18MAB101T- TRANSFORMS AND BOUNDARY VALUE PROBLEMS

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UNITWISE IMPORTANT TOPICS

UNIT-1-PARTIAL DIFFERENTIAL EQUATIONS

Compulsory Topics: Lagrange, Clairaut's form, Second order PDE

	LAGRANGE'S MULTIPLIER METHOD (Most Important)
1.	$x(z^2-y^2)p+y(x^2-z^2)q=z(y^2-x^2)$.
2.	Solve $(mz-ny)p+(nx-lz)q=ly-mx$
3.	$x(y-z)p+y(z-x)q=z(x-y)$.
	CLAIRAUT'S FORM(Most Important)
4.	$z=px+qy+\sqrt{1+p^2+q^2}$
5.	Find the singular solution of $z=px+qy+p^2+q^2$.
6.	Solve: $z=px+qy+p^2+pq+q^2$
7.	Find the singular solution of $z=px+qy+p^2q^2$.
	HOMOGENEOUS DIFFERENTIAL EQUATIONS.(Most Important)
8.	$(D^2-2DD'+D'^2)z=\cos(x-3y)$
9.	$(D^2-DD'^2)z=e^{x+2y}$.
10.	Solve $(D^3-2D^2D')z=\sin(x+2y)+3x^2y$.
11.	Solve $(D^2-2DD')z=x^3y+e^{2x}$.

12.	Solve $\left(D^3-2D^2D'\right)z=\sin(x+2y)+3x^2y$
13.	Solve (i) $\left(D^2+DD'-6D'^2\right)z=x^2y+e^{3x+y}$ (ii) $z^2(p^2+q^2)=x^2+y^2$.
FORM PARTIAL DIFFERENTIAL EQUATIONS	
14.	Form the PDE by eliminating the arbitrary constants 'a' and 'b' from $z=(x^2+a)(y^2+b)$.
15.	Form the partial differential equation by eliminating arbitrary constants 'a' and 'b' from $\log(az-1)=x+ay+b$.
16.	Form the partial differential equation by eliminating the arbitrary function from the relation $\phi\left(x^2+y^2+z^2, lx+my+nz\right)=0$.
17.	Find the partial differential equation of all planes cutting equal intercepts form the x and y axes.
18.	From the Partial Differential Equation by eliminating the arbitrary functions fr. $z=f(x+ct)+\phi(x-ct)$.
19.	Form the Partial Differential Equation, by eliminating arbitrary constants from $(x-a)^2+(y-b)^2+z^2=r^2$, where a and b are arbitrary constants.

UNIT-2-FOURIER SERIES

Compulsory Topic: Harmonic Analysis

1.	Find the Fourier series of $f(x)=x+x^2$ in $(-\pi,\pi)$ of periodicity 2π. Hence deduce $\Sigma\frac{1}{n^2}=\frac{\pi^2}{6}$.
2.	Find the Fourier series expansion of period 2 for the function $f(x)=\begin{cases}\pi x \text{ in } 0\le x\le 1\\ \pi(2-x) \text{ in } 1\le x\le 2\end{cases}$.
3.	Expand $f(x) = x(2l-x)$ in $(0,2l)$ as a Fourier series of period $2l$. Hence deduce the sum $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots$
4.	Expand $f(x)=x(2\pi-x)$ as Fourier series in $(0,2\pi)$ and hence deduce that the sum of $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots$
5.	Find the Fourier series of $f(x)=(\pi-x)^2$ in $(0,2\pi)$ of periodicity 2π.
6.	Obtain the Fourier series of periodicity 3 for $f(x)=2x-x^2$ in $0<x<3$.
7.	Find the Fourier series of x^2 in $-\pi<x<\pi$. Hence deduce the value of $\sum_{n=1}^{\infty}\frac{1}{n^2}$ and $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots=\frac{\pi^4}{90}$.
8.	Obtain the Fourier series to represent the function $f(x)=\lvert x\rvert$, $-\pi<x<\pi$ and deduce $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{8}$.
9.	Obtain the Fourier series for the function $f(x)$ given by $f(x)=\begin{cases}1-x, & -\pi<x<0\\ 1+x, & 0<x<\pi\end{cases}$. Hence deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots=\frac{\pi^2}{8}$.

10.

1. Find the half-range cosine series for $f(x) = x$ in $(0,\pi)$. Hence deduce the value of $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\ldots$.

2. Find the half range cosine series of the function $f(x) = x(\pi - x)$ in the interval $0 < x < \pi$. Hence deduce that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots=\frac{\pi^4}{90}$.

3. Find the half-range Fourier cosine series of $f(x)=(\pi - x)^2$ in the interval $(0,\pi)$. Hence find the sum of the series $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots+\infty$.

11. Find the half-range sine series of $f(x)=4x-x^2$ in the interval $(0,4)$. Hence deduce the value of the series $\frac{1}{1^3}-\frac{1}{3^3}+\frac{1}{5^3}-\frac{1}{7^3}+\ldots\infty$.

HARMONIC ANALYSIS (Most Important)

12. Compute the first two harmonics of the fourier series f(x) given by the following table.

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1

13. The values of x and the corresponding values of f(x) over a period T are given below. Show that $f(x)=0.75+0.37\cos\theta+1.004\sin\theta$ where $\theta=\frac{2\pi x}{T}$.

x	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
f(x)	1.98	1.30	1.05	1.30	–0.88	–0.25	1.98

14. Determine the first three harmonics of the Fourier series for the following data:

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
y	1.96	1.3	1.05	1.3	– 0.88	– 0.25

15. Find the Fourier series upto second harmonic for the following data.

x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$
f(x)	1.0	1.4	1.9	1.7	1.05	1.2

16. Find the Fourier series as far as the second harmonic to represent the function $f(x)$ with the period 6, given in the following table.

x	0	1	2	3	4	5
$f(x)$	9	18	24	28	26	20

UNIT-3 -APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS

Compulsory Topics: One dimensional wave or one dimensional Heat

ONE DIMENSIONAL WAVE EQUATION (Most Important)

1. A tightly stretched string with fixed end point $x = 0$ and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $3x(l-x)$. Find the displacement.

2.	1. A string is stretched and fastened to points at a distance ℓ apart. Motion is started by displacing the string in the form $y = a\sin\left(\frac{\pi x}{\ell}\right)$, $0 < x < \ell$, from which it is released at time $t = 0$. Find the displacement at any time t. 2. A tightly stretched string with fixed end points $x = 0$ and $x = \ell$ is initially in a position given by $y(x,0) = y_0 \sin^3\left(\frac{\pi x}{\ell}\right)$. It is released from rest from this position. Find the expression for the displacement at any time t. 3. A uniform string is stretched and fastened to two points 'ℓ' apart. Motion is started by displacing the string into the form of the curve $y = kx(\ell - x)$ and then released from this position at time $t = 0$. Derive the expression for the displacement of any point of the string at a distance x from one end at time t.
3.	A tightly stretched string between the fixed end points $x = 0$ and $x = \ell$ is initially at rest in its equilibrium position. If each of its points is given a velocity $kx(\ell - x)$, find the displacement $y(x,t)$ of the string.
ONE DIMENSIONAL HEAT EQUATIONS(Most Important)	
4.	A rod of length l has its ends A and B kept at 0°C and 100°C respectively unit steady state conditions prevail. If the temperature at B is reduced suddenly to 0°C and kept so, while that of A is maintained. Find the temperature u(x, t).
5.	A rod, 30 cm long has its ends A and B kept at 20°C and 80°C respectively, until steady state conditions prevail. The temperature at each end is then suddenly reduced to 0°C and kept so. Find the resulting temperature function is a regular function $u(x,t)$ taking $x = 0$ at A.

UNIT -4 FOURIER TRANSFORMS

Compulsory Topics: 1-|x|, 1-x^2, a-x^2, evaluation by using convolution and parsevals

(MOST IMPORTANT)	
1.	Find the Fourier transform of $f(x) = \begin{cases} 1 - \lvert x \rvert & if \lvert x \rvert < 1 \\ 0 & if \lvert x \rvert \ge 1 \end{cases}$ and hence find the value of i) $\int_0^\infty \frac{\sin^2 t}{t^2}\, dt$ ii) $\int_0^\infty \frac{\sin^4 t}{t^4}\, dt$.
2.	Find the Fourier transform of $f(x)$ given by $f(x) = \begin{cases} 1 & \text{for } \lvert x \rvert < a \\ 0 & \text{for } \lvert x \rvert > a > 0 \end{cases}$ and using Parseval's identity prove that $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$.

3.	Find the Fourier transform of $f(x)=\begin{cases}1-x^2 & if\|x\|<1\\ 0 & if\|x\|>1\end{cases}$. Hence evaluate $\int_0^{\infty}\left(\frac{x\cos x-\sin x}{x^3}\right)\cos\frac{x}{2}\,dx$.
4.	Show that the Fourier transform of $f(x)=\begin{cases}a^2-x^2, & \|x\|\le a\\ 0, & \|x\|>a>0\end{cases}$ is $2\sqrt{\frac{2}{\pi}}\left(\frac{\sin as-as\cos as}{s^3}\right)$. Hence deduce that $\int_0^{\infty}\frac{\sin t-t\cos t}{t^3}dt=\frac{\pi}{4}$. Using Perserval's identity show that $\int_0^{\infty}\left(\frac{\sin t-t\cos t}{t^3}\right)^2 dt=\frac{\pi}{15}$.
5.	Evaluate $\int_0^{\infty}\frac{dx}{(x^2+1)(x^2+4)}$ using Fourier transforms. Evaluate $\int_0^{\infty}\frac{dx}{(x^2+a^2)(x^2+b^2)}$ using Fourier cosine transforms
6.	Use transform method to evaluate $\int_0^{\infty}\frac{x^2}{(x^2+a^2)(x^2+b^2)}dx$.

UNIT-5 Z-TRANSFORMS

Compulsory Topics: Solve difference equation, convolution , residue method, partial fraction, formula derivation

(MOST IMPORTANT QUESTIONS)	
1	Find $Z(a^n)$ and $Z(n^2)$.
2	Using residues find the inverse Z-transform of $\frac{z}{(z-1)(z-2)}$.
3	Solve $y_{n+2}+4y_{n+1}+3y_n=2^n$ with $y_0=0$ and $y_1=1$, using Z – transform.
4	Solve: $u_{n+2}+4u_{n+1}+3u_n=3^n$ given that $u_0=0, u_1=1$.
5	Solve $u_{n+2}-3u_{n+1}+2u_n=4^n$, given that $u_0=0, u_1=1$.

6	Find $Z\left(\frac{1}{n}\right)$. Find $Z^{-1}\left\{\frac{2z^2+4z}{(z-2)^3}\right\}$ by using residue theorem. Solve the equation $x_{n+2}-5x_{n+1}+6x_n=36$, given that $x_0=x_1=0$, using Z transform.
7	Find the Z – transform of $\frac{1}{n(n+1)}$, for $n\geq 1$. Find the Z-transform of $\frac{2n+3}{(n+1)(n+2)}$. Find $Z(\cos n\theta)$ and $Z(\sin n\theta)$.
8	Find the inverse Z – transform of $\frac{10z}{z^2-3z+2}$. Find the inverse Z-transform of $\frac{4z^2-2z}{(z-1)(z-2)^2}$ by method of partial fraction. Find the inverse Z-transform of $\frac{z^3}{(z-1)^2(z-2)}$ by method of partial fraction.
9	Using convolution theorem, find $Z^{-1}\left[\frac{z^2}{(z-a)(z-b)}\right]$.
10	Using Convolution theorem, find the inverse Z – transform of $\frac{8z^2}{(2z-1)(4z-1)}$.