

Assignment 14

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Abstract—This document demonstrates the proof of linearity and approach to find the minimal polynomial for P , where P is the operator on R^2 .

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[https://github.com/priya6971/
matrix_theory_EE5609/tree/master/
Assignment14](https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment14)

1 PROBLEM

Let P be the operator on R^2 which projects each vector onto the x-axis, parallel to the y-axis: $P(x, y) = (x, 0)$. Show that P is linear. What is the minimal polynomial for P ?

2 SOLUTION

Given	P be the operator on R^2 , $P : R^2 \rightarrow R^2$ which projects each vector onto the x-axis, parallel to the y-axis: $P(x, y) = (x, 0)$
To Prove	<p style="text-align: center;">P is linear</p> <p>For P to be linear, it should satisfy the properties mentioned in the Given section Consider two vectors (x_1, y_1) and (x_2, y_2)</p>
To find	Minimal Polynomial for P
Matrix	<p>For the projection $P(x, y) = (x, 0)$, the matrix of linear transform is,</p> $P(x, y) = \mathbf{A} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = (x, 0)$ <p style="text-align: center;">So, $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$</p>

Proof	<p>Consider two vectors (x_1, y_1) and (x_2, y_2), then</p> $P((x_1, y_1) + (x_2, y_2)) = P(x_1 + x_2, y_1 + y_2)$ $P((x_1, y_1) + (x_2, y_2)) = P(x_1 + x_2, 0)$ $P((x_1, y_1) + (x_2, y_2)) = P((x_1, 0), (x_2, 0))$ <p>Thus, P satisfied the property of linearity.</p> <p>Now, consider some scalar k, then</p> $P(k(x_1, y_1)) = P((kx_1, ky_1))$ $P(k(x_1, y_1)) = P((kx_1, 0))$ $P(k(x_1, y_1)) = kP(x_1, 0)$ <p>Thus, using above observations we can conclude that P is linear.</p>
Minimal Polynomial	$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ <p>Characteristic Polynomial of A is</p> $\det(xI - A) = \begin{vmatrix} (x-1) & 0 \\ 0 & (x-0) \end{vmatrix}$ $\det(xI - A) = x(x-1)$ <p>Characteristic polynomial is a product of distinct linear terms</p> $m(x) = x(x-1)$ $\implies m(\mathbf{A}) = \mathbf{A}(\mathbf{A} - \mathbf{I}) = 0$ $\implies x(x-1) \text{ is the minimal polynomial}$ <p>Thus, minimal polynomial is equal to $m(x) = x(x-1)$</p>

TABLE 0: Illustration of Proof and finding of minimal polynomial