

Assignment 3

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Abstract—This document solves a problem based on the congruency of a triangles.

Download latex-tikz codes from

https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment3

1 PROBLEM

In right triangle ABC, right angled at C, **M** is the mid-point of hypotenuse AB. **C** is joined to **M** and produced to a point **D** such that $DM = CM$. Point **D** is joined to point **B**. Show that:

$$a) \triangle AMC \cong \triangle BMD \quad (1.0.1)$$

$$b) \angle DBC = 90^\circ \quad (1.0.2)$$

$$c) \triangle DBC \cong \triangle ACB \quad (1.0.3)$$

$$d) CM = \frac{1}{2}AB \quad (1.0.4)$$

2 SOLUTION

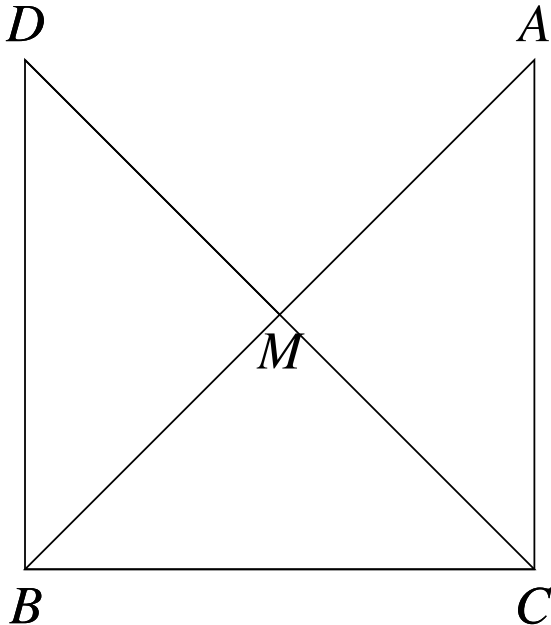


Fig. 1: Triangle ABC and DBC

In $\triangle ABC$, **M** is midpoint of hypotenuse AB, thus

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} \quad (2.0.1)$$

$$2\mathbf{M} = (\mathbf{A} + \mathbf{B}) \quad (2.0.2)$$

$$(\mathbf{A} - \mathbf{M}) = (\mathbf{M} - \mathbf{B}) \quad (2.0.3)$$

$$\|\mathbf{A} - \mathbf{M}\| = \|\mathbf{M} - \mathbf{B}\| \quad (2.0.4)$$

$$\mathbf{M} = \frac{\mathbf{C} + \mathbf{D}}{2} \quad (2.0.5)$$

$$2\mathbf{M} = (\mathbf{C} + \mathbf{D}) \quad (2.0.6)$$

$$(\mathbf{C} - \mathbf{M}) = (\mathbf{M} - \mathbf{D}) \quad (2.0.7)$$

$$\|\mathbf{C} - \mathbf{M}\| = \|\mathbf{M} - \mathbf{D}\| \quad (2.0.8)$$

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} = \frac{\mathbf{C} + \mathbf{D}}{2} \quad (2.0.9)$$

Now it is given that $AC \perp BC$

$$\Rightarrow (\mathbf{A} - \mathbf{C})^T (\mathbf{B} - \mathbf{C}) = 0 \quad (2.0.10)$$

$$(\mathbf{A} - \mathbf{M} + \mathbf{M} - \mathbf{C})^T (\mathbf{B} - \mathbf{C}) = 0 \quad (2.0.11)$$

$$(\mathbf{M} - \mathbf{B} + \mathbf{D} - \mathbf{M})^T (\mathbf{B} - \mathbf{C}) = 0 \quad (2.0.12)$$

$$(\mathbf{D} - \mathbf{B})^T (\mathbf{B} - \mathbf{C}) = 0 \quad (2.0.13)$$

This shows that $DB \perp BC$

Let \mathbf{m}_{CM} and \mathbf{m}_{AB} are direction vectors of CM and AB respectively. Then,

$$\mathbf{m}_{CM} = \mathbf{C} - \mathbf{M} = \mathbf{C} - \frac{\mathbf{C} + \mathbf{D}}{2} \quad [\text{From (2.0.5)}] \quad (2.0.14)$$

$$\mathbf{m}_{CM} = \frac{\mathbf{C} + \mathbf{D}}{2} \quad (2.0.15)$$

$$\mathbf{m}_{CM} = \frac{\mathbf{A} + \mathbf{B}}{2} \quad [\text{From (2.0.9)}] \quad (2.0.16)$$

$$\mathbf{m}_{CM} = \frac{\mathbf{m}_{AB}}{2} \quad (2.0.17)$$

$$\mathbf{C} - \mathbf{M} = \frac{1}{2}(\mathbf{A} - \mathbf{B}) \quad (2.0.18)$$

$$\|\mathbf{C} - \mathbf{M}\| = \frac{1}{2} \|\mathbf{A} - \mathbf{B}\| \quad (2.0.19)$$

Hence from (2.0.19) proved,
 $CM = \frac{1}{2} AB$