Assignment 10

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***Abstract*—This document gives an explicit description of the vectors in R^5 which are linear combination of the vectors.**

Download latex-tikz codes from

https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment10

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1 Problem

Give an explicit description of the type $b_j = \sum_{i=1}^{r} b_{ki} R_{ij}$ for the vectors

$$\beta = (b_1, b_2, b_3, b_4, b_5)$$

in R^5 which are linear combinations of the vectors

$$\alpha_1 = (1, 0, 2, 1, -1), \tag{1.0.1}$$

$$\alpha_2 = (-1, 2, -4, 2, 0), \tag{1.0.2}$$

$$\alpha_3 = (2, -1, 5, 2, 1), \tag{1.0.3}$$

$$\alpha_4 = (2, 1, 3, 5, 2) \tag{1.0.4}$$

2 Solution

Above matrix represented as: $Ax = \beta$

$$\begin{pmatrix} 1 & -1 & 2 & 2 \\ 0 & 2 & -1 & 1 \\ 2 & -4 & 5 & 3 \\ 1 & 2 & 2 & 5 \\ -1 & 0 & 1 & 2 \end{pmatrix} x = \beta \tag{2.0.1}$$

$$\begin{pmatrix} 1 & -1 & 2 & 2 \\ 0 & 2 & -1 & 1 \\ 2 & -4 & 5 & 3 \\ 1 & 2 & 2 & 5 \\ -1 & 0 & 1 & 2 \end{pmatrix} x = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{pmatrix} \tag{2.0.2}$$

Now finding row space of A is equivalent to finding column space of A^T. So, here we do elementary matrix multiplication of the A^T matrix whose rows are given by $\alpha_i' s$:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ -1 & 2 & -4 & 2 & 0 \\ 2 & -1 & 5 & 2 & 1 \\ 2 & 1 & 3 & 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ 0 & 2 & -2 & 3 & -1 \\ 2 & -1 & 5 & 2 & 1 \\ 2 & 1 & 3 & 5 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ -1 & 2 & -4 & 2 & 0 \\ 2 & -1 & 5 & 2 & 1 \\ 2 & 1 & 3 & 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ 0 & 2 & -2 & 3 & -1 \\ 0 & -1 & 1 & 0 & 3 \\ 2 & 1 & 3 & 5 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ -2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ -1 & 2 & -4 & 2 & 0 \\ 2 & -1 & 5 & 2 & 1 \\ 2 & 1 & 3 & 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ 0 & 2 & -2 & 3 & -1 \\ 0 & -1 & 1 & 0 & 3 \\ 0 & 1 & -1 & 3 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ -4 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ -1 & 2 & -4 & 2 & 0 \\ 2 & -1 & 5 & 2 & 1 \\ 2 & 1 & 3 & 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ 0 & 2 & -2 & 3 & -1 \\ 0 & -1 & 1 & 0 & 3 \\ 0 & 0 & 0 & 3 & 7 \end{pmatrix}$$

Now final row reduced echelon form of above matrix is:

$$\begin{pmatrix} 0.67 & -1.67 & -2 & 1.33 \\ 0.5 & -1.5 & -2.5 & 1.5 \\ -0.167 & 1.17 & 1.5 & -0.83 \\ -0.5 & -0.5 & -0.5 & 0.5 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ -1 & 2 & -4 & 2 & 0 \\ 2 & -1 & 5 & 2 & 1 \\ 2 & 1 & 3 & 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.0.3}$$

Now if we check for linear independence of A^T matrix, we can do so by the following method:

$$a_1 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + a_2 \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} + a_3 \begin{pmatrix} 2 \\ -1 \\ 0 \\ 0 \end{pmatrix} + a_4 \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} + a_5 \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.4}$$

Now from (2.0.4), we can find the value of a_1, a_2, a_3, a_4, a_5 and if all the scalar values came out to be 0, then above matrix is linearly independent otherwise linearly dependent:

$$a_1 + 2a_3 = 0 \tag{2.0.5}$$

$$a_2 - a_3 = 0 \tag{2.0.6}$$

$$a_4 = 0 \tag{2.0.7}$$

$$a_5 = 0 \tag{2.0.8}$$

Using above equations, we can find out that the value of a_1, a_2, a_3, a_4, a_5 is equal to 0. Thus, this indicates that column vectors of matrix A are linearly independent. Thus, the required solution of above problem is matrix A as mentioned in (2.0.2).