Assignment 5

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Abstract—This document finds the normal at the given point on the curve

1 PROBLEM

Find the normal at the point $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ on the curve $2y + x^2 = 3$

2 SOLUTION

Given,

$$x^2 + 2y - 3 = 0 \qquad (2.0.1)$$

From (2.0.1),

$$\mathbf{V} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad (2.0.2)$$

$$\mathbf{u} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (2.0.3)$$

$$f = -3 \qquad (2.0.4)$$

From (2.0.2),

$$|V| = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0 \qquad (2.0.5)$$

Now (2.0.5) implies that the curve is a parabola. We can find the Eigen values corresponding to the $\mathbf{V}$,

$$|V - \lambda I| = 0$$

$$\begin{vmatrix} 1 - \lambda & 0 \\ 0 & -\lambda \end{vmatrix} = 0$$

$$\implies \lambda = 0, 1 \qquad (2.0.6)$$

Calculating the Eigen Vectors corresponding to $\lambda = 0, 1$ respectively,

$$\mathbf{Vx} = \lambda \mathbf{x}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} = 0; \implies \mathbf{p}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (2.0.7)$$

$$\begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \mathbf{x} = 0; \implies \mathbf{p}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad (2.0.8)$$

By Eigen decomposition on $\mathbf{V}$,

$$\mathbf{V} = \mathbf{PDP}^T$$

$$where, \mathbf{P} = \begin{pmatrix} \mathbf{p}_1 & \mathbf{p}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad (2.0.9)$$

$$\mathbf{D} = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad (2.0.10)$$

To find the vertex of the parabola,

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p}_1^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p}_1 - \mathbf{u} \end{pmatrix} \qquad (2.0.11)$$

$$where, \eta = \mathbf{u}^T \mathbf{p}_1 = 1 \qquad (2.0.12)$$

Substituting values from (2.0.2), (2.0.7) and (2.0.12) in (2.0.11),

$$\begin{pmatrix} 0 & 2 \\ 1 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix} \qquad (2.0.13)$$

Removing last row and representing (2.0.13) as augmented matrix and then converting the matrix to echelon form,

$$\begin{pmatrix} 0 & 2 & 3 \\ 1 & 0 & 0 \end{pmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \end{pmatrix} \xrightarrow{R_2 \leftarrow \frac{R_2}{2}} \qquad (2.0.14)$$

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{3}{2} \end{pmatrix} \qquad (2.0.15)$$

From (2.0.15) it can be observed that,

$$\mathbf{c} = \begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix} \qquad (2.0.16)$$