

Assignment 4

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Abstract—This document explains QR decomposition with an Example.

Download python codes from

[https://github.com/priya6971/
matrix_theory_EE5609/tree/master/
Assignment6/codes](https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment6/codes)

Download latex-tikz codes from

[https://github.com/priya6971/
matrix_theory_EE5609/tree/master/
Assignment6](https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment6)

1 PROBLEM

Find the QR decomposition of

$$\mathbf{A} = \begin{pmatrix} 7 & 3 \\ 2 & 4 \end{pmatrix} \quad (1.0.1)$$

2 SOLUTION

Let \mathbf{x} and \mathbf{y} be the column vectors of the given matrix.

$$\mathbf{x} = \begin{pmatrix} 7 \\ 2 \end{pmatrix} \quad (2.0.1)$$

$$\mathbf{y} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad (2.0.2)$$

The column vectors can be expressed as follows,

$$\mathbf{x} = k_1 \mathbf{u}_1 \quad (2.0.3)$$

$$\mathbf{y} = r_1 \mathbf{u}_1 + k_2 \mathbf{u}_2 \quad (2.0.4)$$

Here,

$$k_1 = \|\mathbf{x}\| \quad (2.0.5)$$

$$\mathbf{u}_1 = \frac{\mathbf{x}}{k_1} \quad (2.0.6)$$

$$r_1 = \frac{\mathbf{u}_1^T \mathbf{y}}{\|\mathbf{u}_1\|^2} \quad (2.0.7)$$

$$\mathbf{u}_2 = \frac{\mathbf{y} - r_1 \mathbf{u}_1}{\|\mathbf{y} - r_1 \mathbf{u}_1\|} \quad (2.0.8)$$

$$k_2 = \mathbf{u}_2^T \mathbf{y} \quad (2.0.9)$$

The (2.0.3) and (2.0.4) can be written as,

$$\begin{pmatrix} \mathbf{x} & \mathbf{y} \end{pmatrix} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{pmatrix} \begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \quad (2.0.10)$$

$$\begin{pmatrix} \mathbf{x} & \mathbf{y} \end{pmatrix} = \mathbf{QR} \quad (2.0.11)$$

Now, \mathbf{R} is an upper triangular matrix and also,

$$\mathbf{Q}^T \mathbf{Q} = \mathbf{I} \quad (2.0.12)$$

Now using equations (2.0.5) to (2.0.9) we get,

$$k_1 = \sqrt{7^2 + 2^2} = \sqrt{53} \quad (2.0.13)$$

$$\mathbf{u}_1 = \frac{1}{\sqrt{53}} \begin{pmatrix} 7 \\ 2 \end{pmatrix} \quad (2.0.14)$$

$$\mathbf{u}_1 = \begin{pmatrix} \frac{7}{\sqrt{53}} \\ \frac{2}{\sqrt{53}} \end{pmatrix} \quad (2.0.15)$$

$$r_1 = \begin{pmatrix} \frac{7}{\sqrt{53}} & \frac{2}{\sqrt{53}} \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \frac{29}{\sqrt{53}} \quad (2.0.16)$$

$$\mathbf{u}_2 = \frac{1}{\sqrt{53}} \begin{pmatrix} 2 \\ -7 \end{pmatrix} \quad (2.0.17)$$

$$\mathbf{u}_2 = \begin{pmatrix} \frac{2}{\sqrt{53}} \\ -\frac{7}{\sqrt{53}} \end{pmatrix} \quad (2.0.18)$$

$$k_2 = \begin{pmatrix} \frac{2}{\sqrt{53}} & -\frac{7}{\sqrt{53}} \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = -\frac{22}{\sqrt{53}} \quad (2.0.19)$$

Thus putting the values from (2.0.13) to (2.0.19) in (2.0.11) we obtain QR decomposition,

$$\begin{pmatrix} 7 & 3 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} \frac{7}{\sqrt{53}} & \frac{2}{\sqrt{53}} \\ \frac{2}{\sqrt{53}} & -\frac{7}{\sqrt{53}} \end{pmatrix} \begin{pmatrix} \sqrt{53} & \frac{29}{\sqrt{53}} \\ 0 & -\frac{22}{\sqrt{53}} \end{pmatrix} \quad (2.0.20)$$

Which can also be written as,

$$\begin{pmatrix} 7 & 3 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} -\frac{7}{\sqrt{53}} & -\frac{2}{\sqrt{53}} \\ -\frac{2}{\sqrt{53}} & \frac{7}{\sqrt{53}} \end{pmatrix} \begin{pmatrix} -\sqrt{53} & -\frac{29}{\sqrt{53}} \\ 0 & \frac{22}{\sqrt{53}} \end{pmatrix} \quad (2.0.21)$$