Submission 1

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Problem 1: Self avoiding polymer model:

a) Probability distribution of 4 monomer chain:

Possible values for chain length (found by Drawing): $|\boldsymbol{R}_3 - \boldsymbol{R}_0| = 1, \sqrt{5}, 3$.

Probabilities, determined by enumeration and symmetry arguments:

p	Value
$p(\lvert\boldsymbol{R}_3 - \boldsymbol{R}_0\rvert = 1)$	$\frac{2}{9}$
$p(\lvert\boldsymbol{R}_3 - \boldsymbol{R}_0\rvert = \sqrt{5})$	$\frac{6}{9}$
$p(\lvert\boldsymbol{R}_3 - \boldsymbol{R}_0\rvert = 3)$	$\frac{1}{9}$

The probability to find the chain in a straight line is that of $p(|\boldsymbol{R}_3 - \boldsymbol{R}_0| = 3) = \frac{1}{9}$.

b) Mean:

$$\langle|\boldsymbol{R}_3 - \boldsymbol{R}_0|\rangle = \frac{2}{9} + \sqrt{5}\frac{6}{9} + 3\frac{1}{9} \approx 2.04$$

Variance:

$$\sigma = \sqrt{\left\langle|\boldsymbol{R}_3 - \boldsymbol{R}_0|^2\right\rangle} = \sqrt{\frac{2}{9} + 5\frac{6}{9} + 9\frac{1}{9}} \approx 1.91$$

Problem 2: Random walk on a lattice without self-avoidance:

a) Probability distribution $p(X, Y|N)$:

Start with considering random walk in x-direction, assume $N_x = r + l$ steps in this direction, with r the steps in positive and l steps in negative direction, i.e. $X = r - l$.

$\implies r - l = x, N = r + l \implies r = \frac{N_x + x}{2}, l = \frac{N_x - x}{2}$.

Thus, we have

$$p(X|N_x) = \binom{N_x}{\frac{N_x+x}{2}} p^{N_x}$$

in one direction (of course analogously in y-direction), with $p = \frac{1}{2}$.

To get the probability $p(X, Y|N)$, we need to sum over all combinations of possible number of path-steps in each direction, where we know:

$$\begin{gathered} N = N_x + N_y, \\ \forall N_x \in \{X, \dots, N - Y\}, \\ \forall N_y \in \{Y, \dots, N - X\}. \end{gathered}$$

Thus, overall, we have:

$$p(X, Y|N) = \sum_{N_x = X}^{N-Y} \binom{N_x}{\frac{N_x + X}{2}} \binom{N - N_x}{\frac{N - N_x + Y}{2}} \frac{1}{4}^N ,$$

with the condition that $X + Y$ is even (odd), if N is even (odd), otherwise we immediately know $p(X, Y|N) = 0$, because there is no possible path to that point.

I am not sure if there is a more elegant, close form solution to this probability distribution, because this is a bit ugly.

b) Mean value:

One can reason that there is nothing special about the x-direction, i.e. $\langle N_x \rangle = \frac{N}{2}$, thus, we may use the probability distribution $p(X|N_x)$ as defined above in a).

$$\langle X|N \rangle = \sum_X X p(X| \underbrace{N_x}_{=N/2}) = \sum_{X=0}^{N} X \binom{N/2}{\frac{N/2+X}{2}} \frac{1}{2}^{N/2}$$

The same obviously holds for $\langle Y|N \rangle$.

c) The path cannot return to (0,0) for odd number of jumps, as $X + Y = 2\mathbb{Z}$ is only possible for even numbers of steps.

The probability function.