―――――――――――――――――― MODULE *KnuthYao* ――――――――――――――――――

EXTENDS *Reals*

VARIABLES *p*,   The probability we are here
          state, The current state
          flip   The current flip

$vars \;\triangleq\; \langle p,\; state,\; flip \rangle$

$Done \;\triangleq\; \{\text{"1"},\; \text{"2"},\; \text{"3"},\; \text{"4"},\; \text{"5"},\; \text{"6"}\}$
$Flip \;\;\triangleq\; \{\text{"H"},\; \text{"T"}\}$

$One \;\triangleq\; 1$
$Probability \;\triangleq\; \{x \in Real : 0 \le x \wedge x \le One\}$

$Transition \;\triangleq\; [s0 \mapsto [H \mapsto \text{"s1"},\; T \mapsto \text{"s2"}],$
$\qquad\qquad\qquad\;\; s1 \mapsto [H \mapsto \text{"s3"},\; T \mapsto \text{"s4"}],$
$\qquad\qquad\qquad\;\; s2 \mapsto [H \mapsto \text{"s5"},\; T \mapsto \text{"s6"}],$
$\qquad\qquad\qquad\;\; s3 \mapsto [H \mapsto \text{"s1"},\; T \mapsto \text{"1"}],$
$\qquad\qquad\qquad\;\; s4 \mapsto [H \mapsto \text{"2"},\;\; T \mapsto \text{"3"}],$
$\qquad\qquad\qquad\;\; s5 \mapsto [H \mapsto \text{"4"},\;\; T \mapsto \text{"5"}],$
$\qquad\qquad\qquad\;\; s6 \mapsto [H \mapsto \text{"6"},\;\; T \mapsto \text{"s2"}]]$

$TossFairCoin \;\triangleq\; \wedge\; flip' \in Flip$
$\qquad\qquad\qquad\;\; \wedge\; p' \;\; = p/2$

$Init \;\;\triangleq\; \wedge\; state = \text{"s0"}$
$\qquad\quad \wedge\; p \quad\;\; = One$
$\qquad\quad \wedge\; flip \;\;\; \in Flip$

$Next \;\triangleq\; \wedge\; state \;\; \notin Done$
$\qquad\quad \wedge\; state' = Transition[state][flip]$
$\qquad\quad \wedge\; TossFairCoin$

$Spec \;\triangleq\; Init \wedge \square[Next]_{vars} \wedge \mathrm{WF}_{vars}(Next)$

THEOREM $Converges \;\triangleq\; \forall\, e \in Probability \setminus \{0\} : Spec \Rightarrow \Diamond(state \in Done \vee p < e)$

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