Assignment 14

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Abstract

This document demonstrates the proof of linearity and approach to find the minimal polynomial for P, where P is the operator on R^2.

Download the latex-tikz codes from

https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment14

1 Problem

Let P be the operator on R^2 which projects each vector onto the x-axis, parallel to the y-axis: $p(x, y) = (x, 0)$. Show that P is linear. What is the minimal polynomial for P?

2 Definitions

Characteristic Polynomial	For an $n \times n$ matrix $\mathbf{A}$, characteristic polynomial is defined by, $$p(x) = \left\lvert x\mathbf{I} - \mathbf{A} \right\rvert$$
Cayley-Hamilton Theorem	If $p(x)$ is the characteristic polynomial of an $n \times n$ matrix $\mathbf{A}$, then, $$p(\mathbf{A}) = \mathbf{0}$$
Minimal Polynomial	Minimal polynomial $m(x)$ is the smallest factor of characteristic polynomial $p(x)$ such that, $$m(\mathbf{A}) = 0$$ Every root of characteristic polynomial should be the root of minimal polynomial

TABLE 1: Definitions

3 Solution

Proof of P is linear	Consider two vectors (x_1, y_1) and (x_2, y_2), then $P((x_1, y_1) + (x_2, y_2)) = P(x_1 + x_2, y_1 + y_2)$ $P((x_1, y_1) + (x_2, y_2)) = P(x_1 + x_2, 0)$ $P((x_1, y_1) + (x_2, y_2)) = P((x_1, 0), (x_2, 0))$ Now, consider some scalar k, then $P(k(x_1, y_1)) = P((kx_1, ky_1))$ $P(k(x_1, y_1)) = P((kx_1, 0))$ $P(k(x_1, y_1)) = kP(x_1, 0)$ Thus, using above observations we can conclude that P is linear.
Matrix form of Projection	For the projection $P(x, y) = (x, 0)$, the matrix of linear transform is, $P(x, y) = \mathbf{A}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = (x, 0)$ So, $\mathbf{A} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$
Characteristic polynomial	$p(x) = \left\lvert x\mathbf{I} - \mathbf{A} \right\rvert$ $= \begin{vmatrix} x-1 & 0 \\ 0 & x-0 \end{vmatrix}$ $= x(x-1)$
Minimal Polynomial	$p(x) = x^a (x-1)^b, \quad a \leq 1, b \leq 1$
$a = 1, b = 1$	$m(x) = x(x-1)$ $\implies m(\mathbf{A}) = \mathbf{A}(\mathbf{A} - \mathbf{I}) = 0$ $\implies x(x-1)$ is a minimal polynomial
Conclusion	For the given matrix $\mathbf{A}$, $x(x-1)$ is the characteristic polynomial as well as minimal polynomial.

TABLE 2: Illustration of Proof and finding of minimal polynomial