— MODULE Knuth Yao

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EXTENDS Reals, Integers
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VARIABLES
$$p$$
, state, $flip$ The probability we are here The current state The current flip
$$vars \triangleq \langle p, \, state, \, flip \rangle$$

$$Done \triangleq \{\text{"1", "2", "3", "4", "5", "6"}\}$$

$$Flip \triangleq \{\text{"H", "T"}\}$$

$$One \triangleq 1$$

Probability
$$\triangleq \{x \in Real : 0 \le x \land x \le One\}$$

$$\begin{array}{ll} Transition \; \stackrel{\triangle}{=} \; [s0 \mapsto [H \mapsto \text{``s1''}, \; T \mapsto \text{``s2''}], \\ s1 \mapsto [H \mapsto \text{``s3''}, \; T \mapsto \text{``s4''}], \\ s2 \mapsto [H \mapsto \text{``s5''}, \; T \mapsto \text{``s6''}], \\ s3 \mapsto [H \mapsto \text{``s1''}, \; T \mapsto \text{``1''}], \\ s4 \mapsto [H \mapsto \text{``2''}, \; T \mapsto \text{``3''}], \\ s5 \mapsto [H \mapsto \text{``4''}, \; T \mapsto \text{``5''}], \\ s6 \mapsto [H \mapsto \text{``6''}, \; T \mapsto \text{``s2''}] \end{array}$$

$$\begin{array}{ccc} TossFairCoin & \stackrel{\Delta}{=} & \wedge flip' \in Flip \\ & \wedge p' & = p/2 \end{array}$$

$$\begin{array}{ccc} Init & \stackrel{\Delta}{=} & \wedge \ state = \text{``s0''} \\ & \wedge \ p & = \ One \\ & \wedge \ flip & \in \ Flip \end{array}$$

$$Next \triangleq \land state \notin Done \\ \land state' = Transition[state][flip] \\ \land TossFairCoin$$

$$Spec \stackrel{\triangle}{=} Init \wedge \Box [Next]_{vars} \wedge WF_{vars}(Next)$$

Theorem Converges $\stackrel{\Delta}{=} \ \forall \ e \in \textit{Probability} \setminus \{0\} : \textit{Spec} \Rightarrow \Diamond (\textit{state} \in \textit{Done} \lor \textit{p} < e)$