for $\dot{x}_0$ and similar initial conditions as used in the heuristic search. The range for $\dot{x}_0$ is $[1,2]$ (m/s) and that for y_0 is $[0.55,0.75]$ m. The initial orientation of the hopper θ_0 is fixed to 90 deg and angular velocity $\dot{\theta}$ to zero. The fraction for comparing the difference between maximum heights attained between the successive hops is chosen 0.05. After carrying out a series of GA based search runs, the maximum number of consistent hops obtained is found to be 13. The optimal value of $\dot{x}_0$ was 1.00974 m/s and that of y_0 was 0.64994 m, which is very close to that found by heuristic search method ($\dot{x}_0$=1.00974 m/s and y_0 =0.65 m). With these values of $\dot{x}_0$ and y_0, a GA search is carried out for other values of θ_0 and $\dot{\theta}_0$. The range chosen for θ_0 is $[1.3,1.8]$ rad and that for $\dot{\theta}_0$ is $[-1,1]$ rad/s. The optimal value obtained for θ_0 is 1.51211 rad and that for $\dot{\theta}_0$ is 0.3931 rad/s. There is no significant change in the number of consistent hops. Additional results are tabulated in Table 1.

Similarly, a sample GA result for the search initiated from the stance phase is discussed below. The range chosen for θ_0 is $[1.2,1.6]$ rad, while the range chosen for l_0 is $[0.492,0.495]$ m. The maximum number of consistent hops observed is 13 and total number of hops is 19. The optimal value of θ_0 is 1.34 rad and that of l_0 is 0.4943 m. Now with the same values of θ_0 and l_0, a GA search is conducted for nonzero $\dot{l}_0$ and $\dot{\theta}_0$. The range chosen $\dot{l}_0$ is $[-1,1]$ m/s and that for $\dot{\theta}_0$ is $[-1,1]$ rad/s. The optimal value of $\dot{\theta}_0$ is found to be 5.8708×10^{-3} rad/s and that of $\dot{l}_0$ 1.3599×10^{-2} m/s. A marginal increase in number of consistent hops is observed while the total number of hops is 23. Additional results are tabulated in Table 2.

Additional searches with nonzero initial angular velocities did not affect the results significantly.

4 Mechanism of Slom Hopping Motion

At the initiation of the stance phase, the foot is arrested by the ground and results in a negative angular velocity. Stance phase, therefore, begins with a relatively high negative angular velocity and the hopper rotates clockwise. The spring force causes a positive moment throughout the stance phase. At take-off, the hopper must acquire a counter-clockwise rotation as during passive flight, the angular velocity remains constant and the leg needs to reorient to the previous landing angle. Thus, the offset spring force tends to reorient the leg, resulting in SLOMPiSH motion.

Table 1 Results of GA searches for hopping initiated from flight phase

Zero $\dot{\theta}_0$						Nonzero $\dot{\theta}_0$	
Bound $\dot{x}_0$ (m/s)	Bound y (m)	Optimal $\dot{x}_0$ (m/s)	Optimal y_0 (m)	Leg angle θ_0 (deg)	Consistent hops	Optimal $\dot{\theta}_0$ (deg/s)	Consistent hops
0–2	0.55–0.85	0.3780	0.7441	78.69	10	0.2536	11
0–2	0.55–0.85	0.3521	0.7868	81.69	12	0.1503	12

Table 2 Summary of results of GA searches for hopping initiated from stance phase

Zero $\dot{\theta}_0, \dot{l}_0$					Nonzero $\dot{\theta}_0, \dot{l}_0$		
Bound l (m)	Bound θ_0 (rad)	Optimal l (m)	Optimal θ_0 (rad)	Consistent hops	Optimal $\dot{\theta}_0$ (rad/s)	Optimal $\dot{l}_0$ (m/s)	Consistent hops
0.46–0.48	0.8–1.5	0.47989	1.15324	14	0.02882	0.08744	14
0.48–0.49	1.0–1.6	0.48212	1.18762	17	−0.19492	−0.43821	17
0.49–0.494	1.0–1.6	0.49345	1.33102	16	−0.38739	−0.69956	13
0.495–0.50	1.0–1.8	0.49565	1.36419	24	0.02882	0.47987	21

5 Conclusions

For the monopedal hopping robot configuration considered in this work, initial conditions resulting in SLOMPiSH motion have been identified using a heuristic search and an evolutionary strategy. While the former worked in a subspace of the initial state-space, the latter could search predefined ranges of the entire initial state-space. Both techniques yield similar results and confirm the mechanism of sustained hopping due to offset mass.

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