A Theory of Finite Indexing

Clinton P. Mah and Raymond J. D'Amore, formerly of PAR Technology Corporation

1. Introduction

In 1975, Professor Girard Salton of Cornell University published his monograph *A Theory of Indexing*. This became a classic of natural language text analysis, and its ideas are still widely employed in 21st Century information systems.

Much of the knowledge of modern civilization is recorded in natural language text documents. This is hard for computational systems to access because the structure of language is complex, highly unpredictable, and often quite irregular. Different languages, or even dialects in the same language, can operate under different rules and can be hard to map into each other.

For example, Mandarin Chinese and English are the two most widely spoken languages in the world today. They evolved independently over many millennia and have made incompatible choices in grammar that make translations between them problematic. Chinese has no true relative clauses, an awkward passive voice, and odd sentence particles.

Salton's insight was that messy linguistic details are unimportant in text data applications dealing only with general content. To see what a text document is about, it is enough just to see what words occur in the document, regardless of how they are formed or how they are grammatically related.

In particular, Salton would represent a document d in a data collection D as a numerical vector v^d defined as

$$v^d = \langle f_w^d \rangle_{w \in W}$$

where W is the set of all distinct root words in D and f_w^d is the number of times that a word or word root $w \in W$ occurs in document d. Index set W will be unbounded. Whenever new documents enter a data collection, they often contain words or word roots never seen before.

Given another vector u^e for another document e in D, we can compute a raw simlarity score between them as an inner product

$$IP(u^e, v^d) = \sum_{w \in W} f_w^e \cdot f_w^d$$

This formula works only if documents d and e both are finite. The inner product here is still awkward for measuring similarity. It is sensitive to the length of documents being compared and has no upper bound. Salton tried to skirt these problems by normalizing document vectors to a unity length in a Euclidean vector space.

So, instead of working with a vector v^d, Salton used

$$\hat{v}^d = \frac{v^d}{[\ \sum_{w \in W} (f_w^d)^2\ ]^{1/2}}$$

and then defined

$$cosine(u^e, v^d) = IP(\hat{u}^e, \hat{v}^d)$$

This is called the cosine measure of similarity. It can be interpreted as the cosine of the angle between $\hat{u}$ and $\hat{v}$ in an infinite-dimensional Euclidean space of document vectors. We have

$$0 \leq cosine(u^e, v^d) \leq 1$$

where 1 is an exact match and 0 is none at all. The cosine measure is often used in document searches to get the best match in a collection D for a document d for a query q and also to cluster documents by similarity. Such algorithms typically benefit from setting some minimum similarity threshold on similarity to reduce computation; but it is unclear how high such a measure must be to become significant.

2. Finite Modeling

A standard engineering practice is to define a simplified model to expedite the design for a major project. For example, we can reduce a bridge to a structural skeleton on which to check the stresses and strains that it must handle under expected use. Models can also help in problems like finding possible hot spots on a packed silicon chip at various voltage of operation or speeding realistic computer animation of long hair blowing in the wind.

Salton's numerical vector representation of text documents is such a model, which has had success in many information applications. We must remember, however, that a model is only one possible approximation of the real world. As such systems evolve, they often reveal the shortcomings in their initial modeling, forcing re-examination of assumptions. After almost a half century, can we do better than Salton's approach?

One obvious shortcoming with Salton's vector representation is the infinite dimensionality of his vectors. Since individual documents will always have a finite number of words, we can finesse infinity by computing with sparse vector encodings, but this is tricky in document collections with highly dynamic content. Its set of indexing features over time will grow and so must their associated vectors.

Deep neural nets highlight the difficulties here clearly. These were invented early in the 21st Century and have led to tremendous progress in artificial intelligence through unsupervised machine learning from vector data, but the input layers of such networks always have some fixed number M of nodes that all their data has to match up with. With vectors growing uncontrollably in dimensionality, any network can quickly become obsolete.

To transform a vector counting finitely many occurrences of words or word roots $w \in W$ to a new vector counting occurrences of $x \in X$, a finite index set, we have many options. The simplest is a linear transformation

$$T^M : W^\infty \to X^M$$

where T^M might be represented as a rectangular matrix with M rows of unbounded length. That can be hard to compute with, but we can choose a target index set X so that each row will have only a finite number of non-zero entries and so be represented compactly as a sparse vector. If $d \in D$ has a vector v^d of finitely many word counts, it is transformable into a finite indexing vector in X by muliplying it as a column vector with our matrix for T^M.

$$T^M(v^d) = [T^M] \circ v^d$$

There are infinitely many ways to define T^M. The challenge here is to find some right way for particular purposes.

3. Evaluating Finite Index Sets

Our discussion now will shift exclusively to English text written in a Latin alphabet. In non-alphabetic languages like Chinese (ideographic), Korean (syllabic), or Japanese (ideographic plus two different syllabaries), defining a finite set of indexing features for text will be quite different and much harder.

Let us first lay out some general guidelines for setting up finite indexing for English. We shall mainly look at the likelihood that each x in a candidate index set X will occur in a document d in D. With such probabilities $\{ p_x \}$, along with a little basic information science and knowledge of English, we might establish four general goals to aim for:

power Text in widely used languages like English and Chinese often assume that a reader willl understand about only about 10^4 distinct words. In America, this is about the vocabulary of the average educated sixth grader, which makes 10^4 a reasonable minimum goal for the size of a finite index set X. That should be large enough to rival a sixth grader in distinguishing between different kinds of possible content.

c If Salton's vectors distinguish document content well, then indexing with a finite X alternative needs to remain fairly close to indexing with W. We can accomplish this in various ways, but a simple one here is to require that for any word-indexed vector v^d

$$v^d \neq 0 \;\rightarrow\; T^M(v^d) \neq 0^M$$

where 0 is the zero vector of Salton's W space, and 0^M is the zero vector for the finite-dimensional space indexed by X. Note that the implication arrow $\rightarrow$ goes only one way. Mapping of infinite to finite vectors will introduce irreversible indexing noise.

entropy The frequency of a set ofelements in any natural language text tends to follow a statistical power law. This typically means that that a small subset of elements of any X or W will account for most of their occurrences in any data collection. This imbalance leads to inefficiency in indexing, but cannot be completely eliminated. The problem is usually called Zipf's Law and is often expressed mathematically as

$$F_x \propto \frac{1}{rank\ x}$$

where F_x is the total frequency of feature $x \in X$ in a text data, and $rank\, x$ is the ordinal ranking by frequency of x versus all other features in X. (1 = highest, 2 = next highest, and so forth). Zipf's Law seems a bit strange, but it helps to make natural languages more learnable in a home or other informal setting. The imbalance in indexing is often measured by Shannon's measure of information entropy for an index set X.

$$H(X) = -\sum_{x \in X} p_x \cdot log_2 p_x$$

where p_x is the probability of index feature x in a data collection D. A perfectly balanced set of indexing features X would have $p_x = p_y$ for all x and y in X, with $p_x = p_y = 1/M$. In this case, we have

$$-\sum_{x \in X} p_x \cdot log_2 p_x = -log_2 \frac{1}{M} \cdot \sum_{x \in X} p_x = log_2 M \cdot 1 = log_2 M$$

which is maximum information entropy

$$\max H(X) = log_2 M$$

The presence of the log_2 means that entropy will be expressed as bits of information. Minimum entropy is when $p_x = 1$ for some single x.

` $$\min H(X) = p_x \cdot log_2 p_x = 0$$

or zero bits of information.

Maximum entropy is unachievable because of Zipf's Law, but our goal is to make entropy as large as possible.

independence This is a weakness of indexing text with words W. In English, words can be variously synonymous; for example, RED, RUFOUS, SCARLET, CRIMSON, RUBY, CARNELIAN, or CARMINE. We can ignore this in some indexing applications, but with a finite X, every $x \in X$ must pull its own weight. Otherwise, the indexing effectiveness of X will be illusory.

Independence will be useful theoretically. Ideally, we want an indexing pair $x, y \in X$ will be independent property when

$$p_x \cdot p_y = p_{xy} \quad for \quad x \neq y$$

where p_{xy} = the probability of x and y both occurring in a single document. Unfortunately, this condition is hard to verify. For $M = 10^4$, we must compute p_{xy} for $(M - 1)(M / 2)$ instances. Furthermore, since our estimates of probabilities are based on text samples, they will be less reliable for rarer indices $x \in X$. So, our test for independence would have to be a looser inequality

$$|p_x \cdot p_y - p_{xy}| < \epsilon$$

for some fixed small $\epsilon > 0$. Zipf's Law means that, for $x, y \in X$, most probabilities p_x, p_y, and p_{xy} will be quite small for large M. We can also choose our n-gram indices to take advantage of the rule of non-redudundant indexing to reduce some higher probabilities directly. This may seem an odd way to get more independence of $x \in X$, but the mathematics here looks sound, mostly.

We are close. Must we really satisfy our inequality above for all p_x? Can we live with a few exceptions and work with incomplete independence? Can we get by with good enough? Maybe we scan somehow mask out x, for which p_x is unusually high? This could greatly help our multinomial statistics. Our best path forward is to try out various X with actual data and see what happens.

We now have all the basics to launch finite text indexing. Most important is the ability to work with probabilities $\{ p_x \}$ over a finite number dimensions. The next step is to define an actual finite set X of text indexing features for English and to compute their probabilites.

4. The Probabilities of Index Features

Let us clarify what we mean by p_x. Finite indexing lets us estimate such probabilities easily. This will require only summing the occurrences for each $x \in X$ of a finite indexing space over a large, but though, sample D of text data. Our estimated p_x will simply be

$$p_x \approx \frac{\sum_{d \in D} f_x^d}{\sum_{d \in D} \sum_{x \in X} f_x^d}$$

The quality of estimates will of course depend on the collection D, but with a large enough D, probabilities should be fairly close to those for English in general.

A finite set X with about 10^4 features would require a significant amount of computation for probabilities, but modern processors can handle such loads easily. When data collections are dynamic, we can even still afford to update probabilities often. Our current strategy is to use D itself as our sample data for estimates. We then process new text data in small batches and update overall probabilities after indexing each batch.

In Salton's system, a word index set W can grow without bounds, though at any time, it will be finite. One still might estimate probabilities for $w \in W$ as above, but the arrays for keeping partial sums will grow without bounds. Also, most words in W will be rare, resulting in poor estimates of their probabilities. One typically would want at least 10 to 20 occurrences of an indexing features for best results. This is easier to accomplish with finite indexing.

Salton's system avoids working with any probabilities. Functions of the relative frequency of words like inverse document frequency do help to weight query terms; but they never really go anywhere else. Finite indexing, in contrast, will be built around probabilities. Even with estimation error, indexing probabilities will let us better assess the importance of individual text items and to get a handle on the significance of similarity scores for a pair of items.

5. Finite Indexing, Step by Step

The four requirements of Section 3 above mean that

$$X \not\subset W$$

since we would otherwise violate the coverage requirement for finite X. It is possible, however, to put a few whole words into a finite X. With text data in English, a better general indexing alternative overall is to count word fragments instead of full words. This will meet our coverage requirement, though we still much address power, information, and independence. A word fragment approach will have important consequences here.

Consider fragments of single letter (1-grams[1]). Anyone familiar with cryptanalysis in English will know E, T, A, O, N, I, S, H, R, D, L, and U, in descending order of expected frequency in English text; these will be the most common letters in almost any nontrivial collection of English text data, and their statistics are mostly unhelpful for telling documents apart. They do satisfy our coverage requirement though, which is a start.

How about alphanumeric 2-grams? English has 36×36 = 1,296 of them. This is getting closer to our target of 10^4, but English cryptanalysis tables indicate that certain alphabetic 2-grams like TH and AN still have notably high frequency, lowering the information entropy of indexing; independence also remains an issue. Here is how we might count 2-grams in a bit of text.

---

[1] Google uses the term “n-gram” to refer to sequences of whole words Our usage of the term is from cryptanalysis, which long predates Google.

```
resource
re
 es
  so
   ou
    ur
     rc
      ce
```

This is better than indexing with just 1-grams, but documents can still be hard to tell apart. We need a larger X to work with.

We get more indexing features with alphanumeric 3-grams, but will run into a new problem. In English, there will be 36×36×36 = 46,656 alphanumeric 3-grams. This seems out of control, since most will be nonsense like JRJ, HHQ, EEE, or ZKM, There is no reason to have an index set X so bloated. So, why not just continue to index with all 2-grams to meet our coverage requirement, but add on selected alphabetic 3-grams to increase the power of our index set?

The idea is to count a 3-gram whenever possible and fall back on 2-grams only when necessary. For example, if we introduce SOU for indexing, our analysis above becomes

```
resource
re
 es
  sou
    ur
     rc
      ce
```

Such counting of n-grams of different length will be called “the rule of non-redundant indexing.” If a segment of text is already covered by a longer n-gram, we do not count any shorter n-grams falling entirely within that segment

Adding just a single frequent 3-gram to our X will raise indexing entropy. It will also increase independence overall. With only 2-grams for indexing English text, lhe probabilities of SO and OU in a document will show dependence. If adjacent occurrences of SO and OU are counted as a single occurrence of SOU, our non-redundancy rule will mask some of this dependency. That makes SO and OU more useful as individual index features.

In general, the addition of a single frequent (n+1)-gram to an index set X consisting of k-grams (k = 1 to n) will be advantageous. The gain here will be cumulative, which suggests how we might get ultimately to a goal of around 10^4 indexing features. Zipf’s Law will help us here: we shall only have to look at a few frequent n-gram indices to find (n+1)-grams that should make a big difference in indexing.

Selecting a suitable subset of alphabetic 3-grams can be complicated. For expediency, we started with 3-grams formed by appending a single letter to the K most frequent alphabetic 2-grams in English text. The choice of K = 216 was because we wanted 3-grams of the form QU■ in our index set X. We then get 216 × 26 = 5,616 alphabetic 3-grams, giving us a little under 7,000 2- and 3-gram indices, which was good enough for our initial experiments.

6. Adding a Key Theoretical Detail

At some point, we have to think about building practical software achieving important information goals. First, we must to run experiments to test our ideas objectively and rigorously. The results of such experimentation will be presented in subsequent sections, but we must begin by providing a proper framework for evaluation.

Although we have nibbled around the edges of what an index set X might be, we have yet to show any advantage for finite indexing. So, let us get this detail out of the way, for it will be critical to deeper understanding of all text indexing. This has no counterpart in Salton's formulation of text indexing.

If we model a collection of documents D as points in a finite M-dimensional numerical vector space, we like Salton can define an inner product of a pair of vectors as a raw measure of the similarity of their corresponding documents. The finiteness of the vectors, however, will also allow us to estimate the distribution of raw inner products expected by chance, given the estimated probabilities $\{ p_x \}$ for $x \in X$. This allows us to use a multinomial model for inner products computed for vectors with dimensions corresponding to X. A precondition for using a multinomial model is that each $x \in X$ will have a frequency of occurrence across documents that have a binomial distribution, and p_x must be independent of p_y when $x \neq y$. This is hard to meet directly; for example, the occurrences of the most interesting words tend to clump in fewer documents than a binomial assumption would predict. The problem is less with n-gram indexing in that n-gram can occur across many different words.

With a bit of manipulation of random variables, a multinomial distribution lets us model the distribution of raw inner products between finite vectors. Its most important statistics will be

$$E[IP(d', e')] = L_{d'} L_{e'} \cdot \sum_{x \in X} a_x p_x^2$$

$$Var[IP(d', e')] = L_{d'} L_{e'} \cdot [\sum_{x \in X} a_x p_x^2 \ + \ (L_{d'} + L_{e'} - 2) \cdot \sum_{x \in X} a_x p_x^3 \ - \ (L_{d'} + L_{e'} - 1) \cdot [\sum_{x \in X} a_x p_x^4 \ - \sum_{x \in X} \sum_{y \in X \wedge y \neq x} a_x a_y p_x^2 p_y^2 \ ] \ ]$$

where

$$d' = T^M(d)$$

$$e' = T^M(e)$$

$$L_{d'} = \sum_{x \in X} f_x^{d'}$$

$$L_{e'} = \sum_{x \in X} f_x^{e'}$$

$$a_x = 1 \ (by \ default)$$

The full derivation of the expected value and variance of the raw inner product can be found in the Appendix. It assumes that the probabilities $\{ p_x \}$ are independent. As noted above, this condition is hard to verify, but we can compute the distributions of actual inner products for

random pairs of vectors from our finite space in X and compare them to theoretical predictions to see if they are independent enough.

If so, then we then can compute a statistically scaled similarity

$$sim(d', e') = \frac{IP(d', e') - E[IP(d', e')]}{[Var[IP(d', e')]^{1/2}}$$

This similarity score will be in units of standard deviations, easier to interpret than Salton's hyperspace cosines. We do not even have to approximate the multinomial noise distribution as Gaussian to interpret the significance of similarity scores. Current separation of signal versus noise will let us go conservatively with the Chebyshev inequality to estimate p-values. In any event, we no longer have to normalize document vectors to a Euclidean length of 1.

7. Evaluation Framework

We now have all the ingredients necessary for some rigorous testing of finite indexing that counts 2-and 3-grams as features in text. X with about 7,000 indices is shy of our goal of about 10^4, but should be adequate to test our ideas about finite indexing before going to the next stage of R&D, which will require extensive writing of actual software.

Salton evaluated the performance of document search with word indexing by running it on a standard collection of documents D with predefined queries Q. Human judges had to examine each document $d \in D$ to determine which were relevant to each query $q \in Q$. One could then execute every q to get its top K matches. The number of relevant matches *r* for a query would then let us compute its "recall" as *r* / (total number of documents in D relevant to the queries) and its "precision" as *r* / K.

For finite indexing, we could do the same, but our multinomial model of raw similarity allows for a simpler approach. We can eliminate the need for subjective judgments of relevance as well as provide a more extensive test of a system. Such testing can be quickly done for new data collections of text data of interest to current information system users. The key here is finite indexing and the multinomial model for raw similarity scores for document vectors.

Here is how we now can do testing: Get a large representative sample D' of text documents.

- For $d' \in D'$ and some fixed $L \geq 100$, obtain a segment of text that will have an index vector $v_A^{d'}$, where $L_{v_A^{d'}} = L$.
- For $d' \in D'$, obtain a different text segment to get an index vector $v_B^{d'}$ with $L_{v_B^{d'}} = L$.
- Compute raw inner products for random pairs of index vectors $v_A^{d'}$. This will provide an actual noise distribution to compare against multinomial predictions.
- Compute raw inner products for all corresponding vectors $v_A^{d'}$ and $v_B^{d'}$. This will be an actual signal distribution; determine its separation from the noise distribution.

That is all. Any successful finite indexing must have a noise distribution closely modeled with a multinomial assumption and also show good separation between signal and noise distributions. The whole evaluation procedure can be carried out objectively in only a week or two from scratch with any new data set. No human judges are needed.

8. Text Preprocessing

The next section will report actual results from the initial finite indexing experiments. To interpret these results properly, one should be aware that our indexing code incorporated some

legacy text processing tools. These should already be familiar to most people working in text data indexing in general, but should be noted here because they greatly affect the statistics in n-gram finite indexing.

The first tool relates to grammatical function words in Indo-European languages, including English. These help to structure sentences and can account for a third or more of word occurrences in English text; for example, THE, AND, WITH, BY, FOR, IT, WHAT, NEVER. They typically carry no denotative meaning by themselves, and most indexes will omit ("stop") them. There is no consensus, however, on which words to stop.

Some purists in machine learning would have no stop list at all, holding that automata like artificial neural nets are capable of figuring everything out themselves. On the other hand, this requires much more complexity in an automaton, which will only slow down an experimental schedule. Machine learning after all is not what we want to evaluate. Our choice was a maximal chosen stopword list, so that n-grams like THE and AND could be more useful as indices.

A second tool was needed for word suffixes. English has two kinds: inflectional and morphological. English regular inflectional suffixes are -S, -ED, and -ING at the ends of nouns and verbs. They appear everywhere in English text data and can produce n-grams that make documents of different content seem more alike in their indexing vectors.

Common English morphological suffixes like -MENT, -SHIP, -OR, IZE are well known. These usually serve to change the part of speech for a root; for example, the noun IDEAL becomes the verb IDEALIZE. Since parts of speech are a finer distinction than needed in a text index for computer information systems, morphological suffixes tend to be removed. This was maximal in the experiments for evaluating the effectiveness and utility of n-gram indexing.

Proper removal of word suffixes ("stemming") is much more complicated than stopword removal. The problem is that proper detachment of suffixes requires knowledge of English spelling rules. For example, MATING becomes MATE, while MATTING becomes MAT. Such details seem to be a burdensome chore, but can improve the quality of indexing. Fortunately, good stemmers are readily available for download.

Inflectional ending like -S, -ED, and -ING are harder to remove than morphological suffixes. Inflections are much more frequent, and have more special cases to recognize in rules. This effort is necessary, however, because we already have plenty to contend with. Inflectional stemming makes n-grams like TED and ING more helpful for indexing. Inflectional stemmers are widely available, although they will differ in breadth and accuracy.

A third preprocessing decision related to how one should count occurrences of n-gram indices in a document. John Tukey in his own classic book *Exploratory Data Analysis* suggests that, in any statistical analysis of text data, one should replace raw counts with lower transformed counts to lessen the effect of Zipf's Law; for f_x^d in document d, Tukey suggests $\log_2(f_x^d + 1)$ in a document vector.

AW testing showed a transform does improve performance of n-gram indexing by lessening the impact of high-frequency indices in X. It reduces the variance of our model noise distribution. There seems to be no best transform, but we have settled on a rounded integer approximation of square-roots, which seems good enough.

9. Objective Performance Testing

The idea of finite indexing of text data emerged shortly after Salton's original monograph was published. Many information users were dissatisfied with the whole-word indexing systems then being fielded by the U.S. military. This was before the IBM Personal Computer and before the Internet. With hardware extremely limited by small hard drives and random-access memory, information engineers had to think small.

The 2- and 3-gram subset in X used in testing is described in the ActiveWatch User Guide. The 3-grams are inefficient, including many low-value 3-grams like THZ, ANB, and EEJ. We wanted, however, to come up with a reasonable X quickly. Our goal was only to show that n-grams could work; optimization could come later An X of about 7,000 2- and 3-grams would suffice for initial exploratory testing.

To recapitulate, we had two main testing objectives:

- Validate a multinomial model of raw similarity.
- Show discernible separation between signal and noise in raw similarity.

For our data sets, we took whatever was readily available before the Internet:

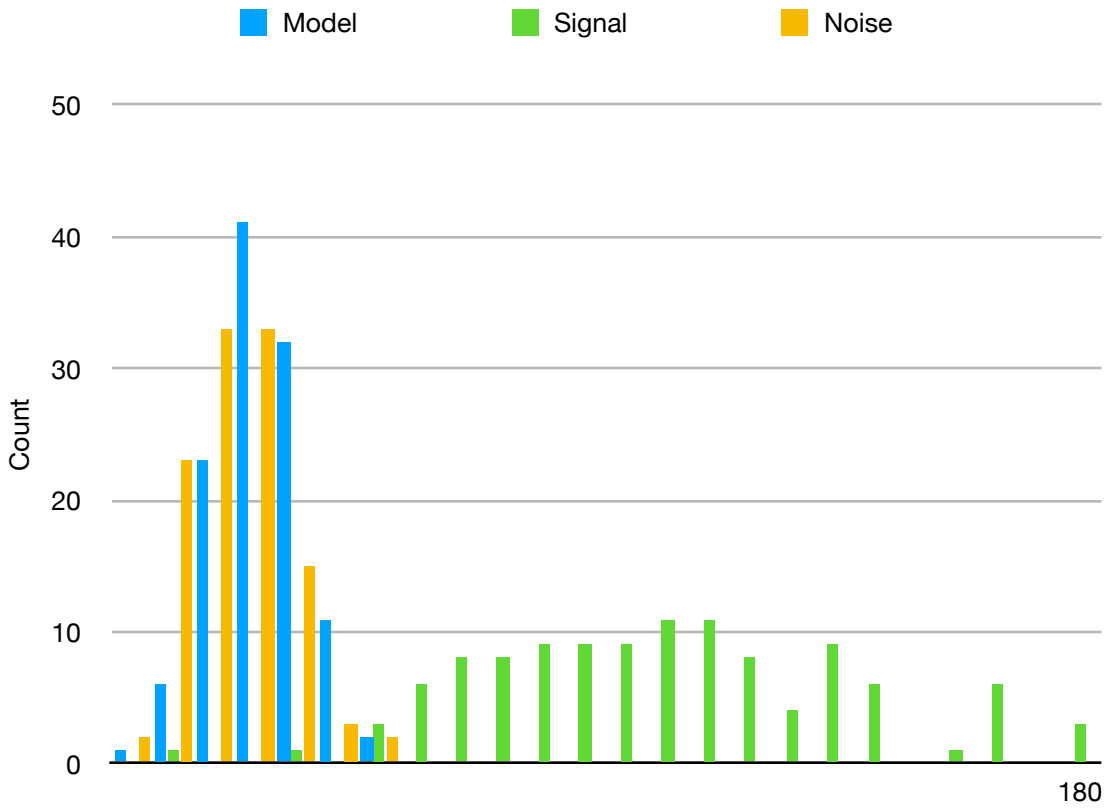
- Newswire stories collected for a previous project by a prospective sponsor (= News),
- Essays from the Norton Anthology, retyped (= Literary).
- A file of abstracts of technical papers in solid-state physics and electronic circuit design; these had been saved on Hollerith punchcards (= Science).

The data sets were indexed separately with our X of about 7,000 2- and 3-grams. It was unclear how well finite indexing would work with diverse data. After 40 years, many things can get lost in changes of employment along with interstate moving: the original software, hardware, data, and raw results. What survives is a summary of results from an unpublished technical paper written shortly after the experimental results were first available. The essential findings are as follows:

Data Set	Noise μ	Noise σ	Model μ	Model σ	Signal μ	Signal σ
News	13.7	6.2	14.7	4.2	34.1	10.6
Literary	12.9	4.6	13.6	4.0	25.6	11.5
Science	19.6	7.9	20.3	5.2	44.0	12.7

This indicates that finite n-gram indexing is succeeding. We have reasonable matches between our our actual noise distributions and multinomial predictions, although the model variance underestimates actual noise variances.

We can get a more dramatic view of these results by putting them into a binned histogram with the raw inner product scores along the X axis and counts along the Y axis. These plots have become iconic for finite indexing and standard for introducing finite indexing to new audiences.



Raw Similarity Scores: 2-, 3-grams with L = 100 for News

This chart is just for the News data collection, showing a good fit of its noise distribution with the a multinomial model and good separation of signal and noise. The shape of similar plots for Literary and Science looked similar, though with less separation of signal and noise. The graphs show plainly what a table of means and variance could not.

Although the multinomial model was an imperfect predictor in our experiments, the obvious separation of signal and noise in all three of our data sets suggested that finite indexing might be good enough already with only 2- and 3-grams for us to start putting together a demonstration application based only on those n-grams.

10. Scaling Document Similarity

The multinomial is an approximate model of raw document similarity scores, providing a consistent and practical way to map out a document collection. Model performance should improve with eventual expansion of our index set X to 10^4 n-grams. The immediate priority, however, was to show how such a mapping might serve current text data analysis.

Making sense of the variance was a first step. Scientists, engineers, and educators are familiar with the interpretation of standard deviations, which is the square-root of variance. It is often used to estimate the probability of some measurement happening by chance relative to an error distribution around some mean value of observed measurements. That distribution is typically Gaussian, popularly known as a bell curve.

There are published tables of significance for particular Gaussian standard deviations. For example, a measurement that is three standard deviations from an expected valuatiom has a

probability of p=.05. A multinomial distribution has one “hump” like a Gaussian distribution, but will be asymmetric with a long tail to the right, unlike the Gaussian. A Gaussian interpretation is nevertheless not unreasonable.

Because of the Law of Large Numbers, a multinomial distribution for inner products of vectors will approach a Gaussian one as dimensionality M increases. So for $M \approx 10^4$, a Gaussian interpretation of standard deviations for inner-product similarity is reasonable. The catch here is a multinomial underestimation of actual variance, but sufficient separation of signal noise should allow some tolerance of that. Here are the numbers for our three data sets.

Data Set	Signal μ - Model μ / Model σ	Comments
News	7.0	the best
Literary	3.0	should be better with bigger L or M
Science	4.5	

11. A Demonstration Application

The development of the multinomial model for the raw similarity of random pairs of document vectors is the cornerstone of finite indexing. It can be only a single brick, however, in the design of pracrtical software. We must able to show how a system with finite indexing will look to an actual user and how it differs from having just another search engine.

A full demonstration of finite indexing still needs to provide the capability of comparing documents in a collection D against a fixed reference profile q, which might serve as a query.

$$q = < q_x >_{x \in X}$$

where the $\{ q_x \}$ are arbitrary real values. This will make q a numerical vector of M dimensions, but this is unlikely to resemble a vector for any document in any collection D. To define a new scaled similarity measure for a document d and reference profile q, we have many options. With finite indexing, this should be simple enough to make modeling its noise distribution easier. We start with

$$Raw(d, q) = \sum_{x \in X} f_x^d \cdot q_x$$

If we rename q_x as a_x and reverse the order of multiplication in the summation, we get

$$Raw(q, d) = \sum_{x \in X} a_x f_x^d$$

which looks almost like raw inner product similarity for a pair of documents. If we do the same mathematics as for computing expected value and variance for pairwise similarity, we get

$$E[Raw(d, q)] = L_d \cdot [\sum_{x \in X} q_x p_x \ ]$$

$$Var[Raw(d, q)] = L_d \cdot [\sum_{x \in X} q_x^2 p_x (1 - p_x) \ - \sum_{x, y \in X \wedge x \neq y} q_x q_y p_x p_y \ ]$$

These noise distribution parameters must be recomputed for each new q; for scaling document for pairwise similarity, we must recompute them only when the probabilities $\{ p_x \}$ change significantly.

The statistically scaled document similarity to q will then be

$$sim(d,q) = \frac{(Raw(d,q) - E(Raw(d,q))}{[Var(Raw(d,q)]^{1/2}}$$

With our two types of statistical scaling, we can envision a real-time text information mapping system quite beyond what conventional engines can support. In particular, this system can identify major areas of content in a data stream and also construct its own standing queries for deetecting them. The system always operates under the control of its users, but can run productively for extended periods of time without any intervention.

12. A Scenario for a Demonstration

We now have two distinct, though similar, ways to calculate a statistically scaled measure of similarity. This is unusual for a text-based information system. It raises the question of how to put such a capability into an actual system implementation and how it might extend what we have already in conventional search engines. Many text information users would seem to be already well-served by software now widely available on any desktop.

There are, however, some kinds of processing with more demanding requirements. Consider a command center gathering streams of data from dozens of different sources arriving day and night at unpredictable times. The incoming information is often highly perishable and might have dire consequences if not identified quickly. So, teams of personnel keep watch in shifts, which could be wasteful when data streams are quiet.

In the not-so-distant past, the command center would have an array of teletype machines clattering away, and those on watch would look at the paper output and tear off each individual document. If obviously important, it will be immediately passed up the command chain. Otherwise, they will be sorted out by subject and snapped onto appropriate clipboards hanging on a wall for later review. A status board will show how subjects are trending.

Such an operation is chaotic. No one person will see all related incoming information. Something can easily slip through, especially when arriving from multiple sources on different shifts to be pieced togther. In modern command centers, everything is digital and stored on networked hard drives, but its basic problems remain. Technology can ease the burden of watch personnel, but will also increase the total amount of data to sift through.

13. Finite Indexing at Work

A system with finite Indexing will never replace search engines, which have proven utility. If someone wants to find text documents cotaining certain words, there is no reason to fragment those words and look for documents containing those fragments. This wlll only add noise into search results. Command centers, however, need tight control over the matching process and could use more support than a search engine usually provides.

Normal search engines operate interactively. For example, Google allows users to enter a query and get immediate ranked results. A searcher has no guarantee, though, that the top retrieved document will be relevant to the query. Someone has to read at least a description ot top results. This is acceptable where a searched collection is largely static. It gets more complicated when live data streams carry documents needing to be identified quickly.

A standing query should check every document as it arrives and notify a user right away about a strong match. This can be hard to do. With Boolean logic queries, setup and tuning will call for coding and debugging skills. System employing numerical similarity scores for a match have an advantage here, but one then has to set the right lower score thresholds for good performance. With the cosine measure, thresholds probably have to be different for each query.

With finite indexing, similarity thresholds should be easier to control. For example, matches below 4σ can be immediately forgotten; matches between 4σ and 6σ will be recorded and sorted, but not reported unless specifically requested; matches between 6σ and 8σ will always show up in immediate search results; matches above 8σ might sound an alert. Notifications might also be set for when unreviewed matched matches above 6σ exceed some limit.

A similar situation holds for link thresholds for automatic clustering even with a different way of scaling similarity. Since any link below 4σ will be too noisy and could produce spurious clusters; it will require trial and error to find a better threshold to produce the number and the type of clusters wanted; but around 6σ seems to work out well. A normalized, but unscaled, cosine measure will be much harder to adjust in such applications.

14. Demonstration Architecture

With only theoretical validation so far, it was premature to start building a complete prototype command center system. Potential users will always want to see and understand any new technology before accepting it. For finite n-gram indexing, this would mean actually processing real text data on hardware familiar to them to give an idea of its complexity of operation, the overhead in its conputations, and the quality of results.

For this purpose, an application was built to focus on automatic clustering. This allows one to generate timely maps of the current contents of a dynamic data collection, helpful to most organizations; and it would emphasize what finite indexing could do beyond conventional search engines or even beyond familiar clustering techniques. Our demonstration application would do two-stage clustering to allow for overlapping sets of documents.

The demonstration would execute a series of modules, each writing out intermediate files to be read by subsequent modules and available for later inspection. These modules are as follows:

(a) Divide a text file into document segments, using string patterns to decide where to cut.

(b) Generate one or more finite n-gram vectors for each segment.

(c) Update estimates of n-gram probabilities for a document collection.

(d) Choose a subset of document segments to cluster.

(e) Precompute multinomial noise model statistics needed for the pairwise similarity, and gather vectors of selected segments into a single compact file for faster computation.

(f) Compute scaled pairwise similarity scores for each unique segment pair and make a link when scaled similarity is above a minimum threshold.

(g) Apply a variation of the minimal spanning tree algorithm to get raw clusters, constraining the maximum size of a raw cluster and requiring a minimum density of linkage within each raw cluster.

(h) Produce an n-gram profile summarizing each raw cluster, which will be used to assign items to final clusters. We need to compute a noise distribution for each profile, which will employ our query version of the multinomial model.

(i) Assign document segments to final clusters by comparing their document vectors to the profiles obtained from cluster seeds; those vectors with scaled similarity above a minimum threshold for any cluster profile will be assigned to that cluster.

(j) Summarize each final cluster by finding words or word roots in document segments of the cluster that contain highly weighted n-grams in its profile.

This is much computation. It tests both of our two ways of scaling similarity scores in steps (e) and (g). Users should be able to see any problems in indexing of content here.

Step (h) will allow a document segment to be in multiple clusters. This is appropriate because a segment of text can relate to multiple subjects.

15. Example of Clustering

After debugging, the clustering application ran on a batch of newswire stories more recent than that in validation testing. This was done many times to determine how to set processing parameters to get the best clusters to show in a demonstration: the final choice had a minimum of 8σ for pairwise links taken in the formation of raw clusters and a minimum of 6σ for a profile match to assign a segment to its final clusters.

The high thresholds imply that we had good separation of signal and noise to work with. The resulting final clusters are not a disappointment and would impress even skeptics of finite indexing. Unfortunately, any records of the actual demonstration have been lost after 40 years, along with the original data and the code that was run.

We can, however, get an idea of what the viewers of the demonstration saw by running the earliest version of current software with only 2-, 3-grams plus a few extra indices (described below). The data was a set of 611 Google News stories from 2017-8, which had been collected for another purpose. There were 88 final clusters. The descriptive words and word roots for top six final clusters according to size of their corresponding seeds were:

```
------- cluster 1
brexit              theresa             minist              vote
parliament          divorce             britain             exit
prime               leave               deal                europe
talk                move                eu                  term
negotiate           transition          pull                bloc
u.k                 memb                support
------- cluster 2
cambridge           analytic            facebook            firm
data                campaign            found               politic
trump               zuckerberg          employ              london
act                 report              broad               committee
mark                profile             tie                 customer
investige           share               brian               work
chief               britain             donald
------- cluster 3
sheriff             columbia            officer             county
police              school              office              report
old                 new                 florida             parent
boy                 coach               search              investige
found               york                rodriguez           canada
teenage             frisine             detain              ria
teen                charge              belie               caitlyn
car                 northeast           soccer
------- cluster 4
aluminum            steel               tariff              preside
import              donald              produce             add
measure             penal               lawmake             set
act                 new                 america             match
threat              familiar            china               industry
impose              lead
------- cluster 5
spokesman           plata               search              know
del                 novemb              short               found
explo               area                country             sub
hope
------- cluster 6
snow                north               heavy               east
massachusett        boston              york                new
coast               philadelphia        nor'east            area
wind                virginia            england             inch
atlantic            travel              flood               jersey
northeast           maine               south               part
baltimore           fall                maryland            move
heav                snowfall            caroline            washington
a.m                 power               weekend             pennsylvania
d.c                 centr
```

The top eight descriptive keys for each cluster are in **bold** here to help readers scan a cluster list more quckly.

In 2017, Britain was still debating on leaving the European Union and a new U.S. president was inaugurated. The cluster keywords seem to be a fair summary of over three-quarters of the Google News sample. A content map of an unknown collection of documents should probably be augmented with analysis of unclustered items, but we have made a good start here.

To get a better assessment of our final clusters, here also are the descriptions for the last six final clusters, which had the smallest raw clusters to start from.

```
------- cluster 83
preside          barack           obama            trump
obamas           repeat           power            donald
help
------- cluster 84
jinp             china            xi               preside
limit            power            term             chin
presiden         lead             allow            lift
vice             extend           constitute
------- cluster 85
hill             capitol          memb             lawmake
harass           sex              female           advance
staff            complain         detail           allege
young            wrong            employ           sen
------- cluster 86
nra              gun              price            world
walmart          rifle
------- cluster 87
candidate        challenge        fail             democrat
elect            party            senate           eager
midterm          novemb           divide           individe
prime            carry            win              lose
year             sign             support          office
------- cluster 88
shoot            police           fatal            gunman
officer          man              shot             act
ent              old
```

Again the clusters are a bit noisy, but have readable and reasonable descriptions. There are fewer descriptive words and word roots for the last clusters, but this may be an advantage. Anyhow, most people had never seen anything like this, and it sparked interest for a prototype system that could run on their own hardware with their own data.

16. Actual Clustered Text

At some point, anyone doing actual cluster analysis will want to see the text of the items being put into particular groups. There is no space here to show all the results of any analysis. Below is a summary of the text clusters obtained for the sample of 611 items with 2-, 3-, 4- , and 5-grams reported above with ActiveWatch release v2.8.1. This produced 78 clusters, sorted by the size of their original seeds; cluster 1 had the biggest, while cluster 78 had the smallest.

Below are the the starts of the top 3 items in cluster 1 and cluster 78. The ranking of items is determined by their significance of their match with the assignment profile for each cluster.

Cluster 1

```
** subsegment 0::159  @37.13σ  length=1079
Emmerson Mnangagwa, who fled into a brief exile after
losing a power struggle less than three weeks ago, became
Zimbabwe's new president on Friday — succeeding Robert
Mugabe, the leader he had backed for decades before helping
oust him last week.

** subsegment 0::95   @30.69σ  length=1102
President Robert Mugabe's own party voted to oust him as
its leader on Sunday, a day after thousands of Zimbabweans
took to the streets to celebrate his stunning fall from
power after a military takeover.
```

```
** subsegment 0::49   @28.06σ   length=1096
Robert Mugabe, the only head of state that Zimbabwe has
ever known in its 37-year existence, is this morning under
house arrest. Although the military insists that this is
not a coup, it has all the hallmarks of one: The army
controls the television station and the airport, and has
confined the president and his family to their mansion.
```

Cluster 78

```
** subsegment 0::281  @22.13σ    length=933
Two men appeared in court Wednesday in London on terror
charges over an alleged plot to kill British Prime Minister
Theresa May, according to British government officials.
Officials say the men conspired to launch a suicide-knife
attack on Downing Street, the official home of the Prime
Minister.

** subsegment 0::217  @21.25σ    length=457
The British backlash to President Trump picked up steam
Thursday with fresh calls to cancel a planned a state visit
and Britain's prime minister standing by her denunciations
of Trump's retweets of a fringe anti-Muslim group.
Prime Minister Theresa May blasted Trump for crossing a
line by posting the inflammatory videos on his Twitter page
Wednesday — and then warning May to essentially mind her
own business and focus on Islamist terror instead of him.

** subsegment 0::42   @18.94σ   length=1239
David Davis is on a determined mission to show global
bankers some love, promising he'll work to protect the City
of London from any trouble caused by Brexit.
The minister for exiting the European Union has made two
speeches in the space of a week setting out his plan to
ensure the U.K. capital keeps its status as a world-leading
financial center.
```

(The listings above are an edited composite of output from two different tools in the prototype ActiveWatch system (DLST and DTXT; see below).

As can be seen, the clusters contain different documents of similar content. Their match scores are shown with their degree of match in standard deviations versus the assignment profiles for each cluster. If the multinomial is valid for modeling the noise distribution of random match random scores, then the cluster assignments must be highly significant. The other 76 Google News clusters show similar results

17. The Effectiveness of Finite Indexing

Finite indexing of text data will always be imperfect and noisy to some degree. The clustering demonstration shows, however, that one can still employ finite indexing to get useful information out of dynamic text data collections by simple means. The exact choice of indices is less important than satisfying our criteria of power, coverage, entropy, and independence. No exotic hardware or software is required. There is no artificial intelligence.

All the steps of the original clustering demonstration together took only minutes to run, allowing viewers to watch its entire operation in real time. Steps (f) and (g) dominated

processing overhead, since they have a running time proportional to the square of the total number of document segments to be clustered.

The total elapsed time to process 611 segments was under 6 seconds on a MacBook Pro (2020) laptop powered by an 8-core M2 chip running at about 2.4 GHz. No attempt was made to speed up computations with multithreading, nor was the M2 built-in GPU engaged for the pairwise inner products of step (f). The software should easily handle much larger data sets.

The latest versions of our software differ from the original ones. They have about 3,700 4- and 5-grams from various sources, including personal linguistic competence. This brought the size of our index set X to about 10^4 n-grams. It revealed some of the shortcomings of earlier systems with only 2- and 3-grams for processing. The Google News data produced only 78 clusters with added 4- and 5-grams in ActiveWatch release v2.8.1. and also had shorter descriptions with words and word roots.

Here are the descriptions of the top 6 clusters obtained by v2.8.1 with expanded indexing for the Google News data:

```
------- cluster 1
independ          harare            capital           rule
vice              lead              zanu              africa
flee              know              serve             new
state             address           head              struggle
took              thousand          decade
------- cluster 2
ventura           santa             county            california
barbara           acre              fire              evacuate
burn              rage              new               people
threat            thoma             communite         flame
south             coast             wind              order
los               montecito         cross             highway
spokesman         gust              destruct          time
push
------- cluster 3
explode           suspect           injury            police
detonate          austin            person            dead
bomb              pack              connect           blast
treat             federal           texas             kill
people            hour              night             device
set               fedex             report            rock
authorit
------- cluster 4
cambridge         facebook          analyt            firm
million           data              politic           company
zuckerberg        mark              found             profile
committee         executive         research          employ
develop           report            inform            chief
social            digit             co                collect
work              app
------- cluster 5
argentine         submarine         crew              juan
navy              san               south             miss
coast             membr             ara               atlantic
plata             communice         locate            search
del               contact           report            know
base              disappear         area              mile
novembr           sub
```

```
------- cluster 6
soldier          defect           korea            south
north            wound            surgery          condition
lee              border           jong             medic
guard            escape           cross            hospital
shot             university       troop            person
milite           lead             cook             fellow
suwon
```

The new top 6 clusters have only one cluster in common with the top 6 reported above for only 2- and 3-gram indexing. The earlier clusters were not wrong, but the new ones seem to be more coherent and cleaner in their descriptions. On the whole, we would probably want to go with the new clusters.

The drop in the number of clusters with added 4- and 5-gram indices is a sign that the earlier clusters were noisy. News stories are more likely to resemble each other with only about 7,000 indices. The higher entropy and greater independence with about 10^4 n-gram indices does make a noticeable difference.

18. A Prototype System

The finite indexing prototype was originally designed to run on a DEC PDP-11 minicomputer with limited primary random access memory and secondary storage. Moore's Law has generally allowed for major increases in processing throughput, but the modular structure introduced for the initial clustering demonstration of the system has been kept throughout migration to different hardware platforms.

The prototype system added new main modules and a set of low-level tools. The major changes overall include:

- Reorganization to process input text in periodic batches so as to allow regular updates of document collection statistics as new data arrives from multiple input streams.
- A front end for user-defined standing profiles. This was integrated with the framework of assignment profiles in the clustering demonstration to provide more complete support for a command center scenario.
- Two modules were added to allow generation of phrases for describing clusters and standing profile matches. This required a new capability to parse the text in a document collection.
- Modules for handling residual items failing to match any standing profile or be assigned to an preexisting cluster.
- Modules for looking at clustering links in various way, including the identification of hubs in the graph formed frim clustering links.
- Many low-level tools accessing intermediate files. In a pinch, these can serve as a basic command-line user interface.

The original prototype system was written in thr C language. Later it was rewritten in Java for better portability, but saw minimal service. It took another twenty years to get it into a deliverable package and made available as open-source code under BSD licensing. The original Java code had a graphical user interface built with its AWT package, but that library has been long deprecated.

19. How N-Grams Differ From Words for Indexing

When the n-gram finite indexing project began, it was unclear if how much would come of it. The idea of using n-grams in searching and other applications had already been explored in various published papers. Notable perhaps was Donald Knuth's experiment with 2-grams for searching as described in Volume III of his series *The Art of Computer Programming*.

None of these efforts showed any practical advantages for text processing, and there was no major followup. It was only after looking more deeply at the problem of finite indexing that a way forward appeared. The idea was to add a subset of alphabetic 3-grams to alphanumeric 2-grams and employ the rule of non-redundant indexing to get a compact set of indices. This produced about 7,000 indexing n-grams as the basis for later experimentation.

Finite indexing provides a way to get reliable probabilities. This led to multinomial models for inner product similarity scores between n-gram vectors allowed for fresh thinking on how to build practical text information systems. Rapid progress followed. N-gram indexing remains noisier than whole word indexing, but will be advantageous for close real-time statistical control of matching.

One has to wonder how everything worked out so nicely. After all, n-grams mostly convey no definite meaning. An answer seems to come out of the noise and signal distributions discussed above for the inner-product similarity scores for pairs of document vectors. The idea is that an information system should provide good separation of noise and signal. One can operate with high overlap, but this yields poor precision and recall.

Typical information systems support some kind of adjustment to increase separation of noise and signal. In Salton's approach, this was done by assigning weights to words in inner-product computations. Those weights tend to correlate with the total frequency of word in in a document collection. High-frequency words will be less selective for particular documents and therefore get lower weights.

The problem with such weighting is that it is a blunt instrument. When we increase the weight of a word, it will have the same effect on a noise distribution as on the signal distribution. To get better separation, one wants to increase the similarity scores in the signal distribution while at least keeping more or less the same noise distribution. This is quite hard to manage without knowledge of the shapes of the various distributions and how their scores arise.

Salton assumed that related documents have distinct subsets of words that characterize them and that frequency weighting can somehow favor them. As it turns out, that idea actually works, although a system dealing with many different possible kinds of relatedness may have find that weights good for one query may be poor for another. We can never cover all cases.

In short, Salton tried to get better separation by focusing on signal distributions while ignoring any noise distribution. N-gram finite indexing does the opposite; it focuses on noise while avoiding the complexity of trying to corral all possible signal distributions into a single theoretical framework. This had not been planned, but once multinomial models for noise matches were developed, signal distributions no longer have to be modeled.

N-grams for finite indexing have to be selected specifically. Alphanumeric 2-grams were to satisfy the coverage requirement for alternate indexing; adding longer n-grams mainly serve to improve the fit of a multinomial model to actual similarity scores for random pairs of document vectors. For this purpose, individual n-grams need not have special semantic importance. Only their relative frequencies in resulting vectors will matter.

Scaled similarity allows the expected contribution of noise to be subtracted from similarity measures with finite indexing. There could be nothing left at all, but indexing coverage with Salton's work now comes to the rescue. It seems that related documents do have subsets of words with frequency above what is expected by chance. Words contain n-grams. We

therefore should also expect an excess of their associated n-gram occurrences in related documents.

The criteria for choosing an n-grams with n > 3 has been been quite simple: (1) it has to occur in at least two familiar English words or names; (2) frequent ones are favored over infrequent ones. Salton's system makes no selection of index words, but assigning weights to words is a fuzzy kind of choosing. Frequent words typically get lower weights than infrequent ones. In the end, however, only the resulting vectors will really matter.

N-gram finite indexing became quite different from word indexing in the end. The multinomial model of similarity is essential. It eliminates the need for any weighting of individual indices and provides a means of interpreting the significance of inner-product similarity scores. Some users remain leary about n-grams having no intrinsic meaning, but they work fine for indexing documents, which is the important thing. Most users should never have to see any n-grams.

20. User Adjustment of Indexing

System builders never know everything about their target pool of users. Yet the success of a system can hinge on details, calling for flexibility in finite indexing. The prototype system lets a user adjust the indexing in three places to process with particular text data better: (1) choosing words to exclude from indexing; (2) changing the rules for morphological stemming; and (3) defining special n-grams to augment the built-in ones.

The first two adjustments are somewhat complicated, serving mostly to hide grammatical words and common functional suffixes from n-gram indexing. Changing them will call for special technical expertise, and so will not be discussed here. For more details, please refer to the ActiveWatch User's Guide available online at the URL given in References.

More easily managed are user-defined n-grams. These can be any sequence of letters and digits at the start of a word or at its end. For example, PSYCH- or -OMETRY where the hyphen stands for the rest of a word, if any. The rule of non-redundant indexing applies to user-defined n-grams. These will show up in an n-gram analysis as follows:

```
pyschometry
pysch-
   cho
    hom
    -ometry
```

User-defined n-grams will give users longer n-grams for indexing, though only at the start or end of a word. They also let usds sneak a few whole words into an index set for more precision in matching up document pairs of particular content. This will affect the noise distribution for scaling similarity scores, but should be negligible for multinomial statistics if they are only a small part of an n-gram index set.

ActiveWatch allows as many as 2,000 user-defined n-grams, although fewer than 1,000 is more typical. They can make a significant contribution for indexing, but built-in 2-, 3-, 4-, and 5-grams still dominate. For ActiveWatch v2.8.2, here is a comparison of total probability of all n-gram indices occurring at least once in the Google News document collection:

```
User-defined n-grams
total prob=0.113775 for 349 indices
min=0.000011, max=0.004700
--
Alphanumeric 2-grams
total prob=0.055125 for 518 indices
min=0.000011, max=0.002282
--
Alphabetic 3-grams
total prob=0.332964 for 2708 indices
min=0.000011, max=0.002407
--
Alphabetic 4-grams
total prob=0.390597 for 2231 indices
min=0.000011, max=0.004372
--
Alphabetic 5-grams
total prob=0.107538 for 562 indices
min=0.000011, max=0.003457
--
6368 non-zero n-gram indices
```

User-defined n-grams had more impact on indexing than either 2-grams or 5-grams. The bulk of indexing, however, is still with 3- and 4-grams. Surprisingly, 4-grams beat out more numerous 3-grams, justifying longer n-grams in an indexi set. Currently built-in 4- and 5-grams account for about half of all n-gram occurrences in a text dataset.

Further expansion of finite indexing is likely, but may require a more systematic approach with more text data. The last few additions of 4- and 5-grams have barely budged the total entropy of our n-gram index set. It takes about 1,000 more built-in n-grams for a measurable difference. This will still be under our 10^4 order of magnitude limit for indexing.

One can add other user-defined n-grams when needed. These will loaded automatically at run time from a text input file (see the AW User's Guide). The dominant indices for our multinomial model, however, will probably remain 3- and 4-grams. Finite indexing in subsequent systems will be mainly with 4- and 5-grams.

21. Recent History

The prototype differs from conventional search engines, which are "Passive" and have to be used interactively. Statistically scaled similarity allows a system to operate reliably on its own and to discover information for users without being asked.

The earlier AW demonstration application experimented with about 2,500 phonetic indices, which reflect the English pronunciation of words according to phonetic categories derived from Soundex. Along with 2- , 3-, and user-defined n-grams, this gives us almost 10^4 indices in all, but increased only the entropy of indexing. The inherent dependence between 2- and 3-gram occurrences was unchanged by phonetic indices.

The prototype system was delivered to a military organization keeping track of weapons development in the Middle East. During its operation, only a single issue arose about the quality of AW output and was immediately addressed in a software update. Generally, the organization was happy with the system, though it never revealed how it used the software.

An extended version of AW with support for matching of user-defined profiles was installed for a business news aggregator. This was called "HawkEye"; it was notable in incorporating ideas of W. Edwards Deming about real-time statistical quality control on industrial assembly lines. HawkEye users can set up control charts to monitor the data matches of profiles and to raise alerts when similarity scores are unusually high or low.

ActiveWatch continued in military use until a Pentagon edict after the end of the Cold War banned any non-shrinkwrap software from their computers. HawkEye hit a different problem when intellectual property issues blocked updates to keep pace with rapidly evolving hardware and software environments. Eventually, HawkEye shut down for lack of support.

There was no more work on finite indexing for about three decades. Then a casual conversation between the two authors of this writeup led to the idea of making AW open-source and in the public domain. A search of a closet unearthed an ancient backup CD-ROM in a jewel box with an almost finished Java reimplemention of AW. The CD-ROM was still readable, but the Java code needed a thorough revamp for 21st Century Java compilers.

The phonetic indices in older code was replaced with alphabetic 4- and 5-grams. This was more in line with our emerging theory of n-gram finite indexing. It also showed how to expand the set of n-gram indices beyond 10^4. Having just n-grams in an index set simplifies AW code and makes it easier to understand.

The entire AW Java prototype system is now downloadable for free from the Web at

`https://github.com/prohippo/ActiveWatch`

Version v2.8 was used to generate the results above from Google News data. The 21st Century Java reimplementation introduced 4- and 5-gram indexing in release v0.7. Work on AW continues as time permits.

22. Conclusion

Finite indexing was never a certainty. In particular, everyone knew that n-gram indexing would be noisy. That could have overwhelmed any signal; but everything turned out better than anyone had expected. It gave us another perspective on text indexing. Finite indexing was workable, if we chose the right text features to count.

Finite indexing is more general than n-gram indexing. Our AW prototype system has shown, however, that n-grams are a good choice for finite indexing. Other options are possible; for example, phonetic indices, indices of non-adjacent letters, or even hash functions on arbitrary pieces of text; but they are more unfamiliar than n-grams and will take extra work to integrate into a system like ActiveWatch.

One can get respectable performance with only 2- and 3-grams, which might be useful for analyzing corrupted text data. Adding 4- and 5- grams, however, increases the power of a finite index set to distinguish between typical documents, increases the overall entropy of indexing, and decreases dependence between n-gram indices for a better fit with multinomial models. Users should decide what is best for them.

The rule of non-redundant indexing for n-gram indices means that vectors will automatically downshift from 4- and 5-grams to 2- and 3-grams when noisy text prevents the longer n-grams from being found. As a finite index set includes longer n-grams, it gets closer to indexing with full words, though remaining finite.

Noise has always been unavoidable for information systems, but we can control it. Here, n-gram finite indexing departs from Salton's approach of weighting whole-word indices to raise the similarity measure for a pair of related documents, but which words to upweight will depend on the subject area. Finite indexing with n-grams is independent of subject; noise models for raw similarity will look only at the statistics of a language in general.

Multinomial modeling eventually leads to statistically scaled similarity, which is the payoff for finite indexing. Information systems can now make more of their own decisions in the analysis of text data. They can be active in discovering information instead of just being passive and relying on users to judge success at each step.

People can also can more easily interpret a statistically scaled numerical similarity measure. We know that six standard deviations for a profile match score should make it a far outlier for any random noise distribution. This is no guarantee that two matching document vectors with a high pairwise scaled similarity will actually be related, but odds are good that a match at six standard deviations is no accident.

The ActiveWatch prototype system is the proof of the pudding. Despite almost 30 years hibernation for over, it remains *sui generis* in its finite indexing and statistically scaled similarity. It has become available as open source under liberal BSD licensing; anyone can test it out with their own document collections. A multnomial approach lets us do this on any text data set without any predefined queries or human judgments of relevance for them.

Our demonstration of automatic clustering at minimum thresholds of eight standard deviations shows plenty of separation between signal and noise. No human is required to evaluate the top matches in every ranking by similarity to make sure that the best in every case will actually be good. The statistical approach entails only a bit of extra computation.

In a sense, our two multinomial models for document similarity measures turn indexing noise into a friend. Salton's concept of text indexing ignores noise, which is fine when a user can closely observe a system and direct its operation. Such supervision is hard to sustain, however, when streaming high volumes of online information in real time.

N-gram indexing will never replace Google or even turn researchers and developers away from Salton's hyperspheres in infinite-dimensonal vector spaces. Finite indexing does have a niche, however, in the command center and in the aggregation of news for customers with known special interests. Its robustness also may be helpful in exploring the content of text evidence obtained in pre-trial judicial discovery or in tracking social media trends on the dark web.

Finally, we should remember that n-gram finite indexing so far is tuned for English text. Even with French and Spanish, which have greatly influenced English, one must construct new index sets. Such specialization might seem unnecessary when deep artificial neural networks already can translate from English to French or Spanish without special preparation. Yet why force a robot to relearn something that we already know well?

Extensive research opportunities remain in n-gram finite indexing. Some directions have already been touched on above, including:

- larger and more efficient sets of n-gram indices, perhaps from a larger alphabet distiguishing the pronunciation of letters by context.
- better understanding of the independence of probabilities for overlapping n-grams.
- exploration of algorithms for raw clustering and for generating n-gram profiles.
- providing information other than clustering in content maps of large text data sets.
- more detailed reporting of unclustered items.
- monitoring major changes of individual n-gram probabilities across batches.
- defining more precise criteria for statistical quality control in profile matching on live data.
- setting guidelines for applying n-gram indexing to extreme text data like tweets.

N-gram finite indexing for English now has a strong foundation. It has come a long way from Salton's original word indexing, although we still generate numerical index vectors and compute inner products. It all invites us to think more broadly about the problems of automatic text processing and to escape well-worn ruts. Zipf's Law can help us if we let it. All in all, our almost half-century journey of discovery has been a great adventure.

References

(1) I. Barton, S. Creasy, M. Lynch, and M. Snell. An Information theoretic approach to text searching in direct access systems. *Communications of the ACM* 17:6 (June, 1974), pp.345-350.

(2) B. Box, W. Hunter, and J. Hunter. *Statistics for Experimenters*. New York: John Wiley, 1978.

(3) H. Gaines. *Cryptanalysis*, New York: Dover, 1956.

(4) R. Kimbrell. Searching for text? Send an n-gram!. *Byte* 13:6 (May, 1988). pp. 297-308.

(5) D. Knuth. *The Art of Computer Programming III: Sorting and Searching*. Reading, MA: Addison-Wesley, 1973.

(6) C. Landauer and C. Mah. Message extraction through estimated relevance. In *Information Retrieval Research*, R. Oddy et al. (eds), London: Butterworths, 1981.

(7) S. Lydell, English bigram and letter pair frequencies from the Google Corpus Data in JSON format, https://gist.github.com/lydell/c439049abac2c9226e53, 2015.

(8) C. Mah. ActiveWatch User's Guide. https://github.com/prohippo/ActiveWatch/AWug.pdf, 2023.

(9) C. Mah and R. D'Amore. The information value of n-grams in characterizing the content of text. PAR Technology Corporation, New Hartford, NY, 1983.

(10) J. Peterson. Computer program for detecting and correcting spelling errors. *Communications of the ACM* 23:12 (December, 1980), pp. 676-687.

(11) B, Rohatgi. *Introduction to Mathematical Statistics*. New York: John Wiley, 1976.

(12) G. Salton. *A Theory of Indexing*. Philadelpha: Society of Industrial and Applied Mathematics, 1975.

(13) E. Shuegraf and H. Heaps. Selection of equifrequent word fragments for information retrieval. *Information Storage and Retrieval* 9 (1973), pp. 697-711.

(14) C. Shannon and W. Weaver. *The Mathematical Theory of Communication*. Urbana, IL: University of Illinois Press, 1949.

(15) J. Tukey. *Exploratory Data Analysis*. Reading, MA: Addison-Wesley, 1977.

(16) E. Yannakoudakis, F. Goyal, and J. Huggill. The generation and use of text fragments for data compression. *Information Processing and Management* 13:1 (1982), pp. 15-21.

Appendix. Modeling Similarity Measure Noise

For a Pair of Document Vectors

Given any finite set X of text features to index with, regardless of how these were obtained, we can define a vector representation for any text document d with the counts f_x^d of each feature $x \in X$. A raw inner product similarity measure for documents d and e can then be defined:

$$s(d,e) = \sum_{x \in X} a_x \cdot f_x^d \cdot f_x^e$$

The a_x are importance weights that someone might assign to each index feature *x* in X. We usually make $a_x = 1$ to avoid any subjective weighting of features.

We still need to scale the measure $s(d,e)$ so that we can interpret similarity scores between pairs of text documents of differing lengths. Instead of converting their vectors to unit length as done for the cosine measure, we can approach this problem probabilistically with some simplifying assumptions: (1) the probability of any index feature $x \in X$ can be estimated; (2) given any text document d, with L_d occurrences of index features, x on the average will be expected $L_d \cdot p_x$ times with a variance of $L_d \cdot p_x \cdot (1 - p_x)$.

These two assumptions basically mean that the features of index set X are independent of each other. Strictly speaking, this is untrue. For example, words in text in particular do not occur independently; if you see “smoke” in some segment of text, then it becomes more likely that you will also see “fire.” Overlapping word fragments are also not entirely independent as features, but since almost all of them have no meaning by themselves, they will start out more independent than whole words.

If our assumptions hold approximately, we can employ the multinomial distribution in statistics to get a handle on our inner product similarity measure. First, let us define an auxiliary random variable

$$g_x = f_x^d \cdot f_x^e$$

It does not matter what d and e are; they just have to be different. We can compute the expected value of g_x from the expected value of f_x^d times the expected value of f_x^e, since f_x^d and f_x^e, are assumed to be be independent. We shall then derive $E[S]$ and $Var[S]$. from $E[g_x]$, $Var[g_x]$ and $Cov[g_x,g_y]$. Let us compute $E[g_x]$ first.

(a) Expected Value

$$\begin{aligned} E[g_x] &= E[f_x^d] \cdot E[f_x^e] \\ &= L_d \cdot p_x \cdot L_e \cdot p_x \\ &= L_d L_e p_x^2 \end{aligned}$$

(b) Variance

By definition,

$$Var[g_x] = E[(g_x)^2] - E[g_x]^2$$

To substitute above on the right, we need

$$Var[(g_x)^2] = E[(f_x^d)^2] \cdot E[(f_x^e)]^2$$

where

$$E[(f_x^d)^2] = E[f_x^d]^2 - Var[f_x^d]$$
$$= L_d[(L_d - 1)]p_x^2 + p_x)]$$
$$E[(f_x^e)^2] = L_e[(L_e - 1)]p_x^2 + p_x)]$$

Now we have that

$$E[(g_x)^2] = E[(f_x^d)^2] \cdot E[(f_x^e)^2]$$
$$= L_d[(L_e - 1)p_x^2 + p_x]$$
$$E[f_x^e]^2 = L_e[(L_e - 1)p_x^2 + p_x]$$

So

$$E[((g_x)^2] = L_d L_e \cdot [(L_d L_e - L_d - L_e + 1)p_x^4 + (L_d + L_e - 2)p_x^3 + p_x^2]$$
$$Var[(g_x)^2] = L_d L_e \cdot [- (L_d + L_e - 1)p_x^4 + (L_d + L_e - 2)p_x^3 + p_x^2]$$

(c) Covariance.

Finally, we have the statistic

$$Cov[g_x g_y] = E[g_x g_y] - E[g_x]E[g_y]$$

Now

$$E[g_x g_y] = E[f_x^d f_y^d] \cdot E[f_x^e f_y^e]$$

and by definition

$$E[f_x^d f_y^d] = E[f_x^d]E[f_y^d] + Cov[f_x^d f_y^d]$$

If index features x and y are independent, which should be approximately true with a large index set X text of 10^4 carefully selected n-grams, we get

$$Cov[f_x^d f_y^d] = - L_d p_x p_y$$

$$Cov[f_x^e f_y^e] = - L_e p_x p_y$$

We can now calculate for our inner product measure in terms of g_x:

$$E[s(d,e)] = \sum_{x \in X} a_x \cdot E[g_x]$$

$$Var[s(d,e)] = \sum_{x \in X} a_x^2 \cdot Var[g_x] + \sum_{x \in X} \sum_{y \in X \wedge, y \neq x} a_x a_y \cdot Cov[g_x g_y]$$

Substituting from above, we finally get

$$E[s(d,e)] = L_d L_e \cdot \sum_{x \in X} a_x \cdot p_x^2$$

$$Var[s(d,e)] = L_d L_e \cdot [\sum_{x \in X} a_x p_x^2 \ + \ (L_d + L_e - 2) \sum_{x \in X} a_x p_x^3 \ -$$

$$(L_d + L_e - 1) \sum_{x \in X} a_x p_x^4 \ - \ (L_d + L_e - 1) \sum_{x \in X} \sum_{y \in X \wedge y \neq x} a_x a_y p_x^2 p_y^2 \ ]$$

All the summations in both $E[s(d,e)]$ and $Var[s(d,e)]$ can be precomputed, since they will be the same for all document vector pairs.

For a Query Versus a Document Vector

The calculations above will simplify when we want to compare many item vectors against a fixed query vector q as in a search over a collection of text items. Our raw similarity measure becomes just

$$S_q(d) = \sum_{x \in X} q_x f_x^d$$

where query vector q has the components $\{ q_x \}$, which do not have to be integer counts.

With cosine similarity, a query match score would be computed the same as simillarity between two item vectors. Our probabilistic approach would have us instead define another model with fewer degrees of freedom

$$E[s_q(d)] = \sum_{x \in X} q_x \cdot E[f_x^d]$$

$$Var[s_q(d)] = \sum_{x \in X} q_x^2 \cdot Var[f_x^d] \ + \ \sum_{x \in X} \sum_{y \in X \wedge y \neq x} q_x q_y \cdot Cov[f_x^d f_y^d]$$

Substituting as before, the multinomial model predictions for similarity to a particular profile will be

$$E[s_q(d)] = L_d \cdot [\sum_{x \in X} q_x p_x \ ]$$

$$Var[s_q(d)] = L_d \cdot [\sum_{x \in X} q_x^2 p_x (1 - p_x) \ - \ \sum_{x \in X} \sum_{y \in x \wedge y \neq x} q_x q_y p_x p_y \ ]$$

The distributional parameters for profile matching are much simpler to compute. The bracketed summations must be precomputed only once for each query, however. Except for automatic clustering, which must compare pairs of document vectors, similarity scaled with a simplified model for profile matching with fewer statistical degrees of freedom will become the workhorse of our text information processing with finite indexing.