Assignment 11

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***Abstract*—This document demonstrate how to find the dimension of the subspace.**

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https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment11

1 Problem

Let F be a subfield of the complex numbers. We define n linear functionals on $F^n(n \geq 2)$ by

$$f_k(x_1, ..., x_n) = \sum_{j=1}^{n}(k-j)x_j, 1 \leq k \leq n. \tag{1.0.1}$$

What is the dimension of the subspace annihilated by $f_1, f_2, ..., f_n$?

2 Solution

Given	F be a subfield of the complex numbers Definition of n linear functionals on $F^n(n \geq 2)$ by $f_k(x_1, ..., x_n)$
To prove	ϵ_i is standard basis vector $\epsilon_i \notin \bigcup_{j=1}^{n-1} N_{f_i}$ $\dim \bigcap_{j=1}^{i} N_{f_i} = n - i$
Basis vector	Let ϵ be the standard basis vector N_{f_k} is the subspace annihilated by f_k
Proof	The dimension of N_{f_k} is given by $\dim N_{f_k}$ = n-1 The standard basis vector ϵ_2 is in N_{f_2} but is not in N_{f_1} $\implies N_{f_1}, N_{f_2}$ are distinct hyperspaces Thus, the intersection of N_{f_1} and N_{f_2} dimension is given by $\dim N_{f_1} \cap N_{f_2}$ = n-2 The standard basis vector ϵ_3 is in N_{f_3} but is not in N_{f_1}, N_{f_2} $\implies N_{f_1}, N_{f_2}, N_{f_3}$ are distinct hyperspaces Thus, the intersection of $N_{f_1}, N_{f_2}, N_{f_3}$ dimension is given by $\dim N_{f_1} \cap N_{f_2} \cap N_{f_3}$ = n-3 Hence using above results, it can be concluded that $\epsilon_i \notin \bigcup_{j=1}^{n-1} N_{f_i}$ $\dim \bigcap_{j=1}^{i} N_{f_i} = n - i$ Hence, proved
Find	The dimension of the subspace annihilated by $f_1, f_2, ..., f_n$ Here, the value of $i = n$, using above proof we can easily determine the dimension $\dim \bigcap_{j=1}^{n} N_{f_i} = n - n = 0$ Hence, according to the definition of n linear functionals on $F^n(n \geq 2)$ the dimension of the subspace annihiliated by $f_1, f_2, ..., f_n = 0$