Assignment 7

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***Abstract*—This document finds the coordinates of foot of perpendicular using Singular Value Decomposition**

Download python codes from

```
https://github.com/priya6971/
    matrix_theory_EE5609/tree/master/
    Assignment7/codes
```

Download latex-tikz codes from

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https://github.com/priya6971/
    matrix_theory_EE5609/tree/master/
    Assignment7
```

1 Problem

Determine the distance from the Z-axis to the plane 5x - 12y - 8 = 0

2 Solution

Equation of plane can be expressed as

$$\mathbf{n}^T\mathbf{x} = c \tag{2.0.1}$$

Rewriting given equation of plane in (2.0.1) form

$$\begin{pmatrix} 5 & -12 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 8 \tag{2.0.2}$$

where the value of

$$\mathbf{n} = \begin{pmatrix} 5 \\ -12 \\ 0 \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{x} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} \tag{2.0.4}$$

$$c = 8 \tag{2.0.5}$$

We need to represent the equation of plane in parametric form,

$$\mathbf{x} = \mathbf{p} + \lambda_1\mathbf{q} + \lambda_2\mathbf{r} \tag{2.0.6}$$

Here p is any point on plane and $\mathbf{q}, \mathbf{r}$ are two vectors parallel to plane and hence $\perp$ to $\mathbf{n}$. Now, we need to find these two vectors $\mathbf{q}$ and $\mathbf{r}$ which are $\perp$ to $\mathbf{n}$

$$\begin{pmatrix} 5 & -12 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = 0 \implies 5a - 12b = 0 \tag{2.0.7}$$

Put $a = 0$ and $c = 1$ in (2.0.7), $\implies b = 0$
Put $a = 1$ and $c = 0$ in (2.0.7), $\implies b = \frac{5}{12}$
Hence $\mathbf{q} = \begin{pmatrix} 1 \\ \frac{5}{12} \\ 0 \end{pmatrix}, \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$
Let us find point $\mathbf{p}$ on the plane. Put $x = 1, z = 0$ in (2.0.2), we get $\mathbf{p} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$
Since given plane is parallel to Z-axis, we can use any point P on Z-axis to compute shortest distance.

$$\mathbf{P} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.8}$$

Let $\mathbf{Q}$ be the point on plane with shortest distance to $\mathbf{P}$. $\mathbf{Q}$ can be expressed in (2.0.7) form as

$$\mathbf{Q} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \lambda_1\begin{pmatrix} 1 \\ \frac{5}{12} \\ 0 \end{pmatrix} + \lambda_2\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{2.0.9}$$

Computation of Pseudo Inverse using SVD in order to determine the value of λ_1 and λ_2 :