Assignment 11

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***Abstract*—This document demonstrate how to find the dimension of the subspace.**

Download latex-tikz codes from

https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment11

1 Problem

Let F be a subfield of the complex numbers. We define n linear functionals on $F^n (n \geq 2)$ by

$$f_k(x_1, ..., x_n) = \sum_{j=1}^{n} (k-j)x_j, 1 \leq k \leq n. \tag{1.0.1}$$

What is the dimension of the subspace annihilated by $f_1, f_2, ..., f_n$?

2 Solution

N_{f_k} is the subspace annihilated by f_k. Then the dimension of N_{f_k} is given by:

$$\dim N_{f_k} = n-1 \tag{2.0.1}$$

Now the standard basis vector ϵ_2 is in N_{f_2} but is not in N_{f_1}.Thus N_{f_1} and N_{f_2} are distinct hyperspaces.Thus, the intersection of N_{f_1} and N_{f_2} dimension is given by:

$$\dim N_{f_1} \cap N_{f_2} = n-2 \tag{2.0.2}$$

Now the standard basis vector ϵ_3 is in N_{f_3} but is not in $N_{f_1} \cup N_{f_2}$.Thus N_{f_1}, N_{f_2} and N_{f_3} are distinct hyperspaces.Thus, the intersection of N_{f_1}, N_{f_2} and N_{f_3} dimension is given by:

$$\dim N_{f_1} \cap N_{f_2} \cap N_{f_3} = n-3 \tag{2.0.3}$$

From (2.0.1),(2.0.2) and (2.0.3) we can conclude the following:

$$\epsilon_i \notin \bigcup_{j=1}^{n-1} N_{f_i} \tag{2.0.4}$$

$$\dim \bigcap_{j=1}^{i} N_{f_i} = n-i \tag{2.0.5}$$

Thus when $i = n$, dimension of the subspace is given by:

$$\dim \bigcap_{j=1}^{n} N_{f_i} = 0 \tag{2.0.6}$$