Assignment 14

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***Abstract*—This document demonstrates the proof of linearity and approach to find the minimal polynomial for P, where P is the operator on R^2.**

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https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment14

1 Problem

Let P be the operator on R^2 which projects each vector onto the x-axis, parallel to the y-axis: $P(x,y)$=$(x,0)$. Show that P is linear. What is the minimal polynomial for P?

2 Solution

Given	P be the operator on R^2, $P : R^2 \to R^2$ which projects each vector onto the x-axis, parallel to the y-axis: $P(x,y)$=$(x,0)$
To Prove	P is linear For P to be linear, it should satisfy the properties mentioned in the Given section Consider two vectors (x_1,y_1) and (x_2,y_2)
To find	Minimal Polynomial for P
Matrix	For the projection $P(x,y) = (x,0)$, the matrix of linear transform is, $P(x,y) = \mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = (x,0)$ So, $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

Proof	Consider two vectors (x_1, y_1) and (x_2, y_2), then $P((x_1, y_1) + (x_2, y_2)) = P(x_1 + x_2, y_1 + y_2)$ $P((x_1, y_1) + (x_2, y_2)) = P(x_1 + x_2, 0)$ $P((x_1, y_1) + (x_2, y_2)) = P((x_1, 0), (x_2, 0))$ Thus, P satisfied the property of linearity. Now, consider some scalar k, then $P(k(x_1, y_1)) = P((kx_1, ky_1))$ $P(k(x_1, y_1)) = P((kx_1, 0))$ $P(k(x_1, y_1)) = kP(x_1, 0)$ Thus, using above observations we can conclude that P is linear.
Minimal Polynomial	$\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ Characteristic Polynomial of A is $\det(xI - A) = \begin{vmatrix} (x-1) & 0 \\ 0 & (x-0) \end{vmatrix}$ $\det(xI - A) = x(x-1)$ Therefore, minimal polynomial can be x or $(x-1)$ or $x(x-1)$ It can be observed that if the minimal polynomial is $p(x) = x$, then $p(A) = A \neq 0$ It can be observed that if the minimal polynomial is $p(x) = (x-1)$, then $p(A) = A - I \neq 0$ Therefore, minimal polynomial is same as characteristic polynomial that is $p(x) = x(x-1)$

TABLE 0: Illustration of Proof and finding of minimal polynomial