

Assignment 12

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Abstract—This document demonstrate how to find the dimension of the subspace.

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[https://github.com/priya6971/
matrix_theory_EE5609/tree/master/
Assignment12](https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment12)

1 PROBLEM

Let V be a vector space over C of all the polynomials in a variable X of degree atmost 3. Let $D : V \rightarrow V$ be the linear operator given by differentiation with respect to X . Let A be the matrix of D with respect to some basis for V . Which of the following are true?

1. A is nilpotent matrix
2. A is diagonalizable matrix
3. the rank of A is 2
4. the Jordan canonical form of A is

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

2 SOLUTION

Given	<p>V be a vector space over C of all the polynomials in a variable X of degree atmost 3</p> <p style="text-align: center;">$D : P_3 \rightarrow P_3$</p> <p style="text-align: center;">$D : V \rightarrow V$ be the linear operator given by differentiation wrt X</p> <p style="text-align: center;">$D(P(x)) \rightarrow P'(x)$</p> <p style="text-align: center;">A be the matrix of D wrt some basis for V</p> <p style="text-align: center;">Assume basis for V be $\{1, x, x^2, x^3\}$</p>
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Matrix	$D(1) = 0 = 0.1 + 0.x + 0.x^2 + 0.x^3$ $D(1) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ $D(x) = 1 = 1.1 + 0.x + 0.x^2 + 0.x^3$ $D(x) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ $D(x^2) = 2x = 0.1 + 2.x + 0.x^2 + 0.x^3$ $D(x^2) = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix}$ $D(x^3) = 3x^2 = 0.1 + 0.x + 3.x^2 + 0.x^3$ $D(x^3) = \begin{pmatrix} 0 \\ 0 \\ 3 \\ 0 \end{pmatrix}$ $\text{Matrix } A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
Nilpotent	$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ <p>All eigen values of matrix A is 0</p> <p>Thus, above matrix is nilpotent matrix</p> <p>Thus, above statement is true</p>

Diagonalizable	$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ <p>$nullity(A) = 1$</p> <p>means there exists only one linearly independent eigen vector corresponding to 0 eigen values</p> <p>Thus, matrix A is not Diagonalizable.</p> <p>Thus, above statement is false</p>
Rank	$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ <p>Rank of matrix A is 3</p> <p>Thus, above statement is false</p>
Jordan CF	$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ <p>Assume minimal polynomial of A is $p_A(x)$</p> <p>$p_A(x) = \{x, x^2, x^3, x^4\}$</p> <p>minimal polynomial always annihilates its matrix. Thus, we see that</p> <p>$A^4 = 0$</p> <p>Thus, Jordan canonical form is</p> $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ <p>which is same as given in the question. Thus, statement is true</p>