Unit 5

→ Computational & complexity theory.

→ Decidable or Undecidable

1. Recursive → TM Halt ⟨ W∈L (accept) / W∉L (reject).

2. Recursively enumerable → TM Halt ⟨ W∈L accept / W∉L reject / loop.

3. Decidable → recursive

4. Partial decidable → RErecursively enumerable

5. Undecidable → No TM exists.

Halting Prblm → Undeciable.

TM → prgm → Halt ⟨ Yes / No

→ i/p

Check prgm with Halt / not with i/p

Given prgm P and an i/p I, will P halt on I?

proof by contradiction

Assume H → TM

H is TM ,

H(P, I) C(x)

if H(x, x) == Halt:

loop forever

else

if {H(x, x) = = not Halt

return.

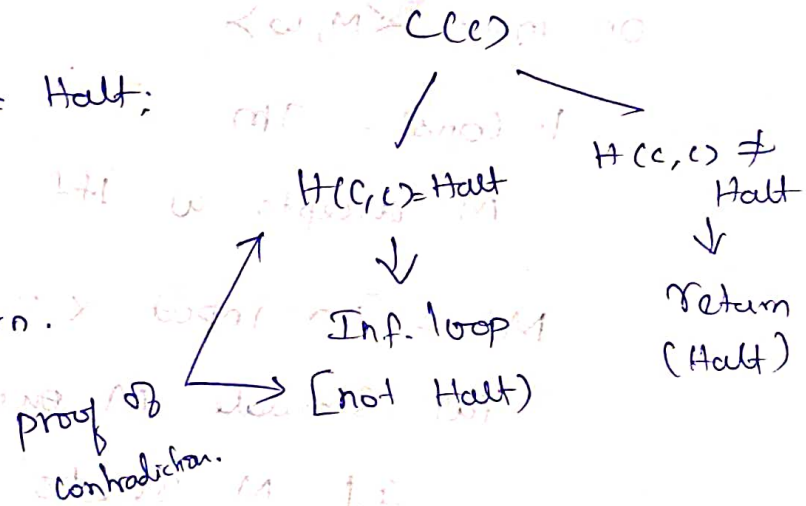
C(C)

if H(C, C) = = Halt:

loop forever

else.

~~if~~ return.



Rice Theorem

A property of TM lang. P is a set of descriptions so that for any two TM's M_1 & M_2 with $L(M_1) = L(M_2)$

(i) $\langle M_1 \rangle, \langle M_2 \rangle \in P$

ii) $\langle M_1 \rangle \langle M_2 \rangle \notin P$

A nontrivial prop. is one statement.

Some $\langle M_1 \rangle \in P$, and some $\langle M_2 \rangle \notin P$.

Rice theorem: Every nontrivial prop is undecidable.

proof: Let P be arbitary nontrivial property

Suppose P is decidable

Build a decider for A_{TM}

$A_{TM} \Rightarrow \langle M, w \rangle$: M is TM
w is string

on input $\langle M, w \rangle$

1. Const. TM

M accepts w iff this m/c $\in P$.

M = on input x:

Can Simulate M on w

If M rejects w then reject x

Post Correspondence Problem – Undecidable Prblm

(eg) $\frac{B}{CA}$ $\frac{A}{AB}$ $\frac{CA}{A}$ $\frac{ABC}{C}$ → Dominos (or) Tiles

we need to find a seq. of dominos such that the top & bottom string are same

* can use domino any no. of times.

$\frac{A}{AB}$ $\frac{B}{CA}$ $\frac{CA}{A}$ $\frac{A}{AB}$ $\frac{ABC}{C}$

→ A B CA A ABC

→ A B CA A A BC

A = $x_1, x_2, x_3 \ldots$

B = $\omega_1, \omega_2, \omega_3 \ldots$

(eg)

	A	B
①	1	111
②	10111	10
③	10	0

A – top

B – Bottom

$\frac{1}{111}$ $\frac{10111}{10}$ $\frac{10}{0}$

② ① ① ②

10111 1 1 10 → 10 11111 10

10 111 1 0 → 10 11111 10

eg.

	A	B	
①	10	101	$\frac{10}{101}$
②	011	11	$\frac{011}{11}$
③	101	011	$\frac{101}{011}$

1 1

10 10

101 101 ×

10 011

101 11 ×

10 101 10 ×

101 011 101

Undecidability of PCP

* Take a prblm that is already proven to be undecidable and try to convert it to PCP

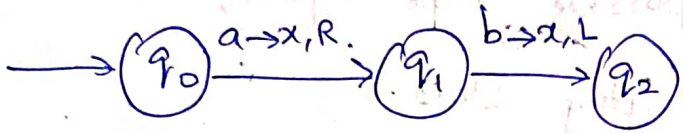
* If successfully convert it to an eq: PCP then we prove PCP is also undecidable.

Acceptance problem of TM (Undecidable prblm)

↓

Convert to PCP (called modified PCP - MPCP)

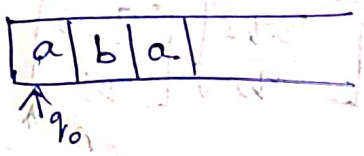
eg)



$\Sigma = \{a, b\}$; $\Gamma = \{a, b, x, B\}$ → blank — tape symbols

i/p: $w = aba$

Step 1:



domino $\frac{\#}{\#q_0 aba\#}$; $\frac{\#}{\#q_0 w \#}$

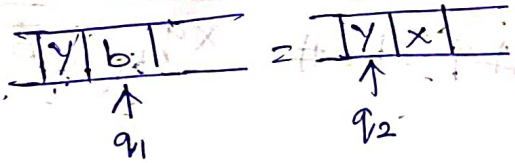
Step 2: $\delta(q_0, a) = (q_1, x, R)$

configuration $q_0 a = x q_1$; $\frac{q_0 a}{x q_1}$

domino $\frac{q_0 a}{x q_1}$

Step 3: $\delta(q_1, b) = (q_2, x, L)$

$q_1 b \neq x q_2$



domino $\frac{q_1 b}{x q_2}$; $\frac{Y q_1 b}{q_2 Y x}$; $Y \in \Gamma$

$\Gamma = \{a, b, x, B\}$

$Y q_1 b = q_2 Y x$

$\left[\frac{a q_1 b}{q_2 a x}\right] \left[\frac{b q_1 b}{q_2 b x}\right] \left[\frac{x q_1 b}{q_2 x x}\right] \left[\frac{B q_1 b}{q_2 B x}\right]$

Step 4: For all possible tape symbols

$\Gamma = \{a, b, x, B\}$

$\left[\frac{a}{a}\right] \left[\frac{b}{b}\right] \left[\frac{x}{x}\right] \left[\frac{B}{B}\right]$

Step 5: For all possible tape symbol, after reaching the accepting state (q_2)

$\left[\frac{a\,q_2}{q_2}\right] \left[\frac{q_2\,a}{q_2}\right], \left[\frac{b\,q_2}{q_2}\right] \left[\frac{q_2\,b}{q_2}\right]$

$\left[\frac{x\,q_2}{q_2}\right] \left[\frac{q_2\,x}{q_2}\right], \left[\frac{B\,q_2}{q_2}\right] \left[\frac{q_2\,B}{q_2}\right]$

Step 6: $\left[\frac{\#}{\#}\right] \left[\frac{\#}{B\#}\right]$ (Blank & Hash Symbol)

↓ blank

Step 7: $\left[\frac{q_2\,\#\#}{\#}\right]$ domino for final st.

$\frac{\#}{\#}$ $\frac{q_0\,a}{x\,q_1}$ $\frac{b}{b}$ $\frac{a}{a}$ $\frac{\#}{\#}$ $\frac{x\,q_1\,b}{q_2\,x\,x}$ $\frac{a}{a}$ $\frac{\#}{\#}$

$\frac{\#}{\#\,q_0\,a\,b\,a\,\#}$

$\frac{\#}{\#}$ $\frac{q_2\,x}{q_2}$ $\frac{x}{x}$ $\frac{a}{a}$ $\frac{\#}{\#}$ $\frac{q_2\,x}{q_2}$ $\frac{a}{a}$

$\frac{}{\#\,q_2\,x\,x\,a\,\#}$

$\frac{}{q_2\,a\,\#}$ $\frac{q_2\,a}{q_2}$ $\frac{\#}{\#}$ $\frac{q_2\,\#\,\#}{\#}$ $\frac{\#}{\#}$

PCP is undecid

MCP to PCP

→ Insert a symbol * after each symbol in list A

→ For List B insert a symbol * before any symbol in list B

→ These inserted symbol should be a new symbol

→ The symbol A0 = B0, So we insert a* symbol infront of A1

→ For end of B. insert $ in A and *$ in B list

eg:

A	B
11	10110
111	000
001	0101
010	0

I	A	B
1.	*1*1	*1*0*1*1*0
2.	1*1*1	*0*0*0
3.	0*0*1*	*0*1*0*1
4.	0*1*0*	*0
5	$	*$

MCP to PCP

→ Insert a symbol * after all symbol in list A.

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3.	0*0*1*	*0*1*0*1
4.	0*1*0*	*0
5	$	*$