Assignment 10

Priya Bhatia

***Abstract*—This document gives an explicit description of the vectors in R^5 which are linear combination of the vectors.**

Download latex-tikz codes from

```
https://github.com/priya6971/
    matrix_theory_EE5609/tree/master/
    Assignment10
```

Download python codes from

```
https://github.com/priya6971/
    matrix_theory_EE5609/tree/master/
    Assignment10/codes
```

1 Problem

Give an explicit description of the type $b_j = \sum_{i=1}^{r} b_{ki} R_{ij}$ for the vectors

$$\beta = (b_1, b_2, b_3, b_4, b_5)$$

in R^5 which are linear combinations of the vectors

$$\alpha_1 = (1, 0, 2, 1, -1), \tag{1.0.1}$$

$$\alpha_2 = (-1, 2, -4, 2, 0), \tag{1.0.2}$$

$$\alpha_3 = (2, -1, 5, 2, 1), \tag{1.0.3}$$

$$\alpha_4 = (2, 1, 3, 5, 2) \tag{1.0.4}$$

2 Solution

Above matrix represented as: $Ax = \beta$

$$\begin{pmatrix} 1 & -1 & 2 & 2 \\ 0 & 2 & -1 & 1 \\ 2 & -4 & 5 & 3 \\ 1 & 2 & 2 & 5 \\ -1 & 0 & 1 & 2 \end{pmatrix} x = \beta \tag{2.0.1}$$

$$\begin{pmatrix} 1 & -1 & 2 & 2 \\ 0 & 2 & -1 & 1 \\ 2 & -4 & 5 & 3 \\ 1 & 2 & 2 & 5 \\ -1 & 0 & 1 & 2 \end{pmatrix} x = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{pmatrix} \tag{2.0.2}$$

Now finding row space of A is equivalent to finding column space of A^T. So, here we do row reduction of the matrix whose rows are given by $\alpha_i' s$:

$$\begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ -1 & 2 & -4 & 2 & 0 \\ 2 & -1 & 5 & 2 & 1 \\ 2 & 1 & 3 & 5 & 2 \end{pmatrix} \xleftrightarrow{R_2=R_2+R_1} \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ 0 & 2 & -2 & 3 & -1 \\ 2 & -1 & 5 & 2 & 1 \\ 2 & 1 & 3 & 5 & 2 \end{pmatrix} \tag{2.0.3}$$

$$\xleftrightarrow{R_3=R_3-2R_1} \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ 0 & 2 & -2 & 3 & -1 \\ 0 & -1 & 1 & 0 & 3 \\ 2 & 1 & 3 & 5 & 2 \end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow{R_4=R_4-2R_1} \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ 0 & 2 & -2 & 3 & -1 \\ 0 & -1 & 1 & 0 & 3 \\ 0 & 1 & -1 & 3 & 4 \end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow{R_4=R_4-2R_1} \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ 0 & 2 & -2 & 3 & -1 \\ 0 & -1 & 1 & 0 & 3 \\ 0 & 1 & -1 & 3 & 4 \end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow{R_4=R_4+R_3} \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ 0 & 2 & -2 & 3 & -1 \\ 0 & -1 & 1 & 0 & 3 \\ 0 & 0 & 0 & 3 & 7 \end{pmatrix} \tag{2.0.7}$$

$$\xleftrightarrow{R_2=R_2+R_3} \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ 0 & 1 & -1 & 3 & 4 \\ 0 & 0 & 0 & -3 & -9 \\ 0 & 0 & 0 & 3 & 7 \end{pmatrix} \tag{2.0.8}$$

$$\xleftrightarrow[R_3=-\frac{R_3}{3}]{R_2=R_2+R_3} \begin{pmatrix} 1 & 0 & 2 & 1 & -4 \\ 0 & 1 & -1 & 0 & -5 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.0.9}$$

$$\xleftrightarrow{R_3=R_3+R_2} \begin{pmatrix} 1 & 0 & 2 & 0 & -4 \\ 0 & 1 & -1 & 0 & -5 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.0.10}$$

$$\xleftrightarrow[R_1=R_1+4R_4]{R_2=R_2+5R_4} \begin{pmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.0.11}$$

So column vector of above matrix is:

$$C_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.12}$$

$$C_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.13}$$

$$C_3 = \begin{pmatrix} 2 \\ -1 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.14}$$

$$C_4 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \tag{2.0.15}$$

$$C_5 = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} \tag{2.0.16}$$

Then the general element that is a linear combination of the $\alpha_i's$ is:

$$(b_1, b_2, 2b_1 - b_2, b_3, b_4) \tag{2.0.17}$$