Assignment 11

Priya Bhatia

***Abstract*—This document demonstrate how to find the dimension of the subspace.**

Download latex-tikz from

https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment11

1 Problem

Let F be a subfield of the complex numbers. We define n linear functionals on $F^n (n \geq 2)$ by

$$f_k(x_1, ..., x_n) = \sum_{j=1}^{n} (k-j)x_j, 1 \leq k \leq n. \tag{1.0.1}$$

What is the dimension of the subspace annihilated by $f_1, f_2, ..., f_n$?

2 Solution

Given	F be a subfield of the complex numbers Definition of n linear functionals on $F^n(n \geq 2)$ by $f_k(x_1, ..., x_n) = \sum_{j=1}^{n}(k-j)x_j; 1 \leq k \leq n$
To find	The dimension of the subspace annihilated by $f_1, f_2, ..., f_n$
f_k	$f_k(x_1, ..., x_n) = \sum_{j=1}^{n}(k-j)x_j$ $f_k(x_1, ..., x_n) = k\sum_{j=1}^{n} x_j$-$\sum_{j=1}^{n} jx_j$ All f_k are linear combinations of the two linear functionals

Vector	$f_k(x_1, ..., x_n) = \sum_{j=1}^{n}(k-j)x_j$ $\mathbf{f} = \begin{pmatrix} f_1 \\ f_2 \\ . \\ . \\ f_n \end{pmatrix}$ $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{pmatrix}$ $f_1(\mathbf{x}) = 0.x_1 - x_2 - 2x_3 - ... - (n-1)x_n$ $f_2(\mathbf{x}) = x_1 + 0.x_2 - 1.x_3 - ... - (n-2)x_n$ . . $f_n(\mathbf{x}) = (n-1)x_1 + (n-2).x_2 + ... + (n-2)x_{n-1} + 0.x_n$ $A_{n*n} = \begin{pmatrix} 0 & -1 & -2 & . & . & -(n-1) \\ 1 & 0 & -1 & . & . & -(n-2) \\ . & . & . & . & . & . \\ (n-1) & (n-2) & . & . & (n-2) & 0 \end{pmatrix}$
Matrix	$AX = 0$ where the i^{th} row is defined by $A_i = (i-1, i-2, ..., i-n)$ $1 \leq i \leq n$ For the $n = 4$, matrix A is: $\begin{pmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -1 & -2 \\ 2 & 1 & 0 & -1 \\ 3 & 2 & 1 & 0 \end{pmatrix}$ For $i \geq 3$, perform the following elementary operations of n linear functionals as defined below $(a)A_i \longrightarrow (1-i)A_2 + A_i$ $A_i = (0, i-2, 2(i-2), 3(i-2), ..., (n-1)(i-2))$ $(b)A_i \longrightarrow \frac{1}{i-2}A_i$ A_i = -A_1 $(c)A_i \longrightarrow A_i + A_1$ $A_i = 0$ Since, A_1 and A_2 are linearly independent Thus, the dimension of the subspace annihiliated $= n-2$