Assignment 3

Priya Bhatia

***Abstract*—This document solves a problem based on the congruency of a triangles.**

Download latex-tikz codes from

https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment3

1 Problem

In right triangle ABC, right angled at **C**, **M** is the mid-point of hypotenuse AB.**C** is joined to **M** and produced to a point **D** such that DM = CM. Point **D** is joined to point **B**. Show that:

$$a)\quad \triangle AMC \cong \triangle BMD \tag{1.0.1}$$
$$b)\quad \angle DBC = 90^\circ \tag{1.0.2}$$
$$c)\quad \triangle DBC \cong \triangle ACB \tag{1.0.3}$$
$$d)\quad CM = \frac{1}{2}AB \tag{1.0.4}$$

2 Solution

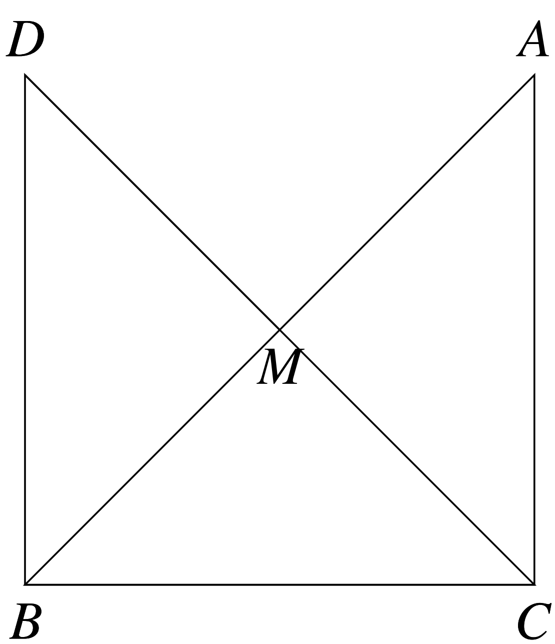

Fig. 1: Triangle ABC and DBC

In $\triangle ABC$, **M** is midpoint of hypotenuse AB, thus

$$AM = BM \tag{2.0.1}$$

and also it is given that DM = CM. To show $\triangle AMC \cong \triangle BMD$, we use

$$\angle AMC = \angle BMD \qquad (VOA) \tag{2.0.2}$$
$$AM = BM \qquad (from\ equation\ 2.0.1) \tag{2.0.3}$$
$$CM = DM \qquad (Given) \tag{2.0.4}$$

Thus, by SAS Congruency Criteria, $\triangle AMC \cong \triangle BMD$. Now we have to show that $\angle DBC$ is right angle. As **M** is the mid point of AB, so MC bisect $\angle C$ in the $\triangle ABC$,

$$\angle MCB = \angle MCA = 45^\circ \tag{2.0.5}$$
$$\angle MCB = \angle DCB \tag{2.0.6}$$
$$\angle DCB = 45^\circ \tag{2.0.7}$$

Now, as we know that $\triangle AMC \cong \triangle BMD$. Thus,

$$\angle ACM = \angle BDM \tag{2.0.8}$$
$$\angle ACM = 45^\circ \qquad (from\ equation\ 2.0.5) \tag{2.0.9}$$
$$\angle BDM = \angle BDC = 45^\circ \qquad (from\ equation\ 2.0.8) \tag{2.0.10}$$

Now, in $\triangle DBC$,

$$\angle CDB + \angle DBC + \angle DCB = 180^\circ \tag{2.0.11}$$
$$45^\circ + 45^\circ + \angle DBC = 180^\circ \tag{2.0.12}$$
$$\angle DBC = 90^\circ \tag{2.0.13}$$

Also, we have to show that $\triangle DBC \cong \triangle ACB$. After proving an equation 1.0.1, we can say that $\triangle AMC \cong \triangle BMD$. Thus,

$$AC = BD \tag{2.0.14}$$

To show $\triangle DBC \cong \triangle ACB$ we use

$$\angle DBC = \angle ACB \quad (Both\ are\ right\ angled) \tag{2.0.15}$$

$$DB = AC \quad (from\ equation\ 2.0.14) \tag{2.0.16}$$

$$BC = CB \quad (Common\ Side) \tag{2.0.17}$$

Thus, by SAS Congruency Criteria, $\triangle DBC \cong \triangle ACB$. Now, we have to show

$$CM = \frac{1}{2}AB \tag{2.0.18}$$

Now, as we already show that $\triangle DBC \cong \triangle ACB$. So,

$$DC = AB \tag{2.0.19}$$

$$\frac{1}{2}DC = \frac{1}{2}AB \tag{2.0.20}$$

$$DM = CM \tag{2.0.21}$$

$$CM = \frac{1}{2}DC \tag{2.0.22}$$

So from equation 2.0.20 and 2.0.22, we can get

$$CM = \frac{1}{2}AB \tag{2.0.23}$$