Assignment 17

Mtech in AI Department
Priya Bhatia
AI20MTECH14015

Abstract

This document illustrates about the properties related to projection and its relevant proof.

Download the latex-tikz codes from

https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment17

1 Problem

Let E be a projection of V and let T be a linear operator on V. Prove that the range of E is invariant under T if and only if $ETE = TE$. Prove that both the range and null space of E are invariant under T if and only if $ET = TE$.

2 Solution

Proof of $ETE = TE$	Any projection E is represented by a matrix that is a part of an identity matrix Assume the Basis can be defined as follows: $B = \{\alpha_1, ..., \alpha_r, ..., \alpha_n\}$ such that $E_{ii} = 1$ for i $\leq$ r and 0 elsewhere $E_B = \begin{pmatrix} \text{I} & 0 \\ 0 & 0 \end{pmatrix}$ where I is the $r \times r$ matrix Let $\alpha = (a_1, ..., a_n)$ $\implies T(E_\alpha) = T(a_1, ..., a_r, ...0) = \beta$ If we assume T to be invariant over the range W of E, then $\beta \in W$ $\beta = (\beta_1, ..., \beta_r, ..., 0)$, $E_\beta = \beta$ Therefore, $ETE = TE$
Proof of $ET = TE$	Consider the same assumption for basis B and projection E as defined above. $B = \{\alpha_1, ..., \alpha_r, ..., \alpha_n\}$ $E_B = \begin{pmatrix} \text{I} & 0 \\ 0 & 0 \end{pmatrix}$ where I is the $r \times r$ matrix Let $TE \neq ET$, then there exists some vector V such that $T(a_1, ..., a_r, ...0) \neq (Ta_1, ..., Ta_r, ..., 0)$ But for this case, T is not an invariant of W. Assuming that T is an invariant of W,$T(a_1, ..., a_r, ...0) \in W$ for all $\alpha \in W$. Therefore, $T(a_1, ..., a_r, ...0) = (Ta_1, ..., Ta_r, ..., 0) \implies ET = TE$

Conclusion	Hence, it is proved that the range of E is invariant under T if and only if $ETE = TE$. And both the range and null space of E are invariant under T if and only if $TE = ET$.

TABLE 1: Illustration of Proof