

Assignment 10

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Abstract—This document demonstrate how to find the dimension of the subspace.

Download latex-tikz codes from

[https://github.com/priya6971/
matrix_theory_EE5609/tree/master/
Assignment11](https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment11)

From (2.0.1),(2.0.2) and (2.0.3) we can conclude the following:

$$\epsilon_i \notin \bigcup_{j=1}^{n-1} N_{f_j} \quad (2.0.4)$$

$$\dim \bigcap_{j=1}^i N_{f_j} = n - i \quad (2.0.5)$$

Thus when $i = n$, dimension of the subspace is given by:

$$\dim \bigcap_{j=1}^n N_{f_j} = 0 \quad (2.0.6)$$

1 PROBLEM

Let F be a subfield of the complex numbers. We define n linear functionals on $F^n (n \geq 2)$ by

$$f_k(x_1, \dots, x_n) = \sum_{j=1}^n (k - j)x_j, 1 \leq k \leq n. \quad (1.0.1)$$

What is the dimension of the subspace annihilated by f_1, f_2, \dots, f_n ?

2 SOLUTION

N_{f_k} is the subspace annihilated by f_k . Then the dimension of N_{f_k} is given by:

$$\dim N_{f_k} = n - 1 \quad (2.0.1)$$

Now the standard basis vector ϵ_2 is in N_{f_2} but is not in N_{f_1} . Thus N_{f_1} and N_{f_2} are distinct hyper-spaces. Thus, the intersection of N_{f_1} and N_{f_2} dimension is given by:

$$\dim N_{f_1} \cap N_{f_2} = n - 2 \quad (2.0.2)$$

Now the standard basis vector ϵ_3 is in N_{f_3} but is not in $N_{f_1} \cup N_{f_2}$. Thus N_{f_1}, N_{f_2} and N_{f_3} are distinct hyperspaces. Thus, the intersection of N_{f_1}, N_{f_2} and N_{f_3} dimension is given by:

$$\dim N_{f_1} \cap N_{f_2} \cap N_{f_3} = n - 3 \quad (2.0.3)$$