Assignment 14

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***Abstract*—This document demonstrates the proof of linearity and approach to find the minimal polynomial for P, where P is the operator on R^2.**

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https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment14

1 Problem

Let P be the operator on R^2 which projects each vector onto the x-axis, parallel to the y-axis: $P(x, y)$=$(x, 0)$. Show that P is linear. What is the minimal polynomial for P?

2 Solution

Given	P be the operator on R^2, $P : R^2 \to R^2$ which projects each vector onto the x-axis, parallel to the y-axis: $P(x, y)$=$(x, 0)$
To Prove	P is linear For P to be linear, it should satisfy the properties mentioned in the Given section Consider two vectors (x_1, y_1) and (x_2, y_2)
To find	Minimal Polynomial for P
Matrix	For the projection $P(x, y) = (x, 0)$, the matrix of linear transform is, $P(x, y) = \mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = (x, 0)$ So, $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

Proof	Consider two vectors (x_1, y_1) and (x_2, y_2), then $P((x_1, y_1) + (x_2, y_2)) = P(x_1 + x_2, y_1 + y_2)$ $P((x_1, y_1) + (x_2, y_2)) = P(x_1 + x_2, 0)$ $P((x_1, y_1) + (x_2, y_2)) = P((x_1, 0), (x_2, 0))$ Thus, P satisfied the property of linearity. Now, consider some scalar k, then $P(k(x_1, y_1)) = P((kx_1, ky_1))$ $P(k(x_1, y_1)) = P((kx_1, 0))$ $P(k(x_1, y_1)) = kP(x_1, 0)$ Thus, using above observations we can conclude that P is linear.
Minimal Polynomial	$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ Characteristic Polynomial of A is $\det(xI - A) = \begin{vmatrix} (x-1) & 0 \\ 0 & (x-0) \end{vmatrix}$ $\det(xI - A) = x(x-1)$ Characteristic polynomial is a product of distinct linear terms and we know that every root of characteristic polynomial should be the root of minimal polynomial thus, it must be equal to the minimal polynomial. Thus, minimal polynomial is equal to $p(x) = x(x-1)$

TABLE 0: Illustration of Proof and finding of minimal polynomial