SUPPLEMENTAL INFORMATION FOR:
Title

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Stochastic simulations from the fitted models

Here we show stochastic simulations from the fitted models. Simulations are all initialized from the same initial conditions, which were estimated as part of model fitting.

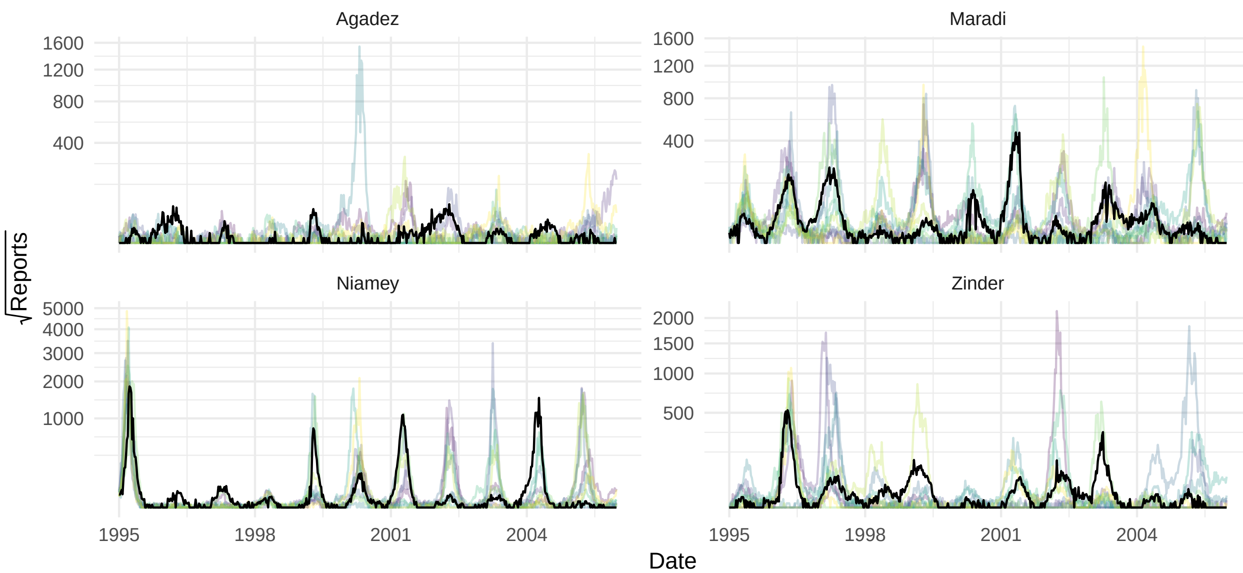

Figure S1: Stochastic simulations from the fitted models. Each colored line is a single realization of the model from the same initial conditions. Ten simulations are shown. The black lines are the observed data. We do not expect model simulations to perfectly align with the data because of the multiple sources of stochasticity present. Thus, these simulations should be judged by their ability to reproduce dynamical features of the data.

Estimating the timing of critical transitions

To define the null and test intervals for our simulations of re-emergence and elimination, we need to know when the critical transition between alternative modes of fluctuation occurs. For re-emergence, we defined the year of the critical transition as the year just after the effective reproduction number (R_E) reaches or exceeds the critical value of 1. For example, if R_E reaches or exceeds 1 at some point during the fifth year of the simulation, then the critical transition year is defined as the sixth year of the simulation. Thus, the simulated data for calculating early warning signals ends at the end of the fifth year. We call this year the "critical year." From this full window, from the beginning of the simulation to the end of the critical year, we defined the null interval as the first half of the window (far from $R_E = 1$) and the test interval as the second half of the full window (near $R_E = 1$). Figure Sx shows a typical example.

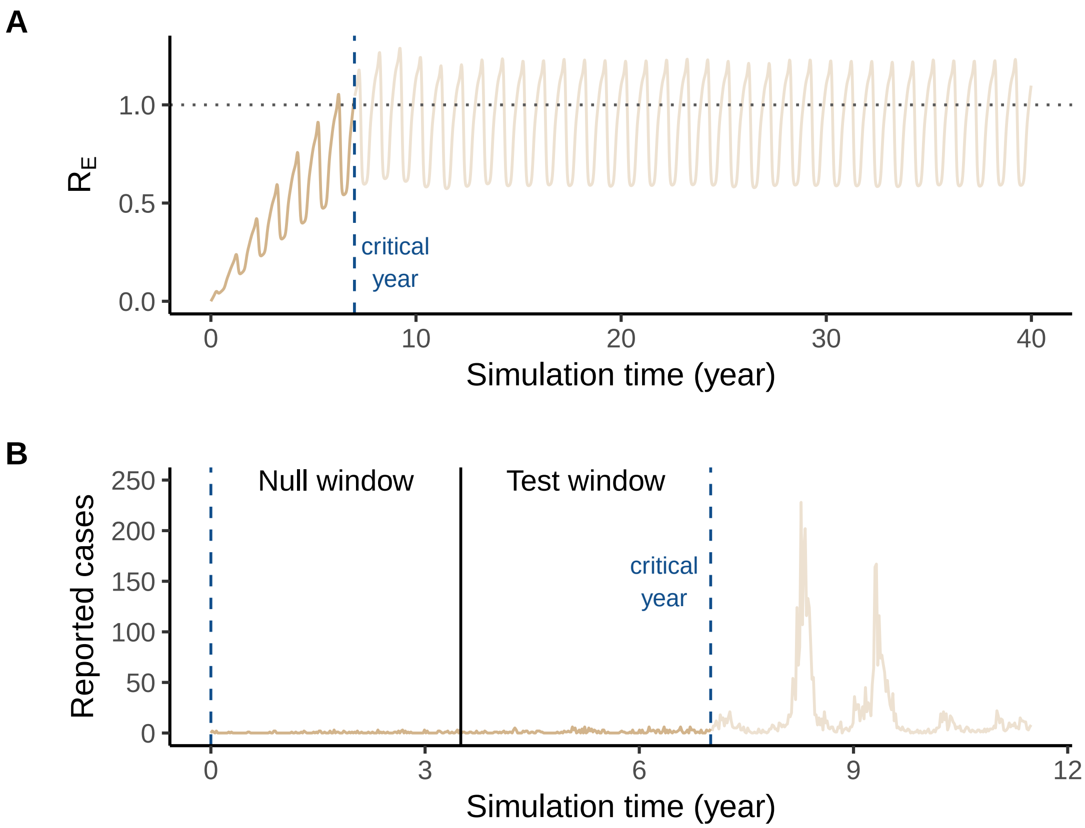

Figure S2: A typical emergence simulation for Maradi where the initial number of susceptibles was discounted by 1e-04. (**A**) The simulated trajectory of R_E and the year in which R_E first reaches the critical value of 1 (denoted by dashed blue line). (**B**) The simulated trajectory of the number of cases. Note that the x-axis has been reduced relative to the top panel. The two vertical blue lines indicate the start (left-most line) and end (line for critical year) of the full window. The black line demarcates the division between the equal-length null and test intervals.

For simulations of disease elimination by increasing the vaccination coverage of the population, we define the critical time as the time at which vaccination coverage reaches the threshold needed for herd immunity. This vaccination threshold is defined as $p = 1 - 1/R_0$. Because our transmission function is seasonal, we first calculated time-specific R_0 as: $R_{0(t)} = \frac{\eta \beta_t \mu}{\nu(\eta+\nu)(\gamma+\nu)}$, where $1/\eta$ is the infectious period, $1/\gamma$ is the recovery period, β_t is the time-specific rate of transmission, μ is the birth rate, and ν is the death rate. Only β_t is estimated by our model. We set $1/\eta$ = 8 days, $1/\gamma$ = 5 days, and $\mu = \nu = 0.05$. Figure 2 in the main text shows these values. We took a conservative approach for calculating the vaccination threshold by using the maximum value of $R_0(t)$, such that: $p = 1 - 1/\max(R_{0(t)})$. We set the time at which vaccination coverage is equal to p as the endpoint for the EWS analysis. All elimination simulations had vaccination campaigns that started at year 50. So, we define the test interval as the times between year 50 and the year at which vaccination coverage is equal to p. We then defined the null interval as window with length equal to test interval and ending at year 49. Figure Sx shows a typical example.

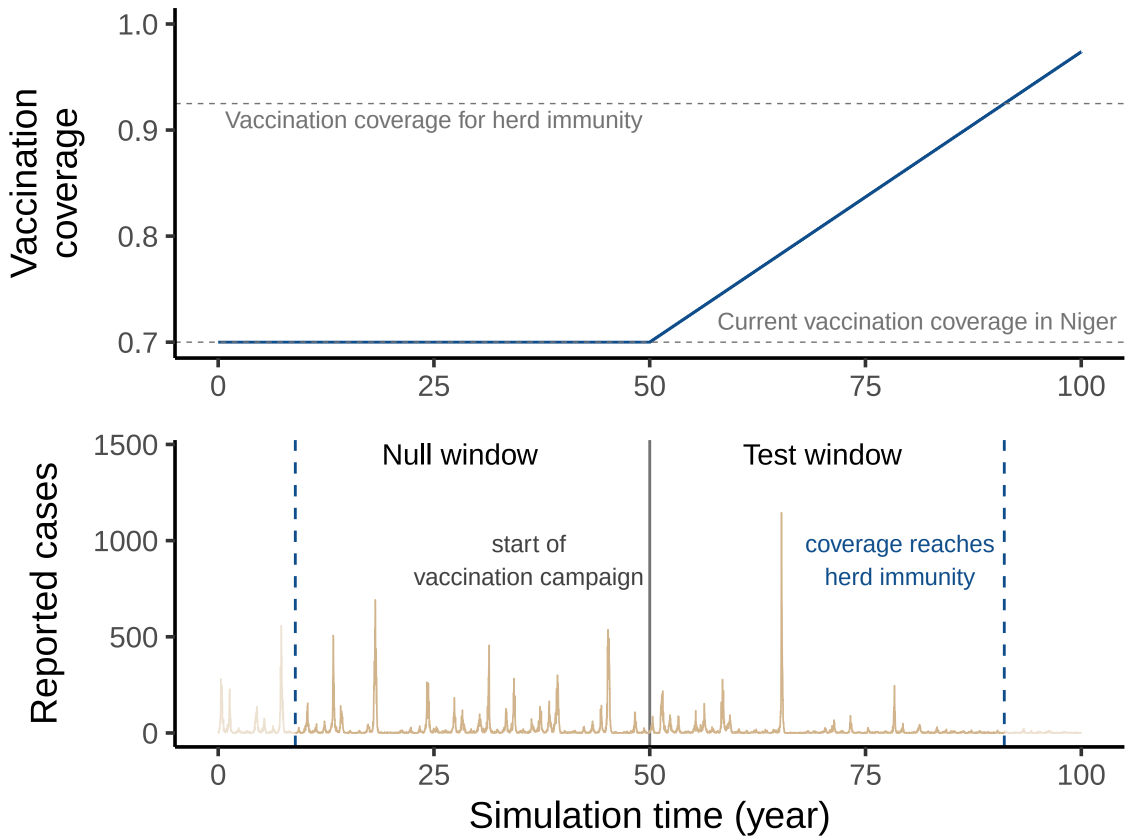

Figure S3: A typical elimination simulation for Maradi where the rate to full vaccination coverage is 1.5e-05. The null and test intervals are defined based on the time at which the vaccination campaign begins (year 50, black line) and the year at which vaccination coverage reaches the vaccination threshold for herd immunity (right-most dashed blue line). The beginning of the null interval is determined by the length of the series from time 50 to the time of herd immunity: the null interval ends at time 50 (when vaccination campaign begins) and starts at whatever time results in a series that is equal in length to the test interval.

Early warning signals

We calculated nine early warning signals using the `spaero::get_stats()` function. Formulas for the early warning signals are in Table S1.

Table S1: List of candidate early warning signals and their estimating equations. Note that b denotes the bandwidth. See (*1*) for details.

EWS	Estimator	Theoretical Correlation with $R_E(t)$
Mean	$\mu_t = \sum_{s=t-(b-1)\delta}^{t+(b-1)\delta} \frac{X_s}{2b-1}$	Positive
Variance	$\sigma_t^2 = \sum_{s=t-(b-1)\delta}^{t+(b-1)\delta} \frac{(X_s-\mu_s)^2}{2b-1}$	Positive
Coefficient of variation	$CV_t = \frac{\sigma_t}{\mu_t}$	Null
Index of dispersion	$ID_t = \frac{\sigma_t^2}{\mu_t}$	Positive
Skewness	$S_t = \frac{1}{\sigma_t^3} \sum_{s=t-(b-1)\delta}^{t+(b-1)\delta} \frac{(X_s-\mu_s)^3}{2b-1}$	Positive

EWS	Estimator	Theoretical Correlation with $R_E(t)$
Kurtosis	$K_t = \frac{1}{\sigma_t^4}\sum_{s=t-(b-1)\delta}^{t+(b-1)\delta}\frac{(X_s-\mu_s)^4}{2b-1}$	Positive
Autocovariance	$\mathrm{ACov}_t = \sum_{s=t-(b-1)\delta}^{t+(b-1)\delta}\frac{(X_s-\mu_s)(X_{s-\delta}-\mu_{s-\delta})}{2b-1}$	Positive
Autocorrelation	$\mathrm{AC}_t = \frac{\mathrm{ACov}_t}{\sigma_t\sigma_{t-\delta}}$	Positive
Decay time	$\overline{\tau}_t = -\delta/\ln[\mathrm{AC}_t(\delta)]$	Positive

Trends of early warning signals

Theory suggests that most EWS should increase as disease dynamics approach a critical transition from below (emergence). When approaching the critical transition from above (elimination), less is known about the behavior of EWS, but theory does tell us that the mean and variance should decrease while the autocorrelation should increase (at least for SIR and SEIR models). To confirm that the EWS are behaving as expected, and to document cases in which they do not, we plotted histograms of the EWS for the test and null intervals for both simulation types, emergence and elimination.

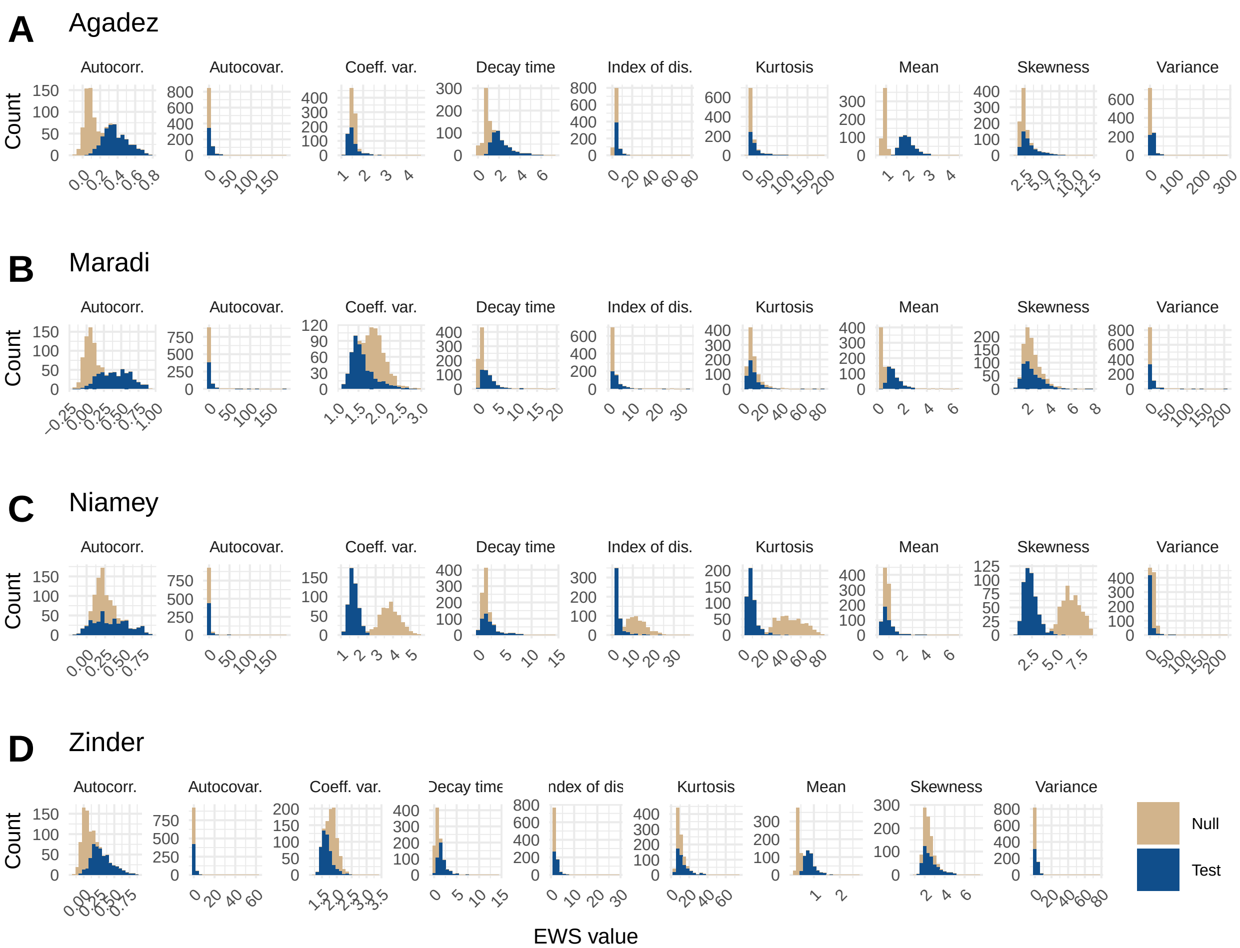

Figure S4: Histograms of EWS from emergence simulations where susceptible depletion fraction was 1e-04.

Figure S5: Histograms of EWS from elimination simulations where rate of vaccination coverage reaching 1 is 1.5e-05.

Bootstrapped parameter confidence intervals

We used a bootstrap approach to estimate approximate 95% confidence intervals for all parameters. Below we show summary statistics for all parameters except the B-splines. For B-splines, and transmission rate more gerenally, we plot the seasonal transmission rate function for all bootstrap replicates.

Table S2: Maximum likelihood estimates and summary statistics of parameters from bootstrapped estimates for Agadez.

Parameter	MLE	Mean	Median	SD	Lower 95% CI	Upper 95% CI
Log likelihood	-961	NA	NA	NA	NA	NA
Log likelihood S.E.	0.114	NA	NA	NA	NA	NA
β	171	187	165	110	43	467
σ	0.0968	0.081	0.0869	0.0327	0.00123	0.133
ψ	7.8	8.71	8.42	2.33	5.14	13.7
ρ	0.77	0.759	0.756	0.166	0.474	0.999
τ	0.107	0.115	0.11	0.0479	0.0426	0.235
$S_{t=0}$	0.23	0.237	0.205	0.166	0.0138	0.736

Parameter	MLE	Mean	Median	SD	Lower 95% CI	Upper 95% CI
$E_{t=0}$	5.53e-06	1.75e-05	1.08e-05	1.66e-05	2.53e-06	6.67e-05
$I_{t=0}$	1.2e-05	9.34e-06	6.94e-06	7.84e-06	1.87e-06	2.94e-05

Table S3: Maximum likelihood estimates and summary statistics of parameters from bootstrapped estimates for Maradi.

Parameter	MLE	Mean	Median	SD	Lower 95% CI	Upper 95% CI
Log likelihood	-1750	NA	NA	NA	NA	NA
Log likelihood S.E.	0.155	NA	NA	NA	NA	NA
β	483	533	529	131	319	813
σ	0.0557	0.0518	0.0518	0.0118	0.0316	0.0729
ψ	24.9	29.7	27.3	10.6	14.9	57.1
ρ	0.333	0.333	0.333	0.0261	0.292	0.387
τ	0.0769	0.0801	0.0805	0.0166	0.054	0.113
$S_{t=0}$	0.104	0.0961	0.0968	0.0172	0.0673	0.13
$E_{t=0}$	8.08e-06	4.01e-05	3.21e-05	3.17e-05	4.13e-06	0.000114
$I_{t=0}$	3.35e-05	2.46e-05	1.92e-05	1.79e-05	3.78e-06	6.84e-05

Table S4: Maximum likelihood estimates and summary statistics of parameters from bootstrapped estimates for Niamey.

Parameter	MLE	Mean	Median	SD	Lower 95% CI	Upper 95% CI
Log likelihood	-1450	NA	NA	NA	NA	NA
Log likelihood S.E.	0.154	NA	NA	NA	NA	NA
β	371	360	374	119	11.2	560
σ	0.0887	0.0818	0.0819	0.0146	0.0594	0.116
ψ	23.3	24.8	24.4	5.14	15.8	35.2
ρ	0.262	0.262	0.255	0.0254	0.219	0.315
τ	0.0526	0.0522	0.0518	0.0132	0.0279	0.0784
$S_{t=0}$	0.11	0.112	0.113	0.0124	0.0897	0.136
$E_{t=0}$	0.000119	0.00024	0.000185	0.000216	1.7e-05	0.000783
$I_{t=0}$	0.000134	0.000111	9.72e-05	7.64e-05	1.74e-05	0.000297

Table S5: Maximum likelihood estimates and summary statistics of parameters from bootstrapped estimates for Zinder.

Parameter	MLE	Mean	Median	SD	Lower 95% CI	Upper 95% CI
Log likelihood	-1420	NA	NA	NA	NA	NA
Log likelihood S.E.	0.0903	NA	NA	NA	NA	NA
β	180	223	214	98.6	78	445
σ	0.0757	0.0682	0.0695	0.0176	0.0269	0.0969
ψ	10.5	11.6	10.9	4.41	5.83	18.9
ρ	0.364	0.346	0.35	0.0727	0.226	0.486
τ	0.0351	0.0394	0.0369	0.0164	0.0167	0.0813
$S_{t=0}$	0.222	0.237	0.222	0.0921	0.116	0.481

Parameter	MLE	Mean	Median	SD	Lower 95% CI	Upper 95% CI
$E_{t=0}$	1.19e-06	3.67e-06	1.84e-06	4.48e-06	6.29e-07	1.32e-05
$I_{t=0}$	9.23e-07	2.16e-06	1.54e-06	1.65e-06	5.03e-07	5.84e-06

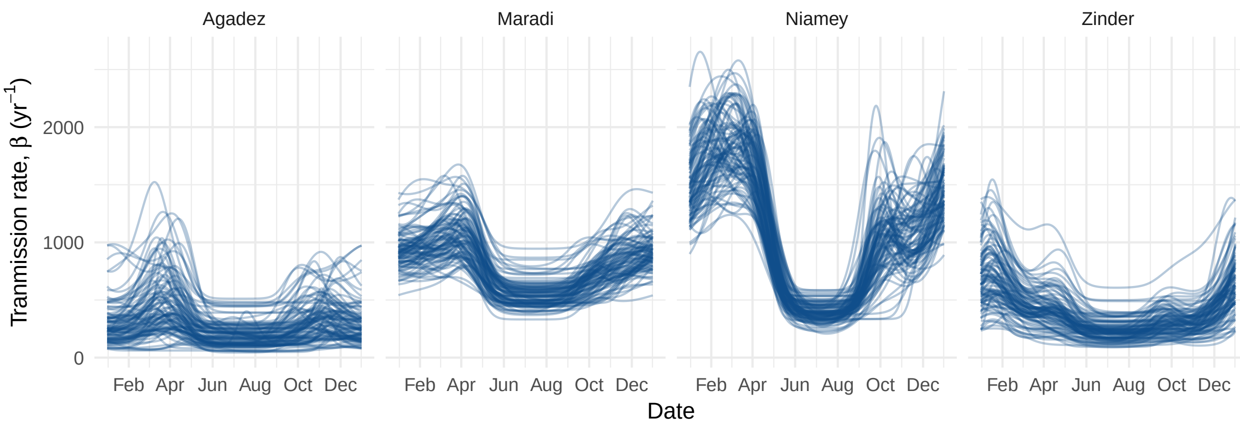

Figure S6: Bootstrap replicates of estimated seasonal transmission functions.

Moving window analysis

To complement our fixed window analysis presented in the main text, we conducted a moving window analysis as might be conducted on real surveilance data. We used the same null and test intervals as described above for the emergence and elimination scenarios. However, instead of calculating a single value for each EWS over the entire window of the interval, we calculated EWS in the intervals over moving windows of 35 weeks. We then calculated the Kendall's rank correlation (τ) between each EWS and time in each of the null and test intervals. We used the distributions of Kendall's τ over the 500 replicate simulations to then calculate the Area Under the Curve (AUC) metric for each EWS. For the approach to elimination, the results are similar to those for the fixed window analysis (Figure S7B). For the approach to emergence, however, the moving window results show much worse performance for Agadez and Zinder, and, on average, the AUC values are lower for all EWS in each city (Figure S7A).

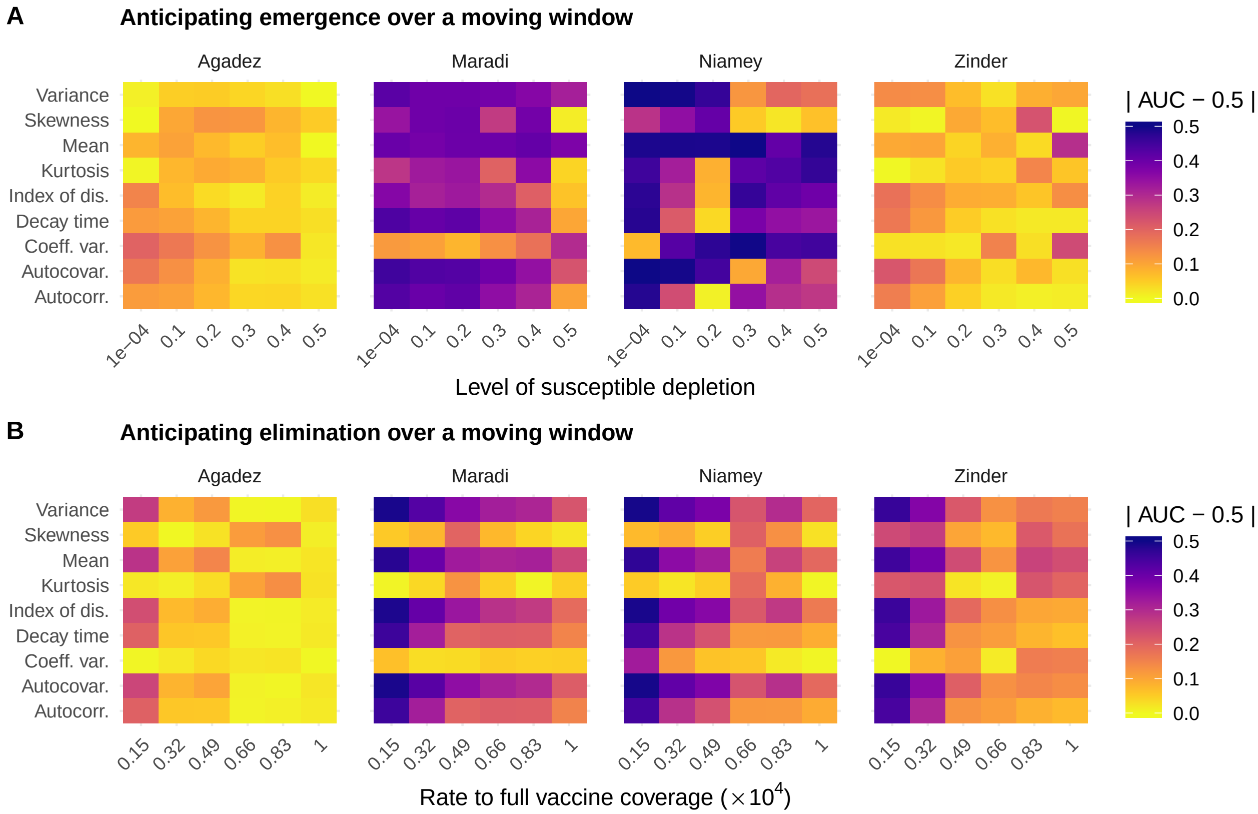

Figure S7: Performance of early warning signals (EWS) over a moving window. Correlations between EWS (calculated over 35 week moving windows) and time were calculated over two intervals, one far from a critical transition and one near, for simulations of re-emergence (A) and elimination (B). EWS performance is quantified using the AUC metric, which we show here as a heatmap of AUC values minus 0.5. AUC values closer to 0.5 indicate higher ability to distinguish among time series near and far from a critical transition.

References

1. T. S. Brett *et al.*, *PLoS Computational Biology*. **14**, e1006204 (2018).