Assignment 4

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***Abstract*—This document explains QR decomposition with an Example.**

Download python codes from

https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment6/codes

Download latex-tikz codes from

https://github.com/priya6971/matrix_theory_EE5609/tree/master/Assignment6

1 Problem

Find the QR decomposition of

$$\mathbf{A} = \begin{pmatrix} 7 & 3 \\ 2 & 4 \end{pmatrix} \tag{1.0.1}$$

2 Solution

Let $\mathbf{x}$ and $\mathbf{y}$ be the column vectors of the given matrix.

$$\mathbf{x} = \begin{pmatrix} 7 \\ 2 \end{pmatrix} \tag{2.0.1}$$

$$\mathbf{y} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \tag{2.0.2}$$

The column vectors can be expressed as follows,

$$\mathbf{x} = k_1\mathbf{u}_1 \tag{2.0.3}$$

$$\mathbf{y} = r_1\mathbf{u}_1 + k_2\mathbf{u}_2 \tag{2.0.4}$$

Here,

$$k_1 = \|\mathbf{x}\| \tag{2.0.5}$$

$$\mathbf{u}_1 = \frac{\mathbf{x}}{k_1} \tag{2.0.6}$$

$$r_1 = \frac{\mathbf{u}_1^T\mathbf{y}}{\|\mathbf{u}_1\|^2} \tag{2.0.7}$$

$$\mathbf{u}_2 = \frac{\mathbf{y} - r_1\mathbf{u}_1}{\|\mathbf{y} - r_1\mathbf{u}_1\|} \tag{2.0.8}$$

$$k_2 = \mathbf{u}_2^T\mathbf{y} \tag{2.0.9}$$

The (2.0.3) and (2.0.4) can be written as,

$$\begin{pmatrix} \mathbf{x} & \mathbf{y} \end{pmatrix} = \begin{pmatrix} \mathbf{u}_1 & \mathbf{u}_2 \end{pmatrix}\begin{pmatrix} k_1 & r_1 \\ 0 & k_2 \end{pmatrix} \tag{2.0.10}$$

$$\begin{pmatrix} \mathbf{x} & \mathbf{y} \end{pmatrix} = \mathbf{QR} \tag{2.0.11}$$

Now, $\mathbf{R}$ is an upper triangular matrix and also,

$$\mathbf{Q}^T\mathbf{Q} = \mathbf{I} \tag{2.0.12}$$

Now using equations (2.0.5) to (2.0.9) we get,

$$k_1 = \sqrt{7^2 + 2^2} = \sqrt{53} \tag{2.0.13}$$

$$\mathbf{u}_1 = \frac{1}{\sqrt{53}}\begin{pmatrix} 7 \\ 2 \end{pmatrix} \tag{2.0.14}$$

$$\mathbf{u}_1 = \begin{pmatrix} \frac{7}{\sqrt{53}} \\ \frac{2}{\sqrt{53}} \end{pmatrix} \tag{2.0.15}$$

$$r_1 = \begin{pmatrix} \frac{7}{\sqrt{53}} & \frac{2}{\sqrt{53}} \end{pmatrix}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \frac{29}{\sqrt{53}} \tag{2.0.16}$$

$$\mathbf{u}_2 = \frac{1}{\sqrt{53}}\begin{pmatrix} 2 \\ -7 \end{pmatrix} \tag{2.0.17}$$

$$\mathbf{u}_2 = \begin{pmatrix} \frac{2}{\sqrt{53}} \\ -\frac{7}{\sqrt{53}} \end{pmatrix} \tag{2.0.18}$$

$$k_2 = \begin{pmatrix} \frac{2}{\sqrt{53}} & -\frac{7}{\sqrt{53}} \end{pmatrix}\begin{pmatrix} 3 \\ 4 \end{pmatrix} = -\frac{22}{\sqrt{53}} \tag{2.0.19}$$

Thus putting the values from (2.0.13) to (2.0.19) in (2.0.11) we obtain QR decomposition,

$$\begin{pmatrix} 7 & 3 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} \frac{7}{\sqrt{53}} & \frac{2}{\sqrt{53}} \\ \frac{2}{\sqrt{53}} & -\frac{7}{\sqrt{53}} \end{pmatrix}\begin{pmatrix} \sqrt{53} & \frac{29}{\sqrt{53}} \\ 0 & -\frac{22}{\sqrt{53}} \end{pmatrix} \tag{2.0.20}$$

Which can also be written as,

$$\begin{pmatrix} 7 & 3 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} -\frac{7}{\sqrt{53}} & -\frac{2}{\sqrt{53}} \\ -\frac{2}{\sqrt{53}} & \frac{7}{\sqrt{53}} \end{pmatrix}\begin{pmatrix} -\sqrt{53} & -\frac{29}{\sqrt{53}} \\ 0 & \frac{22}{\sqrt{53}} \end{pmatrix} \tag{2.0.21}$$