Assignment 12

Priya Bhatia

***Abstract*—This document demonstrate how to check whether the matrix is nilpotent,diagonalizable or not and rank as well as Jordan canonical form of a matrix.**

Download latex-tikz from

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https://github.com/priya6971/
    matrix_theory_EE5609/tree/master/
    Assignment12
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1 Problem

Let V be a vector space over C of all the polynomials in a variable X of degree atmost 3. Let $D : V \to V$ be the linear operator given by differentiation with respect to X. Let A be the matrix of D with respect to some basis for V. Which of the following are true?
1.A is nilpotent matrix
2.A is diagonalizable matrix
3.the rank of A is 2
4.the Jordan canonical form of A is

$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

2 Solution

Given	V be a vector space over C of all the polynomials in a variable X of degree atmost 3 $D : P_3 \to P_3$ $D : V \to V$ be the linear operator given by differentiation wrt X $D(P(x)) \to P'(x)$ A be the matrix of D wrt some basis for V Assume basis for V be $\{1, x, x^2, x^3\}$

Matrix	$D(1) = 0 = 0.1 + 0.x + 0.x^2 + 0.x^3$ $D(1) = \begin{pmatrix}0\\0\\0\\0\end{pmatrix}$ $D(x) = 1 = 1.1 + 0.x + 0.x^2 + 0.x^3$ $D(x) = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$ $D(x^2) = 2x = 0.1 + 2.x + 0.x^2 + 0.x^3$ $D(x^2) = \begin{pmatrix}0\\2\\0\\0\end{pmatrix}$ $D(x^3) = 3x^2 = 0.1 + 0.x + 3.x^2 + 0.x^3$ $D(x^3) = \begin{pmatrix}0\\0\\3\\0\end{pmatrix}$ Matrix $A = \begin{pmatrix}0 & 1 & 0 & 0\\0 & 0 & 2 & 0\\0 & 0 & 0 & 3\\0 & 0 & 0 & 0\end{pmatrix}$
Nilpotent	$A = \begin{pmatrix}0 & 1 & 0 & 0\\0 & 0 & 2 & 0\\0 & 0 & 0 & 3\\0 & 0 & 0 & 0\end{pmatrix}$ All eigen values of matrix A is 0 Thus, above matrix is nilpotent matrix Thus, above statement is true
Diagonalizable	$A = \begin{pmatrix}0 & 1 & 0 & 0\\0 & 0 & 2 & 0\\0 & 0 & 0 & 3\\0 & 0 & 0 & 0\end{pmatrix}$ $nullity(A) = 1$ means there exists only one linearly independent eigen vector corresponding to 0 eigen values Thus,matrix A is not Diagonalizable. Thus, above statement is false
Rank	$A = \begin{pmatrix}0 & 1 & 0 & 0\\0 & 0 & 2 & 0\\0 & 0 & 0 & 3\\0 & 0 & 0 & 0\end{pmatrix}$ Rank of matrix A is 3 Thus, above statement is false
Jordan CF	$A = \begin{pmatrix}0 & 1 & 0 & 0\\0 & 0 & 2 & 0\\0 & 0 & 0 & 3\\0 & 0 & 0 & 0\end{pmatrix}$ Assume characterstic polynomial of matrix A is $c_A(x)$ $c_A(x) = x^4$ Assume minimal polynomial of A is $m_A(x)$ $m_A(x)$ always divide $c_A(x)$ $m_A(x) = \{x, x^2, x^3, x^4\}$ Minimal polynomial always annihilates its matrix.Thus,we see that $m_A(A) = \{A = 0, A^2 = 0, A^3 = 0, A^4 = 0\}$ But we see that neither A is zero matrix nor A^2 and A^3 equal to zero but A^4 is equal to zero.Thus,x^4 is minimal polynomial. Thus, Jordan canonical form is of order 4 $\begin{pmatrix}0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\\0 & 0 & 0 & 0\end{pmatrix}$ which is same as given in the question. Thus,statement is true