MODULE *Prob*

EXTENDS *Integers*

VARIABLES p, *state*

$vars \triangleq \langle p, state \rangle$

$One \triangleq 100000$

$a/b \triangleq$ IF $b \neq 0$ THEN $\langle a, b \rangle$ ELSE CHOOSE $x \in \{\} :$ TRUE

$a \odot b \triangleq (a[1] * b[1])/(a[2] * b[2])$

$Norm(x) \triangleq x[1] \div x[2]$

$$
\begin{aligned}
&MarkovInit(Initial) \triangleq \\
&\quad \land state = Initial \\
&\quad \land p \quad = One/1
\end{aligned}
$$

$$
\begin{aligned}
&MarkovNext(Done,\ Transition) \triangleq \\
&\quad \land state \notin Done \land Norm(p) \neq 0 \\
&\quad \land \exists\, next \in \text{DOMAIN } Transition[state] : \\
&\qquad \land state' = next \\
&\qquad \land p' \quad = p \odot Transition[state][next]
\end{aligned}
$$

$$
\begin{aligned}
Initial &\triangleq \text{“s0”} \\
Accepting &\triangleq \{\text{“I”},\ \text{“II”},\ \text{“III”},\ \text{“IV”},\ \text{“V”},\ \text{“VI”}\} \\
Transition &\triangleq [s0 \mapsto [s1 \mapsto 1/2,\ s2 \mapsto 1/2], \\
&\qquad\ s1 \mapsto [s3 \mapsto 1/2,\ s4 \mapsto 1/2], \\
&\qquad\ s2 \mapsto [s5 \mapsto 1/2,\ s6 \mapsto 1/2], \\
&\qquad\ s3 \mapsto [s1 \mapsto 1/2,\ I \mapsto 1/2], \\
&\qquad\ s4 \mapsto [II \mapsto 1/2,\ III \mapsto 1/2], \\
&\qquad\ s5 \mapsto [IV \mapsto 1/2,\ V \mapsto 1/2], \\
&\qquad\ s6 \mapsto [VI \mapsto 1/2,\ s2 \mapsto 1/2]]
\end{aligned}
$$

$$
\begin{aligned}
Spec \triangleq\ &\land MarkovInit(Initial) \\
&\land \Box[MarkovNext(Accepting,\ Transition)]_{vars} \\
&\land \mathrm{WF}_{vars}(MarkovNext(Accepting,\ Transition))
\end{aligned}
$$

THEOREM $Converges \triangleq Spec \Rightarrow \Diamond(state \in Accepting \lor Norm(p) = 0)$