Monte Carlo Radiation Transfer in Circumstellar Disks

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Systems with Disks

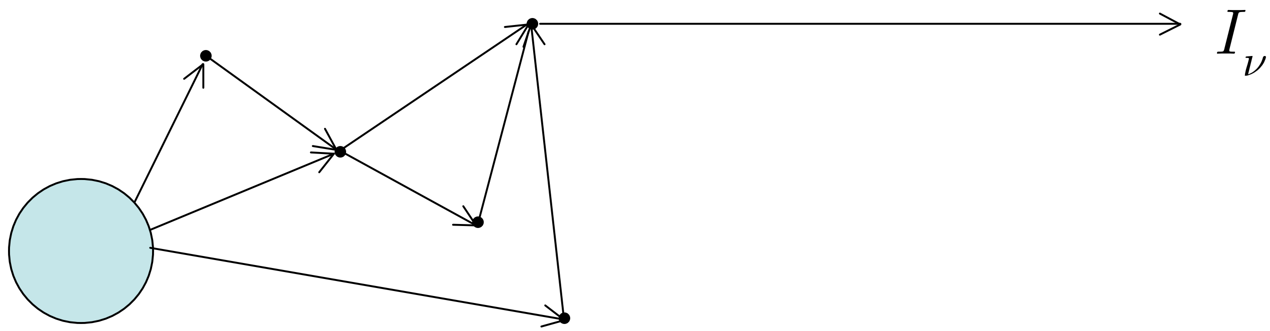
- Infall + Rotation
  - Young Stellar Objects (T Tauri, Herbig Ae/Be)
  - Mass Transfer Binaries
  - Active Galactic Nuclei (Black Hole Accretion Disks)
- Outflow + Rotation (?)
  - AGBs (bipolar planetary nebulae)
  - LBVs (e.g., Eta Carinae)
  - Oe/Be, B[e]
    - Rapidly rotating (V_{rot} = 350 km s^{-1})
    - Hot stars (T = 20000K)
    - Ideal laboratory for studying disks

3-D Radiation Transfer

- Transfer Equation

$$\hat{\mathbf{n}} \cdot \nabla I_\nu = -\chi_\nu \rho I_\nu + j_\nu + \sum_i n_i \int \frac{d\sigma_\nu^i}{d\Omega} I_\nu(\hat{\mathbf{n}}, \hat{\mathbf{n}}') d\Omega'$$

  – Ray-tracing (requires L-iteration)



  – Monte Carlo (exact integration using random paths)
    - *May* avoid L-iteration
    - automatically an adaptive mesh method
      – Paths sampled according to their importance

Monte Carlo Radiation Transfer

- Transfer equation traces flow of energy
- Divide luminosity into equal energy packets ("photons")

$$E_\gamma = L\Delta t \,/\, N_\gamma$$

  - Number of physical photons

$$n = E_\gamma \,/\, h\nu$$

  - Packet may be partially polarized

$$\begin{aligned}
I &= 1 \\
Q &= (E_{\updownarrow} - E_{\leftrightarrow}) \,/\, E_\gamma \\
U &= (E_{\swarrow\!\!\!\!\nearrow} - E_{\nwarrow\!\!\!\!\searrow}) \,/\, E_\gamma \\
V &= (E_{\circlearrowright} - E_{\circlearrowleft}) \,/\, E_\gamma
\end{aligned}$$

Monte Carlo Radiation Transfer

- Pick random starting location, frequency, and direction
  - Split between star and envelope

$$L = L_* + L_{\text{env}}$$

Star

$$\mu I_\nu = \frac{dE \,/\, dt}{dA\, d\nu\, d\Omega}$$

$$\frac{dP}{dA} \propto H$$

$$\frac{dP}{d\nu} \propto H_\nu$$

$$\frac{dP}{d\Omega} \propto \mu I_\nu$$

Envelope

$$\frac{dP}{dV d\Omega} \propto j_\nu$$

Monte Carlo Radiation Transfer

- Doppler Shift photon packet as necessary
  - packet energy is frame-dependent

$$E_\gamma \rightarrow wE_\gamma \qquad w \text{ is photon "weight"}$$

- Transport packet to random interaction location

$$dP = d\tau = \chi_\nu \rho ds \qquad \text{(Poisson Distribution)}$$

$$dN = -N d\tau$$

$$P = 1 - e^{-\tau} \qquad \text{(Cumulative Probability)}$$

$$\tau = -\ln \xi \qquad (\xi \text{ is uniform random number})$$

$$\tau = \int_0^s \chi_\nu \rho ds \qquad (\text{find distance, } s)$$

most CPU time

$$\mathbf{x} = \mathbf{x}_0 + s\,\hat{\mathbf{n}} \qquad \text{(move photon)}$$

Monte Carlo Radiation Transfer

- Randomly scatter or absorb photon packet

$$a = \frac{\sigma_\nu}{\sigma_\nu + \kappa_\nu} \qquad \text{(albedo)}$$

$$\begin{cases} \xi > a & \text{(absorb + reemit)} \qquad \dfrac{dP}{d\Omega\, d\nu} \propto j_\nu \qquad \text{(emissivity)} \\ \xi < a & \text{(scatter)} \qquad \dfrac{dP}{d\Omega} = \dfrac{1}{\sigma_\nu} \dfrac{d\sigma_\nu}{d\Omega} \qquad \text{(phase function)} \end{cases}$$

- If photon hits star, reemit it locally
- When photon escapes, place in observation bin (direction, frequency, and location)

REPEAT 10^6-10^9 times

Sampling and Measurements

- MC simulation produces random events
  - Photon escapes
  - Cell wall crossings
  - Photon motion
  - Photon interactions
- Events are sampled
  - Samples => measurements (e.g., Flux)
  - Histogram => distribution function (e.g., I_n)

SEDs and Images

- Sampling Photon Escapes

$$\frac{F_\nu}{F_*} = \frac{4\pi d^2}{L}\frac{dE}{dt\,dA\,d\nu} = \frac{4\pi d^2}{L}\frac{N_{ij}E_\gamma \,/\, \Delta t}{d^2 d\Omega_i \Delta\nu_j} = \frac{4\pi N_{ij}}{N_\gamma d\Omega_i \Delta\nu_j}$$

where $N_{ij} = \sum w_{ij}$

$$\frac{I_\nu}{F_*} = \frac{4\pi N_{ijkl}}{N_\gamma d\Omega_{ij} d\Omega_k \Delta\nu_l}$$

SEDs and Images

- Source Function Sampling $I_\nu = \int e^{-\tau} S$
  - Photon interactions (scatterings/absorptions)

$$dI \propto \begin{cases} w\left(\dfrac{1}{\sigma}\dfrac{d\sigma}{d\Omega}\right)e^{-\tau_{\text{esc}}} & \text{(scattered)} \\ \dfrac{1}{4\pi}e^{-\tau_{\text{esc}}} & \text{(emitted)} \end{cases}$$

  - Photon motion (path length sampling)

$$dN = d\tau$$

$$dI_{\text{sc}} \propto w\,d\tau_{\text{sc}}\left(\frac{1}{\sigma}\frac{d\sigma}{d\Omega}\right)e^{-\tau_{\text{esc}}}$$

Monte Carlo Maxims

- Monte Carlo is EASY
  - to do wrong (G.W Collins III)
  - code must be tested *quantitatively*
  - being clever is dangerous
  - try to avoid discretization
- The Improbable event WILL happen
  - code must be bullet proof
  - and error tolerant

Monte Carlo Assessment

- Advantages
  - Inherently 3-D
  - Microphysics easily added (little increase in CPU time)
  - Modifications do not require large recoding effort
  - Embarrassingly parallelizable
- Disadvantages
  - High S/N requires large N_{g}
  - Achilles heel = no photon escape paths; i.e., large optical depth

Improving Run Time

- Photon paths are random
  - Can reorder calculation to improve efficiency
- Adaptive Monte Carlo
  - Modify execution as program runs
- High Optical Depth
  - Use analytic solutions in “interior” + MC “atmosphere”
    - Diffusion approximation (static media)
    - Sobolev approximation (for lines in expanding media)
  - Match boundary conditions

MC Radiative Equilibrium

- Sum energy absorbed by each cell
- Radiative equilibrium gives temperature

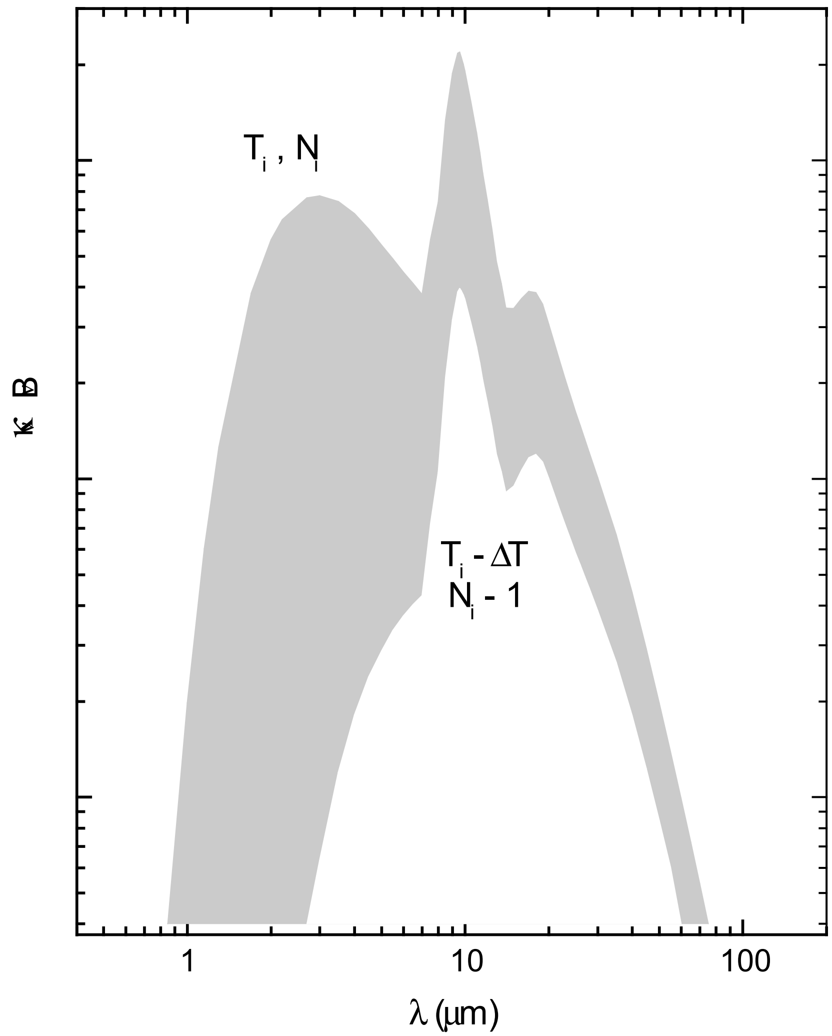
$$E_{\mathrm{abs}} = E_{\mathrm{emit}}$$

$$n_{\mathrm{abs}} E_{\gamma} = 4\pi m_i \kappa_{\mathrm{P}} B(T_i)$$

- When photon is absorbed, reemit at new frequency, depending on T
  - Energy conserved automatically
- Problem: Don't know T *a priori*
- Solution: Change T each time a photon is absorbed and correct previous frequency distribution

avoids iteration

Temperature Correction



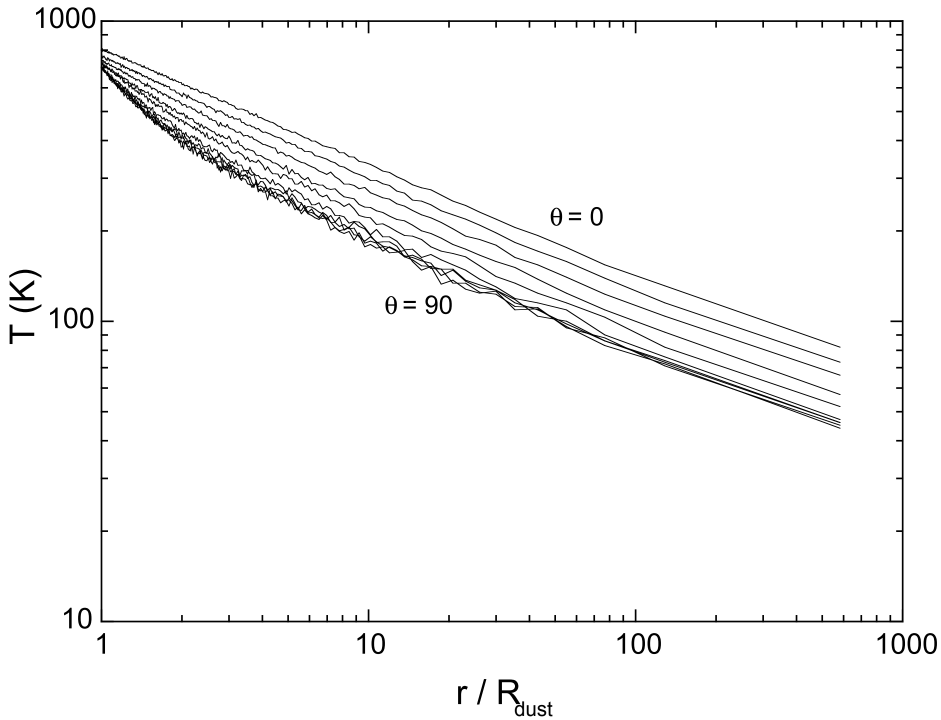
Frequency Distribution:

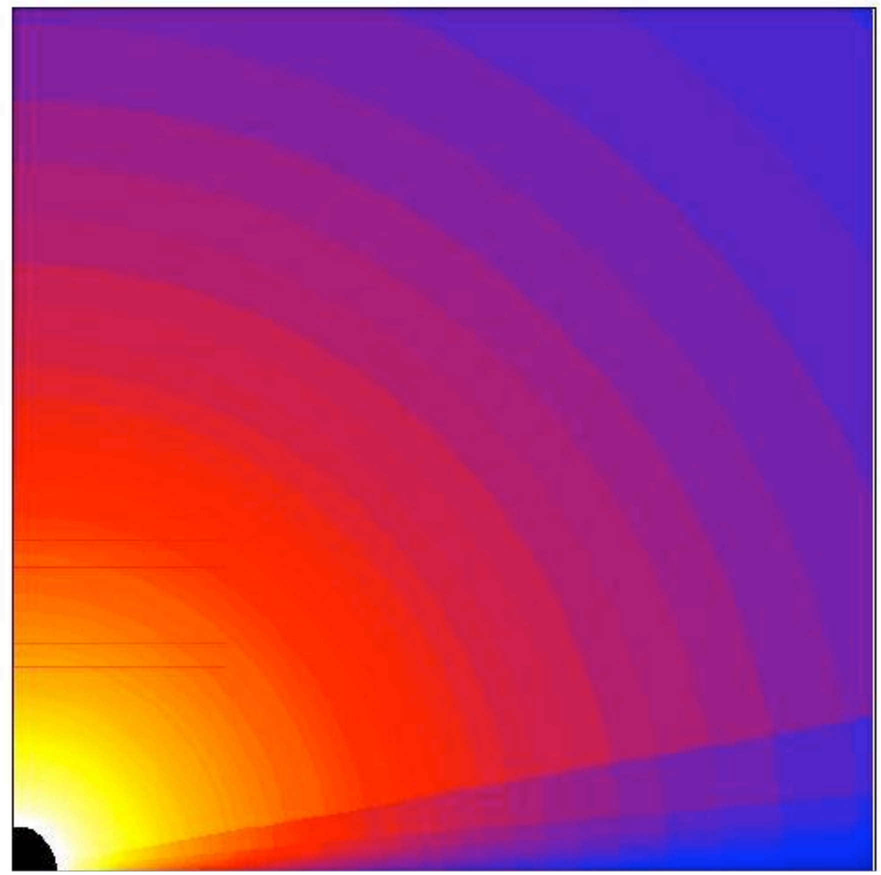
$$\frac{dP}{d\nu} = j_\nu(T + \Delta T) - j_\nu(T)$$

$$= \kappa_\nu \Delta T \frac{dB_\nu}{dT}$$

Bjorkman & Wood 2001

Disk Temperature



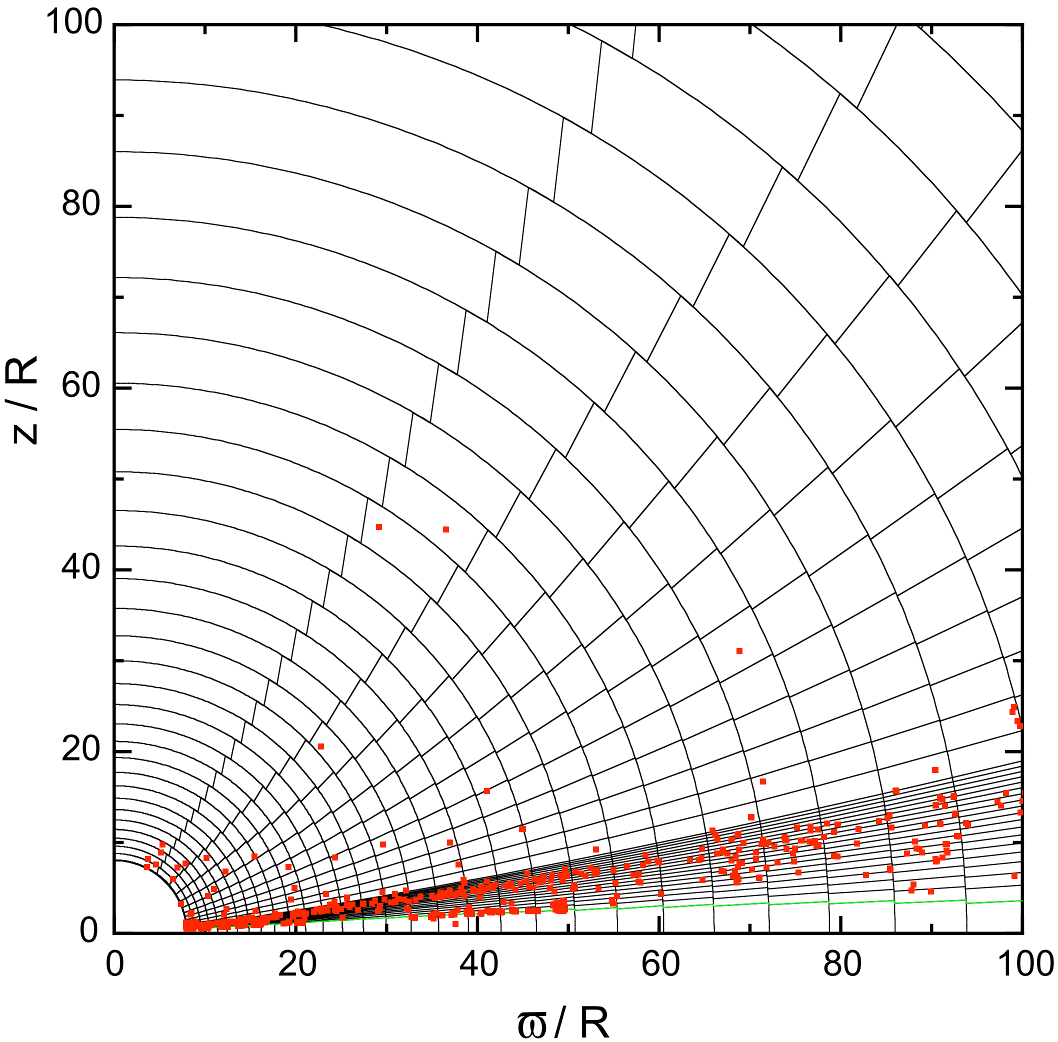


Bjorkman 1998

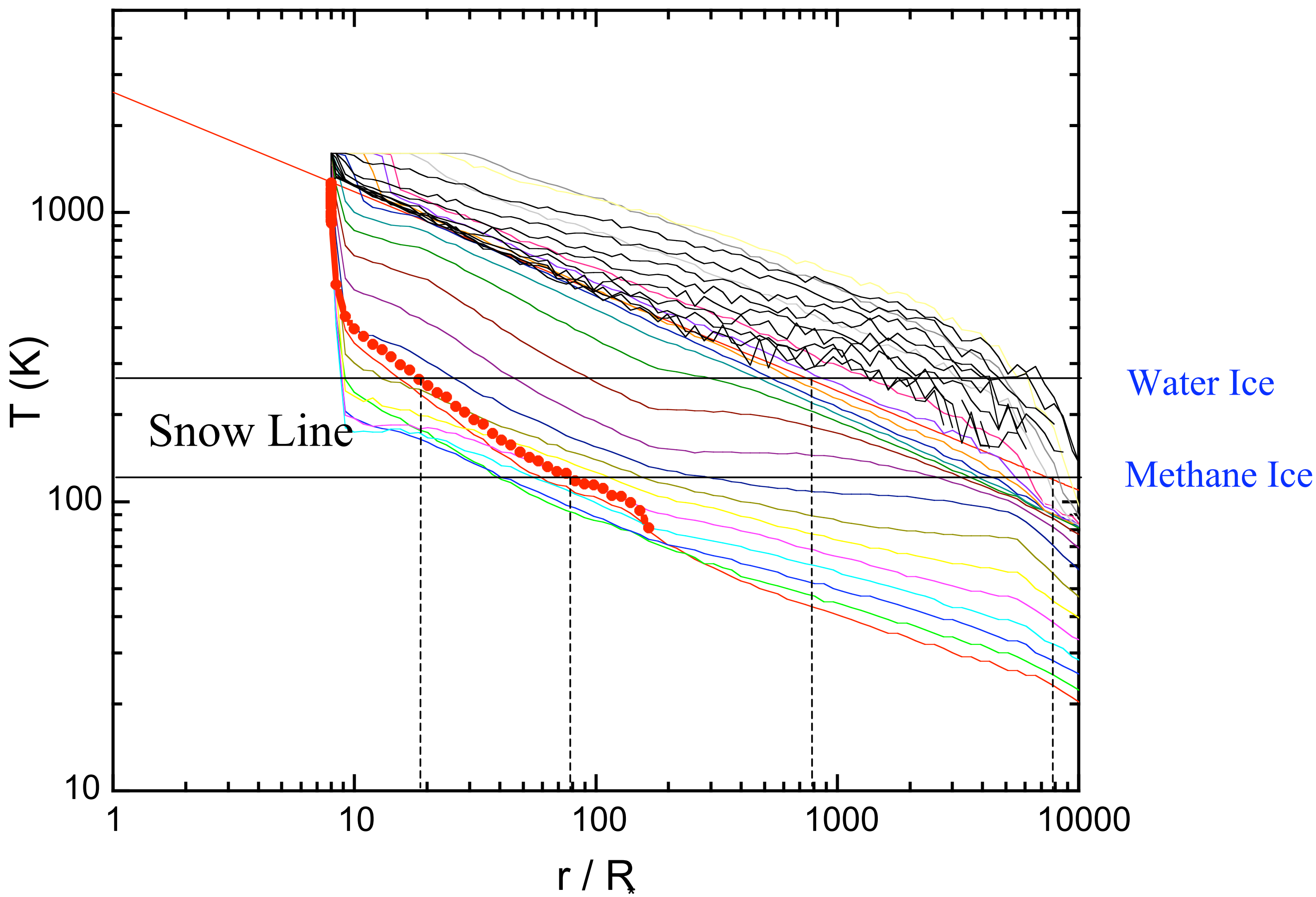
Effect of Disk on Temperature

- Inner edge of disk
  - heats up to optically thin radiative equilibrium temperature
- At large radii
  - outer disk is shielded by inner disk
  - temperatures lowered at disk mid-plane

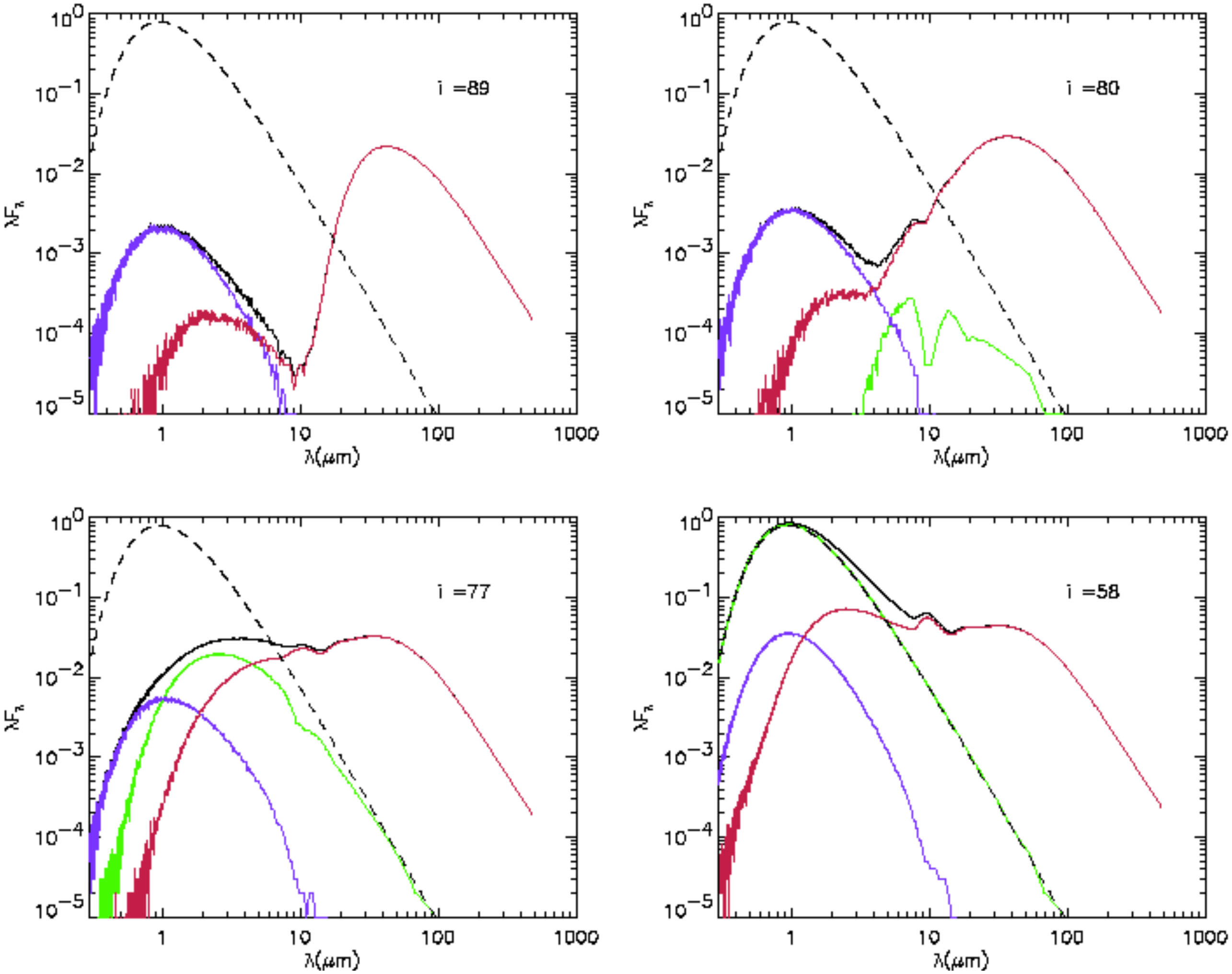
T Tauri Envelope Absorption



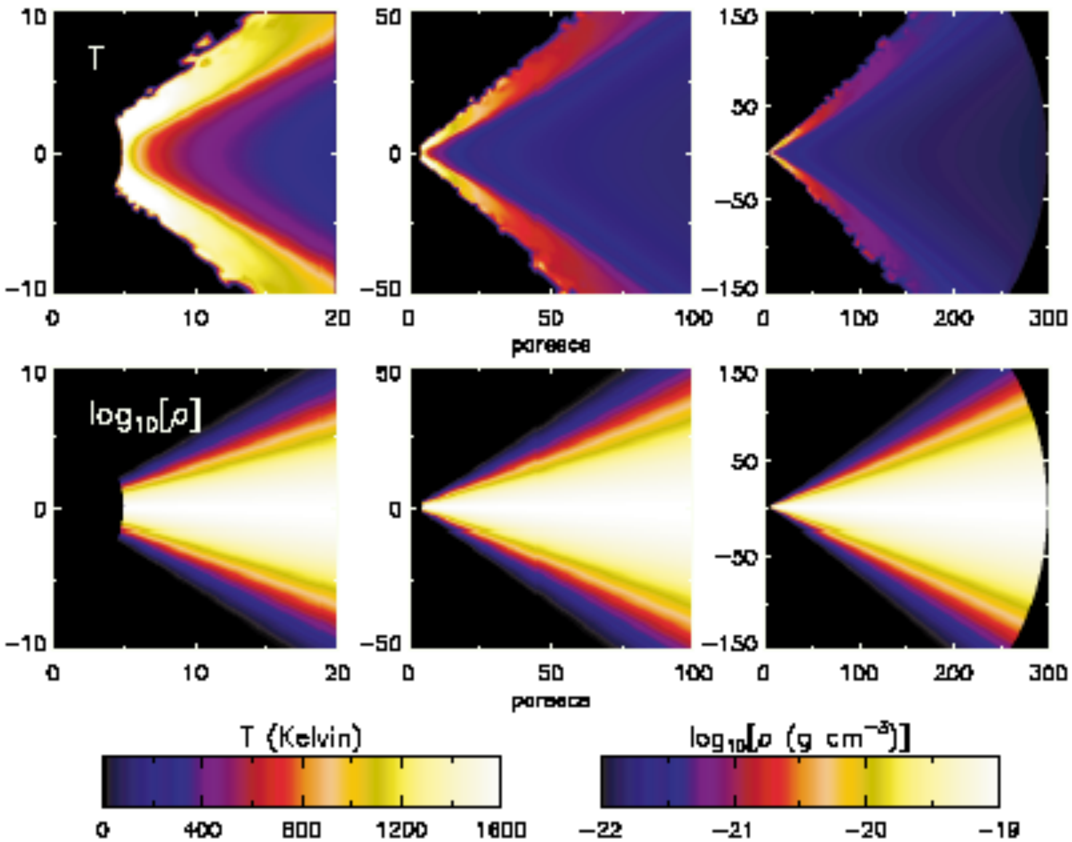
Disk Temperature

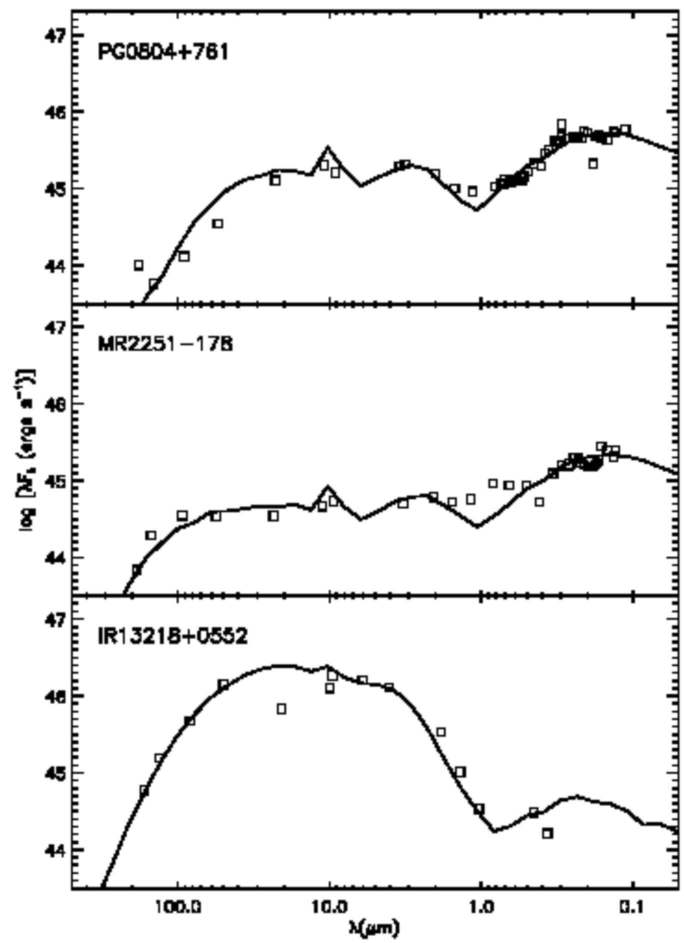


CTTS Model SED



AGN Models





Kuraszkiewicz, et al. 2003

Spectral Lines

- Lines very optically thick
  - Cannot track millions of scatterings
- Use Sobolev Approximation (moving gas)
  - Sobolev length

$$l(\hat{\mathbf{n}}) = \frac{v_D}{\left| dv \,/\, dl \right|} \qquad \frac{dv}{dl} = n^i e_{ij} n^j \qquad e_{ij} = (v_{i;j} + v_{j;i}) \,/\, 2$$

  - Sobolev optical depth

$$\tau_{\text{sob}} = \frac{k_L c}{\nu_0 \left| dv \,/\, dl \right|} \qquad k_L = \frac{\pi e^2}{m_e c} g f \left(\frac{n_l}{g_l} - \frac{n_u}{g_u} \right)$$

  - Assume S, ρ, etc. constant (within l)

Spectral Lines

- Split Mean Intensity

$$J = J_{\rm local} + J_{\rm diffuse}$$

- Solve analytically for $J_{\rm local}$
- Effective Rate Equations

$$\beta_e n_u A_{ul} + \beta_p n_u B_{ul} \bar{J}_{\rm diff} - \beta_p n_l B_{lu} \bar{J}_{\rm diff} + \ldots = 0$$

$$\beta_e = \frac{1}{4\pi} \int \frac{1 - e^{-\tau_{\rm sob}}}{\tau_{\rm sob}} d\Omega \qquad \text{(escape probability)}$$

$$\beta_p = \frac{1}{4\pi} \int \frac{1 - e^{-\tau_{\rm sob}}}{\tau_{\rm sob}} \frac{\bar{I}_{\rm diff}}{\bar{J}_{\rm diff}} d\Omega \qquad \text{(penetration probability)}$$

$$\frac{dP}{d\Omega} \propto j_{\rm esc} = \frac{h\nu n_u A_{ul}}{4\pi} \left(\frac{1 - e^{-\tau_{\rm sob}}}{\tau_{\rm sob}} \right) \qquad \text{(effective line emissivity)}$$

Resonance Line Approximation

- Two-level atom => pure scattering
- Find resonance location

$$\nu_0 = \nu(1 - \mathbf{v} \cdot \hat{\mathbf{n}} \,/\, c)$$

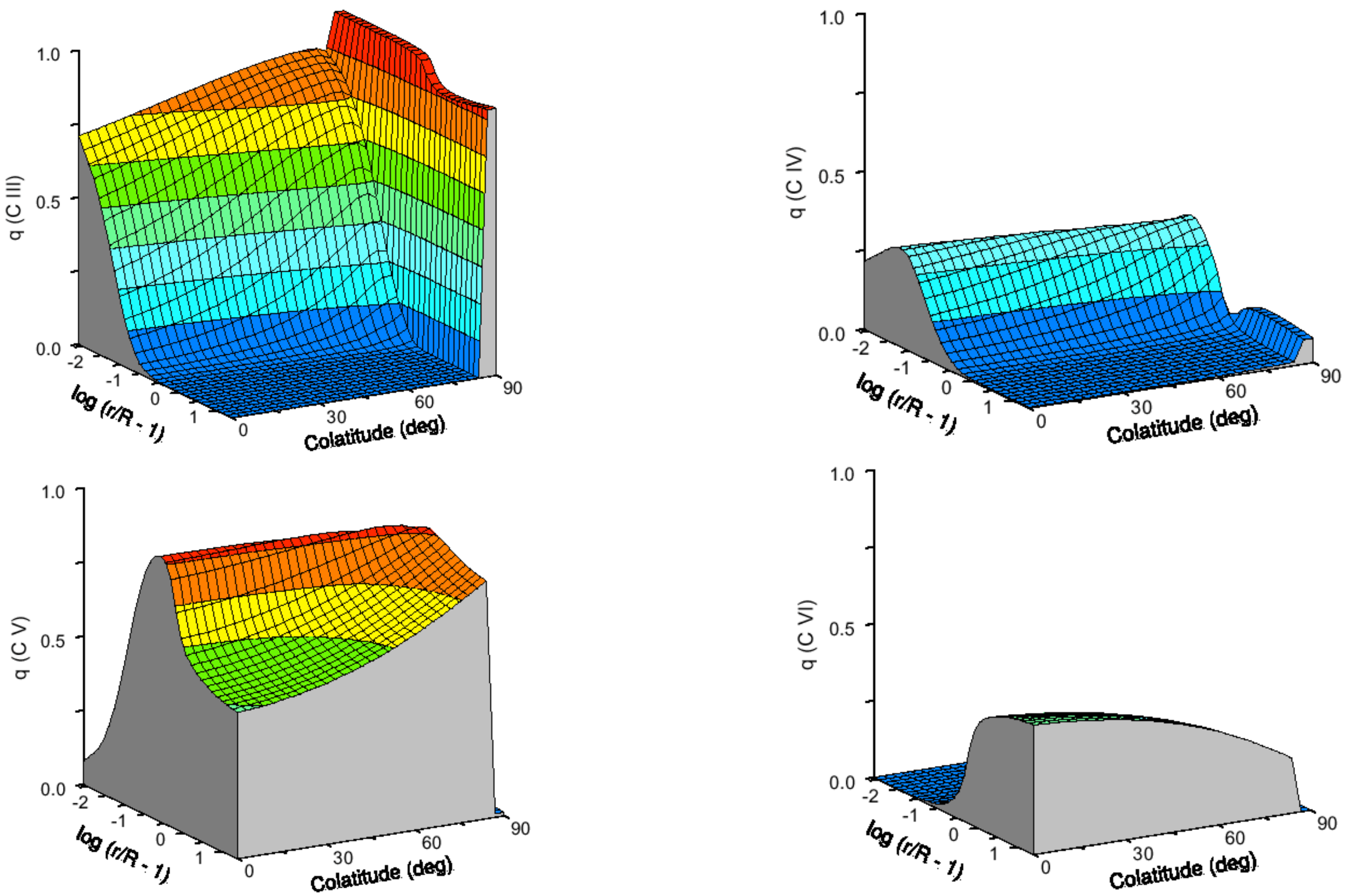
- If photon interacts
  - Reemit according to escape probability

$$\frac{dP}{d\Omega} \propto \frac{1 - e^{-\tau_{\mathrm{sob}}}}{\tau_{\mathrm{sob}}}$$

  - Doppler shift photon; adjust weight

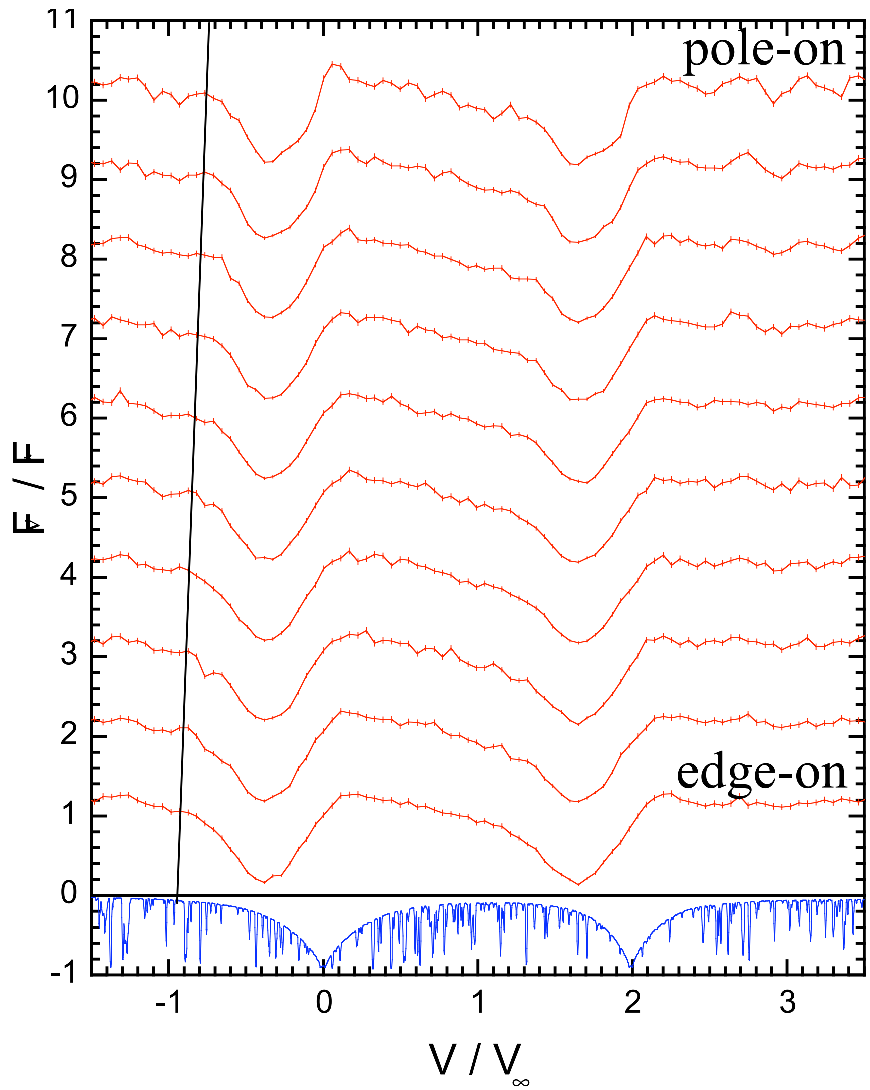
NLTE Ionization Fractions

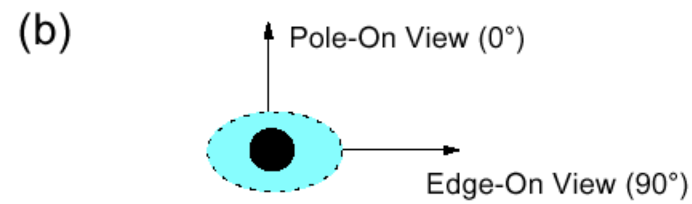
Photoionization ↔ Recombination



Abbott, Bjorkman, & MacFarlane 2001

Wind Line Profiles





Bjorkman 1998

NLTE Monte Carlo RT

- Gas opacity depends on:
  - temperature
  - degree of ionization
  - level populations

determined by radiation field

- During Monte Carlo simulation:
  - sample radiative rates

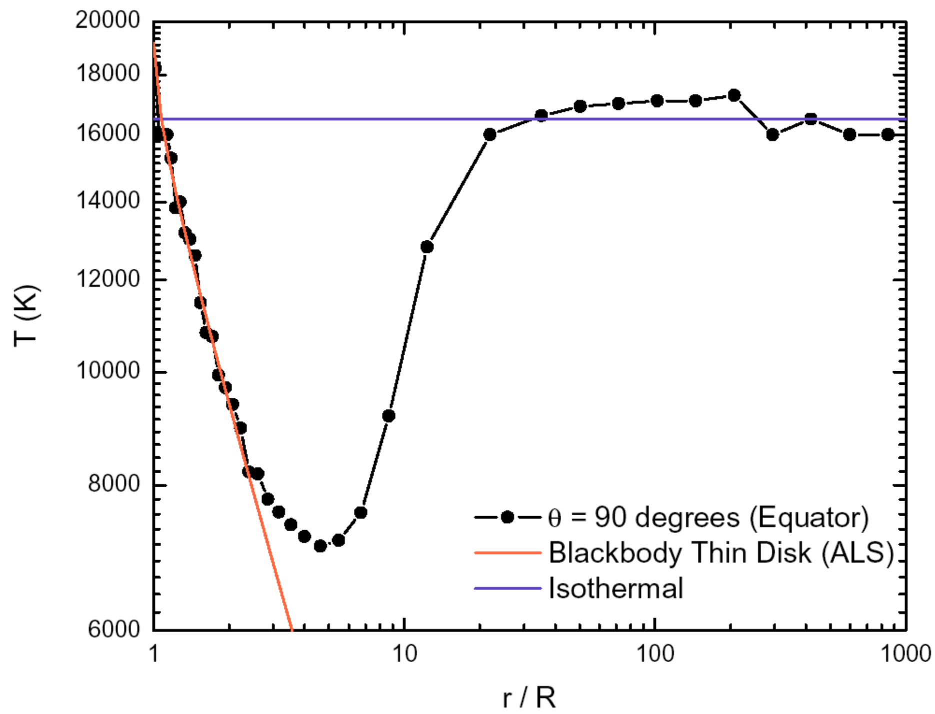
$$dN = n_\gamma d\tau$$

- Radiative Equilibrium
  - Whenever photon is absorbed, re-emit it

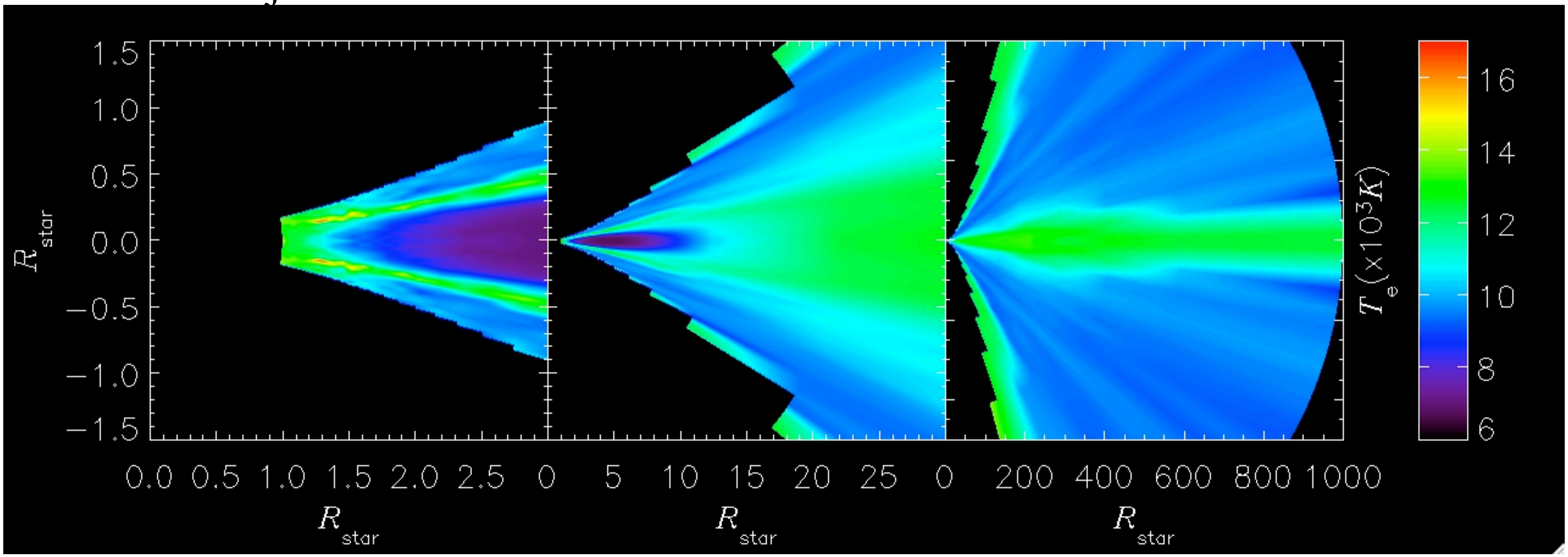
$$\frac{dP}{d\nu d\Omega} \propto j_\nu^{\mathrm{eff}}$$

- After Monte Carlo simulation:
  - solve rate equations
  - update level populations and gas temperature
  - update disk density (integrate HSEQ)

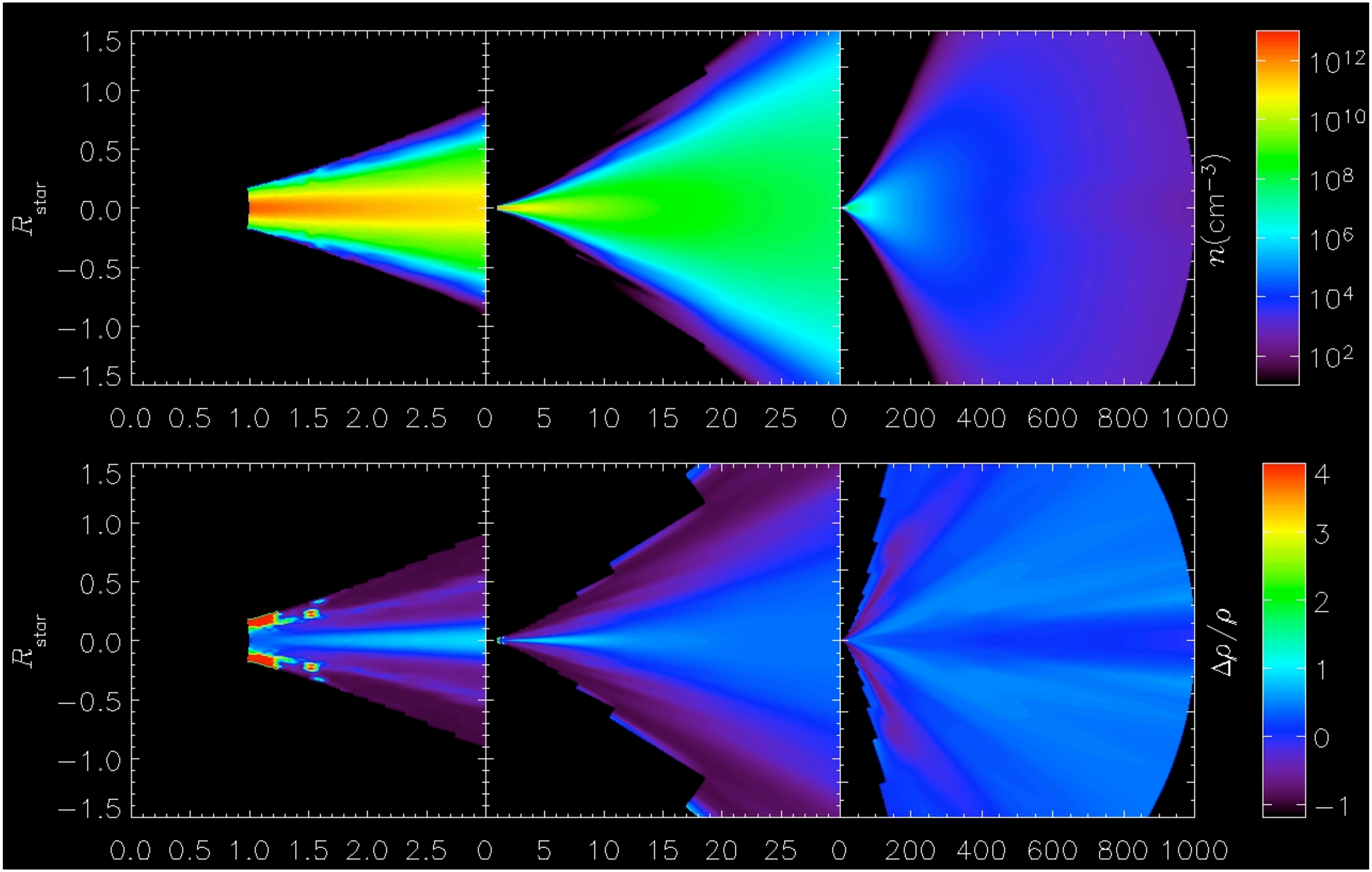
Be Star Disk Temperature



Carciofi & Bjorkman 2004

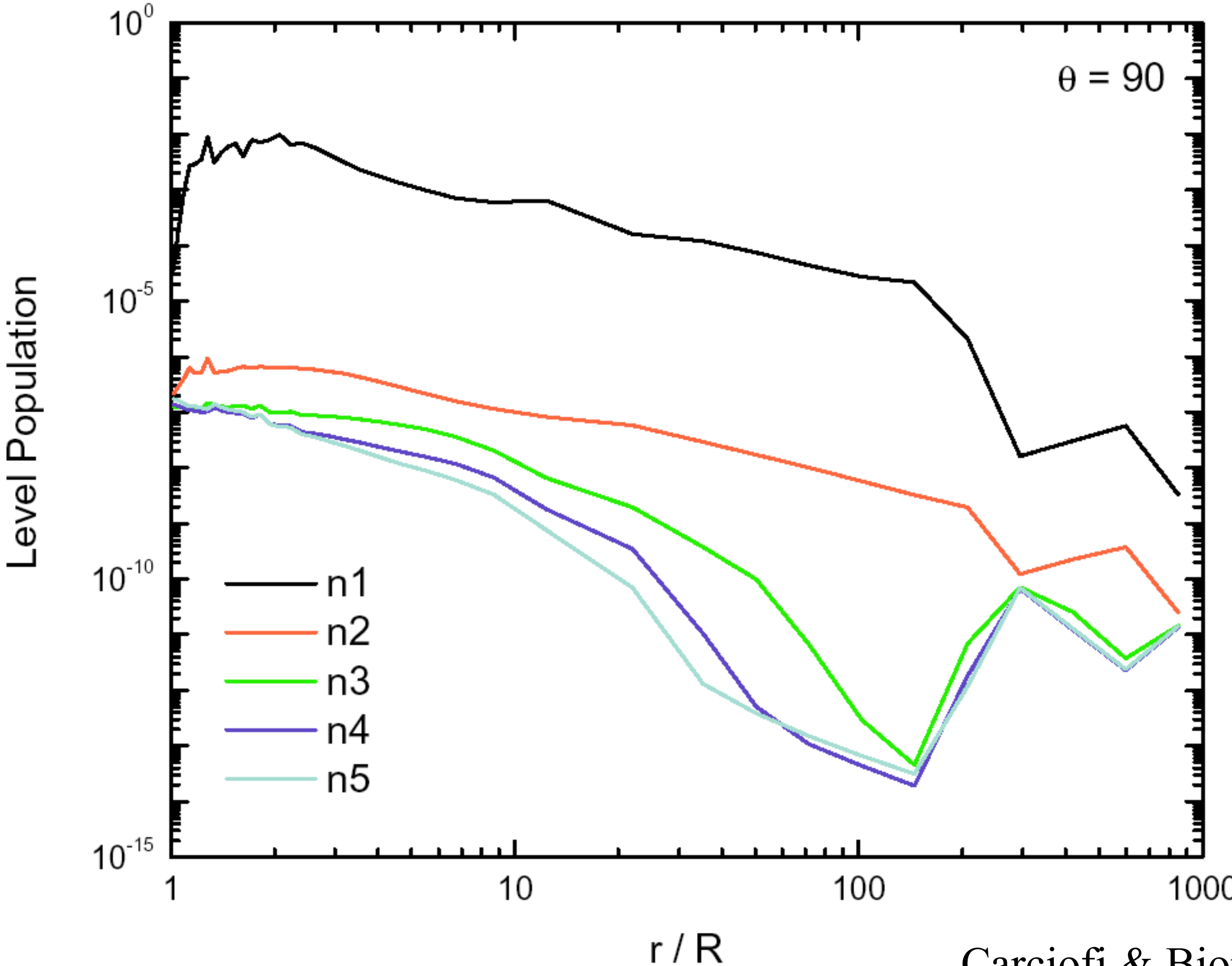


Disk Density



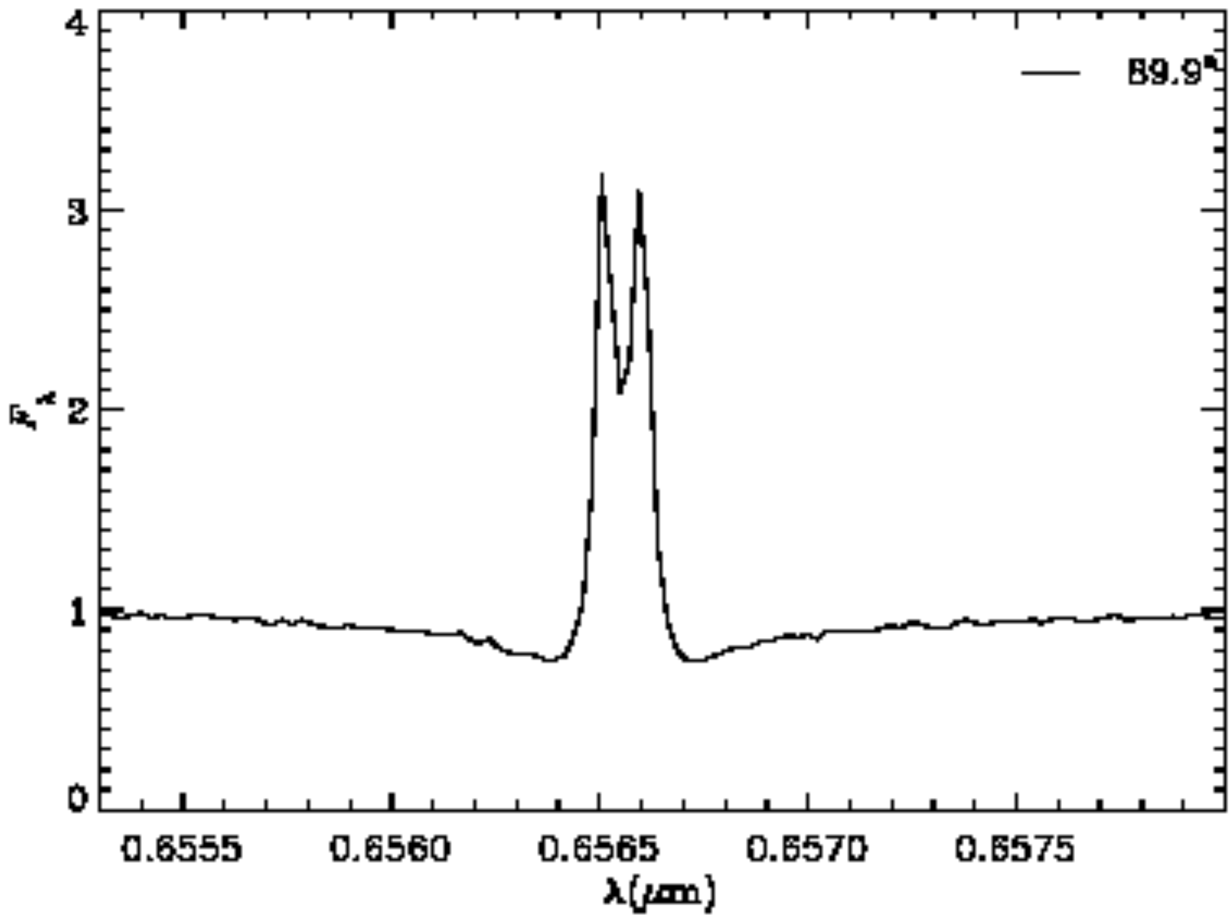
Carciofi & Bjorkman 2004

NLTE Level Populations



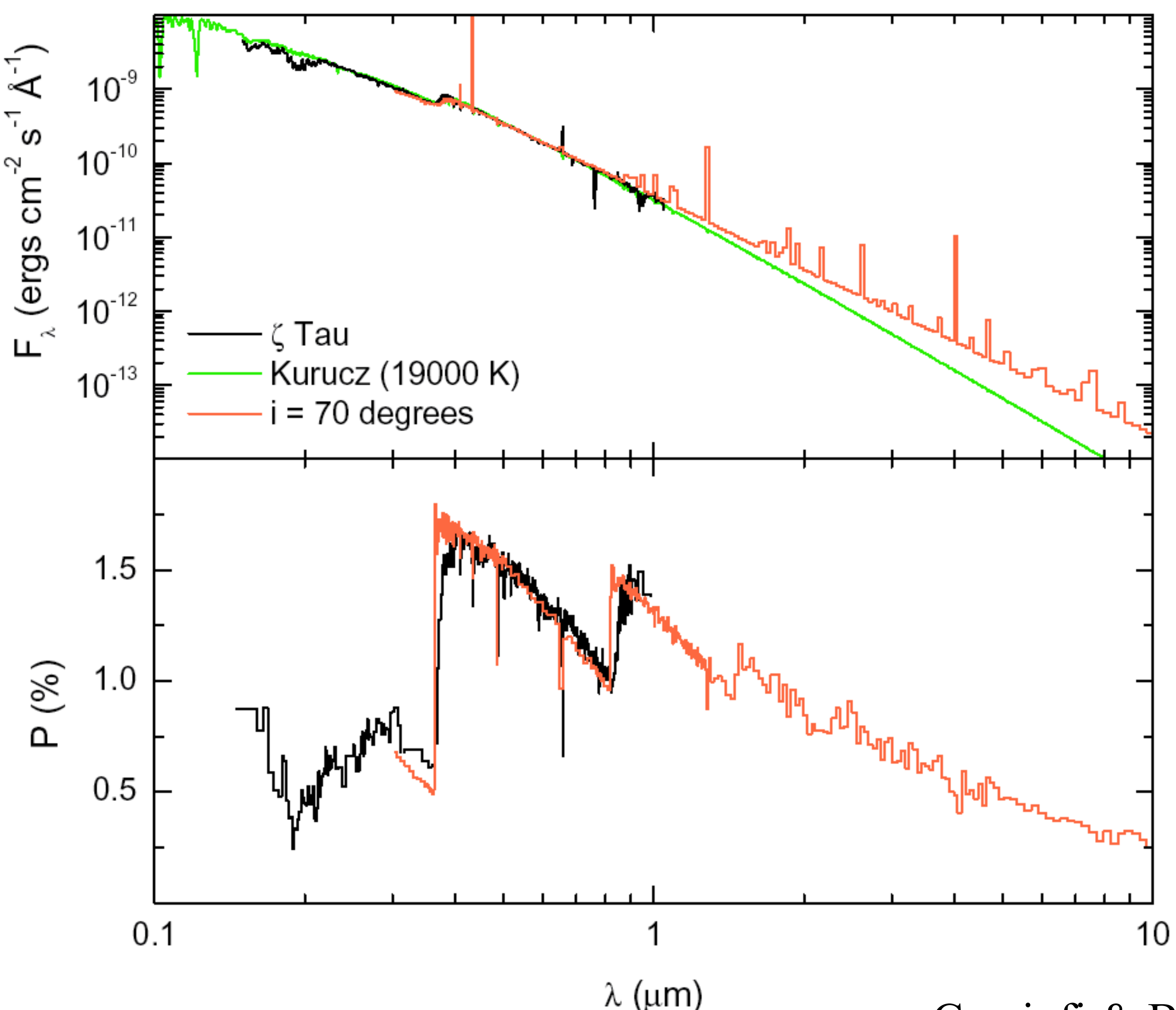
Carciofi & Bjorkman 2004

Be Star Hα Profile



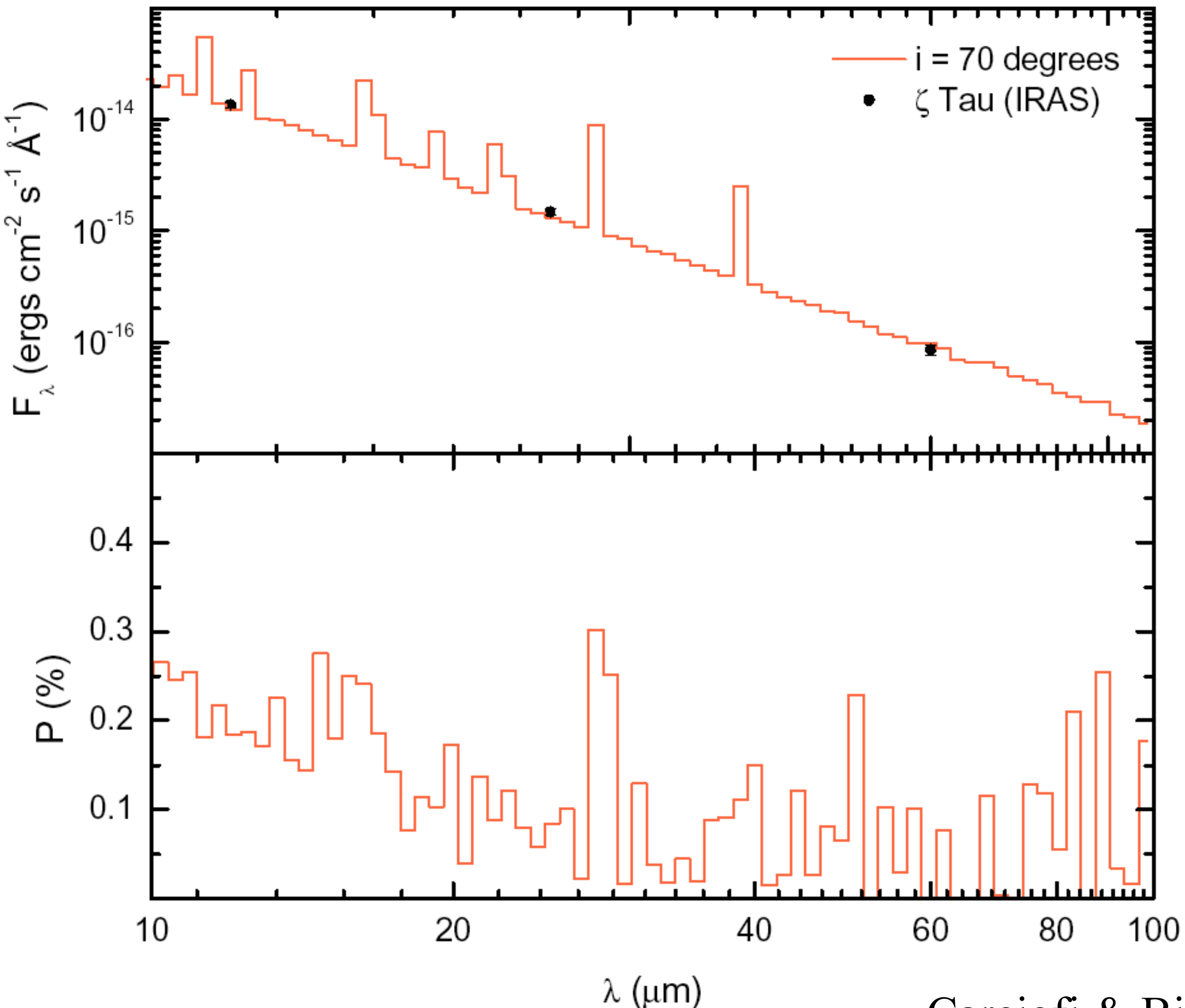
Carciofi and Bjorkman 2003

SED and Polarization



Carciofi & Bjorkman 2004

IR Excess



Carciofi & Bjorkman 2004

Future Work

- Spitzer Observations
  - Detecting high and low mass (and debris) disks
  - Disk mass vs. cluster age will determine disk clearing time scales
  - SED evolution will help constrain models of disk dissipation
  - Galactic plane survey will detect all high mass star forming regions
  - Begin modeling the geometry of high mass star formation
- Long Term Goals
  - Combine dust and gas opacities
  - include line blanketing
  - Couple radiation transfer with hydrodynamics

Acknowledgments

- Rotating winds and bipolar nebulae
  - NASA NAGW-3248
- Ionization and temperature structure
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  - NSF AST-0307686
- Geometry and evolution of low mass star formation
  - NASA NAG5-8794
- Collaborators: A. Carciofi, K.Wood, B.Whitney, K. Bjorkman, J.Cassinelli, A.Frank, M.Wolff
- UT Students: B. Abbott, I. Mihaylov, J. Thomas
- REU Students: A. Moorhead, A. Gault

High Mass YSO

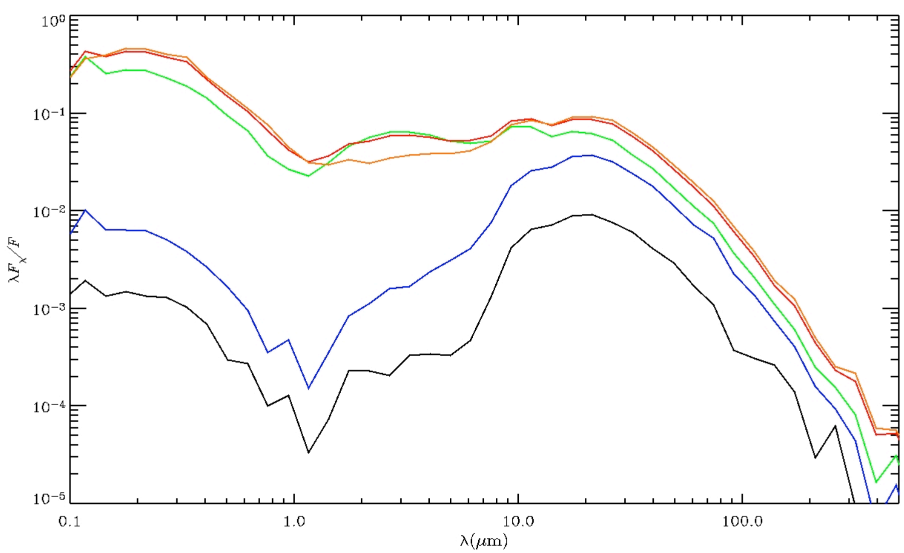
Inner Disk:

- NLTE Hydrogen
- Flared Keplerian
- $h_0 = 0.07$, $b = 1.5$
- $R_* < r < R_{dust}$

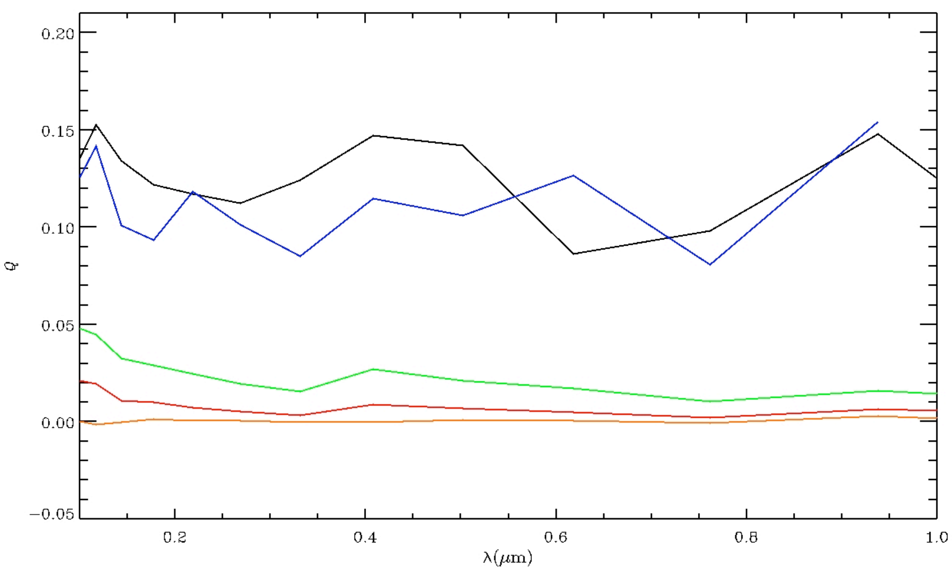
Outer Disk:

- Dust
- Flared Keplerian
- $h_0 = 0.017$, $b = 1.25$
- $R_{dust} < r < 10000\ R_*$

Flux



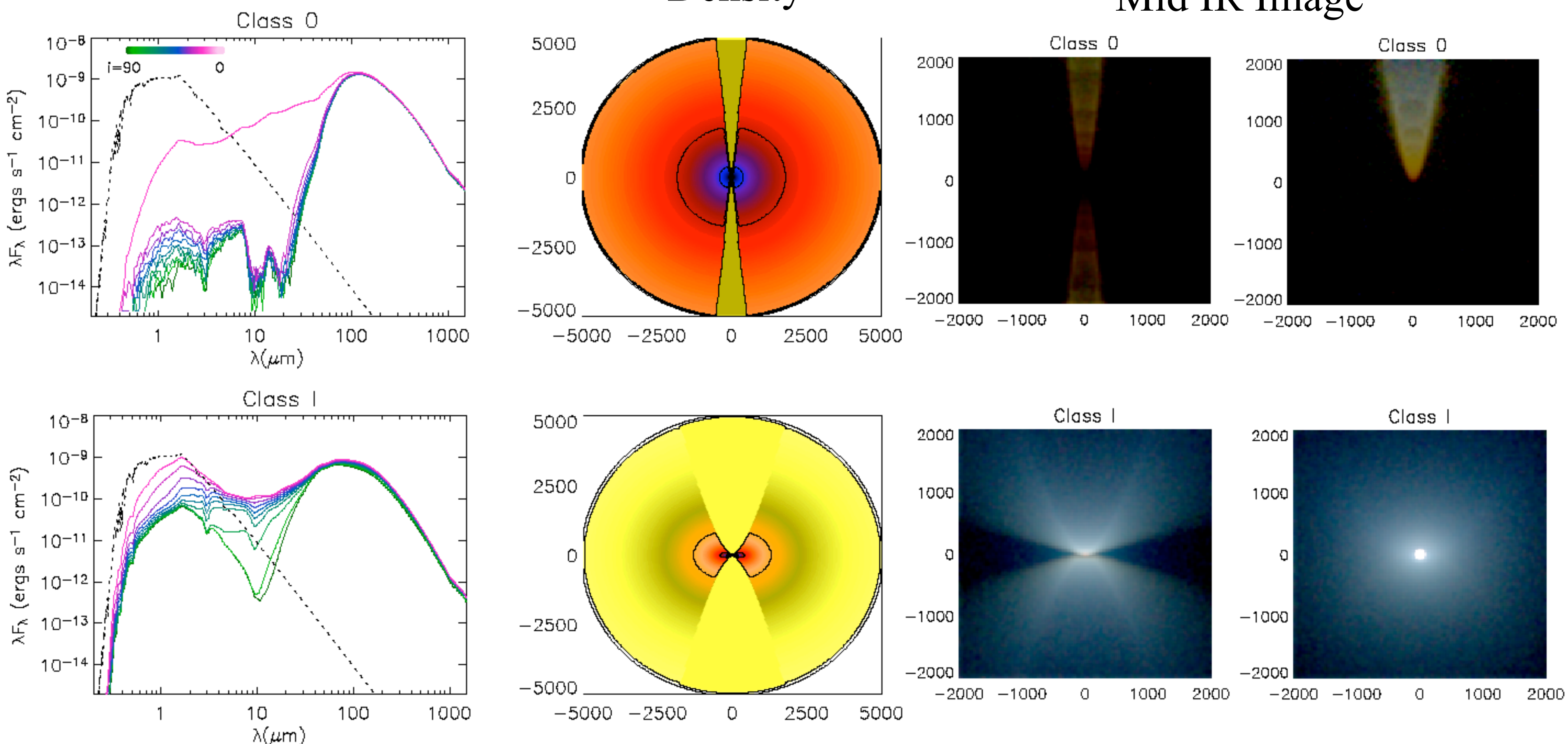
Polarization



Bjorkman & Carciofi 2003

Protostar Evolutionary Sequence

SED | Density | Mid IR Image

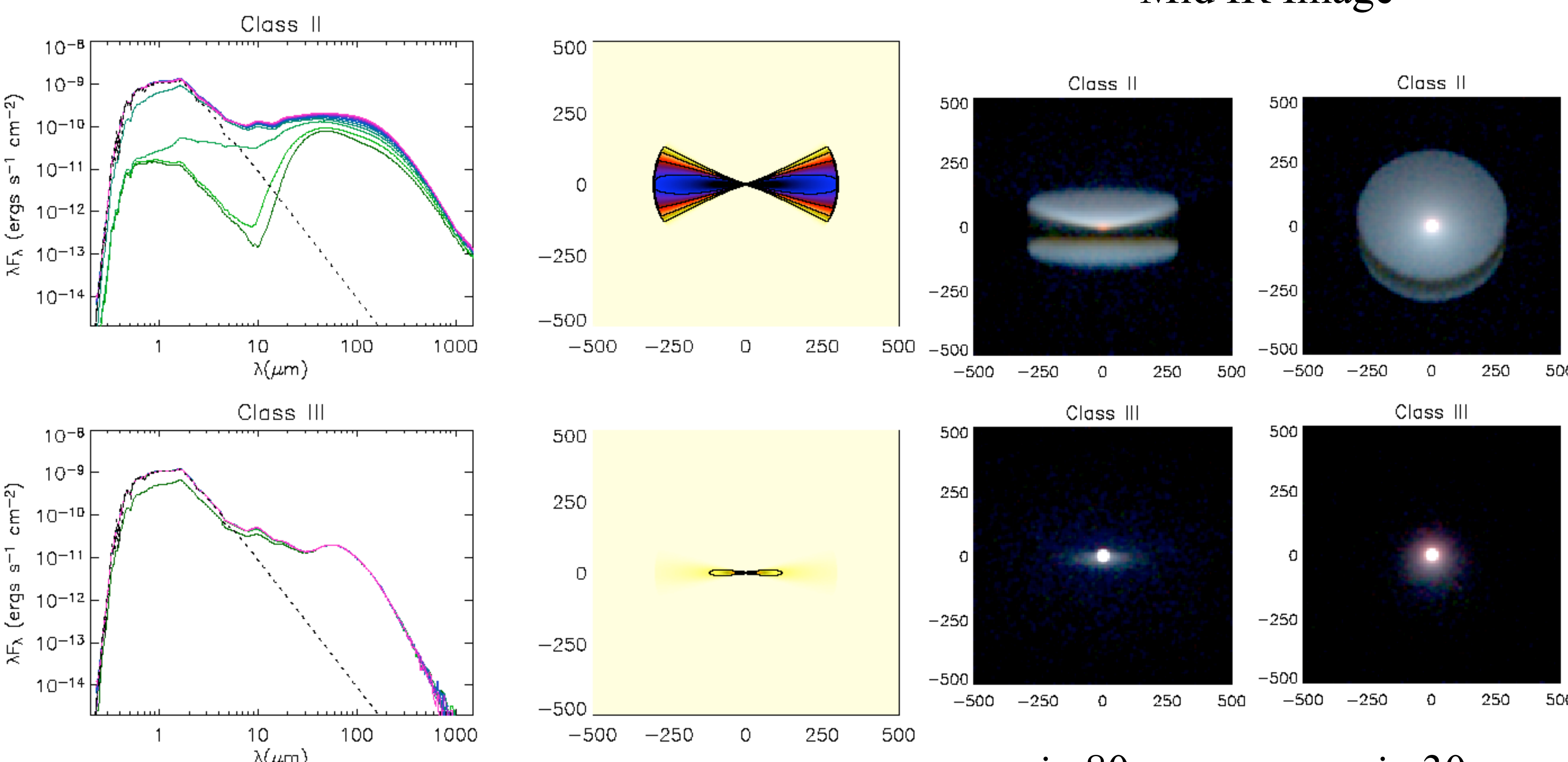


i =80 i =30

Whitney, Wood, Bjorkman, & Cohen 2003

Protostar Evolutionary Sequence

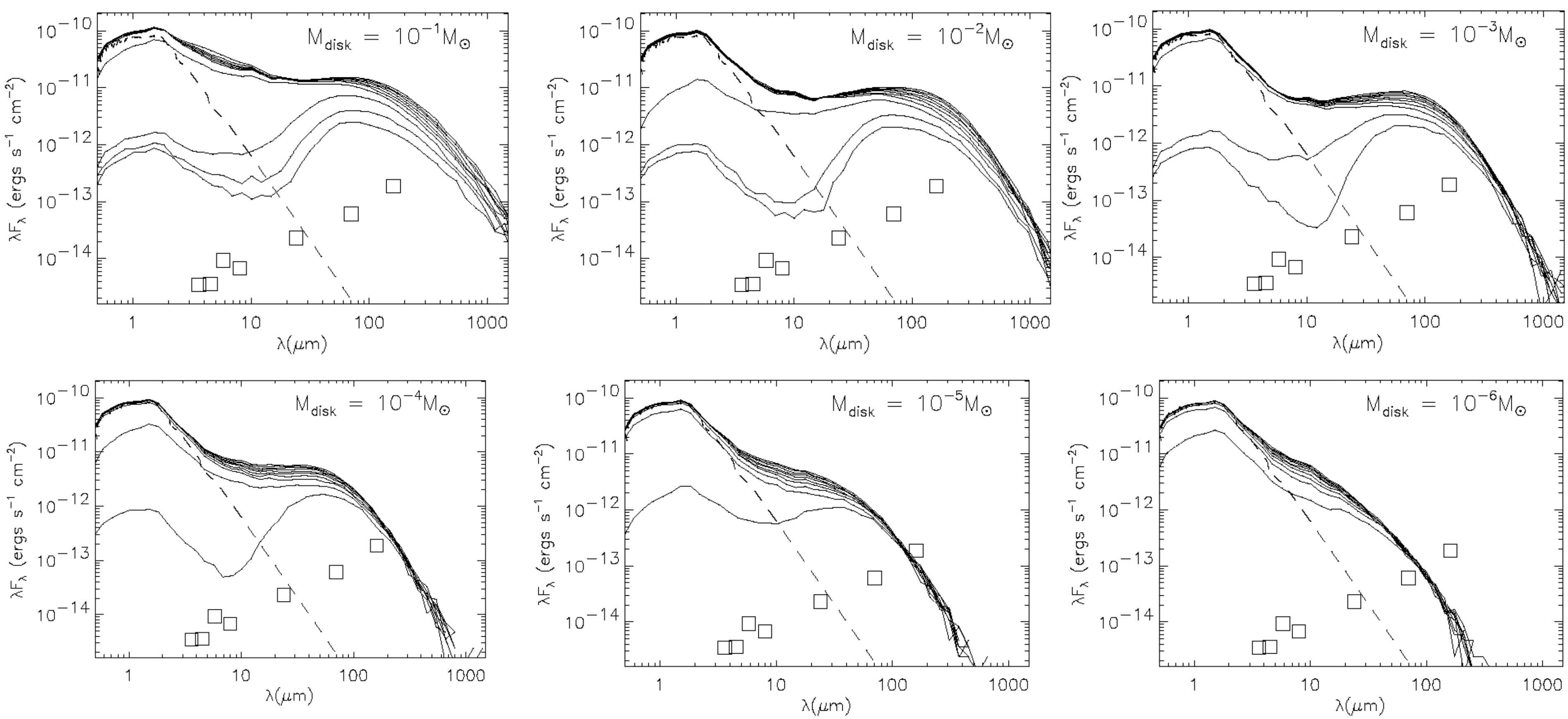
SED Density Mid IR Image



i =80 i =30

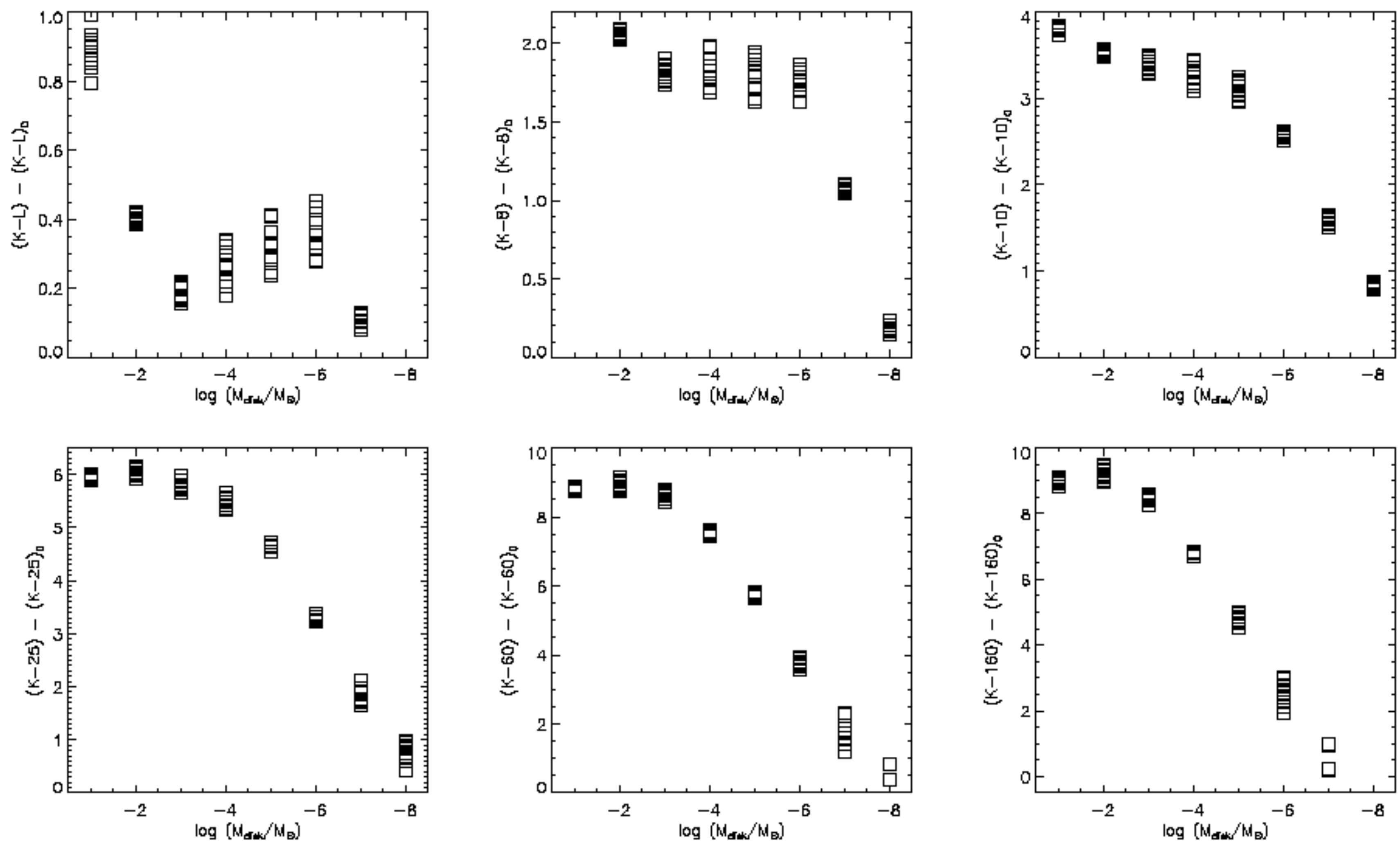
Whitney, Wood, Bjorkman, & Cohen 2003

Disk Evolution: SED



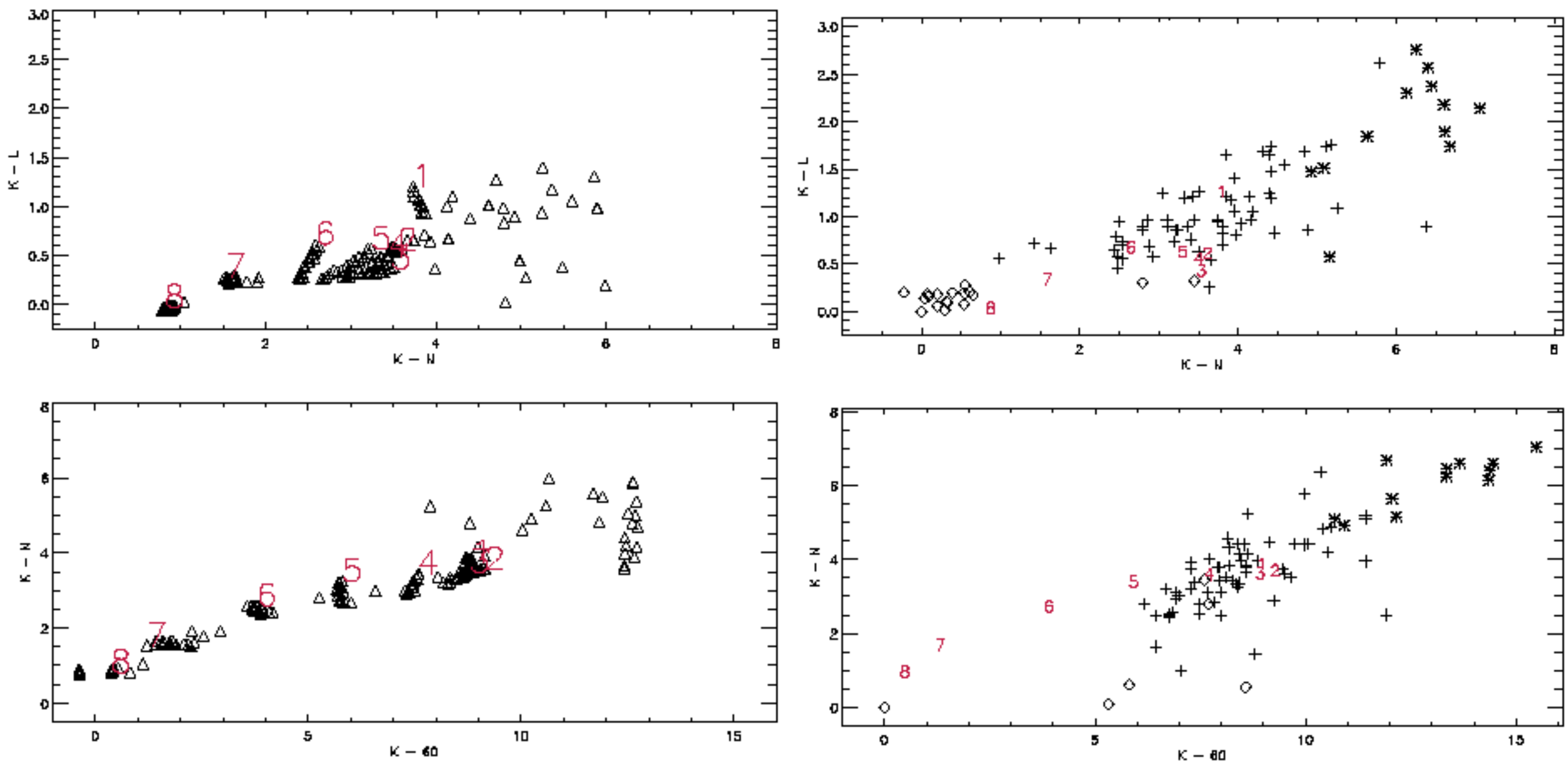
Wood, Lada, Bjorkman, Whitney & Wolff 2001

Disk Evolution: Color Excess



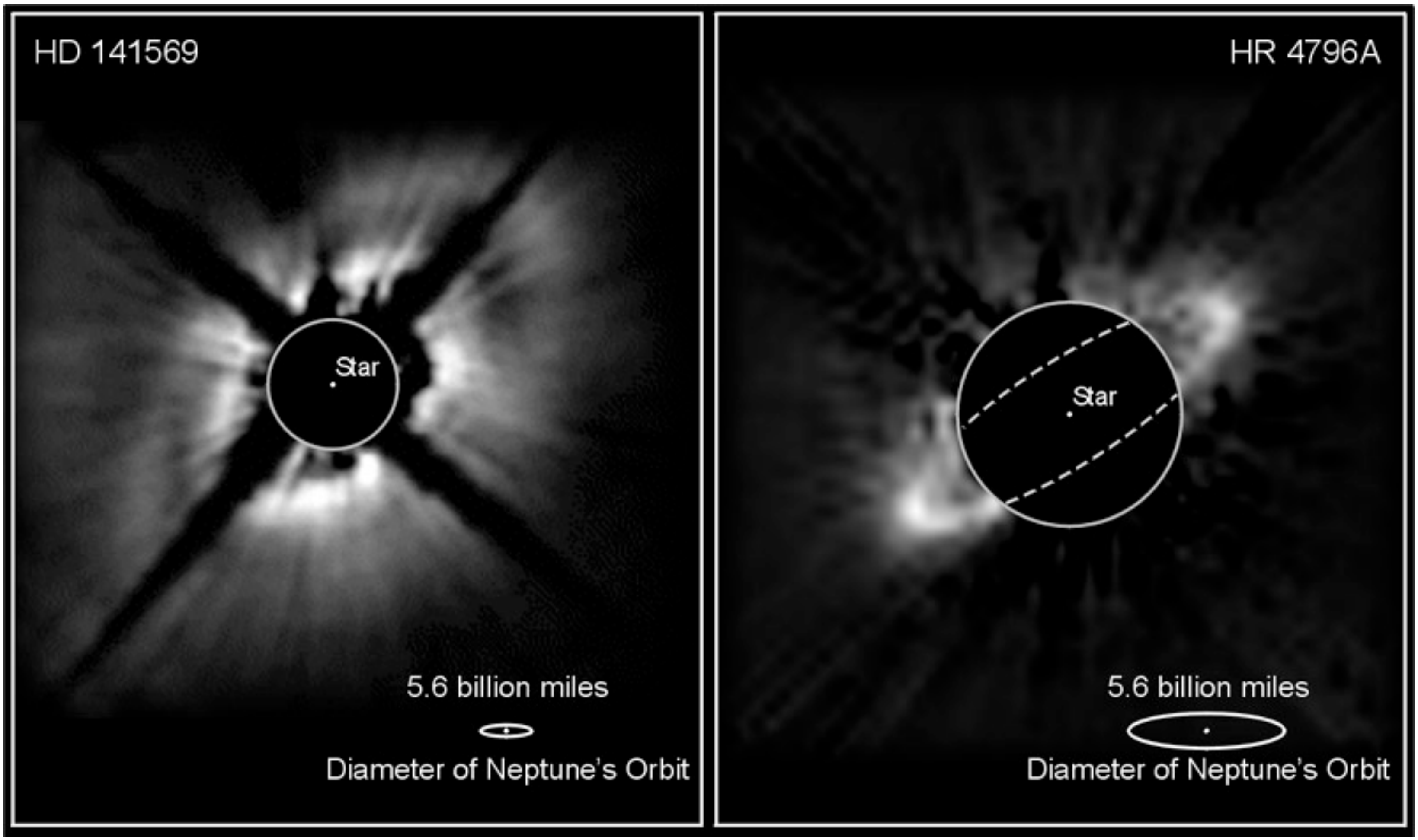
Wood, Lada, Bjorkman, Whitney & Wolff 2001

Determining the Disk Mass



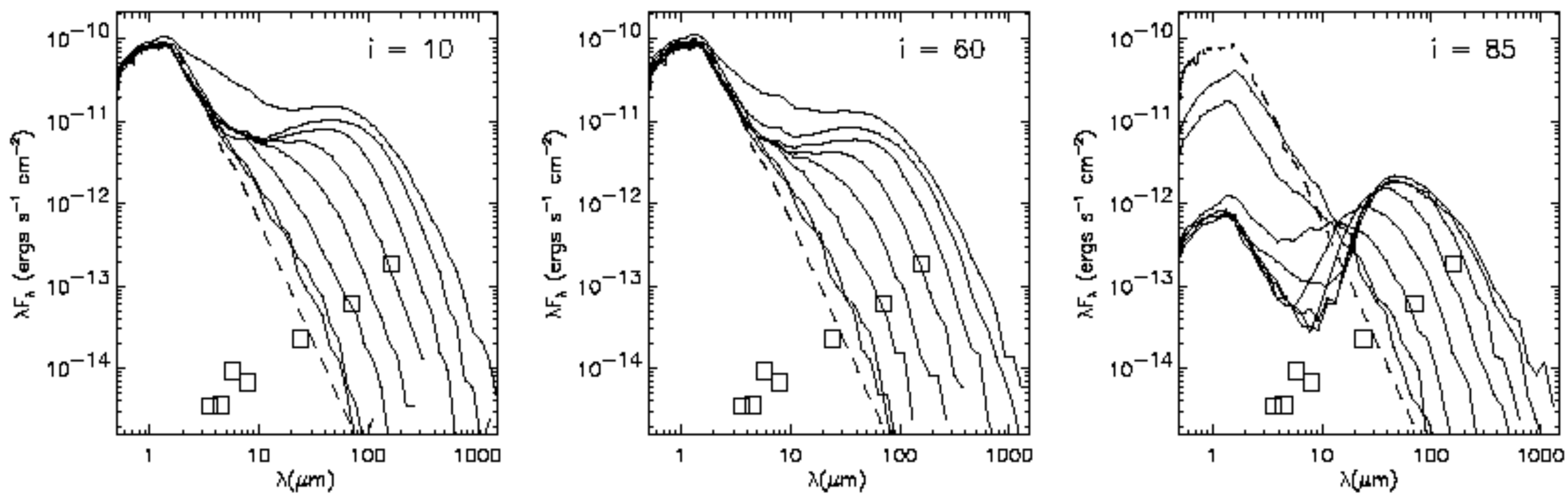
Wood, Lada, Bjorkman, Whitney & Wolff 2001

Gaps in Protoplanetary Disks



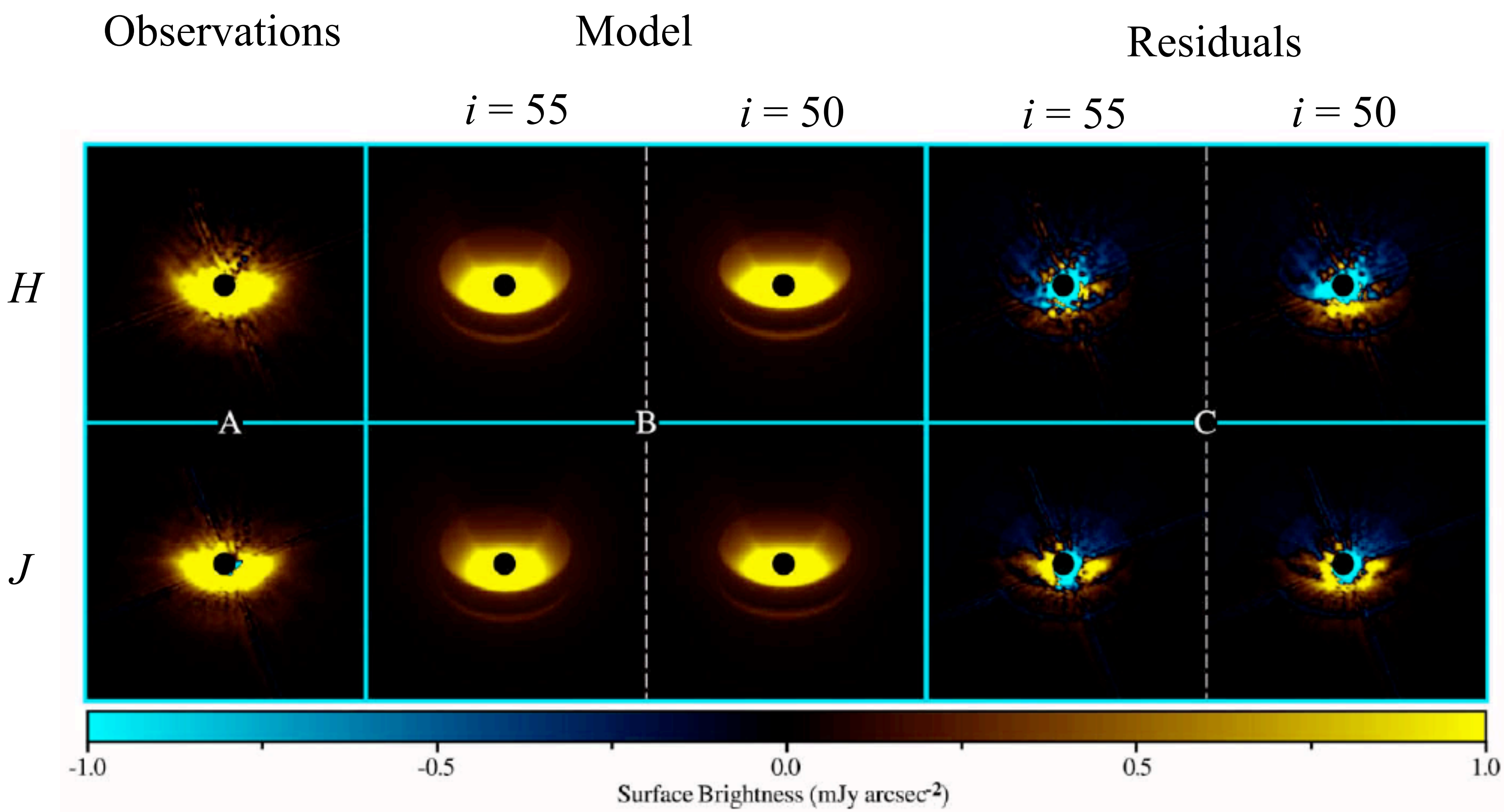
Smith et al. 1999

Disk Clearing (Inside Out)



Wood, Lada, Bjorkman, Whitney & Wolff 2001

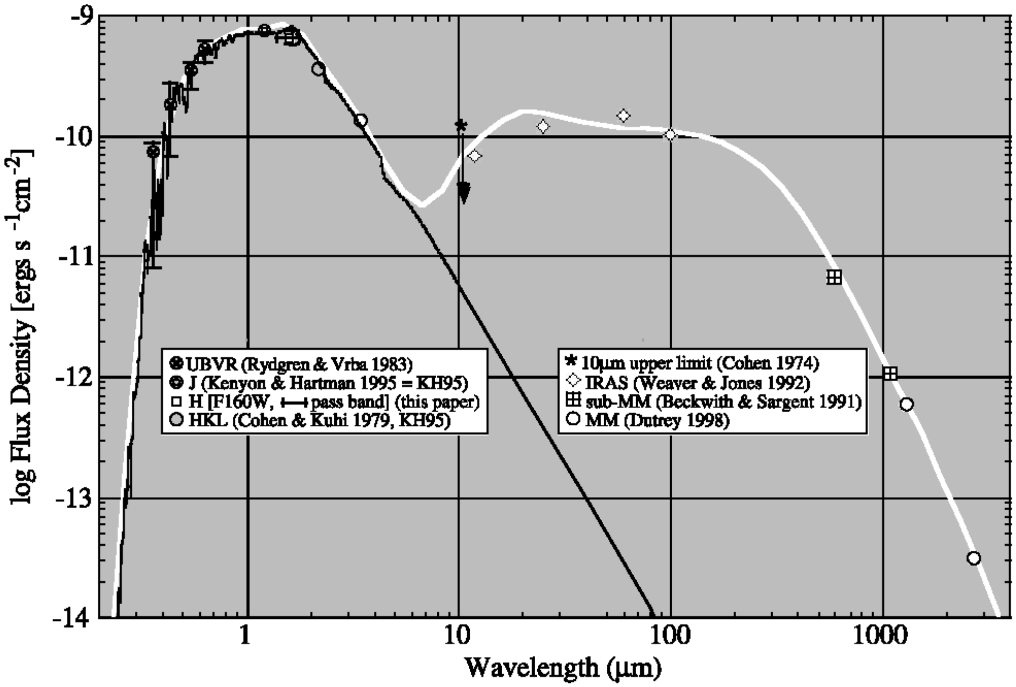
GM AUR Scattered Light Image

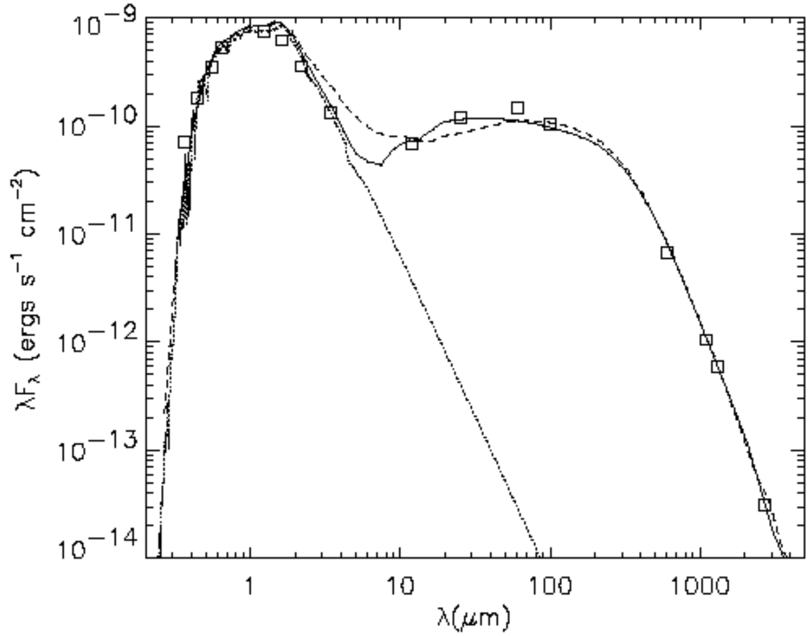


Schneider et al. 2003

GM AUR SED

- Inner Disk Hole = 4 AU

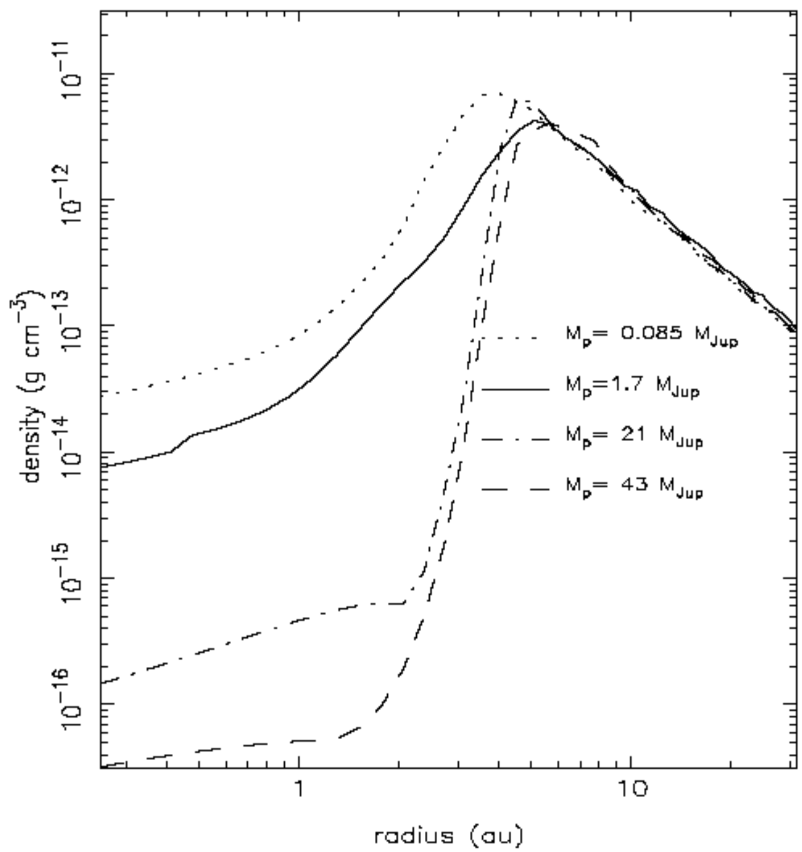


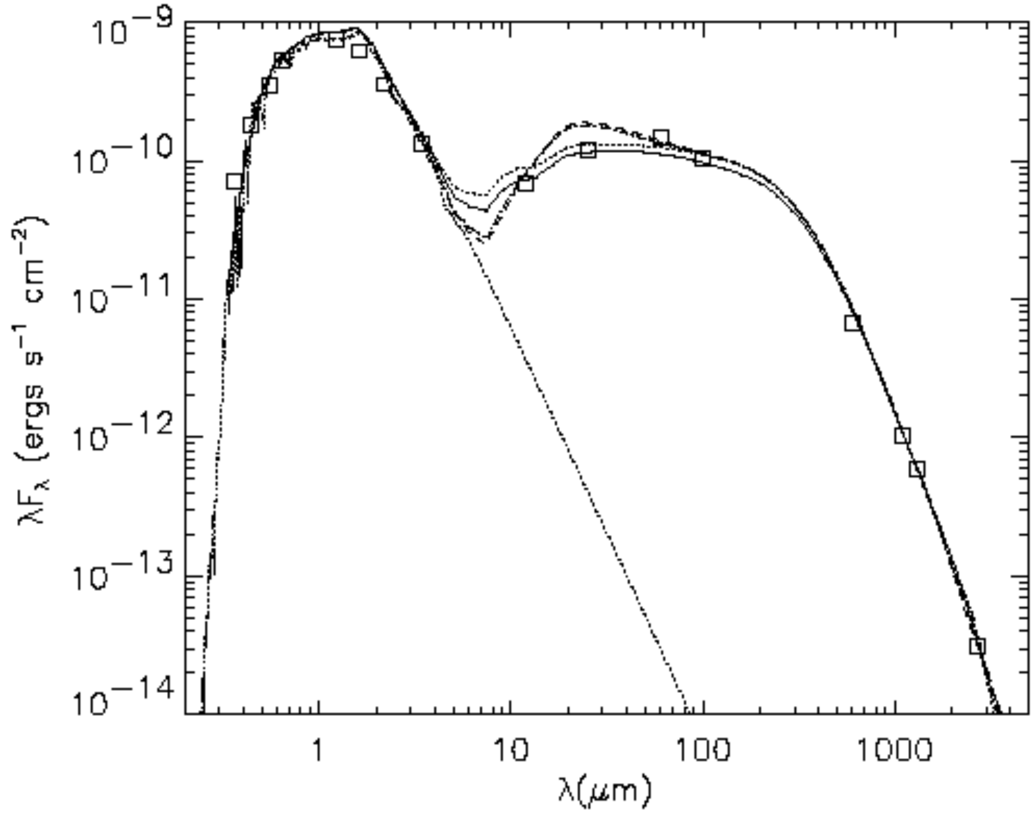


Schneider et al. 2003

Rice et al. 2003

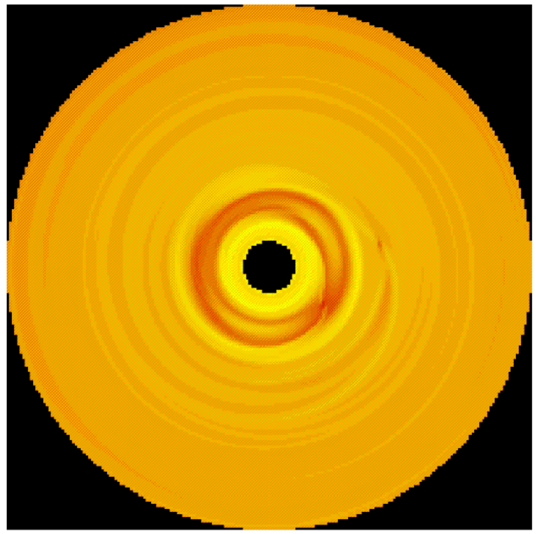
Planet Gap-Clearing Model





Rice et al. 2003

Protoplanetary Disks



Surface Density