Assignment 3

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***Abstract*—This document solves a problem based on the congruency of a triangles.**

Download latex-tikz codes from

https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment3

1 Problem

In right triangle ABC, right angled at $\mathbf{C}$, $\mathbf{M}$ is the mid-point of hypotenuse AB.$\mathbf{C}$ is joined to $\mathbf{M}$ and produced to a point $\mathbf{D}$ such that DM = CM. Point $\mathbf{D}$ is joined to point $\mathbf{B}$. Show that:

$$a) \quad \triangle AMC \cong \triangle BMD \tag{1.0.1}$$
$$b) \quad \angle DBC = 90^\circ \tag{1.0.2}$$
$$c) \quad \triangle DBC \cong \triangle ACB \tag{1.0.3}$$
$$d) \quad CM = \frac{1}{2}AB \tag{1.0.4}$$

2 Solution

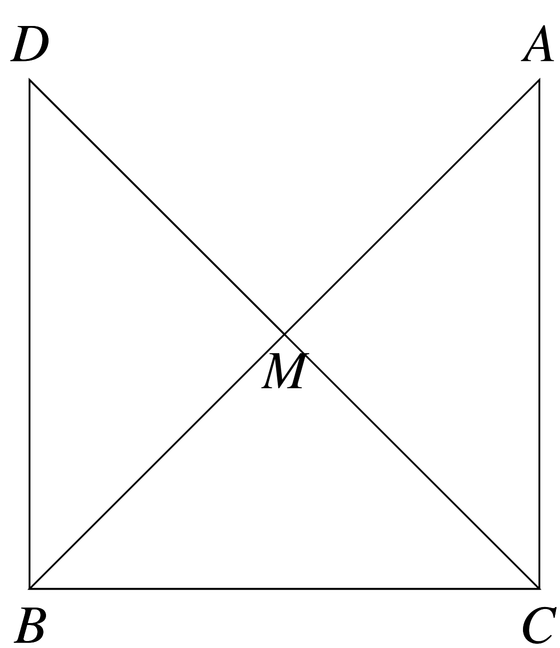

Fig. 1: Triangle ABC and DBC

In $\triangle ABC$, $\mathbf{M}$ is midpoint of hypotenuse AB, thus

$$\mathbf{M} = \frac{\mathbf{A}+\mathbf{B}}{2} \tag{2.0.1}$$
$$2\mathbf{M} = (\mathbf{A}+\mathbf{B}) \tag{2.0.2}$$
$$(\mathbf{A}-\mathbf{M}) = (\mathbf{M}-\mathbf{B}) \tag{2.0.3}$$
$$\|\mathbf{A}-\mathbf{M}\| = \|\mathbf{M}-\mathbf{B}\| \tag{2.0.4}$$
$$\mathbf{M} = \frac{\mathbf{C}+\mathbf{D}}{2} \tag{2.0.5}$$
$$2\mathbf{M} = (\mathbf{C}+\mathbf{D}) \tag{2.0.6}$$
$$(\mathbf{C}-\mathbf{M}) = (\mathbf{M}-\mathbf{D}) \tag{2.0.7}$$
$$\|\mathbf{C}-\mathbf{M}\| = \|\mathbf{M}-\mathbf{D}\| \tag{2.0.8}$$
$$\mathbf{M} = \frac{\mathbf{A}+\mathbf{B}}{2} = \frac{\mathbf{C}+\mathbf{D}}{2} \tag{2.0.9}$$
$$(\mathbf{A}-\mathbf{C}) = (\mathbf{D}-\mathbf{B}) \tag{2.0.10}$$
$$\|\mathbf{A}-\mathbf{C}\| = \|\mathbf{D}-\mathbf{B}\| \tag{2.0.11}$$

Now it is given that AC $\perp$ BC

$$\implies (\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) = 0 \tag{2.0.12}$$
$$(\mathbf{A}-\mathbf{M}+\mathbf{M}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) = 0 \tag{2.0.13}$$
$$(\mathbf{M}-\mathbf{B}+\mathbf{D}-\mathbf{M})^T(\mathbf{B}-\mathbf{C}) = 0 \tag{2.0.14}$$
$$(\mathbf{D}-\mathbf{B})^T(\mathbf{B}-\mathbf{C}) = 0 \tag{2.0.15}$$

This shows that DB $\perp$ BC. Let $\mathbf{m}_{CM}$ and $\mathbf{m}_{AB}$ are direction vectors of CM and AB respectively. Then,

$$\mathbf{A}-\mathbf{B} = \mathbf{A}-\mathbf{C}+\mathbf{C}-\mathbf{B} \tag{2.0.16}$$
$$\mathbf{A}-\mathbf{B} = \mathbf{B}-\mathbf{D}+\mathbf{C}-\mathbf{B} \quad \text{[From (2.0.11)]} \tag{2.0.17}$$
$$\mathbf{A}-\mathbf{B} = \mathbf{C}-\mathbf{D} \tag{2.0.18}$$
$$\mathbf{A}-\mathbf{B} = \mathbf{C}-\mathbf{M}+\mathbf{M}-\mathbf{D} \tag{2.0.19}$$
$$\mathbf{A}-\mathbf{B} = \mathbf{C}-\mathbf{M}+\mathbf{C}-\mathbf{M} \quad \text{[From (2.0.8)]} \tag{2.0.20}$$
$$\mathbf{m}_{AB} = 2\mathbf{m}_{CM} \tag{2.0.21}$$
$$\mathbf{m}_{CM} = \frac{\mathbf{m}_{AB}}{2} \tag{2.0.22}$$
$$\mathbf{C}-\mathbf{M} = \frac{1}{2}(\mathbf{A}-\mathbf{B}) \tag{2.0.23}$$
$$\|\mathbf{C}-\mathbf{M}\| = \frac{1}{2}\|\mathbf{A}-\mathbf{B}\| \tag{2.0.24}$$

Hence from (2.0.24) proved,
$CM = \frac{1}{2}AB$