Assignment 11

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Abstract—**This document demonstrate how to find the dimension of the subspace.**

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https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment11

1 Problem

Let F be a subfield of the complex numbers. We define n linear functionals on $F^n (n \geq 2)$ by

$$f_k(x_1, ..., x_n) = \sum_{j=1}^{n} (k - j)x_j, 1 \leq k \leq n. \quad (1.0.1)$$

What is the dimension of the subspace annihilated by $f_1, f_2, ..., f_n$?

2 Solution

Given	F be a subfield of the complex numbers Definition of n linear functionals on $F^n (n \geq 2)$ by $f_k(x_1, ..., x_n) = \sum_{j=1}^{n} (k - j)x_j$; $1 \leq k \leq n$
To find	The dimension of the subspace annihilated by $f_1, f_2, ..., f_n$
f_k	$f_k(x_1, ..., x_n) = \sum_{j=1}^{n} (k - j)x_j$ $f_k(x_1, ..., x_n) = k \sum_{j=1}^{n} x_j$-$\sum_{j=1}^{n} jx_j$ All f_k are linear combinations of the two linear functionals
Vector	$f_k(x_1, ..., x_n) = \sum_{j=1}^{n} (k - j)x_j$ $\implies f_1(x_1, ..., x_n) = 0.x_1 - x_2 - ... - (n - 1)x_n$ $f_2(x_1, ..., x_n) = x_1 + 0.x_2 - 1.x_3 - ... - (n - 2)x_n$ . . $f_n(x_1, ..., x_n) =$ $(n - 1)x_1 + (n - 2).x_2 + ... + (n - 2)x_{n-1} + 0.x_n$ Dimension of subspace annihilated by $f_i' s$ is the dimension of the solution space of the system $AX = 0$ where the i^{th} row is defined by $A_i = (i - 1, i - 2, ..., i - n)$ $1 \leq i \leq n$
Matrix	$AX = 0$ where the i^{th} row is defined by $A_i = (i - 1, i - 2, ..., i - n)$ $1 \leq i \leq n$ For the $n = 4$, matrix A looks like $\begin{pmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -1 & -2 \\ 2 & 1 & 0 & -1 \\ 3 & 2 & 1 & 0 \end{pmatrix}$ For $i \geq 3$, perform the following elementary operations of n linear functionals $(a)A_i \longrightarrow (1 - i)A_2 + A_i$ $A_i = (0, i - 2, 2(i - 2), 3(i - 2), ..., (n - 1)(i - 2))$ $(b)A_i \longrightarrow \frac{1}{i-2}A_i$ $A_i =$ -A_1 $(c)A_i \longrightarrow A_i + A_1$ $A_i = 0$ Since, A_1 and A_2 are linearly independent Thus, the dimension of the subspace annihiliated $= n - 2$