

Assignment 1

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Abstract—This document solves a problem from Lines and Planes, where we solve the given pair of linear equations.

Download all python codes from

https://github.com/priya6971/matrix_theory_EE5609/tree/master/school/tree/master/training/design/codes

and latex-tikz codes from

https://github.com/priya6971/matrix_theory_EE5609/tree/master/school/tree/master/training/design

1 PROBLEM

Solve the following pair of linear equation

$$\begin{pmatrix} 158 & -378 \end{pmatrix} x = -74$$

$$\begin{pmatrix} -378 & 152 \end{pmatrix} x = -604$$

2 EXPLANATION

Let the matrix is A and b is the vector.
 So, $Ax = b$

Then we can calculate $x = A^{-1} \cdot b$

$$A = \begin{pmatrix} 158 & -378 \\ -378 & 152 \end{pmatrix}$$

$$b = \begin{pmatrix} -74 \\ -604 \end{pmatrix}$$

3 SOLUTION

Solution Approach 1 :

$$A = \begin{pmatrix} 158 & -378 \\ -378 & 152 \end{pmatrix}$$

$$b = \begin{pmatrix} -74 \\ -604 \end{pmatrix}$$

$$x = A^{-1} \cdot b$$

$$x = \begin{pmatrix} 158 & -378 \\ -378 & 152 \end{pmatrix}^{-1} \cdot \begin{pmatrix} -74 \\ -604 \end{pmatrix}$$

$$\text{Adjoint of a given matrix } A = \begin{pmatrix} 152 & 378 \\ 378 & 158 \end{pmatrix}$$

Multiply by 1/Determinant of Matrix A with the adjoint in order to get the final result

$$x = (2.01534475 \quad 1.03815998)$$

Solution Approach 2:

Using Row Reduction Method approach:

$$\text{Augmented Matrix is : } \left(\begin{array}{cc|c} 158 & -378 & -74 \\ -378 & 152 & -604 \end{array} \right)$$

Step1-Multiply Row1 by 378 and Row2 by 152:

$$\left(\begin{array}{cc|c} 59724 & -142884 & -27972 \\ -59724 & 24016 & -95432 \end{array} \right)$$

Step2-Add row1 and row2 in row2:

$$\left(\begin{array}{cc|c} 59724 & -142884 & -27972 \\ 0 & -118868 & -123404 \end{array} \right)$$

Step3-row1 is row1*118868 - row2*142884:

$$\left(\begin{array}{cc|c} 7099272432 & 0 & 1.43 * 10^10 \\ 0 & -118868 & -123404 \end{array} \right)$$

Step4-row1 is row1/7099272432 and row2 is row2/-118868:

$$\left(\begin{array}{cc|c} 1 & 0 & 2.0153 \\ 0 & 1 & 1.03816 \end{array} \right)$$

Step5-Final value of x:

$$x = (2.01534475 \quad 1.03815998)$$

Solution Approach 3 :

Augmented Matrix is : $Ax = b$

$$\text{Assuming } R = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

Now, multiply R on both sides, we get :

$$RAx = Rb$$

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 158 & -378 \\ -378 & 152 \end{pmatrix} x = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -74 \\ -604 \end{pmatrix}$$

After multiplication of matrix:

$$\begin{pmatrix} 378 & -152 \\ -158 & -378 \end{pmatrix} x = \begin{pmatrix} 604 \\ -74 \end{pmatrix}$$

Now, multiply the matrix A on both sides, we get:

$$ARAx = ARb$$

$$\begin{pmatrix} 158 & -378 \\ -378 & 152 \end{pmatrix} \begin{pmatrix} 378 & -152 \\ -158 & -378 \end{pmatrix} x = \begin{pmatrix} 158 & -378 \\ -378 & 152 \end{pmatrix} \begin{pmatrix} 604 \\ -74 \end{pmatrix}$$

Now, after multiplication of above matrices:

$$\begin{pmatrix} 0 & 118,868 \\ -118,868 & 0 \end{pmatrix} x = \begin{pmatrix} 123404 \\ -239560 \end{pmatrix}$$

Now, both the rows is row/118,868:

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} x = \begin{pmatrix} 30851/29717 \\ -59890/29717 \end{pmatrix}$$

As we know $EE^{-1} = I$,

Now inverse of the $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ is $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

Now multiply with the inverse of the Matrix in order to get the identity Matrix :

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} x = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 30851/29717 \\ -59890/29717 \end{pmatrix}$$

So, final matrix is after matrix multiplication:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} x = \begin{pmatrix} 59890/29717 \\ 30851/29717 \end{pmatrix}$$

So, final result is :

$$x = \begin{pmatrix} 59890/29717 & 30851/29717 \end{pmatrix}$$