



## Introduction to computer science

### Lecture 10: Introduction to algorithms Exhaustive & greedy search algorithms Dynamic programming

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## Examen final 2008

Lundi **7 Juillet** de 9h a 11h



- Note finale (HC): 1/3 Pale machine + **2/3** Pale papier
- Le polycopie INF311+transparents sont **autorises**
- Sujet en Francais *or in English* (FR/EN)
- Les EV2s ont le droit a 30 minutes supplementaires
- Plusieurs **parties independantes**



## Examen final 2008

- (re?)Lire le polycopie:  
Chapitres 1 a 11 (130 pages)
- Regarder les annales



INF 311: Introduction à l'informatique, niveau débutant

LIBRE  
Mise à jour: Lundi 16 Juin 2008  
et remis à l'école aux élèves ayant pris un peu de connaissances en informatique.

[Le poly est disponible en pdf](#)

[Les pages des travaux dirigés](#)

[La pale de juillet 2006 et son corrigé](#)

<http://www.enseignement.polytechnique.fr/informatique/INF311/>



## Agenda



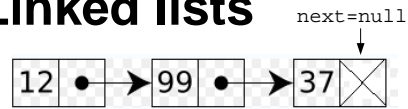
**A few algorithms and paradigms:**

- I/ **Exhaustive search**
- II/ **Greedy algorithm** (set cover problems)
- III/ **Dynamic programming** (knapsack)

+...*Merging two lists...*



# Linked lists



```

public class List
{
    int container;
    List next; // a reference to a cell of a list

    // Constructor List(head, tail)
    // Build a cell so that
    // the reference to the next cell is the tail
    List(int element, List tail)
    {
        this.container=element;
        this.next=tail;
    }
}
  
```

# Merging ordered linked lists

- Two *linked lists*  $u$  and  $v$  of increasing integers
- Build a *new linked list* in increasing order...
- ...using **only cells** of  $u$  and  $v$  (no cell creation, new)

For example:

U	3-->6-->8-->null
V	2-->4-->5-->7-->9-->null
Merge(U,V)	2-->3-->4-->5-->6-->7-->8-->9-->null

# Linked lists

```

class List
{
    int container;
    List next;

    // Constructor List(head, tail)
    List(int element, List tail)
    {
        this.container=element;
        this.next=tail;
    }

    List insert(int el) // insert element at the head of the list
    {
        return new List(el,this);
    }

    void Display()
    {
        List u=this;
        while(u!=null)
        {
            System.out.print(u.container+"-->");
            u=u.next;
        }
        System.out.println("null");
    }
}
  
```

U	3-->6-->8-->null
V	2-->4-->5-->7-->9-->null

```

class MergeList
{
    //
    // Merge two ordered lists
    //
    static List mergeRec(List u, List v)
    {
        if (u==null) return v;
        if (v==null) return u;

        if (u.container < v.container)
        {
            // Recycle cells/ no new
            u.next=mergeRec(u.next,v);
            return u;
        }
        else
        {
            // Recycle cells/no new
            v.next=mergeRec(u,v.next);
            return v;
        }
    }
}
  
```

```

public static void main(String [] args)
{
    List u=new List(8,null);
    u=u.insert(6);u=u.insert(3);
    u.Display();

    List v=new List(9,null);
    v=v.insert(7);v=v.insert(5);
    v=v.insert(4);v=v.insert(2);
    v.Display();

    List w=mergeRec(u,v);
    w.Display();
}
  
```

```

static List sortRec(List u)
{
    int i,l=u.length(), lr;
    List l1, l2, split, psplit; // references to cells

    if (l<=1)
        return u;
    else
    {
        l1=u;

        psplit=split=u;
        i=0;lr=l/2;

        while (i<lr)
        {i++;
         psplit=split;
         split=split.next;}

        l2=split; // terminates with a null
        psplit.next=null;

        return mergeRec( sortRec(l1), sortRec(l2) );
    }
}

```

### Sort in $O(n \log n)$ time:

- Split list in two halves
- Recursively apply sorting
- Merge ordered lists



```

public static void main(String [] args)
{

    List u=new List(3,null);
    u=u.insert(2);
    u=u.insert(9);
    u=u.insert(6);u=u.insert(1);
    u=u.insert(15);u=u.insert(17);
    u=u.insert(23);u=u.insert(21);
    u=u.insert(19);u=u.insert(20);

    u.Display();

    List sortu=sortRec(u);
    System.out.println("Sorted linked list:");
    sortu.Display();
}

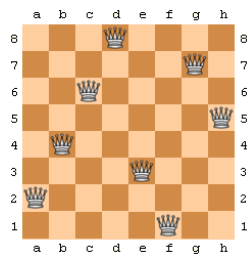
```

20-->19-->21-->23-->17-->15-->1-->6-->9-->2-->3-->null  
Sorted linked list:  
1-->2-->3-->6-->9-->15-->17-->19-->20-->21-->23-->null



## Exhaustive search (Brute force search)

The Eight Queens puzzle:



Max Bezzel (1848, chess player)  
Find safe positions of 8 queens  
on a 8x8 chessboard

- 92 distinct solutions
- 12 non-naïve distinct solutions (rotation/symmetry)
- Good exercise for *designing* algorithms
- Generalize to n-queens



## Exhaustive search & Backtracking

### Brute force (naïve) algorithm:

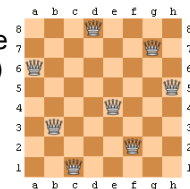
Check all  $64 \times 63 \times \dots \times 57/8! = 283,274,583,552$  ?! solutions...

Easy to check that a **configuration** is not safe  
(check horizontal/vertical/diagonal lines)

→ Two queens cannot be on the same line...

Therefore, incrementally place queen  $i$  (0...7)  
on the  $i$ -th row, on the first **free** column

If there is no more free columns left, then **backtrack...**



# Exhaustive search & Backtracking

Incrementally place queen i (0...7) on the first **free** column  
If there is no more free columns left, then **backtrack**:



Consider the previous queen position and increment its column position, etc., etc., etc.

... until we find a solution  
(=reach a successful location for queen indexed 7)

queen: 1D Array that specifies the column position  
Queen i is located at position (i, queen[i])  
(with i ranging from 0 to 7)

search: Static function that returns a/all solution(s).



```
// Are queen (i1,j1) and queen (i2,j2) safe ?
static boolean WrongPos(int i1, int j1, int i2, int j2)
{ // same row?, same col?, same diag?
  return (i1==i2 ||
          j1==j2 ||
          Math.abs(i1-i2) == Math.abs(j1-j2));
}
```

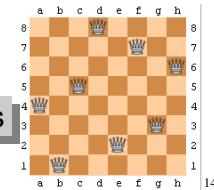
```
// Place safely queen i at column j?
static boolean FreeMove(int i, int j)
{
  boolean result=true;
```

Check for the queens  
placed so far  
on the chessboard

```
for(int k=0;k<i;k++)
  result=result&&!WrongPos(i,j,k,queen[k]);
```

```
return result;
}
```

Static functions to check for collisions



```
static boolean search(int row)
{
  boolean result=false;
```

```
if (row==n)
{ // Terminal case
  DisplayChessboard();
  nbsol++;
}
else
{ // Exhaustive search
  int j=0;
```

```
  while(!result && j<n)
  {
    if (FreeMove(row,j))
    {
      queen[row]=j;
      result=search(row+1); // RECURSION/BACKTRACK
    }
    j++; // explore all columns
  }
}
```

Increment the number  
of found solutions  
(static class variable)



```
static final int n=8;
static int [] queen=new int[n];
static int nbsol;
```

```
static void DisplayChessboard()
{
  int i,j;
```

```
System.out.println("");
```

```
for(i=0;i<n;i++)
{
  for(j=0;j<n;j++)
  {
    if (queen[i]!=j) System.out.print("0");
    else System.out.print("1");
  }
  System.out.println("");
}
```

```
00100000
10000000
00000010
00010000
00000100
00000001
00100000
10000000
00000100
00100000
00001000
00000010
00010000
00000001
00010000
00100000
00000100
01000000
00000010
00001000
00001000
Total number of solution:92
```

```
public static void main(String [] arguments)
{
  nbsol=0;
  search(0); // Call exhaustive search procedure
  System.out.println("Total number of solutions:"+nbsol);
}
```

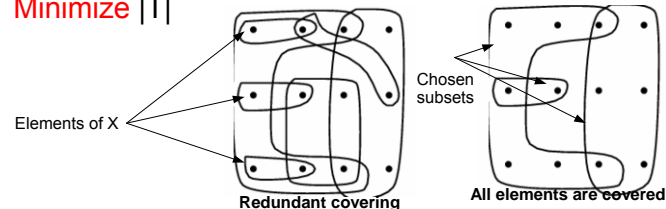


## Optimization: Set Cover Problem (SCP)

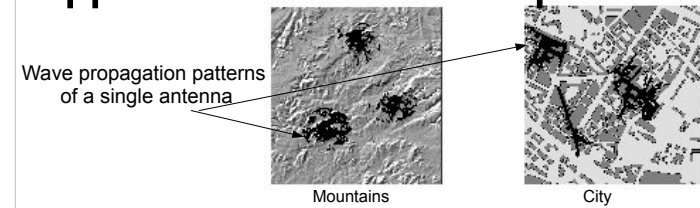
- Given a graph:
  - A **finite set**  $X = \{1, \dots, n\}$
  - A **collection of subsets** of  $S$ :  $S_1, S_2, \dots, S_m$

- Problem:**

- Find a subset  $T$  of  $\{1, \dots, m\}$  such that  $\bigcup_{j \in T} S_j = X$
- Minimize**  $|T|$



## Applications of set cover problems

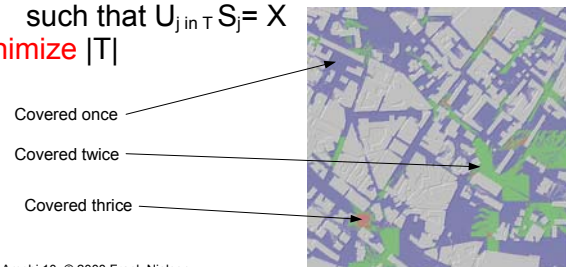


- Choose base stations for quality of service (cell phone network, etc)
- Discretize terrain using a **regular grid** (set  $X$ )
- Each position of antenna  $\rightarrow$  subset of covered grid areas ( $S$ )
- Minimize the number of chosen antennas** ( $X$ )

- Given a graph:
  - A **finite set**  $X = \{1, \dots, n\}$  (=regular grid elements)
  - A **collection of subsets** of  $S$  (=antenna patterns)  $S_1, S_2, \dots, S_m$

- Problem:**

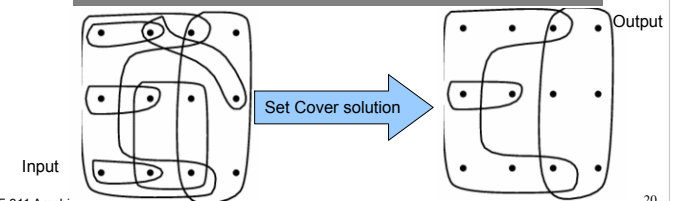
- Find a subset  $T$  of  $\{1, \dots, m\}$  (=subset of antennas) such that  $\bigcup_{j \in T} S_j = X$
- Minimize**  $|T|$



## A greedy algorithm (algorithme glouton)

```

GREEDY-SET-COVER( $X, S$ )
1  $M \leftarrow X$  // all elements must be covered
2  $C \leftarrow \emptyset$  // chosen subsets
3 while  $M \neq \emptyset$  do
4   select an  $Q \in S$  that maximizes  $|Q \cap M|$ 
5    $M \leftarrow M - Q$ 
6    $C \leftarrow C \cup \{Q\}$ 
7 return  $C$ 
    
```



## Visualizing a « range set »

Elements:

$X = \{1, \dots, 6\}$

Subsets:

$S_1 = \{1, 2, 4\}$

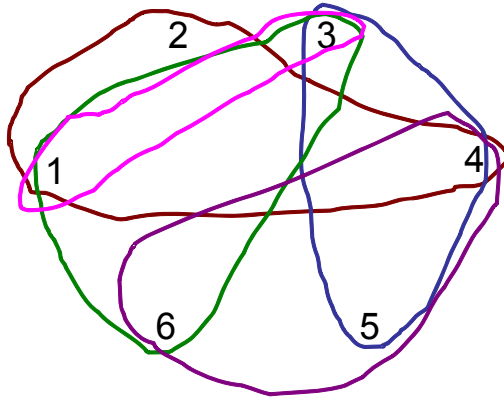
$S_2 = \{3, 4, 5\}$

$S_3 = \{1, 3, 6\}$

$S_4 = \{2, 3, 5\}$

$S_5 = \{4, 5, 6\}$

$S_6 = \{1, 3\}$



## Data-structure for the set cover problem

$X = \{1, \dots, 6\}$

$S_1 = \{1, 2, 4\}$

$S_2 = \{3, 4, 5\}$

$S_3 = \{1, 3, 6\}$

$S_4 = \{2, 3, 5\}$

$S_5 = \{4, 5, 6\}$

$S_6 = \{1, 3\}$

M=6 SUBSETS

	1	2	3	4	5	6	
$S_1$	1	1	0	1	0	0	$S_1$
$S_2$	0	0	1	1	1	0	$S_2$
$S_3$	1	0	1	0	0	1	$S_3$
$S_4$	0	1	1	0	1	0	$S_4$
$S_5$	0	0	0	1	1	1	$S_5$
$S_6$	1	0	1	0	0	0	$S_6$
	↑	↑	↑	↑	↑	↑	
	N=6 ELEMENTS						

**Incidence matrix: boolean matrix**

```
class SetCover
{
    int nbelements;
    int nsubsets;
    boolean [][] incidenceMatrix;

    //Constructor
    SetCover(int nn, int mm)
    {
        this.nbelements=nn; this.nsubsets=mm;

        incidenceMatrix=new boolean[nsubsets][nbelements];

        for(int i=0;i<nsubsets;i++)
            for(int j=0;j<nbelements;j++)
                incidenceMatrix[i][j]=false;
    }

    void SetSubsets(int [][] array) // Set incidence matrix
    {for(int j=0;j<array.length;j++)
        {for(int i=0;i<array[j].length;i++)
            incidenceMatrix[j][array[j][i]]=true;
        }
    }
}
```

```
void Display()
{
    for(int i=0;i<nsubsets;i++){
        for(int j=0;j<nbelements;j++){
            if (incidenceMatrix[i][j]) System.out.print("1");
            else System.out.print("0");
            System.out.println("");
        }
    }
}

public static void main(String [] args)
{
    int [][] subsets={{0,1,3},{2,3,4},{0,2,5},{1,2,4},{3,4,5},{0,2}};

    SetCover setcover=new SetCover(6,6);
    setcover.SetSubsets(subsets);

    System.out.println("Set cover problem:");
    setcover.Display();
}
```

```
Set cover problem:
110100
001110
101001
011010
000111
101000
```

```

static boolean [] GreedySCP(SetCover problem)
{
    boolean [] result=new boolean[problem.nbsubsets];
    int cover=0; int select;

    for(int i=0;i<problem.nbsubsets;i++) // initially no subsets
        result[i]=false;

    while(cover!=problem.nbelements)
    {
        // Choose largest not-yet covered subset
        select=problem.LargestSubset();
        result[select]=true;

        // Update covered matrix
        cover+=problem.Cover(select);

        // Update incidence matrix
        problem.Update(select);

        System.out.println("Selected "+select+" Number of covered
        elements="+cover);
        problem.Display();
    }

    return result;
}

```

### Greedy algorithm

```

// Number of covered element by subset i
int Cover(int i)
{
    int nbEl=0;

    for(int j=0;j<nbelements;j++)
        if (incidenceMatrix[i][j]) ++nbEl;

    return nbEl;
}

// Report the current largest subset
int LargestSubset()
{
    int i, nbEl, max, select;

    max=-1;select=-1;

    for(i=0;i<nbsubsets;i++)
    {
        nbEl=Cover(i);
        if (nbEl>max) {max=nbEl; select=i;}
    }

    return select;
}

```

### Methods of class SetCover

```

// Update the incidence matrix
void Update(int sel)
{

```

```

    int i,j;

    for(i=0;i<nbsubsets;i++)
    {
        if (i!=sel) //use sel below so don't modify it
        {
            for(j=0;j<nbelements;j++)
                if (incidenceMatrix[sel][j])
                    incidenceMatrix[i][j]=false;
        }
    }
    // Remove the chosen subset as well
    for(j=0;j<nbelements;j++)
        incidenceMatrix[sel][j]=false;
}

```

### Methods of class SetCover

```

public static void main(String [] args)
{
    int [] [] subsets ={{0,1,3},{2,3,4},
                        {0,2,5},{1,2,4},
                        {3,4,5},{0,2}};

    SetCover setcover=new SetCover(6,6);

    setcover.SetSubsets(subsets);

    System.out.println("Set cover problem:");
    setcover.Display();

    boolean [] solution=GreedySCP(setcover);

    System.out.print("Solution:");
    for(int i=0;i<setcover.nbsubsets;i++)
        if (solution[i]) System.out.print(" "+i);
    System.out.println("");
}

```

Set cover problem:

```

110100
001110
101001
011010
000111
101000
Selected 0 Number of covered elements=3
000000
001010
001001
001010
000011
001000
Selected 1 Number of covered elements=5
000000
000000
000001
000000
000001
000000
Selected 2 Number of covered elements=6
000000
000000
000000
000000
000000
000000
000000
Solution: 0 1 2

```

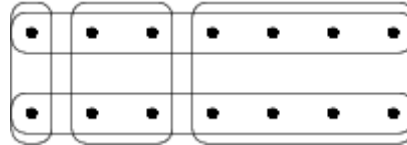
## Optimization bounds for greedy algorithm

$\text{CoptT} \leq \text{Cgreedy} \leq \text{ApproximationFactor} \times \text{Copt}$

### Upper bound:

Approximation factor is at most  $H(n) \leq \log(n)$

2 sets is optimal solution  
but greedy chooses 3 sets here



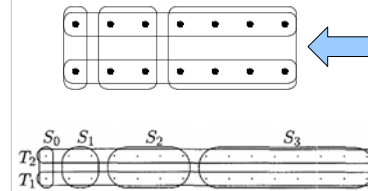
### Lower bound:

Approximation factor can be as big as  $\Omega(\log(n))$

**Difficult to approximate:** cannot beat  $(1-\epsilon)\text{Opt}$  unless  $P=NP$



```
int [] [] subsets={{0,1,2,3,4,5,6},{7,8,9,10,11,12,13},
                  {0,7},{1,2,8,9},{3,4,5,6,10,11,12,13}};
SetCover setcover=new SetCover(14,5);
```



Etc...

Easy to build generic examples  
where greedy does not behave well  
with  $O(\log n)$  approximation ratio

Set cover problem:

```
11111110000000
00000001111111
10000001000000
01100000110000
00011110001111
Selected 4 Number of covered elements=8
11100000000000
00000001110000
10000001000000
01100000110000
00000000000000
Selected 3 Number of covered elements=12
10000000000000
00000001000000
10000001000000
00000000000000
00000000000000
Selected 2 Number of covered elements=14
00000000000000
00000000000000
00000000000000
00000000000000
Solution: 2 3 4
```



## Knapsack problem (sac a dos)

(First version)

Given:

- A set of  $n$  Objects  $o_1, \dots, o_n$  with corresponding weights  $w_1, \dots, w_n$
- And a bag (knapsack) of capacity  $W$

Find:

All the ways of choosing objects to **fully fill** the bag



## Filling the knapsack

Need to enumerate all possibilities:

$n$  objects  $\Rightarrow 2^n$  choices

(2, 4, 8, 16, 32, 64, ...)

How to program this?

$n$  is a variable

(cannot fix the number of nest loops)

Need to enumerate all combinations:

= **Exhaustive search**

$n=4$

$2^4=16$

```
1 1 1 1
1 1 1 0
1 1 0 1
1 1 0 0
1 0 1 1
1 0 1 0
1 0 0 1
1 0 0 0
0 1 1 1
0 1 1 0
0 1 0 1
0 1 0 0
0 0 1 1
0 0 1 0
0 0 0 1
0 0 0 0
```





## Enumerating: A recursive approach

```
static void Display(boolean [] tab)
{
    for(int i=0;i<tab.length;i++)
        if (tab[i]) System.out.print("1 ");
        else
            System.out.print("0 ");

    System.out.println("");
}

static void Enumerate(boolean [] selection, int pos)
{
    if (pos==selection.length-1)
        Display(selection);
    else
    {
        pos++;
        selection[pos]=true;
        Enumerate(selection,pos);
        selection[pos]=false;
        Enumerate(selection,pos);
    }
}

public static void main(String[] args)
{
    int n=4;
    int i;
    boolean [] select=new boolean[n];
    for(i=0;i<n;i++)
        select[i]=false;
    Enumerate(select,-1);
}
```



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## Fully filling the knapsack

```
final static int n=10; // 10 objects
static int [] weight={2,3,5,7,9,11,4,13,23,27};

static void SolveKnapSack(boolean [] chosen, int goal, int
i, int total)
{
    numbercall++; // keep track of total number of calls

    if ((i>=chosen.length)&&(total!=goal)) return;

    if (total==goal)
    {
        Display(chosen, goal);
        numbersol++; // total number of solutions
    }
    else
    {
        chosen[i]=true; // add item first
        SolveKnapSack(chosen,goal,i+1,total+weight[i]);
        chosen[i]=false; // and then remove it
        SolveKnapSack(chosen,goal,i+1,total);
    }
}

}
```



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```
final static int n=10; // 10 objects
static int [] weight={2,3,5,7,9,11,4,13,23,27};
```

```
public static void main(String [] args)
{
    int totalweight=51;
    numbersol=0;
    numbercall=0;
```

```
System.out.println("Knapsack:");
boolean [] chosen=new boolean[n];

SolveKnapSack(chosen, totalweight, 0, 0);
System.out.println("Total number of
solutions:"+numbersol);
System.out.println(" #calls="+numbercall);
}
```

Knapsack:

```
2+3+5+7+11+23+0=51
2+5+7+9+11+4+13+0=51
2+5+4+13+27+0=51
2+7+11+4+27+0=51
2+9+4+13+23+0=51
2+9+13+27+0=51
3+5+7+9+4+23+0=51
3+5+7+9+27+0=51
3+5+7+13+23+0=51
3+5+9+11+23+0=51
7+4+13+27+0=51
9+11+4+27+0=51
11+4+13+23+0=51
11+13+27+0=51
Total number of solutions:14
#calls=2029
```



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## Exhaustive search: Branch & bound

```
static void SolveKnapSack(boolean [] chosen, int goal, int i,
int total)
{
    numbercall++;
```

Stop recursion if we already  
exceed the weight amount

```
if (total>goal) return; // cut
```

```
if ((i>=chosen.length)&&(total!=goal)) return;
```

```
if (total==goal)
{
    Display(chosen, goal);
    numbersol++;
}
else
{
    chosen[i]=true; // add item first
    SolveKnapSack(chosen,goal,i+1,total+weight[i]);
    chosen[i]=false; // and then remove it
    SolveKnapSack(chosen,goal,i+1,total);
}
```

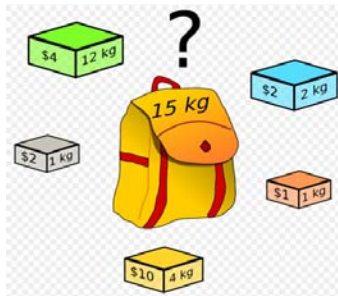
```
Knapsack:
2+3+5+7+11+23+0=51
2+5+7+9+11+4+13+0=51
2+5+4+13+27+0=51
2+7+11+4+27+0=51
2+9+4+13+23+0=51
2+9+13+27+0=51
3+5+7+9+4+23+0=51
3+5+7+9+27+0=51
3+5+7+13+23+0=51
3+5+9+11+23+0=51
7+4+13+27+0=51
9+11+4+27+0=51
11+4+13+23+0=51
11+13+27+0=51
Total number of solutions:14 #calls=1791
```



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## Knapsack: Fundamental optimization problem



Given a bag capacity (15 kg),  
**maximize** the **utility** (price) of selected objects

(NP-hard problem)

## Knapsack optimization problem

Given  $p_1, p_2, \dots, p_n \leftarrow$  weights

$a_1, a_2, \dots, a_n \leftarrow$  utility

$P_{max} \leftarrow$  Maximum weight  
(capacity of bag)

**Optimize**  $\sum_{i \in I} a_i$   
 such that

$$\sum_{i \in I} p_i \leq P_{max}$$

(Maybe there exists several solutions)

## Knapsack: Example

$P_{max} = 12$       8 objects

weight	2	3	5	2	4	6	3	1
utility	5	8	14	6	13	17	10	4

## Dynamic programming (Knapsack)

**Dynamic programming** computes a table...  
 ... from which a **solution** can be retrieved.

Requires a **relational equation** to deduce  
 solutions progressively.

# Dynamic programming (Knapsack)

Let  $u(i,j)$  be the maximum utility by taking objects in  $\{1, \dots, i\}$  with total weight  $\leq j$

- If  $i=1$  then  $u(i,j)=0$  for  $j < p_1$  and  $u(i,j)=A_1$  for  $j \geq p_1$

- If  $i > 1$   

$$u(i,j) = \max(u(i-1,j), u(i-1,j-p_i) + A_i)$$

Do not take object  $O_i$

Take object  $O_i$ :

- gain  $A_i$
- but leave room:  $p_i$

weight	2	3	5	2	4	6	3	1	Pmax= 12	
utility	5	8	14	6	13	17	10	4		

$$u(i,j) = \max(u(i-1,j), u(i-1,j-p_i) + A_i)$$

$u_{i,j}$	0	1	2	3	4	5	6	7	8	9	10	11	12
$i=1$	0	0	5	5	5	5	5	5	5	5	5	5	5
$i=2$	0	0	5	8	8	13	13	13	13	13	13	13	13
$i=3$	0	0	5	8	8	14	14	19	22	22	27	27	27
$i=4$	0	0	6	8	11	14	14	20	22	25	28	28	33
$i=5$	0	0	6	8	13	14	19	21	24	27	28	33	35
$i=6$	0	0	6	8	13	14	19	21	24	27	30	33	36
$i=7$	0	0	6	10	13	16	19	23	24	29	31	34	37
$i=8$	0	4	6	10	14	17	20	23	27	29	33	35	38

Optimization result: Maximum utility, given Pmax

## Reading back: Solution

	2	3	5	2	4	6	3	1	
	5	8	14	6	13	17	10	4	

$u_{i,j}$	0	1	2	3	4	5	6	7	8	9	10	11	12	
$i=1$	0	0	5	5	5	5	5	5	5	5	5	5	5	Choose (38=33+5, 2-2=0)
$i=2$	0	0	5	8	8	13	13	13	13	13	13	13	13	5=5 do not choose
$i=3$	0	0	5	8	8	14	14	19	22	22	27	27	27	5=5 do not choose
$i=4$	0	0	6	8	11	14	14	20	22	25	28	28	33	Choose (33=27+6, 4-2=2)
$i=5$	0	0	6	8	13	14	19	21	24	27	28	33	35	Choose (27=14+13, 8-4=4)
$i=6$	0	0	6	8	13	14	19	21	24	27	30	33	36	24=24, do not choose
$i=7$	0	0	6	10	13	16	19	23	24	29	31	34	37	Choose (14, 11-3=8)
$i=8$	0	4	6	10	14	17	20	23	27	29	33	35	38	Choose (4, 11)

Choose (Utility=0, Pmax=12)

From the table, chosen solution: O8, O7, O5, O4, O1

```
static int nbObjects=8;
static int [] weight={2,3,5,2,4,6,3,1};
static int [] utility={5,8,14,6,13,17,10,4};
static int weightmax=12;
static int [][] array;

static void SolveDP()
{
    int i,j;
    array=new int[nbObjects][weightmax+1];

    // initialize the first row
    for(j=0;j<=weightmax;j++)
        if (j<weight[0]) array[0][j]=0;
        else array[0][j]=utility[0];
    // for all other rows
    for(i=1;i<nbObjects;i++)
    {
        for(j=0;j<=weightmax;j++)
            if (j-weight[i]<0) array[i][j]=array[i-1][j];
            else
                array[i][j]=max( array[i-1][j],
                                array[i-1][j-weight[i]]+utility[i]);
    }
}
```

```

static void InterpretArray()
{
    int i,u,w;
    u=0;
    w=weightmax;

    for(i=nbObjects-1;i>=1;i--)
    {
        if (array[i][w]!=array[i-1][w])
        {System.out.print((i+1)+" ");
         w=w-weight[i];
         u=u+utility[i];
        }
    }

    if (array[0][w]!=0);
    {System.out.println("1");
     w=w-weight[0];
     u=u+utility[0];
    }
    System.out.println("Cross check:"+u+" remaining weight "+w);
}

```

```

public static void main(String[] args)
{
    System.out.println("Solving knapsack using the dynamic
        programming paradigm.");

    SolveDP();
    Display();
    System.out.println("Reading solution:");
    InterpretArray();
}

```

```

Solving knapsack using the dynamic programming paradigm.
0 0 5 8 8 13 13 13 13 13 13 13
0 0 5 8 8 14 14 19 22 27 27 27
0 0 6 8 11 14 14 20 22 25 28 33
0 0 6 8 13 14 19 21 24 27 33 35
0 0 6 8 13 14 19 21 24 27 30 36
0 0 6 10 13 16 19 23 24 29 31 34
0 4 6 10 14 17 20 23 27 29 33 35

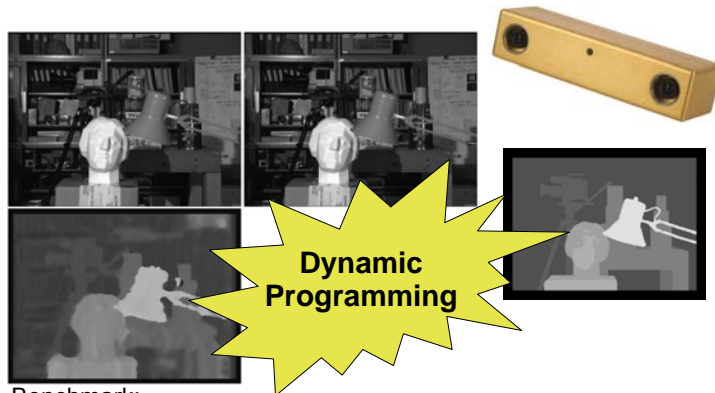
```

```

Reading solution:
2 2 5 4 1
Cross check:38 remaining weight 0

```

## Dynamic programming: binocular stereo matching



Benchmark:  
<http://vision.middlebury.edu/~schar/stereo/web/results.php>

## Optimization: A brief summary

- Exhaustive search: recursion but  $O(2^n)$  complexity
- Can be improved by backtracking (cuts)
- Greedy algorithm: Polynomial  $O(n^3)$   
but yields an approximation
- Dynamic programming yields an exact solution  
but requires  $O(\text{weight} \times \text{objects})$  time  
(weights should not be too big)

# Last but not least: Java applets!

**Applets** are special java programs...  
...that can run into your favorite Internet **browser**

You need to:

- (1) write a web page with <APPLET> </APPLET> tags
- (2) write and compile the Java applet code (javac)

Advantages of applets are:

- (1) to be accessible worldwide
- (2) to provide graphics



```
import java.awt.*;
import java.applet.*;

class Point2D
{double x,y;
Point2D()
{this.x=300.0*Math.random();
this.y=300.0*Math.random();}}

public class AppletINF311 extends Applet {
final static int n=100;
static Point2D [] set;

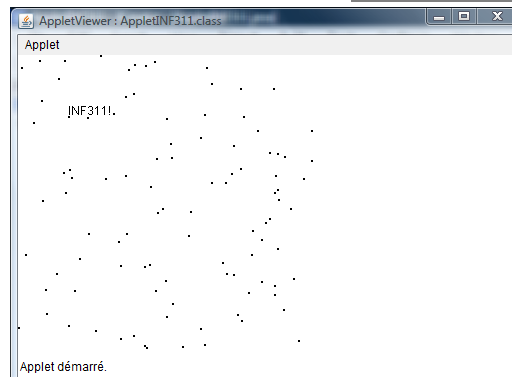
    public void init() {
        int i;
        set=new Point2D[n];
        for(i=0;i<n;i++)
            set[i]=new Point2D();}

    public void paint(Graphics g) {int i;

        for(i=0;i<n;i++)
        {
            int xi, yi;
            xi=(int)set[i].x; yi=(int)set[i].y;
            g.drawRect(xi, yi,1,1);
        }
        g.drawString("INF311!", 50, 60 );
    }
}
```

## Java applets

```
<APPLET
  code = "AppletINF311.class"
  width = "500"
  height = "300"
>
</APPLET>
```



## Java applets for online demos...



Hallucination (mirage)

Two technologies in use:

- Image **segmentation**
- Texture **synthesis** (hole filling)

Try it !!! <http://www.sonycsl.co.jp/person/nielsen/ClickRemoval/>

F. Nielsen, R. Nock ClickRemoval: interactive pinpoint image object removal. *ACM Multimedia 2005*

# There is much more in Java!

Java Language		Java Language											
Tools & Tool APIs	java	javac	javadoc	apt	jar	javap	JPDA	jconsole					
	Security	Int'l	RMI	IDL	Deploy	Monitoring	Troubleshoot	Scripting	JVM TI				
Deployment Technologies	Deployment			Java Web Start				Java Plug-in					
	AWT			Swing				Java 2D					
User Interface Toolkits	Accessibility	Drag n Drop	Input Methods		Image I/O		Print Service		Sound				
	IDL	JDBC~	JNDI~	RMI	RMI-IIOP		Scripting						
JDK Integration Libraries	Beans	Intl Support	I/O	JMX	JNI	Math							Java SE API
	Networking	Override Mechanism	Security	Serialization	Extension Mechanism		XML JAXP						
JRE Other Base Libraries	lang and util	Collections	Concurrency Utilities	JAR	Logging	Management							
	Preferences API	Ref Objects	Reflection	Regular Expressions	Versioning	Zip	Instrument						
Java Virtual Machine	Java Hotspot™ Client VM					Java Hotspot™ Server VM							
	Solaris™		Linux		Windows		Other						
Platforms													

**JDK™ 6 Documentation**

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# A glimpse at object inheritance

All objects inherit from the topmost object: `Object`  
...meaning some methods are already **predefined**

java.lang  
**Class Object**  
java.lang.Object

Method Summary	
protected Object	<code>clone()</code> Creates and returns a copy of this object.
boolean	<code>equals(Object obj)</code> Indicates whether some other object is "equal to" this one.
protected void	<code>finalize()</code> Called by the garbage collector on an object when garbage collection determines that there are no more references to the object.
Class	<code>getClass()</code> Returns the runtime class of this <code>Object</code> .
int	<code>hashCode()</code> Returns a hash code value for the object.
void	<code>notify()</code> Wakes up a single thread that is waiting on this object's monitor.
void	<code>notifyAll()</code> Wakes up all threads that are waiting on this object's monitor.
String	<code>toString()</code> Returns a string representation of the object.

Can overwrite the method `toString`

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# A glimpse at object inheritance

```

class Point2D
{
    double x,y;
    // Constructor
    Point2D(double xi, double yi)
    {
        this.x=xi;this.y=yi;
    }
    // Overrides default method
    public String toString()
    {
        return "["+x+" "+y+"]";
    }
}

class SuperClass
{
    public static void main(String [] a)
    {
        Point2D myPoint=new Point2D(Math.PI, Math.E);
        System.out.println("Point: "+myPoint);
    }
}
    
```

```

Point: [3.141592653589793 2.718281828459045]
    
```

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<http://www.enseignement.polytechnique.fr/profs/informatique/Robert.Cori/Questionnaire/resultatSondage.html>

**Sondage sur l'enseignement INF 311 (débutants), Promotion X2006: I résultats (mer jui 4 13:14:23 CEST 2007)**

Nombre de réponses: 241

Les renseignements que vous fournirez dans ce sondage seront très utiles pour l'amélioration des enseignements futurs.

**Qui êtes-vous?**

- Votre nom (facultatif)
- Filière d'entrée à l'X: MPI 2 MPI 58 PC 112 PSI 36 PT 8 TSI 2 UNIV 2 EVI 3 EV2 22
- Votre numéro de groupe de TD: 1 20 2 16 3 19 4 13 5 20 6 21 7 21 8 22 9 19 10 18 11 24 12 21

**Opinions générales sur le cours**

- Intérêt de la matière enseignée: Grand 59 Assez grand 140 Assez faible 44 Faible 6
- Qualité globale de l'enseignement: Grand 50 Assez grand 175 Assez faible 70 Faible 1

**Please answer the poll INF311 (in TD10)**

Plume Wolligung, tu feras informatique!  
Wolligung, tu feras informatique!

Inspired by Rober Cori...

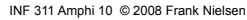
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# Merci et a bientot!

Année	Septembre	Octobre	Novembre	Décembre	Janvier	Février	Mars	Avril	Mai	Jun	Juillet	Août
1	Incorporation										<b>RF 311:</b> Introduction à l'informatique <b>RF 321:</b> Principes des langages de programmation	
2	<b>RF 421:</b> Programmation et Algorithmique		<b>RF 421:</b> Programmation et Algorithmique				<b>RF 431:</b> Algorithmes et Programmation: du séquentiel au distribué					
							<b>RF 444:</b> Modex	<b>RF 444:</b> Modex				
3			Programmes d'approfondissement (Master 1)					Stage de recherche (Master 1)				
4			Année professionnalisante (Master 2)									

<http://www.enseignement.polytechnique.fr/informatique/>

- M1 Informatique, thematique Image
- Colloquium Nov. on visual computing.

[illegible]