Assignment 11

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***Abstract*—This document demonstrate how to find the dimension of the subspace.**

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https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment11

1 Problem

Let F be a subfield of the complex numbers. We define n linear functionals on $F^n (n \geq 2)$ by

$$f_k(x_1, ..., x_n) = \sum_{j=1}^{n} (k-j)x_j, 1 \leq k \leq n. \quad (1.0.1)$$

What is the dimension of the subspace annihilated by $f_1, f_2, ..., f_n$?

2 Solution

Given	F be a subfield of the complex numbers Definition of n linear functionals on $F^n (n \geq 2)$ by $f_k(x_1, ..., x_n) = \sum_{j=1}^{n} (k-j)x_j$; $1 \leq k \leq n$
To find	The dimension of the subspace annihilated by $f_1, f_2, ..., f_n$
f_k	$f_k(x_1, ..., x_n) = \sum_{j=1}^{n} (k-j)x_j$ $f_k(x_1, ..., x_n) = k \sum_{j=1}^{n} x_j$-$\sum_{j=1}^{n} jx_j$ All f_k are linear combinations of the two linear functionals

Vector	The two linear functionals defined below $g_1(x_1, ..., x_n) = \sum_{j=1}^{n} x_j$ $g_2(x_1, ..., x_n) = \sum_{j=1}^{n} jx_j$ Dimension of subspace annihilated by $f_i's$ is the dimension of the solution space of the system $AX = 0$ where the i^{th} row is defined by $A_i = (i-1, i-2, ..., i-n)$ $1 \leq i \leq n$
Matrix	$AX = 0$ where the i^{th} row is defined by $A_i = (i-1, i-2, ..., i-n)$ $1 \leq i \leq n$ For the $n = 4$, matrix A looks like $\begin{pmatrix} 0 & -1 & -2 & -3 \\ 1 & 0 & -1 & -2 \\ 2 & 1 & 0 & -1 \\ 3 & 2 & 1 & 0 \end{pmatrix}$ For $i \geq 3$, perform the following elementary operations of n linear functionals $(a) A_i \longrightarrow (1-i)A_2 + A_i$ $A_i = (0, i-2, 2(i-2), 3(i-2), ..., (n-1)(i-2))$ $(b) A_i \longrightarrow \frac{1}{i-2} A_i$ A_i = -A_1 $(c) A_i \longrightarrow A_i + A_1$ $A_i = 0$ Since, A_1 and A_2 are linearly independent Thus, the dimension of the subspace annihiliated = $n-2$