Assignment 4

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***Abstract*—This document finds the area bounded by curves**

Download python codes from

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https://github.com/priya6971/
    matrix_theory_EE5609/tree/master/
    Assignment4/codes
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Download latex-tikz codes from

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https://github.com/priya6971/
    matrix_theory_EE5609/tree/master/
    Assignment4
```

1 Problem

Find the area bounded by curves $\left\|\mathbf{x}-\begin{pmatrix}1\\0\end{pmatrix}\right\|$=1 and $\|\mathbf{x}\|$=1.

2 Solution

General equation of circle is $\mathbf{x}^T\mathbf{x}+2\mathbf{u}^T\mathbf{x}+$ f = 0 Taking equation of the first curve to be,

$$\left\|\mathbf{x}-\begin{pmatrix}1\\0\end{pmatrix}\right\|^2 = 1^2 \tag{2.0.1}$$

$$\mathbf{x}^T\mathbf{x}+2\mathbf{u_1}^T\mathbf{x}=0 \tag{2.0.2}$$

$$\mathbf{u_1}=\begin{pmatrix}-1\\0\end{pmatrix} \tag{2.0.3}$$

$$f_1=0 \tag{2.0.4}$$

$$\mathbf{O_1}=\begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.5}$$

Taking equation of the second curve to be,

$$\|\mathbf{x}\|^2+2\mathbf{u}_2^T\mathbf{x}+f_2=0 \tag{2.0.6}$$

$$\mathbf{x}^T\mathbf{x}-1=0 \tag{2.0.7}$$

$$\mathbf{u_2}=\begin{pmatrix}0\\0\end{pmatrix} \tag{2.0.8}$$

$$f_2=-1 \tag{2.0.9}$$

$$\mathbf{O_2}=\begin{pmatrix}0\\0\end{pmatrix} \tag{2.0.10}$$

Now, subtracting equation (2.0.2) from (2.0.7) We get,

$$\mathbf{x}^T\mathbf{x}+2\mathbf{u_1}^T\mathbf{x}-\mathbf{x}^T\mathbf{x}-f_2=0 \tag{2.0.11}$$

$$2\mathbf{u}^T\mathbf{x}=-1 \tag{2.0.12}$$

$$\begin{pmatrix}-2 & 0\end{pmatrix}\mathbf{x}=-1 \tag{2.0.13}$$

which can be written as:-

$$\begin{pmatrix}1 & 0\end{pmatrix}\mathbf{x}=1/2 \tag{2.0.14}$$

$$\mathbf{x}=\begin{pmatrix}1/2\\0\end{pmatrix}+\lambda\begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.15}$$

$$\mathbf{x}=\mathbf{q}+\lambda\mathbf{m} \tag{2.0.16}$$

$$\mathbf{q}=\begin{pmatrix}1/2\\0\end{pmatrix} \tag{2.0.17}$$

$$\mathbf{m}=\begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.18}$$

Substituting (2.0.16) in (2.0.6)

$$\|\mathbf{x}\|^2+2\mathbf{u}_2^T\mathbf{x}+f_2=0 \tag{2.0.19}$$

$$\|\mathbf{q}+\lambda\mathbf{m}\|^2+f_2=0 \tag{2.0.20}$$

$$(\mathbf{q}+\lambda\mathbf{m})^T(\mathbf{q}+\lambda\mathbf{m})+f_2=0 \tag{2.0.21}$$

$$\mathbf{q}^T(\mathbf{q}+\lambda\mathbf{m})+\lambda\mathbf{m}^T(\mathbf{q}+\lambda\mathbf{m})+f_2=0 \tag{2.0.22}$$

$$\|\mathbf{q}\|^2+\lambda\mathbf{q}^T\mathbf{m}+\lambda\mathbf{m}^T\mathbf{q}+\lambda^2\|\mathbf{m}\|^2+f_2=0 \tag{2.0.23}$$

$$\|\mathbf{q}\|^2+2\lambda\mathbf{q}^T\mathbf{m}+\lambda^2\|\mathbf{m}\|^2+f_2=0 \tag{2.0.24}$$

Taking λ as common :

$$\lambda(\lambda\|\mathbf{m}\|^2+2\mathbf{q}^T\mathbf{m})=-f_2-\|\mathbf{q}\|^2 \tag{2.0.25}$$

$$\lambda^2\|\mathbf{m}\|^2=-f_2-\|\mathbf{q}\|^2 \tag{2.0.26}$$

$$\lambda^2=\frac{-f_2-\|\mathbf{q}\|^2}{\|\mathbf{m}\|^2} \tag{2.0.27}$$

$$\lambda^2=\frac{3}{4} \tag{2.0.28}$$

$$\lambda=+\sqrt{\frac{3}{4}},-\sqrt{\frac{3}{4}} \tag{2.0.29}$$

$$\lambda=+\frac{\sqrt{3}}{2},-\frac{\sqrt{3}}{2} \tag{2.0.30}$$

Substituting the value of λ in (2.0.16)

$$\mathbf{x} = \mathbf{q} + \lambda \mathbf{m} \tag{2.0.31}$$

$$\mathbf{A} = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \tag{2.0.32}$$

$$\mathbf{B} = \begin{pmatrix} \frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{pmatrix} \tag{2.0.33}$$

Now finding the direction vector $\mathbf{m}_{O_1A}$, $\mathbf{m}_{O_1B}$,$\mathbf{m}_{O_2A}$ and $\mathbf{m}_{O_2B}$.

$$\mathbf{m}_{O_1A} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{pmatrix} \tag{2.0.34}$$

$$\mathbf{m}_{O_1B} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \tag{2.0.35}$$

$$\mathbf{m}_{O_2A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{pmatrix} \tag{2.0.36}$$

$$\mathbf{m}_{O_2B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - \begin{pmatrix} \frac{1}{2} \\ -\frac{\sqrt{3}}{2} \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} \tag{2.0.37}$$

Now finding the angle $\angle O_1AB$.

$$\mathbf{m}_{O_1A}^T \mathbf{m}_{O_1B} = \left\| \mathbf{m}_{O_1A} \right\| \left\| \mathbf{m}_{O_1B} \right\| \cos \theta_1 \tag{2.0.38}$$

$$\frac{\mathbf{m}_{O_1A}^T \mathbf{m}_{O_1B}}{\left\| \mathbf{m}_{O_1A} \right\| \left\| \mathbf{m}_{O_1B} \right\|} = \cos \theta_1 \tag{2.0.39}$$

$$\frac{-2}{4} = \cos \theta_1 \tag{2.0.40}$$

$$\frac{-1}{2} = \cos \theta_1 \tag{2.0.41}$$

$$\theta_1 = 120^\circ \tag{2.0.42}$$

Now finding the angle $\angle O_2AB$.

$$\mathbf{m}_{O_2A}^T \mathbf{m}_{O_2B} = \left\| \mathbf{m}_{O_2A} \right\| \left\| \mathbf{m}_{O_2B} \right\| \cos \theta_2 \tag{2.0.43}$$

$$\frac{\mathbf{m}_{O_2A}^T \mathbf{m}_{O_2B}}{\left\| \mathbf{m}_{O_2A} \right\| \left\| \mathbf{m}_{O_2B} \right\|} = \cos \theta_2 \tag{2.0.44}$$

$$\frac{-2}{4} = \cos \theta_2 \tag{2.0.45}$$

$$\frac{-1}{2} = \cos \theta_2 \tag{2.0.46}$$

$$\theta_2 = 120^\circ \tag{2.0.47}$$

Finding area of $\mathbf{O_1AB}$ and $\mathbf{O_2AB}$.

$$A_{O_1AB} = \frac{\pi\theta_1}{360} r^2 - \frac{1}{2} 2\sqrt{3} \tag{2.0.48}$$

$$= \frac{120}{360}\pi - \frac{1}{2} 2\sqrt{3} \tag{2.0.49}$$

$$A_{O_2AB} = \frac{\pi\theta_2}{360} r^2 - \frac{1}{2} 2\sqrt{3} \tag{2.0.50}$$

$$= \frac{120}{360}\pi - \frac{1}{2} 2\sqrt{3} \tag{2.0.51}$$

Area of $\mathbf{O_1AO_2B}$

$$A_{O_1AO_2B} = \frac{120}{360}\pi - \frac{1}{2} 2\sqrt{3} + \frac{120}{360}\pi - \frac{1}{2} 2\sqrt{3} \tag{2.0.52}$$

$$= \frac{2\pi}{3} - 2\sqrt{3} \tag{2.0.53}$$

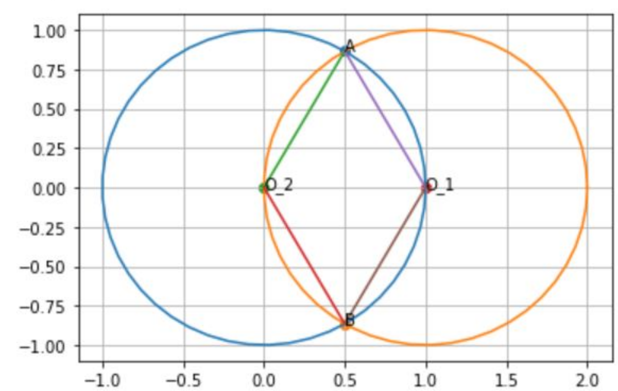

Fig. 0: Figure depicting intersection points of circle