Submission 1

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Problem 1: Trajectory Thermodynamics for driven systems.

a) We know that K_{IJ} is defined as the instantaneous transition probability and that Γ_I, the escape rate is the sum of all $K_{I\cdot}$. Thus, the probability to remain in state I for some time interval $[t_1, t_2)$ is given by

$$p_{\mathrm{rem.}}(t) = \exp\left(-\int\limits_{t_1}^{t_2} \sum_J K_{IJ}(\lambda^{t\prime})\mathrm{d}t'\right).$$

Following the derivation of the path weight of the lecture, we thus get

$$p[I(t)|I^0] = \left[\prod_{l=1}^{L} \exp\left(-\int\limits_{t_{l-1}}^{t_l} \mathrm{d}t' \sum_J K_{I_l^- J}(\lambda^{t'})\right) K_{I_l^- I_l^+}(\lambda^{t_l})\right] \cdot$$
$$\cdot \exp\left(-\int\limits_{t_{L-1}}^{T} \mathrm{d}t' \sum_J K_{I_L^- J}(\lambda^{t\prime})\right)$$

Problem 2: Driven Hamiltonian System:

a) Solve Hamilton's equations for the initial condition, which read

$$\dot{p}(t) = -m\omega^2(q - \lambda^t), \qquad \dot{q}(t) = \frac{p}{m},$$

i.e.

$$\ddot{q}(t) = \omega^2(q - \lambda^t) = \omega^2(q - vt)$$

This differential equation has the solutions:

$$q(t) = c_1 e^{\omega t} + c_2 e^{-\omega t} + vt, \qquad p(t) = \frac{\omega(c_1 e^{\omega t} - c_2 e^{-\omega t}) + v}{m}.$$

Inserting the initial conditions

$$q(0) = q_0 = c_1 + c_2; \qquad p(0) = p_0 = \frac{\omega}{m}(c_1 - c_2),$$

which gives us

$$c_1 = \frac{mp_0}{2\omega} + \frac{q_0}{2}; \qquad c_2 = -\frac{mp_0}{2\omega}.$$