Assignment 11

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Abstract—This document demonstrate how to find the dimension of the subspace.

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1 PROBLEM

Let F be a subfield of the complex numbers. We define n linear functionals on $F^n(n \geq 2)$ by

$$f_k(x_1, ..., x_n) = \sum_{j=1}^{n}(k-j)x_j, 1 \leq k \leq n. \quad (1.0.1)$$

What is the dimension of the subspace annihilated by $f_1, f_2, ..., f_n$?

2 SOLUTION

Given	F be a subfield of the complex numbers
	Definition of n linear functionals on $F^n(n \geq 2)$ by $f_k(x_1, ..., x_n) = \sum_{j=1}^{n}(k-j)x_j;$ $1 \leq k \leq n$
To find	The dimension of the subspace annihilated by $f_1, f_2, ..., f_n$
f_k	$f_k(x_1, ..., x_n) = \sum_{j=1}^{n}(k-j)x_j$ $f_k(x_1, ..., x_n) = k\sum_{j=1}^{n} x_j \text{-} \sum_{j=1}^{n} jx_j$ All f_k are linear combinations of the two linear functionals

Vector	The two linear functionals defined below $g_1(x_1, ..., x_n) = \sum_{j=1}^{n} x_j$ $g_2(x_1, ..., x_n) = \sum_{j=1}^{n} jx_j$ Dimension of subspace annihilated by $fi's$ is the dimension of the solution space of the system $AX = 0$ where the ith row is defined by $A_i = (i-1, i-2, ..., i-n)$ $1 \leq i \leq n$
Matrix	$AX = 0$ where the ith row is defined by $A_i = (i-1, i-2, ..., i-n)$ $1 \leq i \leq n$ For $i \geq 3$, perform the following elementary operations of n linear functionals $(a)A_i \longrightarrow (1-i)A_2 + A_i$ $A_i = (0, i-2, 2(i-2), 3(i-2), ..., (n-1)(i-2))$ $(b)A_i \longrightarrow \frac{1}{i-2}A_i$ $A_i = \text{-}A_1$ $(c)A_i \longrightarrow A_i + A_1$ $A_i = 0$ Since, A_1 and A_2 are linearly independent Thus, the dimension of the subspace annihiliated $= n-2$