MODULE *KnuthYao*

EXTENDS *Reals*, *Integers*

VARIABLES p, The probability we are here
$state$, The current state
flip The current flip

$vars \triangleq \langle p, state, flip \rangle$

$Done \triangleq \{\text{“1”}, \text{“2”}, \text{“3”}, \text{“4”}, \text{“5”}, \text{“6”}\}$
$Flip \triangleq \{\text{“H”}, \text{“T”}\}$

$One \triangleq 1$
$Probability \triangleq \{x \in Real : 0 \leq x \land x \leq One\}$

$$\begin{aligned} Table \triangleq\ & [s0 \mapsto [H \mapsto \text{“s1”},\ T \mapsto \text{“s2”}],\\ & s1 \mapsto [H \mapsto \text{“s3”},\ T \mapsto \text{“s4”}],\\ & s2 \mapsto [H \mapsto \text{“s5”},\ T \mapsto \text{“s6”}],\\ & s3 \mapsto [H \mapsto \text{“s1”},\ T \mapsto \text{“1”}],\\ & s4 \mapsto [H \mapsto \text{“2”},\ \ T \mapsto \text{“3”}],\\ & s5 \mapsto [H \mapsto \text{“4”},\ \ T \mapsto \text{“5”}],\\ & s6 \mapsto [H \mapsto \text{“6”},\ \ T \mapsto \text{“s2”}]] \end{aligned}$$

$$\begin{aligned} Init \triangleq\ & \land\ state = \text{“s0”}\\ & \land\ p = One\\ & \land\ flip \in Flip \end{aligned}$$

$$\begin{aligned} Next \triangleq\ & \land\ state \notin Done\\ & \land\ flip' \in Flip\\ & \land\ p' = p/2\\ & \land\ state' = Table[state][flip] \end{aligned}$$

$Spec \triangleq Init \land \Box[Next]_{vars} \land \mathrm{WF}_{vars}(Next)$

THEOREM $Converges \triangleq \forall\, e \in Probability \setminus \{0\} : Spec \Rightarrow \Diamond(state \in Done \lor p < e)$