b.i. Find the inverse transform of $\frac{z}{z^2+7z+10}$ by long division method.

ii. Find the z- transform of $\sin n\theta$ and $\cos n\theta$.

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Reg. No.														

B.Tech. (PT) DEGREE EXAMINATION, DECEMBER 2018

First Semester

17MAP201 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

(For the candidates admitted from the academic year 2017-2018 onwards)

Time: Three hours Max. Marks: 100

Answer **ALL** Questions

PART – A (10 × 2 = 20 Marks)

1. Form a partial differential equation by eliminating the arbitrary function from $z = f(x^2 - y^2)$.
2. Solve $\left(4D^2 - D'^2\right)z = 0$.
3. State the Dirichlet's condition for a function $f(x)$ to be expanded as a Fourier series.
4. Find the half range sine series for $f(x) = k$ in $0 < x < \pi$.
5. Write down the three possible solutions of one-dimensional heat equation.
6. In the wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$, what does a^2 stand for.
7. Write Fourier transform in pairs.
8. State and prove the change of scale property of Fourier transform.
9. Find the z-transform of $\left\{\frac{1}{n}\right\}$.
10. Find the z-transform of $u(n)$.

PART – B (5 × 16 = 80 Marks)

11. a.i. Find the singular integral of the partial differential equation $z = px + qy + p^2 - q^2$.

ii. For the partial differential equation by eliminating the arbitrary functions f and g in $z = f\left(x^3 + 2y\right) + g\left(x^3 - 2y\right)$.

(OR)

b.i. Solve $\left(D^2 - DD' - 30D'^2\right)z = xy + e^{6x+y}$.

ii. Solve $x(y-z)p + y(z-x)q = z(x-y)$.

12.a. Find Fourier series to represent $f(x) = x - x^2$ from $x = -\pi \text{ to } x = \pi$ and hence deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$.

(OR)

b. Find the first three harmonic to represent $f(x)$ as a Fourier series

x:	0°	60°	120°	180°	240°	800°
y:	1.98	1.3	1.05	1.3	–0.88	–0.25

13. a. A uniform string is stretched and fastened to two points 'l' a parts. Motion is started by displacing the string into the form of the curve $y = kx(l - x)$ and then released from this position at time t=0. Find the displacement of any point of the string at a distance x from one end at any time 't'.

(OR)

b. Solve $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ subject to

$$\left.\begin{array}{l}(i)\quad u(0,t) = 0\\(ii)\quad u(l,t) = 0\end{array}\right\} for\ t \ge 0$$

$$(iii)\ u(x,0) = \begin{cases} x, & for\ 0 \le x \le \frac{l}{2} \\ l - x, & \frac{l}{2} \le x \le l \end{cases}$$

14. a. Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & 0 < x < 1 \\ 0, & otherwise \end{cases}$ hence prove that $\int_0^\infty \left\{\frac{\sin x - x\cos x}{x^3}\right\}\cos\left(\frac{x}{2}\right)dx = \frac{3\pi}{16}$.

(OR)

b. Find Fourier sine and cosine transform of $e^{-ax}, a > 0$ hence evaluate

(i) $\int_0^\infty \frac{dx}{\left(x^2 + a^2\right)^2}$

(ii) $\int_0^\infty \frac{x^2 dx}{\left(x^2 + a^2\right)^2}$

15. a.i. Using Convolution theorem, find inverse z-transform of $\frac{z^2}{(z-a)(z-b)}$.

ii. Solve: $y_{n+2} + 6y_{n+1} + 9y_n = 2^n\ if\ y_0 = y_1 = 0$.

(OR)