──────────────────── MODULE *KnuthYao* ────────────────────

EXTENDS *Reals*, *Integers*

VARIABLES p,   The probability we are here
          $state$,   The current state
          $flip$ The current flip

$vars \triangleq \langle p,\ state,\ flip \rangle$

$Done \triangleq \{\ \text{"1"},\ \text{"2"},\ \text{"3"},\ \text{"4"},\ \text{"5"},\ \text{"6"}\ \}$
$Flip \triangleq \{\ \text{"H"},\ \text{"T"}\ \}$

$One \triangleq 1$
$Probability \triangleq \{x \in Real : 0 \le x \wedge x \le One\}$

$$Transition \triangleq [s0 \mapsto [H \mapsto \text{"s1"},\ T \mapsto \text{"s2"}],$$
$$s1 \mapsto [H \mapsto \text{"s3"},\ T \mapsto \text{"s4"}],$$
$$s2 \mapsto [H \mapsto \text{"s5"},\ T \mapsto \text{"s6"}],$$
$$s3 \mapsto [H \mapsto \text{"s1"},\ T \mapsto \text{"1"}],$$
$$s4 \mapsto [H \mapsto \text{"2"},\quad T \mapsto \text{"3"}],$$
$$s5 \mapsto [H \mapsto \text{"4"},\quad T \mapsto \text{"5"}],$$
$$s6 \mapsto [H \mapsto \text{"6"},\quad T \mapsto \text{"s2"}]]$$

$Init \triangleq\ \wedge\ state = \text{"s0"}$
$\qquad\quad \wedge\ p \quad\ = One$
$\qquad\quad \wedge\ flip \quad \in Flip$

$Next \triangleq\ \wedge\ state\ \notin Done$
$\qquad\quad \wedge\ flip' \quad \in Flip$
$\qquad\quad \wedge\ p' \quad\ = p/2$
$\qquad\quad \wedge\ state' = Transition[state][flip]$

$Spec \triangleq Init \wedge \Box[Next]_{vars} \wedge \mathrm{WF}_{vars}(Next)$

THEOREM $Converges \triangleq \forall\, e \in Probability \setminus \{0\} : Spec \Rightarrow \Diamond(state \in Done \vee p < e)$

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