Assignment 5

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***Abstract*—This document finds the normal at the given point on the curve**

Download python codes from

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https://github.com/priya6971/
    matrix_theory_EE5609/tree/master/
    Assignment5/codes
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Download latex-tikz codes from

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https://github.com/priya6971/
    matrix_theory_EE5609/tree/master/
    Assignment5
```

1 Problem

Find the normal at the point $\begin{pmatrix}1\\1\end{pmatrix}$ on the curve $2y + x^2 = 3$

2 Solution

Given,

$$x^2 + 2y - 3 = 0 \tag{2.0.1}$$

From (2.0.1),

$$\mathbf{V} = \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix} \tag{2.0.2}$$

$$\mathbf{u} = \begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.3}$$

$$f = -3 \tag{2.0.4}$$

From (2.0.2),

$$\left|V\right| = \begin{vmatrix}1 & 0\\0 & 0\end{vmatrix} = 0 \tag{2.0.5}$$

Now (2.0.5) implies that the curve is a parabola. We can find the Eigen values corresponding to the $\mathbf{V}$,

$$\left|V - \lambda I\right| = 0$$

$$\begin{vmatrix}1-\lambda & 0\\0 & -\lambda\end{vmatrix} = 0$$

$$\implies \lambda = 0, 1 \tag{2.0.6}$$

Calculating the Eigen Vectors corresponding to $\lambda = 0, 1$ respectively,

$$\mathbf{V}\mathbf{x} = \lambda\mathbf{x}$$

$$\begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}\mathbf{x} = 0; \implies \mathbf{p}_1 = \begin{pmatrix}0\\1\end{pmatrix} \tag{2.0.7}$$

$$\begin{pmatrix}0 & 0\\0 & -1\end{pmatrix}\mathbf{x} = 0; \implies \mathbf{p}_2 = \begin{pmatrix}1\\0\end{pmatrix} \tag{2.0.8}$$

By Eigen decomposition on $\mathbf{V}$,

$$\mathbf{V} = \mathbf{P}\mathbf{D}\mathbf{P}^T$$

$$where, \mathbf{P} = \begin{pmatrix}\mathbf{p}_1 & \mathbf{p}_2\end{pmatrix} = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} \tag{2.0.9}$$

$$\mathbf{D} = \begin{pmatrix}\lambda_1 & 0\\0 & \lambda_2\end{pmatrix} = \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix} \tag{2.0.10}$$

To find the vertex of the parabola,

$$\begin{pmatrix}\mathbf{u}^T + \eta\mathbf{p}_1^T\\ \mathbf{V}\end{pmatrix}\mathbf{c} = \begin{pmatrix}-f\\ \eta\mathbf{p}_1 - \mathbf{u}\end{pmatrix} \tag{2.0.11}$$

$$where, \eta = \mathbf{u}^T\mathbf{p}_1 = 1 \tag{2.0.12}$$

Substituting values from (2.0.2), (2.0.7) and (2.0.12) in (2.0.11),

$$\begin{pmatrix}0 & 2\\1 & 0\\0 & 0\end{pmatrix}\mathbf{c} = \begin{pmatrix}3\\0\\0\end{pmatrix} \tag{2.0.13}$$

Removing last row and representing (2.0.13) as augmented matrix and then converting the matrix to echelon form,

$$\begin{pmatrix}0 & 2 & 3\\1 & 0 & 0\end{pmatrix} \xleftrightarrow{R_1 \leftrightarrow R_2} \begin{pmatrix}1 & 0 & 0\\0 & 2 & 3\end{pmatrix} \xleftrightarrow{R_2 \leftarrow \frac{R_2}{2}} \tag{2.0.14}$$

$$\begin{pmatrix}1 & 0 & 0\\0 & 1 & \frac{3}{2}\end{pmatrix} \tag{2.0.15}$$

From (2.0.15) it can be observed that,

$$\mathbf{c} = \begin{pmatrix}0\\ \frac{3}{2}\end{pmatrix} \tag{2.0.16}$$

Normal vector is obtained,

$$\mathbf{n} = \mathbf{V}\mathbf{q} + \mathbf{u} \tag{2.0.17}$$

$$\mathbf{n} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.18}$$

$$\mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.19}$$

$$\mathbf{n} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{2.0.20}$$