Assignment 12

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***Abstract*—This document demonstrate how to check whether the matrix is nilpotent,diagonalizable or not and rank as well as Jordan canonical form of a matrix.**

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https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment12

1 Problem

Let V be a vector space over C of all the polynomials in a variable X of degree atmost 3. Let $D : V \to V$ be the linear operator given by differentiation with respect to X. Let A be the matrix of D with respect to some basis for V. Which of the following are true?
1.A is nilpotent matrix
2.A is diagonalizable matrix
3.the rank of A is 2
4.the Jordan canonical form of A is

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

2 Solution

Given	V be a vector space over C of all the polynomials in a variable X of degree atmost 3 $D : P_3 \to P_3$ $D : V \to V$ be the linear operator given by differentiation wrt X $D(P(x)) \to P'(x)$ A be the matrix of D wrt some basis for V Assume basis for V be $\{1, x, x^2, x^3\}$

Matrix	$D(1) = 0 = 0.1 + 0.x + 0.x^2 + 0.x^3$ $D(1) = \begin{pmatrix}0\\0\\0\\0\end{pmatrix}$ $D(x) = 1 = 1.1 + 0.x + 0.x^2 + 0.x^3$ $D(x) = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$ $D(x^2) = 2x = 0.1 + 2.x + 0.x^2 + 0.x^3$ $D(x^2) = \begin{pmatrix}0\\2\\0\\0\end{pmatrix}$ $D(x^3) = 3x^2 = 0.1 + 0.x + 3.x^2 + 0.x^3$ $D(x^3) = \begin{pmatrix}0\\0\\3\\0\end{pmatrix}$ Matrix $A = \begin{pmatrix}0&1&0&0\\0&0&2&0\\0&0&0&3\\0&0&0&0\end{pmatrix}$
Inference	An $n \times n$ matrix with λ as diagonal elements, ones on the super diagonal and zeroes in all other entries is nilpotent with minimal polynomial $(A - \lambda I)^n$
Nilpotent	$A = \begin{pmatrix}0&1&0&0\\0&0&2&0\\0&0&0&3\\0&0&0&0\end{pmatrix}$ All eigen values of matrix A is 0 Thus, above matrix is nilpotent matrix Thus, above statement is true

Diagonalizable	$A = \begin{pmatrix}0&1&0&0\\0&0&2&0\\0&0&0&3\\0&0&0&0\end{pmatrix}$ $Rank(A) + nullity(A) =$ no of column $Rank(A) = 3$, no of column $= 4$ $nullity(A) = 4 - 3 = 1$ means there exists only one linearly independent eigen vector corresponding to 0 eigen values Thus,matrix A is not Diagonalizable. Thus, above statement is false
Rank	$A = \begin{pmatrix}0&1&0&0\\0&0&2&0\\0&0&0&3\\0&0&0&0\end{pmatrix}$ Rank of matrix A is 3 Thus, above statement is false
Jordan CF	Assume characteristic polynomial of matrix A is $c_A(x)$ $c_A(x) = x^4$ Assume minimal polynomial of A is $m_A(x)$ $m_A(x)$ always divide $c_A(x)$ $m_A(x) = \{x, x^2, x^3, x^4\}$ Minimal polynomial always annihilates its matrix.Thus,we see that $m_A(A) = \{A = 0, A^2 = 0, A^3 = 0, A^4 = 0\}$ But we see that neither A is zero matrix nor A^2 and A^3 equal to zero but A^4 is equal to zero.Thus,x^4 is minimal polynomial. Algebraic Multiplicity $= a_M(\lambda = 0) = 4$ Geometric Multiplicity $= g_M(\lambda = 0) = nullity(A - 0I) = nullity(A) = 1$ Hence, Jordan form of block size 4 Using Inference, $\mathbf{J} = \begin{pmatrix}\lambda&1&0&0\\0&\lambda&1&0\\0&0&\lambda&1\\0&0&0&\lambda\end{pmatrix}$ $\lambda = 0$ $\begin{pmatrix}0&1&0&0\\0&0&1&0\\0&0&0&1\\0&0&0&0\end{pmatrix}$ which is same as given in the question. Thus,statement is true