Challenge Problem

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***Abstract*—This document show that Orthogonal vectors are linearly independent**

Download latex-tikz codes from

https://github.com/priya6971/matrix_theory_EE5609/tree/master/ChallengeProblem

1 Problem

Show that the set of Orthogonal vectors is Linear independent.

2 Proof

$$c_1\mathbf{v_1} + c_2\mathbf{v_2} + \cdots + c_n\mathbf{v_n} = 0 \tag{2.0.1}$$

We have to show that in (2.0.1), $c_1 = 0$, $c_2 = 0$ and so on upto $c_n = 0$. We begin by taking only two orthogonal vectors say $\mathbf{v_1}$ and $\mathbf{v_2}$ are the two orthogonal vectors.

And we know that $\mathbf{v_1}$ and $\mathbf{v_2}$ are Linearly Independent if and only if the value of $c_1 = 0$, $c_2 = 0$ in below equation:

$$c_1\mathbf{v_1} + c_2\mathbf{v_2} = 0 \tag{2.0.2}$$

To prove this, we can take the dot product of $\mathbf{v_1}$ on both side in $\mathbf{v_1}$

$$c_1\mathbf{v_1}\mathbf{v_1} + c_2\mathbf{v_1}\mathbf{v_2} = 0 \tag{2.0.3}$$

Now as $\mathbf{v_1}$ and $\mathbf{v_2}$ are orthogonal vectors so dot product $\mathbf{v_1}$ and $\mathbf{v_2}$ is 0. Therefore we get from (2.0.3)

$$c_1\mathbf{v_1}\mathbf{v_1} = 0 \tag{2.0.4}$$

Now $\mathbf{v_1}$ cannot be zero as $\mathbf{v_1}$ is from a set of non-zero orthogonal vectors. Therefore we get $c_1 = 0$, from (2.0.4). And simlarly we can proof that the value of $c_2 = 0$, by taking dot product of vector $\mathbf{v_2}$ in equation (2.0.2)

Thus, orthogonal vectors $\mathbf{v_1}$ and $\mathbf{v_2}$ satisfy the condition of linear independence.

2.1 General Case

Consider, the expression

$$c_1\mathbf{v_1} + c_2\mathbf{v_2} + ... + c_n\mathbf{v_n} = 0 \tag{2.1.1}$$

Take the dot product of 2.1.1 with $\mathbf{v_1}$, we get

$$c_1\left\|\mathbf{v_1}\right\|^2 + c_2\mathbf{v_2}^T\mathbf{v_1} + ... + c_n\mathbf{v_n}^T\mathbf{v_1} = 0 \tag{2.1.2}$$

$$c_1\left\|\mathbf{v_1}\right\|^2 = 0 \quad (\mathbf{v_i}^T\mathbf{v_j} = 0 \quad \forall i \neq j) \tag{2.1.3}$$

$$\left\|\mathbf{v_1}\right\|^2 = 0 \quad \iff \mathbf{v_1} = 0 \tag{2.1.4}$$

Hence, $c_1 = 0$ Similarly, taking the dot product of 2.1.1 with $\mathbf{v_2}$, ...,$\mathbf{v_n}$, we find out $c_2 = 0, ..., c_n = 0$. Thus, the set of Orthogonal vectors $\mathbf{v_1}, \mathbf{v_2}, ..., \mathbf{v_n}$ is Linear independent.

So, we can proof that if $\mathbf{v_1}, \mathbf{v_2}$ upto $\mathbf{v_n}$ are Orthogonal vectors that forms an equation (2.0.1).

Then, the value of $c_1 = 0$, $c_2 = 0$ and so on upto $c_n = 0$.