SRM UNIVERSITY

DEPARTMENT OF MATHEMATICS

FACULTY OF ENGINEERING &TECHNOLOGY

CYCLE TEST – 2

MA1003 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

(COMMON TO EEE, ECE, CSE, SW, EIE, ITCE)

Duration: 2 Period **Max. Marks: 50**

Answer all the Questions

Part – A(5 x 4 = 20)

1. Find the Fourier series expansion for f(x) with period 2

$$f(x)=\begin{cases} 0 & \text{for } 0<x<1 \\ 1 & \text{for } 1<x<2 \end{cases}$$

2. Find the Co-efficient a_n of the Fourier series for the function $f(x) = x^2$ in $(-\pi, \pi)$.
3. Find the Half Range cosine series for the function $f(x) = k$ in $0<x<\pi$.
4. Find the Half Range Sine series for the function $f(x) = x$ in $0<x<\pi$.
5. Define R.M.S Value and find R.M.S Value for the function $f(x) = x-x^2$ in $-1<x<1$.

Part – B (3 x 10 = 30)

6. Find the Fourier series for the periodic function of period 2L defined by

$$f(x)=\begin{cases} L+x & -L\leq x\leq 0 \\ L-x & 0\leq x\leq L \end{cases}$$

7. Express $f(x) = (\pi - x)^2$ as a Fourier cosine series in $(0,\pi)$.
8. The values of x and the corresponding value of f(x) over a period T are given below. Find the first two harmonic of f(x) where $\theta = \frac{2\pi x}{T}$.

x	O	T/6	T/3	T/2	2T/3	5T/6	T
f(x)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

MA1003 Cycle Test - 2 (Answer Key) Set A

1) $f(x) = \frac{a_0}{2} + \sum_1^\infty a_n \cos\frac{n\pi x}{l} + \sum_1^\infty b_n \sin\frac{n\pi x}{l}$

$a_0 = 1, \quad a_n = 0, \quad b_n = \begin{cases} 0 & n \text{ even} \\ \frac{-2}{n\pi} & n \text{ odd} \end{cases}$ — 3M.

$f(x) = \frac{1}{2} + \sum_{n\,odd} \frac{-2}{n\pi} \sin n\pi x$

2) $f(-x) = f(x)$ — 1M

$a_n = \frac{2}{\pi}\int_0^\pi x^2 \cos nx\, dx$ — 1M

$= \frac{4(-1)^n}{n^2}$ — 2M

3) $f(x) = \frac{a_0}{2} + \sum_1^\infty a_n \cos nx$ — 1M

$a_0 = 2k$

$a_n = 0$ } — 2M

— 1M

$f(x) = k$

4) $f(x) = \sum b_n \sin nx$ — 1M

$b_n = \frac{2}{\pi}\int_0^\pi x \sin nx\, dx = \frac{2(-1)^{n+1}}{n}$ — 2M

$f(x) = \sum \frac{2(-1)^{n+1}}{n} \sin nx$ — 1M

5) RMS Value $\bar{y} = \sqrt{\frac{\int_a^b y^2 dx}{b-a}}$ — 1M

$\bar{y} = \sqrt{\frac{\int_{-1}^{1}(x-x^2)^2 dx}{2}}$ — 1M

$= \sqrt{8/15}$ — 2M

6) $\phi_1(-x) = \phi_2(x)$ $f(x)$ is even — 1M

$a_0 = \frac{2}{L}\int_0^L (L-x)\,dx = L$ — 3M

$a_n = \frac{2}{L}\int_0^L (L-x)\cos\frac{n\pi x}{L}\,dx$ — 1M

$= \frac{2L}{n^2\pi^2}[(-1)^n - 1] = \begin{cases} \frac{4L}{n^2\pi^2} & n \text{ odd} \\ 0 & n \text{ even} \end{cases}$ — 4M

$f(x) = \frac{L}{2} + \sum_{n\,odd} \frac{4L}{n^2\pi^2}\cos\frac{n\pi x}{L}$ — 1M

7) $f(x) = \frac{a_0}{2} + \sum a_n \cos nx$ — 1M.

$a_0 = \frac{2}{\pi}\int_0^\pi (\pi - x)^2 dx = \frac{2\pi^2}{3}$ — 3M

$a_n = \frac{2}{\pi}\int_0^\pi (\pi - x)^2 \cos nx\, dx$ — 1M

— 4M

$= 4/n^2$

$f(x) = \frac{\pi^2}{3} + \sum_1^\infty \frac{4}{n^2}\cos nx$ — 1M

8)

θ	y	$\cos\theta$	$\sin\theta$	$\cos 2\theta$	$\sin\theta$	$y\cos\theta$	$y\sin\theta$	$y\cos 2\theta$	$y\sin 2\theta$
0	1.98	1	0	1	0	1.98	0	1.98	0
$\pi/3$	1.30	0.5	0.866	-0.5	0.866	0.65	1.1258	-0.65	1.258
$2\pi/3$	1.05	-0.5	0.866	-0.5	-0.866	-0.53	0.9093	-0.525	-0.9093
π	1.30	-1	0	1	0	-1.3	0	1.3	0
$4\pi/3$	-0.88	-0.5	-0.866	-0.5	0.866	0.44	0.762	0.44	-0.7620
$5\pi/3$	-0.25	0.5	-0.866	-0.5	-0.866	-0.125	0.2165	0.125	0.2165
Σ	4.6					1.12	3.013	2.67	-0.3290

— 6M

$a_0 = 1.5, \quad a_1 = 0.3734, \quad a_2 = 0.89$

$b_1 = 1.0045 \quad b_2 = -0.1096$

$f(x) = 0.75 + 0.37\cos\theta + 0.89\cos 2\theta + 1.004\sin\theta - 0.1096\sin 2\theta$ — 4M

SET B

Reg No. | | | | | | | | | | | | | | |

SRM UNIVERSITY
FACULTY OF ENGINEERING & TECHNOLOGY
DEPARTMENT OF MATHEMATICS
CYCLE TEST-2
MA1003 – TRANSFORMS AND BOUNDARY VALUE PROBLEMS

3rd Semester

Date: 02/09/2015

Time: 100 Min

Max. Marks: 50

Part – A (5 x 4 = 20 Marks)
Answer ALL Questions

1. Find the a_n in the expansion Fourier series for $f(x) = (\pi - x)^2$ in the interval $(0, 2\pi)$. $4/n^2$

2.. Find a_0 for the function $f(x) = |\cos x|$ in the interval $(-\pi, \pi)$. $4/\pi$

3. Express $f(x) = \pi - x$ as a half range cosine series in $0 < x < \pi$. $a_0 = \pi$ $a_n = \frac{2}{n^2\pi}(1-(-1)^n)$

4. Find the Root Mean Square value of $f(x) = x + x^2$ in the interval $-2 < x < 2$. $8/\sqrt{15}$

5. If the fourier series of $f(x) = x^2$ in $(-\pi, \pi)$ is given by $f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4(-1)^n}{n^2} \cos nx$,

Then deduce that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots = \frac{\pi^4}{90}$ 8

PART – B(3 x 10 = 30 Marks)
Answer ALL Questions

6. Obtain the Fourier series expansion of $f(x)$ given by $f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & -\pi \le x \le 0 \\ 1 - \frac{2x}{\pi}, & 0 \le x \le \pi \end{cases}$

and hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$.

7. Find the cosine series for $f(x) = \begin{cases} x, & 0 \le x \le l/2 \\ l - x & l/2 \le x \le l \end{cases}$

8..Compute the first two harmonics of Fourier cosine series for the function $f(x)$ from the following data.

x	0	30	60	90	120	150	180
y	0	5224	8097	7850	5499	2626	0

SET B

MA1003 - Transforms & Boundary Value problems

Answer key

1. $a_n = \frac{1}{\pi}\int_0^{2\pi} (\pi - x)^2 \cos nx \, dx$ (1) mark

$= \frac{1}{\pi}\left[(\pi - x)^2 \frac{\sin nx}{n} + 2(\pi - x)\left(-\frac{\cos nx}{n^2}\right) + 2\left(-\frac{\sin nx}{n^3}\right)\right]_0^{2\pi}$ (2M)

$a_n = \frac{4}{n^2}$ (1M)

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2. $a_0 = \frac{2}{\pi}\left[\int_0^{\pi/2} \cos x \, dx + \int_{\pi/2}^{\pi} [-\cos x] \, dx \right]$ (1) mark

$= \frac{2}{\pi}\left[[\sin x]_0^{\pi/2} - [\sin x]_{\pi/2}^{\pi} \right]$ (2) mark

$a_0 = \frac{4}{\pi}$ (1) mark

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3. $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx$ (1) m

$a_0 = \frac{2}{\pi}\left[\int_0^{\pi} (\pi - x) \right] dx = \pi$ (1) m

$a_n = \frac{2}{\pi}\int_0^{\pi} (\pi - x) \cos nx \, dx = \frac{2}{\pi}\left\{ (\pi - x)\left(\frac{\sin nx}{n}\right) - (-1)\left(-\frac{\cos nx}{n^2}\right)\right\}_0^{\pi}$

$a_n = \frac{2}{n^2 \pi}\left[1 - (-1)^n\right]$

$a_n = \begin{cases} \frac{4}{n^2\pi} & n \text{ is odd} \\ 0 & n \text{ is even,} \end{cases}$ (2) mark

$\boxed{f(x) = \frac{\pi}{2} + \sum_{n=1,3,5..}^{\infty} \frac{4}{n^2 \pi} \cos nx}$

4. R.M.S value of $f(x) = \sqrt{\frac{\int_{-2}^{2} (x + x^2)^2 dx}{4}}$ (1) m

$= \sqrt{\frac{1}{4} \times 2 \int_0^2 [x^2 + x^4 + 2x^3] \, dx}$ (2m)

$= \sqrt{\frac{1}{2} \times \frac{256}{15}} = \sqrt{\frac{64}{15}} = \frac{8}{\sqrt{15}}$ (1m)

5.
$$\frac{1}{2\pi}\int_{-\pi}^{\pi}(f(x))^2\,dx = \frac{a_0^2}{4} + \frac{1}{2}\sum_{n=1}^{\infty} a_n^2 \qquad \text{1m}$$

$$\frac{1}{\pi}\int_0^{\pi} x^4\,dx = \frac{\pi^4}{36} + \frac{1}{2}\sum_{n=1}^{\infty}\frac{16}{n^4} \qquad \text{1m}$$

$$8\sum_{n=1}^{\infty}\frac{1}{n^4} = \frac{\pi^4}{5} - \frac{\pi^4}{9} = \frac{4\pi^4}{45} \qquad \text{2m}$$

$$\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\cdots = \pi^4/90$$

Part-B

6. $f(x)$ is an even function. $\therefore b_n = 0$ (1m)

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx, \quad a_0 = \frac{2}{\pi}\int_0^{\pi}\left(1-\frac{2x}{\pi}\right)dx = 0 \quad \text{— 2m}$$

$$a_n = \frac{2}{\pi}\int_0^{\pi}\left(1-\frac{2x}{\pi}\right)\cos nx\,dx = \frac{4}{n^2\pi^2}\left[1-(-1)^n\right] \quad \text{— 3m}$$

$$a_n = \begin{cases}\frac{8}{n^2\pi^2}, & n \text{ is odd}\\ 0 & n \text{ is even}\end{cases}, \quad f(x) = \sum_{n=1,3..}^{\infty}\frac{8}{n^2\pi^2}\cos nx \quad \text{— 2m}$$

Put $x=0$ is point of continuity,

$$1 = \frac{8}{\pi^2}\left[\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\right], \quad \therefore \boxed{\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots = \frac{\pi^2}{8}}$$

2m

7.
$$a_0 = \frac{2}{l}\left[\int_0^{l/2} x\,dx + \int_{l/2}^{l}(l-x)\,dx\right]$$

$$= \frac{2}{l}\left[\left(x^2/2\right)_0^{l/2} + \left(lx - x^2/2\right)_{l/2}^{l}\right] = l/2 \quad \text{— 3m}$$

$$a_n = \frac{2}{l}\left\{\int_0^{l/2} x\cos\frac{n\pi x}{l}\,dx + \int_{l/2}^{l}(l-x)\cos\frac{n\pi x}{l}\,dx\right\}$$

$$= \frac{2}{l}\left[\left[x\left(\frac{\sin\frac{n\pi x}{l}}{n\pi/l}\right) - \left(-\frac{\cos n\pi x/l}{n^2\pi^2/l^2}\right)\right]_0^{l/2} + \left[(l-x)\frac{\sin n\pi x/l}{n\pi/l} - \frac{\cos n\pi x/l}{n^2\pi^2/l^2}\right]_{l/2}^{l}\right.$$

3m

$$a_n = \frac{2}{l}\left[\frac{l}{2}\sin\frac{n\pi}{2}\left(\frac{l}{n\pi}\right) + \cos\frac{n\pi}{2}\left(\frac{l^2}{n^2\pi^2}\right) - (-1)^n\frac{l^2}{n^2\pi^2} - (-1)^n\frac{l^2}{n^2\pi^2}\right.$$
$$\left. - \frac{l}{2}\sin\frac{n\pi}{2}\left(\frac{l}{n\pi}\right) + \cos\frac{n\pi}{2}\left(\frac{l^2}{n^2\pi^2}\right)\right]$$

$$= \frac{2}{l}\left\{\frac{2l^2}{n^2\pi^2}\cos n\pi/2 - 2(-1)^n\frac{l^2}{n^2\pi^2}\right]$$

$$= \frac{4l}{n^2\pi^2}\left[\cos n\pi/2 - (-1)^n\right] \quad \text{— 3m}$$

$$f(x) = l + \sum_{n=1}^{\infty}\frac{4l}{n^2\pi^2}\left[\cos\frac{n\pi}{2} - (-1)^n\right] \quad \text{— 1m}$$

8.

x	y	$y\cos x$	$y\sin x$	$y\cos 2x$	$y\sin 2x$
0	0	0	0	0	0
30	5224	4524.12	2612	2612	4524.12
60	8097	4048.5	7012.2	−4048.5	7012.2
90	7850	0	7850	−7850	0
120	5499	−2749.5	4762.27	−2749.5	−4762.2
150	2626	−2274.18	1313	1313	−2274.18
180	0	0	0	0	0
	$\Sigma y = 29296$	= 3548.94	= 23549.47	= −107.23	4499.94

5m

$$a_0 = \frac{2\Sigma y}{n} = 9765.33$$

$$a_1 = \frac{2\Sigma y\cos x}{n} = 1182.98 \quad \text{— 2m}$$

$$\cancel{b_1 = \frac{2\Sigma y\sin x}{n} = 7849.82}$$

$$a_2 = \frac{2\Sigma y\cos 2x}{n} = -3574.33 \quad \text{— 1m}$$

$$\cancel{b_2 = \frac{2\Sigma y\sin 2x}{n} = 1499.98}$$

$$f(x) = \frac{a_0}{2} + a_1 \cos x + a_2 \cos 2x \cancel{+ b_1 \, si}$$

$$f(x) = \frac{9765.33}{2} + 1182 \cos x + (-3574.33) \cos 2x$$

2m

SRM UNIVERSITY
DEPARTMENT OF MATHEMATICS

MA-1003-TRANSFORMS AND BOUNDARY VALUE PROBLEMS

CYCLE TEST -II

DATE:02.08.2015 **DURATION :2 Periods**

MAX.MARKS:50

Answer ALL Questions SETC

PART-A (5x4=20)

1. Find a_n in the Fourier expansion of $f(x)=x^2$ in $-\pi < x < \pi$.
2. Find the cosine series of $f(x)= e^x$ in(0,1)
3. Find b_n in the expansion of $f(x)=(\pi - x)^2, 0 < x < 2\pi$.
4. Find the R.M.S value of $f(x) = x + x^3$ in $(0, 2\pi)$
5. Express f(x)=k as a half range sine series in $0 < x < 2$

Part-B(3x10=30)

1. Find the Fourier series upto two harmonics for y=f(x) in $(0, 2\pi)$ defined by the table of values given below.

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
f(x)	1	1.4	1.9	1.7	1.5	1.2	1

2. Find the cosine series of f(x) =x sinx in (0, L)
3. Find the Fourier series for the function $f(x)= 1+x+x^2$ in $(-\pi, \pi)$ hence deduce that

$$\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\cdots \ldots\ldots\ldots\ldots\ldots \infty = \frac{\pi^2}{6}.$$

Answer key SET C Cycle Test - II MA 100

Part - A:

1) $a_n = \frac{2}{\pi}\int_0^{\pi} x^2 \cos nx\,dx = \frac{4(-1)^n}{n^2} \rightarrow$ 1 mark

$\hookrightarrow$ 1 mark

Calculation $\rightarrow$ 2 ma

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2) $f(x) = e^x \quad l = 1$

$a_0 = \frac{2}{1}\int_0^1 f(x)\,dx = 2(e-1) \rightarrow$ 1 mark

$a_n = \frac{2}{1}\int_0^1 e^x \cos n\pi x\,dx = \frac{2}{n^2\pi^2+1}\left(e(-1)^n - 1\right) \rightarrow$ 2 m

$\therefore f(x) = (e-1) + \sum_{n=1}^{\infty} \frac{2}{n^2\pi^2+1}\left(e(-1)^n - 1\right)\cos(n\pi x) \longrightarrow$ 1 ma

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3) $b_n = \frac{1}{\pi}\int_0^{2\pi} f(x) \sin nx\,dx \rightarrow$ 1 mark

$b_n = \frac{1}{\pi}\int_0^{2\pi} (\pi - x)^2 \sin nx\,dx = 0 \rightarrow$ 1 mark

Calculation Part $\rightarrow$ 2 mar

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4) R.M.S Value:

$\bar{y} = \sqrt{\frac{\int_a^b y^2\,dx}{b-a}} \rightarrow$ 1 mark

$f(x) = x + x^3$

$\bar{y} = \frac{4\pi^2}{3} + \frac{64\pi^6}{7} + \frac{32\pi^4}{5} \longrightarrow$ 1 ma

Calculation Part $\rightarrow$ 2 mar

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5. $\boxed{l = 2}$ $b_n = \frac{2}{l}\int_0^{l} f(x) \sin\left(\frac{n\pi x}{l}\right)dx \longrightarrow$ 1 mark

$$b_n = \begin{cases} \frac{4k}{n\pi}, & \text{when } n \text{ is odd} \\ 0 & \text{when } n \text{ is even} \end{cases} \rightarrow 2 \text{ marks}$$

$f(x) = \sum_{n=1}^{\infty} \frac{4k}{n\pi} \sin\left(\frac{n\pi x}{2}\right) \rightarrow$ 1 mark

Part B:

1. $\sum y = 8.7$

$\sum y\cos x = -1.1$ + table

$\sum y\sin x = 0.524$

$\sum y\cos 2x = -0.3$

$\sum y\sin 2x = -0.178$

$\Big\}\rightarrow$ 5 marks

$a_0 = 2.9$

$a_1 = -0.367$

$a_2 = -0.1$

$b_1 = 0.175$

$b_2 = -0.0593$

$\Big\}$ 4 marks including formula

$y = 1.45 - 0.367\cos x - 0.1\cos 2x + 0.175\sin x - 0.0593\sin 2x$

$\rightarrow$ 1 mark.

2) $f(x) = x\sin x \ (0, L)$

$a_0 = 2\left(\dfrac{\sin L - L\cos L}{L}\right) \rightarrow$ 4 marks (formula).

$a_n = \frac{2}{L}\left[\left(\dfrac{1}{n\pi/L+1}\right)\left(-L\cos\left(\frac{n\pi}{L}+1\right)L + \sin\left(\frac{n\pi}{L}+1\right)L\right) - \dfrac{1}{n\pi/L-1}\left(-L\cos\left(\frac{n\pi}{L}-1\right)L + \sin\left(\frac{n\pi}{L}-1\right)L\right)\right]$

$\rightarrow$ 5 marks (formula)

$\therefore f(x) = \left(\dfrac{\sin L - L\cos L}{L}\right) + \sum_{n=1}^{\infty}\frac{2}{L}\left[\left(\dfrac{1}{n\pi/L+1}\right)\left(-L\cos\left(\frac{n\pi}{L}+1\right)L + \sin\left(\frac{n\pi}{L}+1\right)L\right) - \dfrac{1}{(n\pi/L-1)}\left(-L\cos\left(\frac{n\pi}{L}-1\right)L + \sin\left(\frac{n\pi}{L}-1\right)L\right)\right]$

$\rightarrow$ 1 mark

3. $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx \rightarrow$ 1 mark $\quad a_0 = 2(1+\pi^2/3) \rightarrow$ 1 mark.

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx \rightarrow$ 1 mark $\quad a_n = \dfrac{4(-1)^n}{n^2} \rightarrow$ 2 marks

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx \rightarrow$ 1 mark $\quad b_n = -\frac{2}{n}(-1)^n \rightarrow$ 2 marks

At $x=\pi$ $\quad \dfrac{f(\pi)+f(-\pi)}{2} = 1+\pi^2 = 1+\pi^2/3 + 4\sum_{n=1}^{\infty} 1/n^2 \rightarrow$ 1 mark

$\therefore \sum_{n=1}^{\infty} 1/n^2 = \pi^2/6 \rightarrow$ 1 mark.

Reg.No														

Set : D

SRM UNIVERSITY
DEPARTMENT OF MATHEMATICS

MA1003- Transforms and Boundary Value Problems

Cycle Test - II

Date: 02.09.2015 Duration: Two Periods Max.marks:50

(Answer all the questions)

PART-A (5x4=20 marks)

1. Find the half range sine series for $f(x) = k$ in $(0, \pi)$
2. Define Root Mean Square Value of a function in (a,b). Also state the Parsevals Identity for f(x) in (c ,c+2l).
3. Evaluate a_0 *and* a_n for $f(x) = \begin{cases} 1 & -1 < x < 0 \\ 0 & 0 < x < 1 \end{cases}$ if f(x) is periodic with period 2.
4. Find the cosine series for $f(x) = x$ in $(0,L)$.
5. If the values of u_0 *and* u_n are respectively π and $\frac{2}{\pi n^2}((-1)^n - 1)$ in the Fourier series for $f(x) = |x|$ in $(-\pi, \pi)$, find the sum $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$

PART-B (3x10=30 Marks)

1. Find the Fourier Series of period 2L for $f(x) = x(2L - x)$ in (0,2L).
2. Find the half range cosine series for $f(x) = (x-1)^2$, $0 < x < 1$.
3. Determine the first two harmonics of the Fourier series for the following data.

x :	0°	60°	120°	180°	240°	300°	360°
y:	10	12	15	20	17	11	10

CT-2

Set-D

MA1003- Answer key

Part-A

① $L=\pi$ $\quad b_n = \frac{2}{\pi}\int_0^{\pi} k\sin nx\,dx$ ——— (1)

$= \frac{2k}{n\pi}\left(1-(-1)^n\right) = \begin{cases} 4k/n\pi, & n \text{ is odd} \\ 0 & n \text{ is even}\end{cases}$ ——— (2)

$$\boxed{f(x) = \frac{4k}{\pi}\sum_{1,3,5}^{\infty}\frac{1}{n}\sin nx}$$ ——— (1)

② $\bar{y} = \sqrt{\frac{1}{(b-a)}\int_a^b (f(x))^2\,dx}$ ——— (1)

$\bar{y}^2 = \frac{a_0^2}{4} + \frac{1}{2}\left(\sum a_n^2 + \sum b_n^2\right)$

if $f(x)$ is defined in $(c, c+2l)$

where $a_0 = \frac{1}{l}\int_c^{c+2l} f(x)\,dx \quad a_n = \frac{1}{l}\int_c^{c+2l} f(x)\cos\frac{n\pi x}{l}\,dx$ ——— (3)

$b_n = \frac{1}{l}\int_c^{c+2l} f(x)\sin\frac{n\pi x}{l}\,dx$

③ $2L=2$ $\boxed{L=1}$

$a_0 = \frac{1}{1}\int_{-1}^{1} f(x)\,dx = \boxed{1=a_0}$ ——— (2)

$a_n = \frac{1}{1}\int_{-1}^{0} 1\cos\frac{n\pi x}{1}\,dx + \int_0^1 0$

$\boxed{a_n = 0}$ ——— (2)

④ $a_0 = \frac{2}{L}\int_0^L x\,dx = \boxed{L=a_0}$ ——— (1)

$a_n = \frac{2}{L}\int_0^L x\cos\frac{n\pi x}{L}\,dx = \begin{cases} -\frac{4L}{n^2\pi^2}, & n \text{ is odd} \\ 0, & n \text{ is even}\end{cases}$ ——— (2)

$f(x) = \frac{L}{2} - \frac{4L}{\pi^2}\sum_{1,3,5}^{\infty}\frac{1}{n^2}\cos\frac{n\pi x}{L}$ ——— (1)

$2L = 2\pi \quad \boxed{L=\pi}$

(5) $f(x)$ is even. $\therefore \boxed{b_n = 0}$ ——— (1)

F.S is $f(x) = \frac{\pi}{2} + \sum_{n=1,3,5} \frac{-4}{\pi n^2} \cos nx$ ——— (1)

Put $x = 0$

$$0 = \frac{\pi}{2} - \frac{4}{\pi} \sum_{1,3,5}^{\infty} \frac{1}{n^2}$$ ——— (1)

$$\boxed{\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} = \pi^2/8}$$ ——— (1)

Part - B

(1) $a_0 = \frac{1}{L} \int_0^{2L} (2Lx - x^2)\,dx = \frac{4l^2}{3}$ ——— (3)

$a_n = -\frac{4l^2}{n^2\pi^2}$ ——— (3)

$\boxed{b_n = 0}$

Sol. $f(x) = \frac{2l^2}{3} - \frac{4l^2}{\pi^2} \sum \frac{1}{n^2} \cos \frac{n\pi x}{L}$ (4)

(2) $\cancel{a}$ $\boxed{L=1}$

$a_0 = \frac{2}{L} \int_0^L f(x)\,dx = \frac{2}{3}$ ——— (4)

$a_n = \frac{-\cancel{4}}{L n^2\pi^2} = \frac{-4}{n^2\pi^2}$ ——— (4)

$f(x) = \frac{1}{3} + \sum \frac{-4}{n^2\pi^2} \cos n\pi x$ ——— (2)

(3)

x	y	$\cos x$	$\sin x$	$\sin 2x$	$y\cos x$	$y\sin x$	$\cos 2x$	$y\cos 2x$	$y\sin 2x$
0	10	1	0	0	10	0	1	10	0
60	12	0.5	0.866	0.866	6	10.39	-0.5	-6	10.39
120	15	-0.5	0.866	-0.866	-7.5	12.99	-0.5	-7.5	-12.99
180	20	-1	0	0	-20	0	1	20	0
240	17	-0.5	-0.866	0.866	-8.5	-14.72	-0.5	-8.5	14.72
300	10	0.5	-0.866	-0.866	5.5	-9.526	-0.5	-5.5	-9.526

→ Table (4) marks

$N = 6$

$\sum y = 85$
$\sum y\cos x = -14.5$
$\sum y\cos 2x = 2.5$

$\sum y\sin x = -0.866$
$\sum y\sin 2x = 2.598$

$a_0 = 28.33$
$a_1 = -4.833$
$a_2 = 0.833$
$b_1 = -0.288$
$b_2 = 0.866$

→ 5 marks

$f(x) = 14.16 - 4.833\cos x + 0.833\cos 2x - 0.288\sin x + 0.866\sin 2x$

Sol (1) mark