

Dynamic Programming

① What is Dynamic Programming?



Enhanced Recursion

$$n = 0! = 1$$

$$1! = 1$$

$$5! = 5 \times 4!$$

fact(n) ↳

① Base case condition

if ($n \leq 1$)

return 1;

}

② Recursive function call

return $n * \underline{\text{fact}(n-1)}$;

}

fact(5)

|
5 * fact(4)

|
4 * fact(3)

|
3 * fact(2)

|
2 * fact(1) → 1

② Why DP??

O(n)

0	1	2	3	4	5	6	7
0	1	1	2	3	5	8	13

$\text{fib}(n) \propto$

① Base case condition

$T(n)$

$c \left\{ \begin{array}{l} \text{if } (n == 0 \text{ || } n == 1) \\ \propto \text{return } n; \\ \end{array} \right.$

② Recursive function call

$$\text{return } \frac{\text{fib}(n-1) + \text{fib}(n-2)}{T(n-1) + T(n-2)}$$

Time complexity \rightarrow

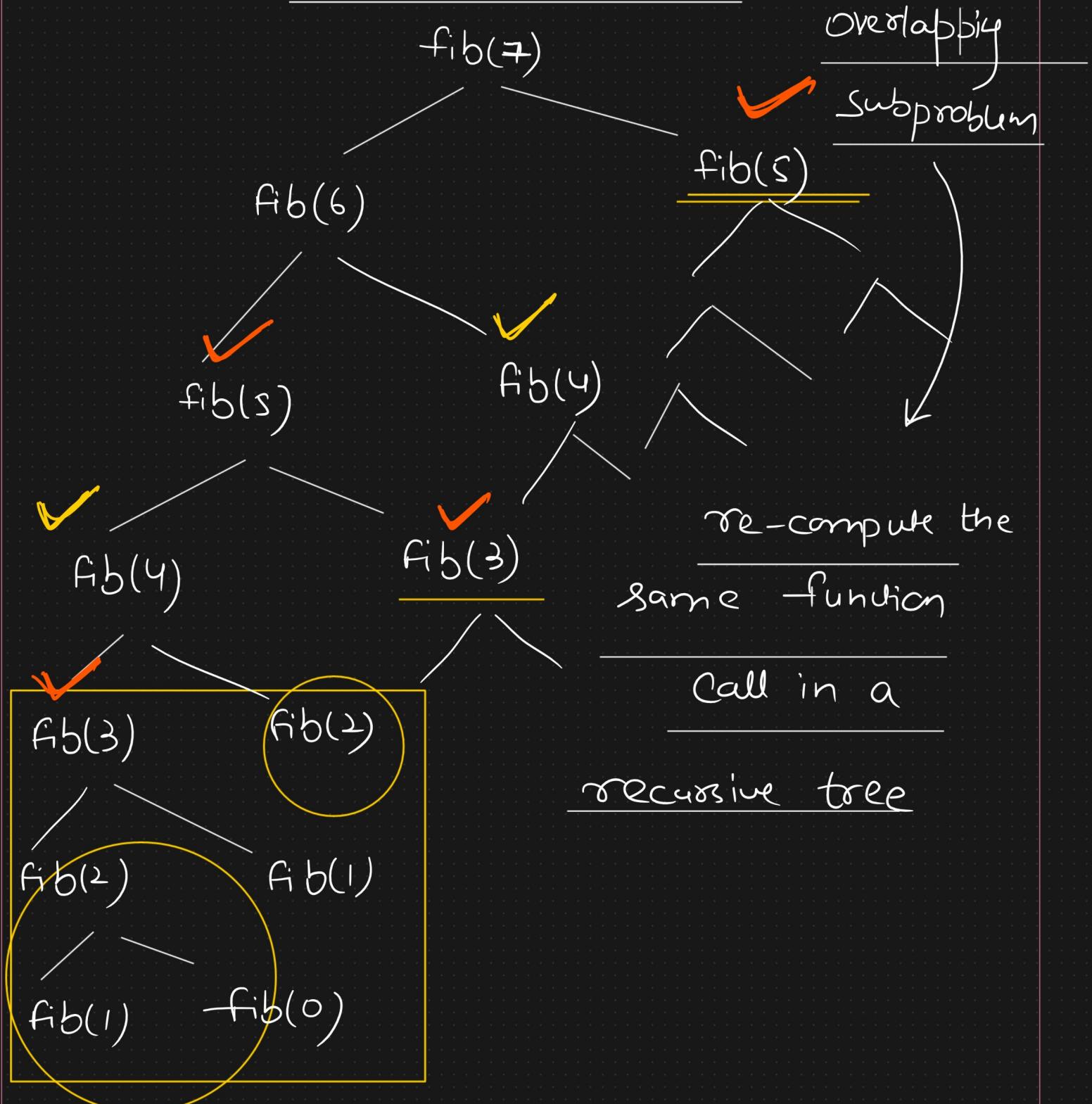
$$T(n) = T(n-1) + T(n-2) + c$$

\hookrightarrow Recursive tree method

$$= \mathcal{O}(2^n)$$

Exponential time complexity

Problem in Recursive code



$$n = 50/100$$

more re-computation

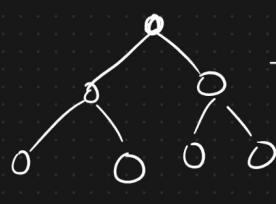
TLE (Time Limit

Exceeded)

Dynamic Programming

How —— DP Problem

①



Overlapping
Subproblems

②

Maximum/Minimum

(Optimal Solution)



DP/Greedy Approach

DP - Approach

① Memoization (Enhanced Recursion)

Hashtable (memo) time complexity

unique function call

$O(n)$



$$n = 7$$



computation

of

only 7

unique

function

call

key	value
$\text{fib}(7)$	13
$\text{fib}(6)$	8
$\text{fib}(5)$	5
$\text{fib}(4)$	3
$\text{fib}(3)$	2
$\text{fib}(2)$	1
$\text{fib}(1)$	1



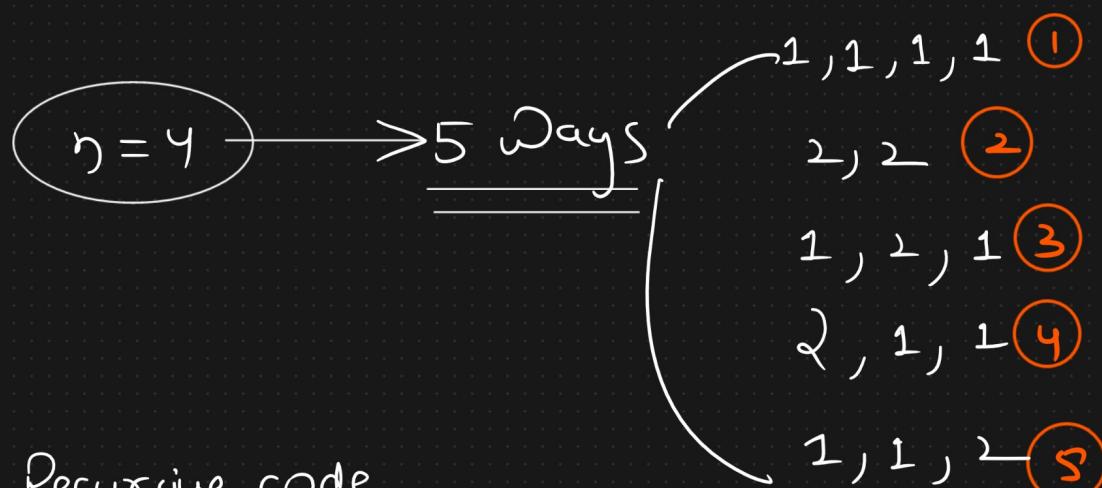
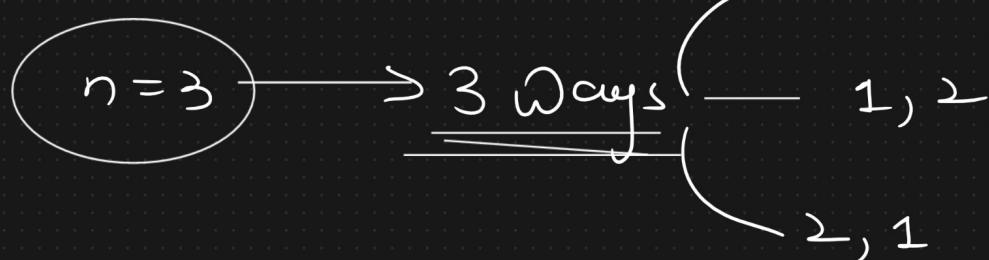
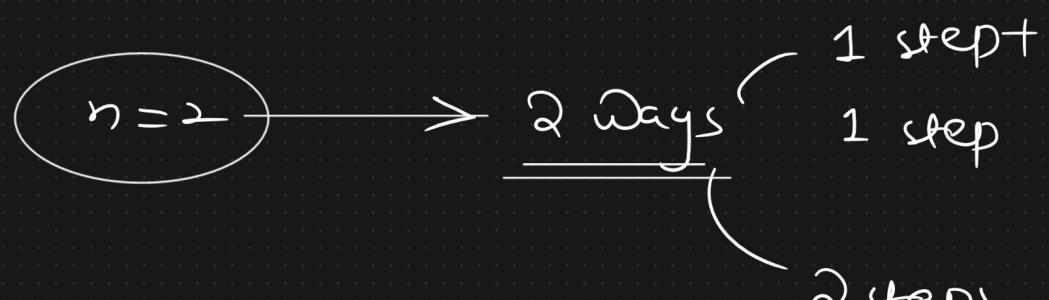
Exponential \rightarrow Linear

time complexity

Space complexity \rightarrow $O(n)$

Climbing Stairs

1 or 2 step



① Recursive code

② Memoization

③ Tabulation

n	1	2	3	4	5	6	7	8	9	10
ways	1	2	3	5	8	13	21	34	55	89

$\uparrow \uparrow$
 $\text{numways}(n)$

① Base case

if $(n == 1 \text{ || } n == 2)$

return n ;

}

else

return $\text{numways}(n-1) +$

$\text{numways}(n-2);$

}

Tabulation

→ No Recursion

$cs()$

\approx

1	2	3	4	5	6	7	8	9
1	2	3	5	8	13	21	34	55

climbStairs(int n) ↗ → $O(n)$

for (i=3 to n) ↗

$$cs(i) = \underline{cs(i-1) + cs(i-2)}$$

}

return cs(n);

}