

Challenge Problem

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Abstract—This document show that Orthogonal vectors are linearly independent

and latex-tikz codes from

[https://github.com/priya6971/
matrix_theory_EE5609/tree/master/
ChallengeProblem](https://github.com/priya6971/matrix_theory_EE5609/tree/master/ChallengeProblem)

1 PROBLEM

Show that the set of Orthogonal vectors is Linear independent.

2 PROOF

$$c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 + \cdots + c_n \mathbf{v}_n = 0 \quad (2.0.1)$$

We have to show that in (2.0.1), $c_1 = 0$, $c_2 = 0$ and so on upto $c_n = 0$.

Let \mathbf{v}_1 and \mathbf{v}_2 are orthogonal vectors:

\mathbf{v}_1 and \mathbf{v}_2 are linearly independent if and only if the value of $c_1 = 0$, $c_2 = 0$ in below equation:

$$c_1 \mathbf{v}_1 + c_2 \mathbf{v}_2 = 0 \quad (2.0.2)$$

To prove this, we can take the dot product of \mathbf{v}_1 on both side in \mathbf{v}_1

$$c_1 \mathbf{v}_1 \mathbf{v}_1 + c_2 \mathbf{v}_1 \mathbf{v}_2 = 0 \quad (2.0.3)$$

Now as \mathbf{v}_1 and \mathbf{v}_2 are orthogonal vectors so dot product \mathbf{v}_1 and \mathbf{v}_2 is 0. Therefore we get from (2.0.3)

$$c_1 \mathbf{v}_1 \mathbf{v}_1 = 0 \quad (2.0.4)$$

Now \mathbf{v}_1 cannot be zero as \mathbf{v}_1 is from a set of non-zero orthogonal vectors Therefore we get $c_1 = 0$ from (2.0.4). And similarly we can proof that the value of $c_2 = 0$ by taking dot product of vector \mathbf{v}_2 in equation (2.0.2)

Thus, orthogonal vectors \mathbf{v}_1 and \mathbf{v}_2 satisfy the condition of linear independence.

Similarly, we can proof that if $\mathbf{v}_1, \mathbf{v}_2$ upto \mathbf{v}_n are Orthogonal vectors that forms an equation (2.0.1).

Then, the value of $c_1 = 0$, $c_2 = 0$ and so on upto $c_n = 0$.