Assignment 13

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***Abstract*—This document finds the characteristic value and for each characteristic value find its basis.**

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https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment13

1 Problem

In each of the following cases, let T be the linear operator on R^2 which is represented by matrix A in the standard ordered basis for R^2, and let U be the linear operator on C^2 represented by A in the standard ordered basis. Find the characteristic polynomial for T and that for U, find the characteristic value of each operator, and for each characteristic value c find a basis for the corresponding space of characteristic vectors.

$$\mathbf{A_1} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{1.0.1}$$

$$\mathbf{A_2} = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix} \tag{1.0.2}$$

$$\mathbf{A_3} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \tag{1.0.3}$$

2 Solution

Given	T be the linear operator on R^2 which is represented by matrix A in the standard ordered basis for R^2 U be the linear operator on C^2 which is represented by matrix A in the standard ordered basis In all cases, denoting B_c the basis for the subspace corresponding to characteristic value c
To find	Characteristic value of each operator For each characteristic value c find a basis for the corresponding space of characteristic vectors

Matrix	Characteristic Polynomial	Basis
$\mathbf{A_1} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	$\det(xI - A)= \begin{vmatrix} (x-1) & 0 \\ 0 & x \end{vmatrix}$ Characteristic Polynomial $= x(x-1)$ To find characteristic values of the operator $\det(xI - A) = 0$,which gives $c_1 = 0$ and $c_2 = 1$ Both c_1 and c_2 are the characteristic values Assume B_1 and B_2 are Basis for c_1 and c_2	Basis for characteristic value $c_1 = 0$ will be obtained by solving homogeneous equation,$(A - c_1 I)x = 0$ After solving, basis for characteristic value c_1 is $B_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ Similarly,we can find out the Basis for $c_2 = 1$ which is $B_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$
$\mathbf{A_2} = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$	$\det(xI - A)= \begin{vmatrix} (x-2) & -3 \\ 1 & (x-1) \end{vmatrix}$ Characteristic Polynomial $= x^2 - 3x + 5$ To find characteristic values of the operator $\det(xI - A) = 0$,which gives $c_1 = \frac{3+i\sqrt{11}}{2}$ and $c_2 = \frac{3-i\sqrt{11}}{2}$ Both c_1 and c_2 are the characteristic values Assume B_1 and B_2 are Basis for c_1 and c_2	Basis for characteristic value $c_1 = \frac{3+i\sqrt{11}}{2}$ will be obtained by solving homogeneous equation,$(A - c_1 I)x = 0$ After solving, basis for characteristic value c_1 is $B_1 = \begin{pmatrix} \frac{1+i\sqrt{11}}{2} \\ -1 \end{pmatrix}$ Similarly,we can find out the Basis for c_2 which is $B_2 = \begin{pmatrix} \frac{1-i\sqrt{11}}{2} \\ -1 \end{pmatrix}$
$\mathbf{A_3} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$\det(xI - A)= \begin{vmatrix} (x-1) & -1 \\ -1 & (x-1) \end{vmatrix}$ Characteristic Polynomial $= x(x-2)$ To find characteristic values of the operator $\det(xI - A) = 0$,which gives $c_1 = 0$ and $c_2 = 2$ Both c_1 and c_2 are the characteristic values Assume B_1 and B_2 are Basis for c_1 and c_2	Basis for characteristic value $c_1 = 0$ will be obtained by solving homogeneous equation,$(A - c_1 I)x = 0$ After solving, basis for characteristic value c_1 is $B_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ Similarly,we can find out the Basis for $c_2 = 2$ which is $B_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

TABLE 0: Finding of Characteristic Polynomial, Characteristic value and corresponding Basis