Assignment 3

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***Abstract*—This document solves a problem based on the congruency of a triangles.**

Download latex-tikz codes from

https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment3

1 Problem

In right triangle ABC, right angled at **C**, **M** is the mid-point of hypotenuse AB.**C** is joined to **M** and produced to a point **D** such that DM = CM. Point **D** is joined to point **B**. Show that:

$$a) \quad \triangle AMC \cong \triangle BMD \tag{1.0.1}$$

$$b) \quad \angle DBC = 90^\circ \tag{1.0.2}$$

$$c) \quad \triangle DBC \cong \triangle ACB \tag{1.0.3}$$

$$d) \quad CM = \frac{1}{2}AB \tag{1.0.4}$$

2 Solution

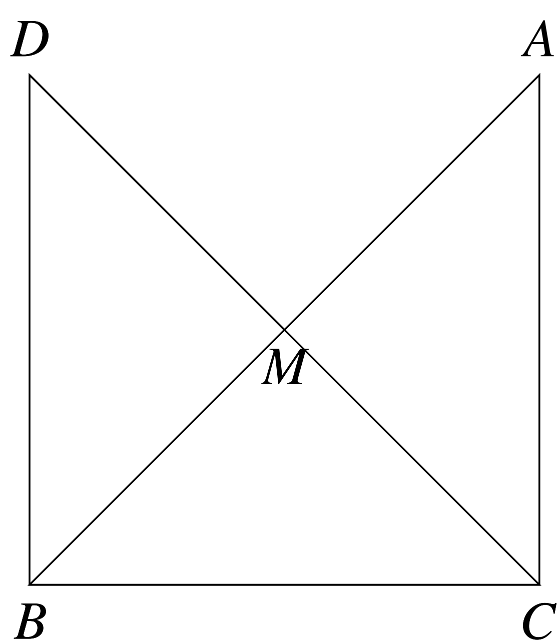

Fig. 1: Triangle ABC and DBC

In $\triangle ABC$, **M** is midpoint of hypotenuse AB, thus

$$\mathbf{M} = \frac{\mathbf{A}+\mathbf{B}}{2} \tag{2.0.1}$$

$$2\mathbf{M} = (\mathbf{A}+\mathbf{B}) \tag{2.0.2}$$

$$(\mathbf{A}-\mathbf{M}) = (\mathbf{M}-\mathbf{B}) \tag{2.0.3}$$

$$||\mathbf{A}-\mathbf{M}|| = ||\mathbf{M}-\mathbf{B}|| \tag{2.0.4}$$

$$\mathbf{M} = \frac{\mathbf{C}+\mathbf{D}}{2} \tag{2.0.5}$$

$$2\mathbf{M} = (\mathbf{C}+\mathbf{D}) \tag{2.0.6}$$

$$(\mathbf{C}-\mathbf{M}) = (\mathbf{M}-\mathbf{D}) \tag{2.0.7}$$

$$||\mathbf{C}-\mathbf{M}|| = ||\mathbf{M}-\mathbf{D}|| \tag{2.0.8}$$

$$\mathbf{M} = \frac{\mathbf{A}+\mathbf{B}}{2} = \frac{\mathbf{C}+\mathbf{D}}{2} \tag{2.0.9}$$

$$(\mathbf{A}-\mathbf{C}) = (\mathbf{D}-\mathbf{B}) \tag{2.0.10}$$

$$||\mathbf{A}-\mathbf{C}|| = ||\mathbf{D}-\mathbf{B}|| \tag{2.0.11}$$

Now it is given that AC $\perp$ BC

$$\implies (\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) = 0 \tag{2.0.12}$$

$$(\mathbf{A}-\mathbf{M}+\mathbf{M}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) = 0 \tag{2.0.13}$$

$$(\mathbf{M}-\mathbf{B}+\mathbf{D}-\mathbf{M})^T(\mathbf{B}-\mathbf{C}) = 0 \tag{2.0.14}$$

$$(\mathbf{D}-\mathbf{B})^T(\mathbf{B}-\mathbf{C}) = 0 \tag{2.0.15}$$

This shows that DB $\perp$ BC.

$$\mathbf{A}-\mathbf{B} = \mathbf{A}-\mathbf{C}+\mathbf{C}-\mathbf{B} \tag{2.0.16}$$

$$\mathbf{A}-\mathbf{B} = \mathbf{B}-\mathbf{D}+\mathbf{C}-\mathbf{B} \quad \text{[From (2.0.11)]} \tag{2.0.17}$$

$$\mathbf{A}-\mathbf{B} = \mathbf{C}-\mathbf{D} \tag{2.0.18}$$

$$\mathbf{A}-\mathbf{B} = \mathbf{C}-\mathbf{M}+\mathbf{M}-\mathbf{D} \tag{2.0.19}$$

$$\mathbf{A}-\mathbf{B} = \mathbf{C}-\mathbf{M}+\mathbf{C}-\mathbf{M} \quad \text{[From (2.0.8)]} \tag{2.0.20}$$

$$\mathbf{A}-\mathbf{B} = 2(\mathbf{C}-\mathbf{M}) \tag{2.0.21}$$

$$\mathbf{C}-\mathbf{M} = \frac{1}{2}(\mathbf{A}-\mathbf{B}) \tag{2.0.22}$$

$$||\mathbf{C}-\mathbf{M}|| = \frac{1}{2}\,||\mathbf{A}-\mathbf{B}|| \tag{2.0.23}$$

Hence from (2.0.23) proved,
CM = $\frac{1}{2}$ AB