Assignment 15

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Abstract

This document show that matrix A such that $A^2 = A$ is similar to a diagonal matrix.

Download the latex-tikz codes from

https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment15

1 Problem

Every matrix A such that $A^2 = A$ is similar to a diagonal matrix.

2 Definitions

Characteristic Polynomial	For an $n \times n$ matrix $\mathbf{A}$, characteristic polynomial is defined by, $$p(x) = \left|x\mathbf{I} - \mathbf{A}\right|$$
Cayley-Hamilton Theorem	If $p(x)$ is the characteristic polynomial of an $n \times n$ matrix $\mathbf{A}$, then, $$p(\mathbf{A}) = \mathbf{0}$$
Minimal Polynomial	Minimal polynomial $m(x)$ is the smallest factor of characteristic polynomial $p(x)$ such that, $$m(\mathbf{A}) = 0$$ Every root of characteristic polynomial should be the root of minimal polynomial
Theorem	Let $\mathbf{V}$ be a finite-dimensional vector space over the field $\mathbf{F}$ and let $\mathbf{T}$ be a linear operator on $\mathbf{V}$ Then $\mathbf{T}$ is diagonalizable if and only if the minimal polynomial for $\mathbf{T}$ has the form p = $(x - c_1)...(x - c_k)$,where $c_1, c_2, ..., c_k$ are distinct elements of F.

TABLE 1: Definitions

3 Solution

Proof	$A^2 = A$ (Given) $\implies A$ satisfies the polynomial $x^2 - x = x(x-1)$
Minimal Polynomial	$p(x) = x^a(x-1)^b, \quad a \leq 1, b \leq 1$ Minimal Polynomial $m_A(x)$, of A divides $x^2 - x$, that is $m_A(x)=x$ or $m_A(x)=x-1$ or $m_A(x)=x(x-1)$ If $m_A(x)=x$, then $A = 0$. If $m_A(x)=x-1$, then $A = I$. If $m_A(x)=x(x-1)$ In all above three cases, the minimal polynomial factors into distinct linears. So, it follows A is diagonisable.
Conclusion	In all three cases, the minimal polynomial splits into distinct linear factors, so it follows that **A** is diagonalisable according to the Theorem mentioned in definitions section.

TABLE 2: Illustration of Proof