MODULE *KnuthYao*

EXTENDS *Reals*, *Integers*

VARIABLES p, The probability we are here
$state$, The current state
flip The current flip

$vars \triangleq \langle p, state, flip \rangle$

$Done \triangleq \{\text{“1”}, \text{“2”}, \text{“3”}, \text{“4”}, \text{“5”}, \text{“6”}\}$
$State \triangleq Done \cup \{\text{“s0”}, \text{“s1”}, \text{“s2”}, \text{“s3”}, \text{“s4”}, \text{“s5”}, \text{“s6”}\}$

$Flip \triangleq \{\text{“H”}, \text{“T”}\}$

$One \triangleq 1$
$Probability \triangleq \{x \in Real : 0 \leq x \wedge x \leq One\}$

$$TypeOK \triangleq \begin{array}{ll} \wedge\, p & \in Probability \\ \wedge\, state & \in State \\ \wedge\, flip & \in Flip \end{array}$$

$$\begin{array}{ll} Table \triangleq [s0 \mapsto [H \mapsto \text{“s1”}, & T \mapsto \text{“s2”}], \\ \quad s1 \mapsto [H \mapsto \text{“s3”}, & T \mapsto \text{“s4”}], \\ \quad s2 \mapsto [H \mapsto \text{“s5”}, & T \mapsto \text{“s6”}], \\ \quad s3 \mapsto [H \mapsto \text{“s1”}, & T \mapsto \text{“1”}], \\ \quad s4 \mapsto [H \mapsto \text{“2”}, & T \mapsto \text{“3”}], \\ \quad s5 \mapsto [H \mapsto \text{“4”}, & T \mapsto \text{“5”}], \\ \quad s6 \mapsto [H \mapsto \text{“6”}, & T \mapsto \text{“s2”}]] \end{array}$$

$$Init \triangleq \begin{array}{l} \wedge\, state = \text{“s0”} \\ \wedge\, p = One \\ \wedge\, flip \in Flip \end{array}$$

$$Next \triangleq \begin{array}{l} \wedge\, state \notin Done \\ \wedge\, flip' \in Flip \\ \wedge\, p' = p/2 \\ \wedge\, state' = Table[state][flip] \end{array}$$

$Spec \triangleq Init \wedge \Box[Next]_{vars} \wedge \mathrm{WF}_{vars}(Next)$

THEOREM $Converges \triangleq \forall\, e \in Probability \setminus \{0\} : Spec \Rightarrow \Diamond(state \in Done \vee p < e)$