MODULE *KnuthYao*

EXTENDS *Reals*, *Integers*

VARIABLES p, The probability we are here
$state$, The current state
flip The current flip

$vars \triangleq \langle p, state, flip \rangle$

$Done \triangleq \{\text{“1”}, \text{“2”}, \text{“3”}, \text{“4”}, \text{“5”}, \text{“6”}\}$
$State \triangleq Done \cup \{\text{“s0”}, \text{“s1”}, \text{“s2”}, \text{“s3”}, \text{“s4”}, \text{“s5”}, \text{“s6”}, \text{“-”}\}$

$Flip \triangleq \{\text{“H”}, \text{“T”}\}$

$One \triangleq 1$
$Epsilon \triangleq One/1000$
$Probability \triangleq \{x \in Real : 0 \leq x \land x \leq One\}$

$$\begin{aligned} TypeOK \triangleq\ & \land p \in Probability \\ & \land state \in State \\ & \land flip \in Flip \end{aligned}$$

$$\begin{aligned} Table \triangleq\ & [s0 \mapsto [H \mapsto \text{“s1”}, T \mapsto \text{“s2”}], \\ & s1 \mapsto [H \mapsto \text{“s3”}, T \mapsto \text{“s4”}], \\ & s2 \mapsto [H \mapsto \text{“s5”}, T \mapsto \text{“s6”}], \\ & s3 \mapsto [H \mapsto \text{“s1”}, T \mapsto \text{“1”}], \\ & s4 \mapsto [H \mapsto \text{“2”}, T \mapsto \text{“3”}], \\ & s5 \mapsto [H \mapsto \text{“4”}, T \mapsto \text{“5”}], \\ & s6 \mapsto [H \mapsto \text{“6”}, T \mapsto \text{“s2”}]] \end{aligned}$$

$$\begin{aligned} Init \triangleq\ & \land state = \text{“s0”} \\ & \land p = One \\ & \land flip \in Flip \end{aligned}$$

$$\begin{aligned} Next \triangleq\ & \land state \notin Done \cup \{\text{“-”}\} \\ & \land flip' \in Flip \\ & \land p' = p/2 \\ & \land state' = \text{IF } p \leq Epsilon \text{ THEN “-” ELSE } Table[state][flip] \end{aligned}$$

$Spec \triangleq Init \land \Box[Next]_{vars} \land \mathrm{WF}_{vars}(Next)$

$Terminates \triangleq state \in Done \cup \{\text{“-”}\}$