

CARe Boot Camp

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US CASUALTY EXPOSURE RATING

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Introduction

The purpose of this white paper is to give an overview of possible exposure rating processes for US casualty risks. Details vary between treaty and fac. Where there are significant differences, these will be pointed out. The information in this white paper is current as of July 1, 2004.

The goal of exposure rating is to get an estimate of the expected losses to a reinsured layer. This is nominal (not discounted for investment income) losses, with no provision added for expenses or profit. For both facultative and treaty, it is usual to then combine the exposure rating estimate with an estimate (or estimates) based on the actual experience of the reinsured risk or portfolio. This paper only discusses “pure” exposure rating. It does not discuss how the exposure rating estimate is combined with the experience-based estimate(s). Nor does it discuss hybrid methods that estimate a higher layer based on a lower layer.

Because they are in common usage in the USA, there are a lot of abbreviations in this report. The last page of the report gives a list of abbreviations and their expansions.

Details of exposure rating vary from line to line for various reasons. A major difference is caused by rating bureaus. Rating bureaus collect, analyze, and disseminate insurance information. See the appendix “Rating Bureaus” for a fuller description. The most important US rating bureaus are the Insurance Services Office (ISO) and the National Council on Compensation Insurance (NCCI). The NCCI publishes information about Workers Compensation (WC) insurance. ISO publishes information about a variety of lines of business.

Rating bureaus create severity distributions that are useful for exposure rating. The NCCI model and the ISO models are quite different from one another. In addition, ISO has different models for different lines of business. In any case, it is vital to understand the strengths and limitations of the rating bureau models to use them intelligently. There are also lines of business where no useful rating bureau information is available. An example of such a line is Directors and Officers liability.

Throughout this paper, we will assume that we are talking about a General Liability (GL) risk—more specifically, a US Products liability risk. US Products is a line of business covered by ISO. We will make excursions from this assumption when appropriate, but in the interests of brevity, will not delve extensively into other lines of business. The reader must be especially wary of extrapolating to WC and non-ISO lines what is said about ISO lines. The ISO line that most resembles GL is CAL (commercial auto liability). But even CAL has some material differences from GL.

Standard primary GL policies have two major kinds of limits: per-occurrence and annual aggregate. In this report, we ignore the effect of aggregate limits. Insurance producers are exposed to lawsuits if they sell a policy with too low a limit. For this reason, the general assumption is that the individual primary policy aggregate limit is high enough, and the expected ground-up frequency for that policy is low enough, relatively, that the aggregate limit will have minimal effect on the losses covered under the primary policy. Throughout this paper, when we say “limit,” we mean “per-occurrence limit.” Also, we use the terms “claim” and “occurrence” more or less interchangeably throughout the paper. “Claim” is the more common terminology in reinsurance practice, but, for various annoying reasons involving

lawyers, ISO uses “occurrence.” Typically, reinsurance contracts are defined in terms of occurrences as ISO defines them. In any case, the distinction between an occurrence and a claim is minor in most cases for casualty lines.

Typical casualty treaties cover certain kinds of losses for which exposure rating does not provide an estimate. For instance, if a trucker hurts himself and someone in another vehicle in an accident, the WC loss from the trucker and the CAL loss from the other hurt person are often added together in defining “loss” for purposes of the treaty. The individual WC loss and CAL loss separately might each be below the retention, but the sum of the two exceeds the retention. This kind of loss is called “clash.” The person doing the pricing needs to be aware that the standard exposure rate includes no provision for clash and certain other kinds of loss exposure.

Exposure rating can be used to price an excess layer on an excess contract. This is not discussed in this paper.

Outline of the process

You can estimate the expected losses to a layer by following a series of steps. The basic steps are conceptually as follows.

Table 1
Steps in exposure rating

0.	Subdivide the data into categories.
1.	Estimate ground-up losses.
2.	Get a pure loss severity distribution.
3.	Use the pure loss severity distribution to estimate the average severity per ground-up claim.
4.	Divide to get an estimate of the expected number of ground-up claims.
5.	From the severity distribution read off the probability of a ground-up claim exceeding the retention.
6.	Also read off the average severity in the layer from the severity distribution.
7.	Load for excess ALAE (Allocated Loss Adjustment Expense).

The exposure-rating estimate of expected losses to the layer is (the expected number of ground-up claims) times (the probability of a ground-up claim exceeding the retention) times (the average severity in the layer). This is equivalent to (loss to layer) = (ground-up frequency) x [severity to layer (per ground-up claim)]. This latter formulation is the one actually used most often in practice. But it is useful conceptually to realize that the exposure-rating process gives you estimates of the excess frequency and severity.

Below, we give more details about each of the steps.

0. Subdividing the data

The information for the risk being exposure-rated needs to be subdivided into categories corresponding to the severity distributions available and by policy limit. The “Rating Bureaus” appendix sketches the ILF (Increased Limits Factor) table structure that ISO uses. Treaty

submission data is rarely provided in the full detail corresponding to the ISO ILF table structure.

Treaty submission data also comes subdivided into categories that need to be combined. For instance, GL coverage is provided under Commercial Multi-Peril (package policies =CMP) and also on a monoline basis. The GL portion of CMP and the monoline GL correspond to the same ISO tables and need to be combined for exposure rating.

In subdividing and re-combining the treaty submission data, practitioners often use simple ratio estimates. Obviously, you should use company information if known. Otherwise, you can use defaults based on bureau information or external information (such as A M Best's).

While there is not much to say technically about this topic, the actual creation and maintenance of this feature in the exposure rating tools takes a substantial effort.

1. Getting the ground-up losses

Getting the ground-up losses is the step where fac and treaty differ the most. For treaty, you typically estimate the ground-up losses by multiplying the subject premium by a loss ratio pick. For fac, you typically multiply the exposures by a loss cost rate to estimate the ground-up losses and ALAE, and then strip out the ground-up ALAE.

The choice of method is dictated by the information available. Historically, treaty submissions have not included exposure information by class, so the practitioner was forced to use the subject premium by default. Fac submissions usually include the premium the client is charging. But the exposure information is more detailed than client premium.

Also, the adequacy of the client premium will vary from fac risk to fac risk. When the client is underwriting on an account basis, (e.g., GL + CAL + WC), the premium charged for one part of the account (e.g., WC only) might be inadequate. The client does not care, as long as the sum of the premiums is adequate for the all the lines being covered. But that one line may be the one the client is seeking reinsurance on.

Treaty

There are several methods for estimating the client loss ratio. The most commonly used method is to estimate a loss ratio based on client-specific data for the program. Another method uses Annual Statement data to estimate the client loss ratio. Pure judgment is also used, but this is rare. In any case, the premium we multiply the loss ratio by is the client's projected subject premium for the treaty.

Using specific program data

Ground-up client losses should be developed and trended. These are compared to on-leveled premiums to get an estimate of the loss ratio in the coverage year. For WC, law amendment factors also need to be considered. You might also compare the result to an estimate of the industry loss ratio and select a compromise estimate.

Issues

The accuracy of client rate change information is sometimes problematic. Rate changes can be hard to estimate because of schedule credits etc.; also changing terms, deductibles, mix of business, and policy limits may make it hard to estimate the true rate change.

The credibility of client data may be low.

The estimate of ground-up losses is based on historical data and is vulnerable to shifts in mix of business and the insurance environment over time.

The experience provided by the client is typically for all policy limits. What we really want is a loss ratio estimate for the policy limits that expose the layers we are pricing.

Clients sometimes provide “as-if” information. In the data they provide, they exclude all losses (and, sometimes, the corresponding premium) arising from a certain category of business. Their justification for this is that they no longer write that category of business. Categories might be: a class of business, business written by an underwriter who was fired, a state, producer, etc. The practitioner needs to examine the totality of experience and the as-if experience to make a judgment about how much credibility to give the as-if experience.

Using Annual Statement data

It is possible to estimate a loss ratio based on Annual Statement data. The Annual Statement is a standard accounting statement that virtually all non-life insurers in the US must complete. The most useful AS data is on “Schedule P.” Schedule P has loss triangles by Annual Statement line of business (ASLOB). The triangles include ALAE. There is also corresponding premium information. Annual Statement Schedule P data is (with one exception) net of reinsurance. For the part of the data which is gross of reinsurance, only the latest diagonal is available. A vendor called OneSource compiles and sells Annual Statement data electronically for virtually every insurer in the US.

Usually, we want to estimate a pure gross loss ratio for each ASLOB. (More generally, we want to estimate the loss ratio net of any reinsurance that inures to the benefit of the layer being exposure-rated.) The loss ratios in the AS are historical. They need to be adjusted to projected conditions for the reinsurance coverage period.

Issues

Annual Statement Schedule P data is by ASLOB which does not necessarily match the reinsurance program LOBs (Lines Of Business). For instance, the Annual Statement line of business "Other Liability" includes GL—an ISO line of business—as well as D&O (Directors and Officers liability) —a non-bureau line of business—among other things. Even a homogenous ASLOB such as CAL may contain information for things such as national accounts mixed with "Main Street" business. These two programs may have different loss ratios, and may or may not be subject of the reinsurance.

Annual Statement Schedule P data is (with one exception) net of reinsurance. For the part of the data which is gross of reinsurance, only the latest diagonal is available.

Annual Statement Schedule P data is on an Accident Year basis for most ASLOBs. When the treaty being contemplated is on a losses-occurring basis, there is no mismatch, but when it is on a policies-written basis, there is.

Judgment

Judgment is used when there is no possibility of using other methods. Obviously, the accuracy of pure judgment can be low.

Issues

People tend to be overoptimistic and overconfident in their judgments. Also, using pure judgment in the very first step makes accuracy in the rest of the process look silly.

Other Information

It is common to ask for and get a client's actuarial reserve report. This can contain information about reserves, loss development, loss ratios, ground-up and excess ALAE ratios, etc. Depending on the information provided and its credibility, the information may be used to supplement the analysis described here.

Fac

There are various methods for getting the ground-up losses and ALAE for fac. The most common method is to multiply the exposures by a manual loss cost rate. A loss cost rate is an estimate of the ratio of certain costs, including losses, to exposures. A rating bureau such as ISO can be the source of manual loss cost rates. A vendor called Silver Plume supplies the ISO manuals. The ISO manuals have the information necessary to create the manual loss cost rates. It is also possible to use internal rates, if the reinsurance department is part of a larger company that also writes primary business.

An alternative method is to estimate the expected fgu losses as the Expected Loss Ratio (ELR) x Client Premium. This method resembles the method used for treaties. Because the individual Line of Business premium adequacy is problematic, the method may not be very accurate.

Finally, it may be necessary to use pure judgment to estimate the ground-up losses and ALAE.

ISO manual loss cost rate method

In most states, the ISO manual loss cost rate contains provisions for expected basic limits loss costs, ALAE, and unallocated loss adjustment expenses (ULAE). ULAE represents claim expenses that cannot be directly traced to an individual claim. Examples would be things like office rental costs, overhead charged by IT, and so forth.

The inclusion of ULAE makes it necessary to have an additional initial step for fac certs in order to get the ground-up loss and ALAE: stripping out the ULAE. Luckily, ISO uses a simple percentage allocation formula: $ULAE = K\% \times (Loss + ALAE)$, so it is simple to divide out the ULAE from the manual loss cost rate.

Typically, CAL and GL policies are sold with relatively low policy limits. In the ISO data base, roughly 80% or more of policies have a limit of \$1 million. Another big chunk of the business is sold with a limit of \$500,000. Only a small percentage is sold with limits above \$1 million. [This may be done for regulatory reasons. Insureds get coverage above their primary policy from excess or umbrella policies. These policies are subject to much less regulation regarding pricing or terms and conditions. Thus, insurers may prefer to write relatively low primary policy limits, to keep more of the total premium out of the regulator's domain.]

Basic Limits and Increased Limits To be able to use data simultaneously from policies with different limits, ISO defines the concept of a "basic limit." This is an artificial policy limit selected by ISO. It is chosen to be low enough that virtually all policies have limits at or above the basic limit. ISO then caps all losses at the basic limit for their main analysis. The result of the main analysis is a set of basic limit loss cost rate estimates (\$ per unit of exposure). They have a separate analysis to find the increased limits factors that adjust the basic limit loss cost rate to the policy limit actually sold.

ISO changes the basic limit, but only rarely. Unfortunately, now (2004) is one of those times. The CAL basic limit was \$25,000 but is being raised to \$100,000. Because the change is implemented state by state, the current manual shows CAL loss cost rates that are for a mix of \$25,000 and \$100,000 basic limits. The GL basic limit is \$100,000.

Issues

For various regulatory reasons, ISO does not publish loss costs rates for every ISO line in all states. In some states, the components of the ISO published rate may differ from those described here.

Not all risks are in the ISO database. This is especially a problem for large risks.

The rates ISO publishes represent the average risk in the ISO database. The risk you are rating differs from average in several ways. Notable is that frequency per unit of exposure tends to drop as the size of the risk increases, probably because larger risks can afford to spend more on loss control. Larger risks are more likely to be facultatively reinsured, so the ISO exposure rate tends to be too high for them.

The loss cost rate published by ISO is one that has been approved by a state regulator. Because regulation varies from state to state and over time, the adequacy of the published loss costs may vary from state to state and from time to time. ISO also changes its methodology occasionally (so that the ISO loss cost rate inherent adequacy changes), but this is less frequent.

Your client does not necessarily rate risks the same way ISO does, so the submission data might not allow you to get an exact ISO rate. For example, consider commercial auto "zone-rated risks." The ISO manual loss cost rate is based on starting and ending zones (e.g. New York City to Detroit). Some clients do not zone-rate, so they can't tell you which zones to use.

As a risk gets bigger, it becomes harder and harder to get all the manual loss costs rates. At the same time, the credibility applied to the exposure rate gets smaller and smaller. This makes it tempting to use shortcuts in getting the exposure rate. An example would be a large commercial auto risk with exposures in many territories around the country. It is very tempting for the person doing the exposure rating to pick a few "typical" territories to rate in order to save time. It is unclear how good people are at picking "typical" territories.

When ISO files loss costs rates, they assume that they will be effective for one year after the filing effective date. They make the assumption that the average policy will incept six months after the filing effective date and the average loss will occur six months after the average policy inception date: one year after the filing effective date. This chain of assumptions allows ISO to figure out how far to trend losses from the historical period. The actual policy effective date for a fac cert will differ from the ISO assumption by up to six months.

Stripping out the ground-up ALAE

When this step is necessary, you can strip out the ALAE from the estimated ground-up loss and ALAE by multiplying the estimated ground-up loss and ALAE by a factor. The factor represents the proportion of the total that is pure loss. You can get the factor from severity distributions based on ISO information and the ISO estimated ALAE per ground-up claim.

You can calculate the average limited loss per ground-up claim using the severity distribution. In American actuarial jargon, this value is usually called the Limited Expected Value (LEV). [The term Limited Average Severity (LAS) is also sometimes used. LAS is identical to LEV.] So, we might speak of LEV(1,000), meaning, the limited expected value when losses are limited to 1,000. In calculating ILFs, ISO creates an estimate of the expected ALAE per ground-up claim. You can use this value and the LEV to find the factor you need.

The formula for the LEV is given in section 3: Getting the average severity per ground-up claim.

It is easy to see that when there is a single policy with policy limit K, the proportion of ground-up loss and ALAE that is pure loss is, on the average, $LEV(K) / (ALAESEV + LEV(K))$, where ALAESEV represents the expected ALAE per ground-up claim. You

can multiply your estimate of this proportion times the ground-up loss and ALAE to get an estimate of the pure loss.

2. Getting the severity distributions

Severity distributions can be used several ways in exposure rating. The source of the severity distributions varies by line. For lines where there is a rating bureau, you can use rating bureau information in some way, although not necessarily by directly adopting the rating bureau severity distribution.

ISO creates severity distributions in order to calculate increased limits factors. The following is a description of the ISO method for getting ILFs. Emphasis is put on getting the severity distributions, with the other parts of the ILFs only mentioned in passing.

ISO ILFs

Purpose

ISO creates increased limits factors (ILFs) in order to rate primary policy limits higher than the basic limit. ISO ILFs are the ratio of two numbers. The numerator represents costs at the policy limit. The denominator represents costs at the basic limit. The costs are expected limited loss costs, all ALAE, ULAE, and risk load. In ISO's methodology, the expected ALAE per claim is assumed to be the same for all policy limits, because the legal obligation to defend the claim exists regardless of the policy limit. The other costs increase as the policy limit increases. See the appendix "Constructing an ISO ILF" for a sample calculation. In the ILF calculation, all values are expressed as severities per ground-up claim. This means that the expected limited loss costs are LEV(policy limit) and LEV(basic limit).

Categories of ILFs (ILF tables)

ISO has separate increased limits tables for different categories of business. The tables are created to reflect differences in severity and ALAE per ground-up claim, keeping in mind the necessity of having a credible body of data. For instance, ISO has broken Products liability into three increased limits tables: Table A, B, and C, where A is least severe, C most severe. See the "Rating Bureaus" appendix for a description of the other ISO tables. For Products liability, the class of product determines the increased limits table for the risk.

Structure of severity distributions

The ISO severity distributions are mixtures of between 4 and 8 exponential distributions. The individual means and weights do not by themselves have distinct interpretations. For instance, there is not a separate category of claim corresponding to the first mean and weight. The largest mean is capped at \$10,000,000.

The exponential distribution can be defined by the equation $S(x) = \exp(-x/\mu)$.

Here $S(x)$ is the survival function: $S(x)$ is the probability that a claim exceeds x . The parameter μ is the mean of the exponential distribution. A mixed exponential distribution is the weighted sum of individual distributions. Here

$$S(x) = \sum_{i=1}^n w_i S_i(x), \text{ where each } S_i(x) \text{ is the survival function of an individual exponential}$$

distribution and $S(x)$ is the overall survival function. Each w_i is a fixed weight (with $\sum_{i=1}^n w_i = 1$).

The table below gives an example of parameters similar to an ISO mixed exponential distribution.

Table 2
ISO-like mixed exponential distribution

Distribution	Mean	Weight
1	1,500	38%
2	5,000	31%
3	25,000	18%
4	90,000	9%
5	400,000	3%
6	1,700,000	0.9%
7	10,000,000	0.1%

Weights and means are fake.

Below is a description of how ISO estimates the parameters of their severity distributions. After describing the sources of the data and the data collected, we describe the steps in the process: trend, development, construction of empirical distributions, tails, and curve fitting.

Sources of Data

ISO uses two sources of data for doing an increased limits review. The first source of data is individual loss information from primary policies. The second source of data is a special call for individual loss information from umbrella and excess policies. The primary policy information underlying the 2003 review for ISO Products table B consists of information about 189,174 closed occurrences with accident dates ranging from 1984 to 2001. The corresponding Products excess and umbrella data consists of 2,040 occurrences with accident dates from 1980 to 2001. Note that the excess and umbrella data does not have information that allows identification of the increased limits table .

Primary policies in the United States—at least those reported to ISO—tend to have fairly low policy limits. In Table 3 below is a policy limit distribution similar to that from ISO for Products liability. For getting the distribution of claims above \$3 million, the ISO database is only 2% as big as it first appears. ISO collects information on the insured loss amount, not the total loss amount, so the scarcity of higher limits significantly diminishes the amount of large loss information available.

Table 3
Policy Limit Distribution Similar to that for ISO Products Liability

Policy Limit (\$,000)	Basic Limit Loss Weight
100	1%
300	2%
500	6%
750	0%
1,000	80%
1,500	1%
2,000	8%
3,000	1%
5,000	2%
<i>TOTAL</i>	<i>100%</i>

Because the low primary policy limits cut out so much of the large loss information, ISO collects data from excess and umbrella policies. However, there are some complications. Participation in the excess and umbrella call is voluntary and an insurer that reports primary policy loss information does not necessarily report excess and umbrella information. In addition, the losses reported in the excess and umbrella call are not matched up with losses from the underlying primary policy. In some cases, the primary insurer does not report to ISO so matching up is not even theoretically possible. But even when the same insurer wrote the primary policy and the umbrella policy, the losses are not reported with fields that would allow the parts of the loss to be matched up.

Data Collected

ISO collects paid loss amounts, accident date, payment date, class code (for Products, this is based on the kind of product, such as paper products), location (state) of loss, deductible or retention (if any), and various other fields. The information is reported on a transaction basis. This means that when there is more than one payment on an occurrence, ISO must reconstruct the occurrence from the individual payment records. There is an occurrence ID field on the records to allow this to be done. In all cases, ISO works with closed claim information. This way, they do not have to deal with the vagaries of case reserves whose adequacy varies from time to time and insurer to insurer.

For certain risks, ISO does not receive class information. Instead, they receive less precise information. Unfortunately, these risks, called Composite-Rated Risks (CRR), are typically the largest risks in the ISO database. Currently, it is not possible to map the CRR information directly to an increased limits table. ISO does a Bayesian analysis to map the data (with the odd result that a given claim might be mapped 31% to table A, 44% to table B, etc.). Table 4 below is similar to data from a 2000 ISO presentation and shows the importance of CRR data.

Table 4
Composite Rated Risk Data
Percent of Increased Limits Data
Typical, albeit fake, values

	Total	Policies with limits above \$1,000,000
Auto	25%	75%
Premises/Operations	25%	50%
Products/Completed Operations	30%	80%

Trend

By trend we mean here severity trend. Other kinds of trend should have been treated in the estimation of ground-up losses and ALAE. ISO assumes that there is uniform trend—that is, that (ignoring policy limits) claims trend at a uniform rate, regardless of size. ISO also assumes that there is a single uniform trend that applies over the entire time span of the data they use. Thus, they assume that, except for the capping effect of policy limits, each claim would grow by (say) 7% per annum, for every claim in the data and for each year. They pick the value (7%) after looking at various limited trends. They then trend each individual claim in the data. They also trend the policy limit and any retention or deductible by the same factor. The trend period is from the midpoint of the year of the accident to a common point one year after the planned effective date of the increased limits review.

Development

You can conceive of the ISO data as forming a triangle, with accident year and payout lag defining the vertical and horizontal dimensions of the triangle. ISO defines payout lag as

$$\text{Payout lag} = (\text{payout year} - \text{accident year}) + 1.$$

For occurrences that encompass several partial payments, the payout year is the dollar-weighted average of the payout years of the partial payments.

ISO uses closed claims in the analysis, so each claim appear in only one place in the triangle.

After the claims are trended, the claims in the same payout lag column are on a comparable basis (each is trended to one year past the assumed effective date of the increased limits review). Thus, trending allows ISO to collapse each payout lag column in the triangle into a single point containing trended claims from all the accident years—all the claims with the same payout lag.

For combining the various payout lags, ISO analyzes the from-ground-up information to get ultimate percentages of claim counts falling into each payout lag.

As described below, they create empirical survival functions for each payout lag. These survival functions are based on the trended claims from all accident years. They then combine the empirical survival function for the different payout lags using the ultimate payout count percentages. Thus the construction of the empirical distribution across accident years and payout lags includes the adjustments for trend and development.

Construction of empirical distributions

ISO constructs empirical survival functions for each payout lag, with all lags from lag 7 and beyond combined together. ISO divides the loss domain into numerous small intervals and uses the claim information to create empirical estimates of conditional survival probabilities: the probability that a given claim exceeds the top of a interval, once it exceeds the bottom of the interval. The notation $\text{Prob}(X > 750 | X > 500) = 50\%$ means “the probability that X (the severity) exceeds 750, given that X is larger than 500, is 50%.” Once ISO has the individual conditional survival probabilities, they chain them together to get the whole empirical survival function. See the table below for an example of how the probabilities are chained together.

Table 5
Chaining conditional survival probabilities to get the survival function

Interval number	Interval Top="T"	Interval Bottom= "B"	Prob($X > T X > B$)	S(T)
1	250	0	30%	30%
2	500	250	40%	12% = 40% x 30%
3	750	500	50%	6% = 50% x 12%

Table 6 below gives an example showing how ISO constructs empirical conditional survival probabilities from individual claim information. Consider the following data and the interval 250 x 500.

Table 6
Example showing how ISO constructs empirical conditional survival probabilities

Claim No.	Retention	Ground-up loss	Insured Loss	Policy Limit	Claim eligible?	Eligible and Ground-up loss > 750?
1.	-	400	400	1,000	No (loss < 500)	-
2.	-	600	600	700	No (PL < 750)	-
3.	-	600	600	1,000	Yes	No
4.	-	800	800	1,000	Yes	Yes
5.	300	700	400	1,000	Yes	No
6.	300	1,000	700	1,000	Yes	Yes
7.	600	1,599	999	1,000	No (Ret > 500)	-

ISO counts the number of eligible claims: Those for which there is sufficient information to see that they *could have* exceeded the top, given that they exceeded the bottom of the interval. And they also count the number of eligible claims which actually *did* exceed the top of the interval.

To be eligible, a claim must satisfy three conditions.

1. The retention must be at or below the bottom of the interval ($\text{ret} \leq 500$).
2. The primary exit point (=policy limit + retention) must be at or above the top of the interval (exit point ≥ 750).

3. The ground-up loss amount must exceed the bottom of the interval (ground-up loss > 500).

In this case, ISO would count as eligible claims 3, 4, 5, and 6. They would count claims 4 and 6 as actually exceeding the interval top. Thus ISO would boil the individual claim information down to: $\text{Prob}(X > 750 | X > 500) = 2/4 = 50\%$, and this is the value that appears in the row for interval number 3 in Table 5.

Tails

Because the data is sparse in the tail, ISO uses a Pareto distribution above a truncation point to smooth the empirical distribution. For instance, for Products, the empirical distribution above \$4 million is replaced by a Pareto distribution. The parameter of the Pareto distribution is selected after looking at percentile-matching information. Selecting the parameter allows ISO to preserve table relationships.

Fitting

ISO uses the minimum distance method to fit their mixture of exponential distributions to the empirical distribution. In the minimum distance method, the distance criterion is defined as the weighted sum of the absolute differences between the empirical distribution function and the fitted distribution function. The parameters of the fitted distribution are juggled until the distance criterion is minimized (usually using Newton-Raphson or some similar algorithm). The points at which the distribution functions are evaluated, and the weights given to the absolute differences, are picked judgmentally, considering the purpose for which the curve is being fitted.

Possible modifications of the ISO severity distributions

Trend

The ISO severity distribution is trended to a certain assumed loss date. You can trend the distribution backwards or forwards to match the average loss date that will be suffered under the reinsurance contract you are pricing. This is accomplished simply by adjusting each μ (mean of the exponential distribution).

Parameter uncertainty

Because the parameters underlying the ISO severity distributions are only estimates, you can adjust the distributions to reflect parameter uncertainty. Generally, this adjustment will thicken the tails of the distributions.

Anti-selection

ISO assumes that there is no anti-selection. That is, within an increased limits table, all losses are assumed to be drawn from the same underlying severity distribution, regardless of the policy limit. This allows ISO to combine data from different policy limits.

There is a feeling on the part of some actuaries that risks that buy higher policy limits do so for a reason: Their severity distribution is more severe than average. The reasoning is

plausible, because larger risks are viewed as “deep pockets” and so may have to pay more punitive and emotional damages than a smaller risk causing the same injuries.

An argument against this view is that the larger risks have more money for loss control. But loss control is generally thought to be more effective on frequency than severity. Another argument used is that buying higher limits is a sign of responsibility, so the risk buying smaller limits is the one more likely to suffer huge claims.

It is possible to adjust the severity distributions, based on the actual primary policy limit, to reflect possible anti-selection.

3. Getting the average severity per ground-up claim

The formula for LEV(Policy Limit) is $LEV(PL) = \int_0^{PL} x \cdot f(x) dx + (PL) \cdot [1 - F(PL)] = \int_0^{PL} S(x) dx$.

Here LEV is the Limited Expected Value function. PL is the Policy Limit. The variable x represents the severity (\$) of an individual claim. F(x) is the distribution function of the severity ($F(x) = 1 - S(x)$) and f(x) is the density function of the severity.

4. Dividing to get the estimate of ground-up claims

This step needs no explanation.

5. Getting the probability of a ground-up claim exceeding the retention

This is simply S(Retention), where S(x) is the survival function after all the adjustments have been made.

6. Getting the average severity of a claim in the layer

There is a formula for finding the average severity of a claim, given that it exceeds the retention. The formula is not given here.

Note that in many cases, the practitioner need not actually carry out steps 5 and 6. Instead, he/she calculates the combined effect. The expected loss to the layer, given a ground-up claim, is the product of the result of step 5 and step 6. But it is also the difference in the LEVs at the top and bottom. One should take care about the definition of “top” and “bottom” for this calculation. For example, for a 4 x 1 layer, you would compute $LEV(5) - LEV(1)$ for a policy limit of 5. For a policy limit of 2, you would compute $LEV(2) - LEV(1)$.

7. Loading for excess ALAE

Typically, ALAE in a reinsurance contract is covered on a pro-rata basis or is added to the loss in determining the reinsurance loss. Also, the primary policy can cover ALAE in two ways. The primary policy may cover ALAE in a basically unlimited way (this is what the

standard ISO policy forms do) or ALAE may be subject to a limit (for instance, added to the loss before application of a joint limit).

With two ways for reinsurance to cover ALAE and two ways for primary insurance to cover ALAE, there are four total cases to consider. While there are methods for handling all these cases, in this paper we will assume that for the primary coverage ALAE is unlimited, since that is what ISO does.

We take slightly different paths depending on whether the reinsurance coverage is pro-rata or added to.

Pro-rata

When ALAE is covered pro-rata, you can estimate of the ratio of the covered ALAE to the covered loss. Multiply the pure loss exposure rate by the estimated ratio to estimate the reinsured ALAE.

Added to

In this case you can use a ratio adjustment like that for the pro-rata case. Depending on the layer, you might need to make more sophisticated adjustments. This is because ALAE “added to” creates exposure where there is no pure loss exposure. For instance, when you have a 4 x 1 layer, a policy limit of 1 normally does not expose the layer. But adding the ALAE to the loss means that some exposure does exist. In the interests of brevity we will not delve into the adjustments. They involve adjusting the policy limits and the severity distribution. The most theoretically correct method is to create a joint loss and ALAE distribution and integrate it against a function representing the reinsurance amount that would be payable for each combination of loss and ALAE. ISO provides a tool that essentially does this.

Appendix: Rating Bureaus

Rating bureaus fill a variety of functions. Rating bureaus collect data from primary insurance companies and file reports based on that data with state regulators. They also analyze the data they collect and file loss cost rates and related items—such as increased limits factors—with the regulators. In order to ensure uniformity in the data, rating bureaus define standard policy forms, which they file with the state regulators for approval. They also have elaborate standards (“statistical plans”) for reporting the data.

Reporting data to a rating bureau is voluntary, as is the decision to use—partially or wholly—the rates and factors, etc. filed by the bureau. Insurers can and do use bureau services and report their data for some parts of their business while ignoring the bureaus for other parts of their business.

Table A1
Lines of Business covered by ISO

1.	AL = CAL = Commercial Auto Liability
2.	GL = General Liability
3.	PAL = Personal Auto liability
4.	HO = the liability portion of Homeowners policies
5.	FO = the liability portion of Farmowners policies
6.	Medical Malpractice
7.	Lawyers
8.	Personal Umbrella
9.	Commercial Umbrella
10.	Plus various non-casualty lines of business

Table A2
Lines of Business covered by the NCCI

1.	WC = Workers Compensation
2.	EL = Employers Liability
3.	USL&H = Coverage under the US Longshoremen's and Harbor Workers' Act

Table A3
ISO Products/Completed Operations ILF Tables

Table A	Table B	Table C
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Table A is least severe, C is most severe. Tables are countrywide—they do not vary by state

Table A4
ISO Premises/Operations ILF Tables

	Table 1	Table 2	Table 3
State Group 1	SG 1 T1	SG1 T2	SG1 T3
State Group 2	Etc.		
...			
State Group 10			

Table 1 is least severe, 3 is most severe

Table A5
ISO Commercial Auto Liability ILF Tables

	Light & Medium	Heavy	Extra Heavy	All Other	Zone-rated Risks
State Group 1	SG1 L&M	SG1 H	SG 1 XH	SG 1 AO	ZRR (only one countrywide table)
State Group 2	Etc.	Etc.			
...					
State Group 8					

All Other includes Private Passenger Types, Publics [= Taxis, Limos, Buses], Garages, Trailers, Fire/Ambulance.

Zone-Rated Risks are long-haul trucks.

Table A6
ISO Medical Malpractice ILF Tables

Physicians
Surgeons
Hospitals
Nurses and other allied health professionals
Dentists
Nursing Homes

Tables are countrywide—they do not vary by state

Appendix: Constructing an ISO ILF

Table A7

PRODUCTS/COMPLETED OPERATIONS LIABILITY – MULTISTATE Increased Limit Factors Mixed Exponential Distribution

Table B

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Policy	Limited			ILF	Process	Parameter	ILF
Limit	Average	ALAE per	ULAE per	Without	Risk	Risk	With
(\$,000)	Severity	Occurrence	Occurrence	Risk Load	Load	Load	Risk Load
100	15,918	16,380	2,099	1.00	347	3779	1.00
1,000	32,837	16,380	3,199	1.52	3,757	7,837	1.66

Source: ISO 2003 Increased Limits Review

ILF without risk load = [(2)+(3)+(4) at policy limit] / [(2)+(3)+(4) at basic limit].

ILF with risk load = [(2)+(3)+(4)+(6)+(7) at policy limit] / [(2)+(3)+(4)+(6)+(7) at basic limit].

Abbreviations

ALAE	Allocated Loss Adjustment Expense
ASLOB	Annual Statement Line of Business
CAL	Commercial Auto Liability
CMP	Commercial Multi-Peril (Package Policies that combine property and casualty)
CRR	Composite-Rated Risks
D&O	Directors and Officers
ELR	Expected Loss Ratio
GL	General Liability
ILF	Increased Limits Factor
ISO	Insurance Services Office
LAS	Limited Average Severity
LDF	Loss Development Factor
LEV	Limited Expected Value
LOBs	Lines of Business
NCCI	National Council on Compensation Insurance
ULAE	Unallocated Loss Adjustment Expense
WC	Workers Compensation
ZRR	Zone-Rated Risks