Assignment 1

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***Abstract*—This document solves a problem from Lines and Planes,where we solve the given pair of linear equations.**

Download all python codes from

https://github.com/priya6971/matrix_theory_EE5609/tree/master/school/tree/master/training/design/codes

and latex-tikz codes from

https://github.com/priya6971/matrix_theory_EE5609/tree/master/school/tree/master/training/design

1 Problem

Solve the following pair of linear equation

$$\begin{pmatrix} 158 & -378 \\ -378 & 152 \end{pmatrix} x = \begin{pmatrix} -74 \\ -604 \end{pmatrix} \tag{1.0.1}$$

2 Explanation

Let the matrix is A and b is the vector. So,Ax = b Then we can calculate x = A^{-1} . b

$$\mathbf{A} = \begin{pmatrix} 158 & -378 \\ -378 & 152 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{b} = \begin{pmatrix} -74 \\ -604 \end{pmatrix} \tag{2.0.2}$$

3 Solution

$$\mathbf{A} = \begin{pmatrix} 158 & -378 \\ -378 & 152 \end{pmatrix} \tag{3.0.1}$$

$$\mathbf{b} = \begin{pmatrix} -74 \\ -604 \end{pmatrix} \tag{3.0.2}$$

Augmented Matrix is : Ax = b

$$\mathbf{R} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{3.0.3}$$

Now,multiply R on both sides, we get : RAx = Rb

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 158 & -378 \\ -378 & 152 \end{pmatrix} x = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} -74 \\ -604 \end{pmatrix} \tag{3.0.4}$$

After multiplication of matrix:

$$\begin{pmatrix} 378 & -152 \\ -158 & -378 \end{pmatrix} x = \begin{pmatrix} 604 \\ -74 \end{pmatrix} \tag{3.0.5}$$

Now, multiply the matrix A on both sides,we get: ARAx = ARb

$$\begin{pmatrix} 158 & -378 \\ -378 & 152 \end{pmatrix}\begin{pmatrix} 378 & -152 \\ -158 & -378 \end{pmatrix} x = \begin{pmatrix} 158 & -378 \\ -378 & 152 \end{pmatrix}\begin{pmatrix} 604 \\ -74 \end{pmatrix} \tag{3.0.6}$$

Now, after multiplication of above matrices:

$$\begin{pmatrix} 0 & 118,868 \\ -118,868 & 0 \end{pmatrix} x = \begin{pmatrix} 123404 \\ -239560 \end{pmatrix} \tag{3.0.7}$$

Now, both the rows is row/118,868:

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} x = \begin{pmatrix} 30851/29717 \\ -59890/29717 \end{pmatrix} \tag{3.0.8}$$

As we know EE^{-1} = I, Now inverse of the below matrix is:

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{3.0.9}$$

Now multiply with the inverse of the Matrix in order to get the identity Matrix :

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} x = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 30851/29717 \\ -59890/29717 \end{pmatrix} \tag{3.0.10}$$

So, final matrix is after matrix multiplication:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} x = \begin{pmatrix} 59890/29717 \\ 30851/29717 \end{pmatrix} \tag{3.0.11}$$

So, final result is :

$$x = \begin{pmatrix} 59890/29717 & 30851/29717 \end{pmatrix} \quad (3.0.12)$$

Approach 2 :

RARA x = RAR b where RARA = I, where I is the Identity Matrix

Now as we know Ax = b,

RARA x = RAR b But as we know RARA = I,

Ix = RAR b So, x = RAR b

Above expression illustrates that A^{-1} = RAR because x = A^{-1}b

So, by putting the values of R,A and b we can easily find out the value of x as follows:

RARA x = RAR b

Matrix Multiplication of RA is :

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 158 & -378 \\ -378 & -152 \end{pmatrix} = \begin{pmatrix} 378 & -152 \\ 158 & -378 \end{pmatrix} \quad (3.0.13)$$

Now, using above resultant matrix RA we can evaluate RARA x = RAR b

$$\begin{pmatrix} 378 & -152 \\ 158 & -378 \end{pmatrix}\begin{pmatrix} 378 & -152 \\ 158 & -378 \end{pmatrix} x = \begin{pmatrix} 378 & -152 \\ 158 & -378 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} -74 \\ -604 \end{pmatrix} \quad (3.0.14)$$

After doing matrix multiplication in LHS and RHS,

$$\begin{pmatrix} 118868 & 0 \\ 0 & 118868 \end{pmatrix} x = \begin{pmatrix} 239560 \\ 123404 \end{pmatrix} \quad (3.0.15)$$

Now, divide both the rows by 118868:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} x = \begin{pmatrix} 239560/118868 \\ 123404/118868 \end{pmatrix} \quad (3.0.16)$$

After further calculations in the fractional result:

$$x = \begin{pmatrix} 59890/29717 \\ 30851/29717 \end{pmatrix} \quad (3.0.17)$$