Assignment 3

Priya Bhatia

***Abstract*—This document solves a problem based on the congruency of a triangles.**

Download latex-tikz codes from

https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment3

1 Problem

In right triangle ABC, right angled at $\mathbf{C}$, $\mathbf{M}$ is the mid-point of hypotenuse AB.$\mathbf{C}$ is joined to $\mathbf{M}$ and produced to a point $\mathbf{D}$ such that DM = CM. Point $\mathbf{D}$ is joined to point $\mathbf{B}$. Show that:

$$a) \quad \triangle AMC \cong \triangle BMD \tag{1.0.1}$$

$$b) \quad \angle DBC = 90^\circ \tag{1.0.2}$$

$$c) \quad \triangle DBC \cong \triangle ACB \tag{1.0.3}$$

$$d) \quad CM = \frac{1}{2}AB \tag{1.0.4}$$

2 Solution

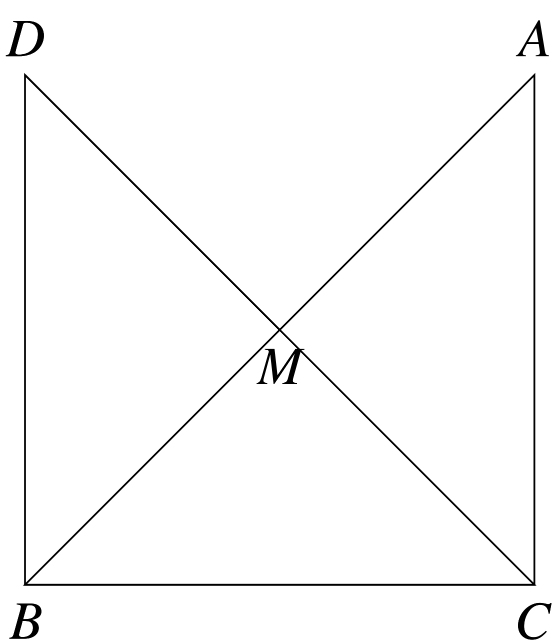

Fig. 1: Triangle ABC and DBC

In $\triangle ABC$, $\mathbf{M}$ is midpoint of hypotenuse AB, thus

$$\mathbf{M} = \frac{\mathbf{A}+\mathbf{B}}{2} \tag{2.0.1}$$

$$2\mathbf{M} = (\mathbf{A}+\mathbf{B}) \tag{2.0.2}$$

$$(\mathbf{A}-\mathbf{M}) = (\mathbf{M}-\mathbf{B}) \tag{2.0.3}$$

$$||\mathbf{A}-\mathbf{M}|| = ||\mathbf{M}-\mathbf{B}|| \tag{2.0.4}$$

$$\mathbf{M} = \frac{\mathbf{C}+\mathbf{D}}{2} \tag{2.0.5}$$

$$2\mathbf{M} = (\mathbf{C}+\mathbf{D}) \tag{2.0.6}$$

$$(\mathbf{C}-\mathbf{M}) = (\mathbf{M}-\mathbf{D}) \tag{2.0.7}$$

$$||\mathbf{C}-\mathbf{M}|| = ||\mathbf{M}-\mathbf{D}|| \tag{2.0.8}$$

$$\mathbf{M} = \frac{\mathbf{A}+\mathbf{B}}{2} = \frac{\mathbf{C}+\mathbf{D}}{2} \tag{2.0.9}$$

$$\mathbf{A}-\mathbf{C} = \mathbf{A}-\mathbf{M}+\mathbf{M}-\mathbf{C} \tag{2.0.10}$$

$$\mathbf{A}-\mathbf{C} = \mathbf{M}-\mathbf{B}+\mathbf{D}-\mathbf{M} \tag{2.0.11}$$

$$(\mathbf{A}-\mathbf{C}) = k(\mathbf{D}-\mathbf{B}) \quad \text{[k value is 1]} \tag{2.0.12}$$

Now from equation (2.0.12) we can say that

$$AC \parallel DB \tag{2.0.13}$$

$$||\mathbf{A}-\mathbf{C}|| = ||\mathbf{D}-\mathbf{B}|| \tag{2.0.14}$$

Now it is given that AC $\perp$ BC, using this we can prove that DB $\perp$ BC.

$$(\mathbf{A}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) = 0 \tag{2.0.15}$$

$$(\mathbf{A}-\mathbf{M}+\mathbf{M}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) = 0 \tag{2.0.16}$$

$$(\mathbf{M}-\mathbf{B}+\mathbf{D}-\mathbf{M})^T(\mathbf{B}-\mathbf{C}) = 0 \tag{2.0.17}$$

$$(\mathbf{D}-\mathbf{B})^T(\mathbf{B}-\mathbf{C}) = 0 \tag{2.0.18}$$

$$\implies DB \perp BC \tag{2.0.19}$$

$$\mathbf{A}-\mathbf{B} = \mathbf{A}-\mathbf{C}+\mathbf{C}-\mathbf{B} \tag{2.0.20}$$

$$\mathbf{A}-\mathbf{B} = \mathbf{B}-\mathbf{D}+\mathbf{C}-\mathbf{B} \quad \text{[From (2.0.14)]} \tag{2.0.21}$$

$$\mathbf{A}-\mathbf{B} = \mathbf{C}-\mathbf{D} \tag{2.0.22}$$

$$\mathbf{A}-\mathbf{B} = \mathbf{C}-\mathbf{M}+\mathbf{M}-\mathbf{D} \tag{2.0.23}$$

$$\mathbf{A}-\mathbf{B} = \mathbf{C}-\mathbf{M}+\mathbf{C}-\mathbf{M} \quad \text{[From (2.0.8)]} \tag{2.0.24}$$

$$\mathbf{A} - \mathbf{B} = 2(\mathbf{C} - \mathbf{M}) \tag{2.0.25}$$

$$\mathbf{C} - \mathbf{M} = \frac{1}{2}(\mathbf{A} - \mathbf{B}) \tag{2.0.26}$$

$$\|\mathbf{C} - \mathbf{M}\| = \frac{1}{2}\|\mathbf{A} - \mathbf{B}\| \tag{2.0.27}$$

Hence from (2.0.27) proved,
CM $= \frac{1}{2}$ AB