Assignment 10

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***Abstract*—This document gives an explicit description of the vectors in R^5 which are linear combination of the vectors.**

Download latex-tikz codes from

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https://github.com/priya6971/
    matrix_theory_EE5609/tree/master/
    Assignment10
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Download python codes from

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https://github.com/priya6971/
    matrix_theory_EE5609/tree/master/
    Assignment10/codes
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1 Problem

Give an explicit description of the type $b_j = \sum_{i=1}^{r} b_{ki} R_{ij}$ for the vectors

$$\beta = (b_1, b_2, b_3, b_4, b_5)$$

in R^5 which are linear combinations of the vectors

$$\alpha_1 = (1, 0, 2, 1, -1), \tag{1.0.1}$$

$$\alpha_2 = (-1, 2, -4, 2, 0), \tag{1.0.2}$$

$$\alpha_3 = (2, -1, 5, 2, 1), \tag{1.0.3}$$

$$\alpha_4 = (2, 1, 3, 5, 2) \tag{1.0.4}$$

2 Solution

Above matrix represented as: $Ax = \beta$

$$\begin{pmatrix} 1 & -1 & 2 & 2 \\ 0 & 2 & -1 & 1 \\ 2 & -4 & 5 & 3 \\ 1 & 2 & 2 & 5 \\ -1 & 0 & 1 & 2 \end{pmatrix} x = \beta \tag{2.0.1}$$

$$\begin{pmatrix} 1 & -1 & 2 & 2 \\ 0 & 2 & -1 & 1 \\ 2 & -4 & 5 & 3 \\ 1 & 2 & 2 & 5 \\ -1 & 0 & 1 & 2 \end{pmatrix} x = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{pmatrix} \tag{2.0.2}$$

Now finding row space of A is equivalent to finding column space of A^T. So, here we do elementary matrix multiplication of the A^T matrix whose rows are given by $\alpha_i's$, consider matrix X and Y which is equal to A^T, we get row reduced echelon form XY defined below:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ -1 & 2 & -4 & 2 & 0 \\ 2 & -1 & 5 & 2 & 1 \\ 2 & 1 & 3 & 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ 0 & 2 & -2 & 3 & -1 \\ 2 & -1 & 5 & 2 & 1 \\ 2 & 1 & 3 & 5 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ -1 & 2 & -4 & 2 & 0 \\ 2 & -1 & 5 & 2 & 1 \\ 2 & 1 & 3 & 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ 0 & 2 & -2 & 3 & -1 \\ 0 & -1 & 1 & 0 & 3 \\ 2 & 1 & 3 & 5 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ -2 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ -1 & 2 & -4 & 2 & 0 \\ 2 & -1 & 5 & 2 & 1 \\ 2 & 1 & 3 & 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ 0 & 2 & -2 & 3 & -1 \\ 0 & -1 & 1 & 0 & 3 \\ 0 & 1 & -1 & 3 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -2 & 0 & 1 & 0 \\ -4 & 0 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ -1 & 2 & -4 & 2 & 0 \\ 2 & -1 & 5 & 2 & 1 \\ 2 & 1 & 3 & 5 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ 0 & 2 & -2 & 3 & -1 \\ 0 & -1 & 1 & 0 & 3 \\ 0 & 0 & 0 & 3 & 7 \end{pmatrix}$$

Now the final row reduced echelon form of above matrix is:

$$X = \begin{pmatrix} \frac{67}{100} & -\frac{167}{100} & -2 & \frac{133}{100} \\ \frac{1}{2} & -\frac{3}{2} & -\frac{5}{2} & \frac{3}{2} \\ -\frac{4}{25} & \frac{117}{100} & \frac{3}{2} & -\frac{83}{100} \\ -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} \tag{2.0.3}$$

$$Y = \begin{pmatrix} 1 & 0 & 2 & 1 & -1 \\ -1 & 2 & -4 & 2 & 0 \\ 2 & -1 & 5 & 2 & 1 \\ 2 & 1 & 3 & 5 & 2 \end{pmatrix} \tag{2.0.4}$$

$$XY = \begin{pmatrix} 1 & 0 & 2 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \tag{2.0.5}$$

Now since the columns of the above matrix are linearly independent, b is described by (2.0.2)