

# Assignment 12

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**Abstract**—This document demonstrate how to check whether the matrix is nilpotent, diagonalizable or not and rank as well as Jordan canonical form of a matrix.

Download latex-tikz from

[https://github.com/priya6971/  
matrix\\_theory\\_EE5609/tree/master/  
Assignment12](https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment12)

## 1 PROBLEM

Let  $V$  be a vector space over  $C$  of all the polynomials in a variable  $X$  of degree atmost 3. Let  $D : V \rightarrow V$  be the linear operator given by differentiation with respect to  $X$ . Let  $A$  be the matrix of  $D$  with respect to some basis for  $V$ . Which of the following are true?

1.  $A$  is nilpotent matrix
2.  $A$  is diagonalizable matrix
3. the rank of  $A$  is 2
4. the Jordan canonical form of  $A$  is

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

## 2 SOLUTION

Given	<p><math>V</math> be a vector space over <math>C</math> of all the polynomials in a variable <math>X</math> of degree atmost 3</p> <p style="text-align: center;"><math>D : P_3 \rightarrow P_3</math></p> <p style="text-align: center;"><math>D : V \rightarrow V</math> be the linear operator given by differentiation wrt <math>X</math></p> <p style="text-align: center;"><math>D(P(x)) \rightarrow P'(x)</math></p> <p style="text-align: center;"><math>A</math> be the matrix of <math>D</math> wrt some basis for <math>V</math></p> <p style="text-align: center;">Assume basis for <math>V</math> be <math>\{1, x, x^2, x^3\}</math></p>
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Matrix	$D(1) = 0 = 0.1 + 0.x + 0.x^2 + 0.x^3$ $D(1) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ $D(x) = 1 = 1.1 + 0.x + 0.x^2 + 0.x^3$ $D(x) = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ $D(x^2) = 2x = 0.1 + 2.x + 0.x^2 + 0.x^3$ $D(x^2) = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \end{pmatrix}$ $D(x^3) = 3x^2 = 0.1 + 0.x + 3.x^2 + 0.x^3$ $D(x^3) = \begin{pmatrix} 0 \\ 0 \\ 3 \\ 0 \end{pmatrix}$ $\text{Matrix } A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$
Nilpotent	$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ <p>All eigen values of matrix A is 0</p> <p>Thus, above matrix is nilpotent matrix</p> <p>Thus, above statement is true</p>

Diagonalizable	$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ <p><math>nullity(A) = 1</math></p> <p>means there exists only one linearly independent eigen vector corresponding to 0 eigen values</p> <p>Thus, matrix A is not Diagonalizable.</p> <p>Thus, above statement is false</p>
Rank	$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ <p>Rank of matrix A is 3</p> <p>Thus, above statement is false</p>
Jordan CF	$A = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ <p>Assume minimal polynomial of A is <math>p_A(x)</math></p> <p><math>p_A(x) = \{x, x^2, x^3, x^4\}</math></p> <p>minimal polynomial always annihilates its matrix. Thus, we see that</p> <p><math>A^4 = 0</math></p> <p>Thus, Jordan canonical form is</p> $\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ <p>which is same as given in the question. Thus, statement is true</p>