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Introduction to Magnetic Resonance Imaging

Michael Bock

Dt. Sektion der ISMRM: PulSeq Course

Magnetic Dipole

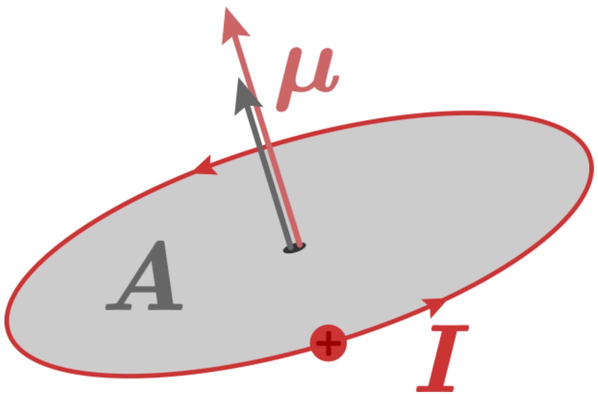
Current Loop

Electron

- electron on circular trajectory
- directed area $A = \pi r^2$
- electron mass and charge: m_e, e
- angular momentum: $L = m_e\, r^2 \omega$
- current: $I = \frac{e}{T_{rot}} = \frac{-e\omega}{2\pi}$
- magnetic moment:

$$\mu = I \cdot A = \frac{-e\omega}{2\pi} \pi r^2 = -\frac{e}{2m_e} L = -\frac{e\hbar}{2m_e}\, \ell = -\mu_B \ell$$

- Bohr‘s magneton: $\mu_B = \frac{e\hbar}{2m_e} = 9{,}27 \cdot 10^{-24} J/T$



Nucleon

- Bohr‘s nuclear magnetic moment: $\mu_N = \frac{e\hbar}{2m_p}$
- Measured values:

$$\mu_{proton} = 2{,}7928\, \mu_N\ ,\mu_{neutron} = -1{,}913\, \mu_N$$

Nucleus

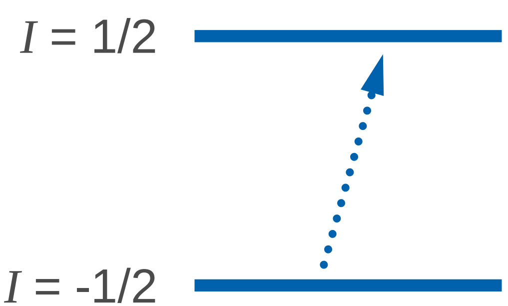
Magnetic Properties

Nuclear Spin

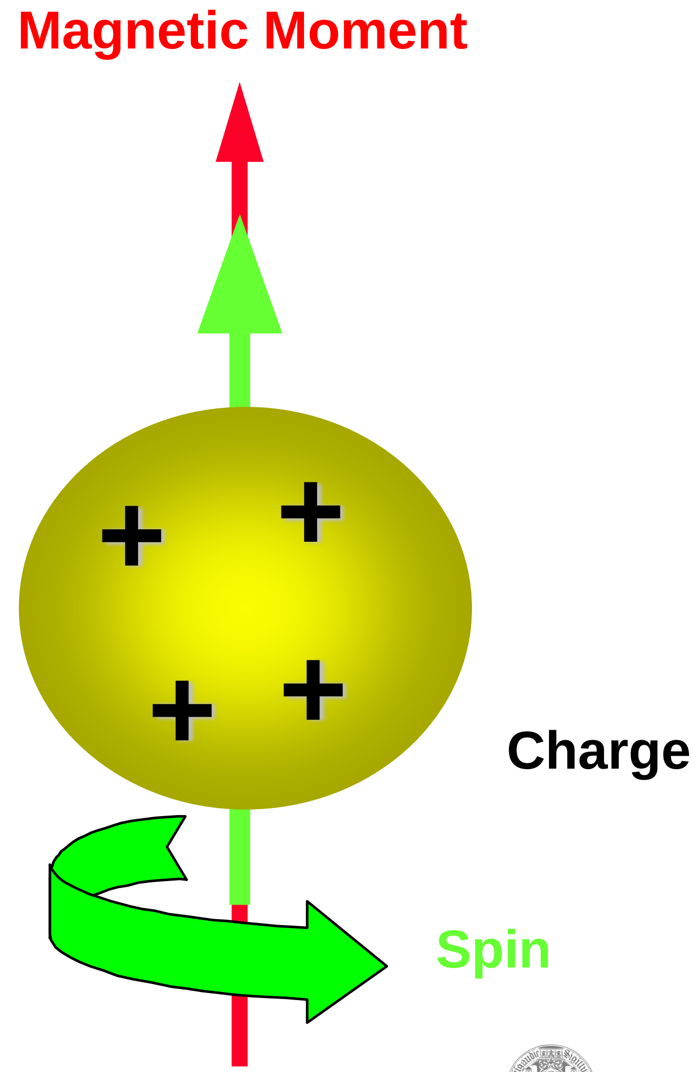
- „rotation of nuclear charges"
- ^{1}H: $I = 1/2$

Magnetic Moment

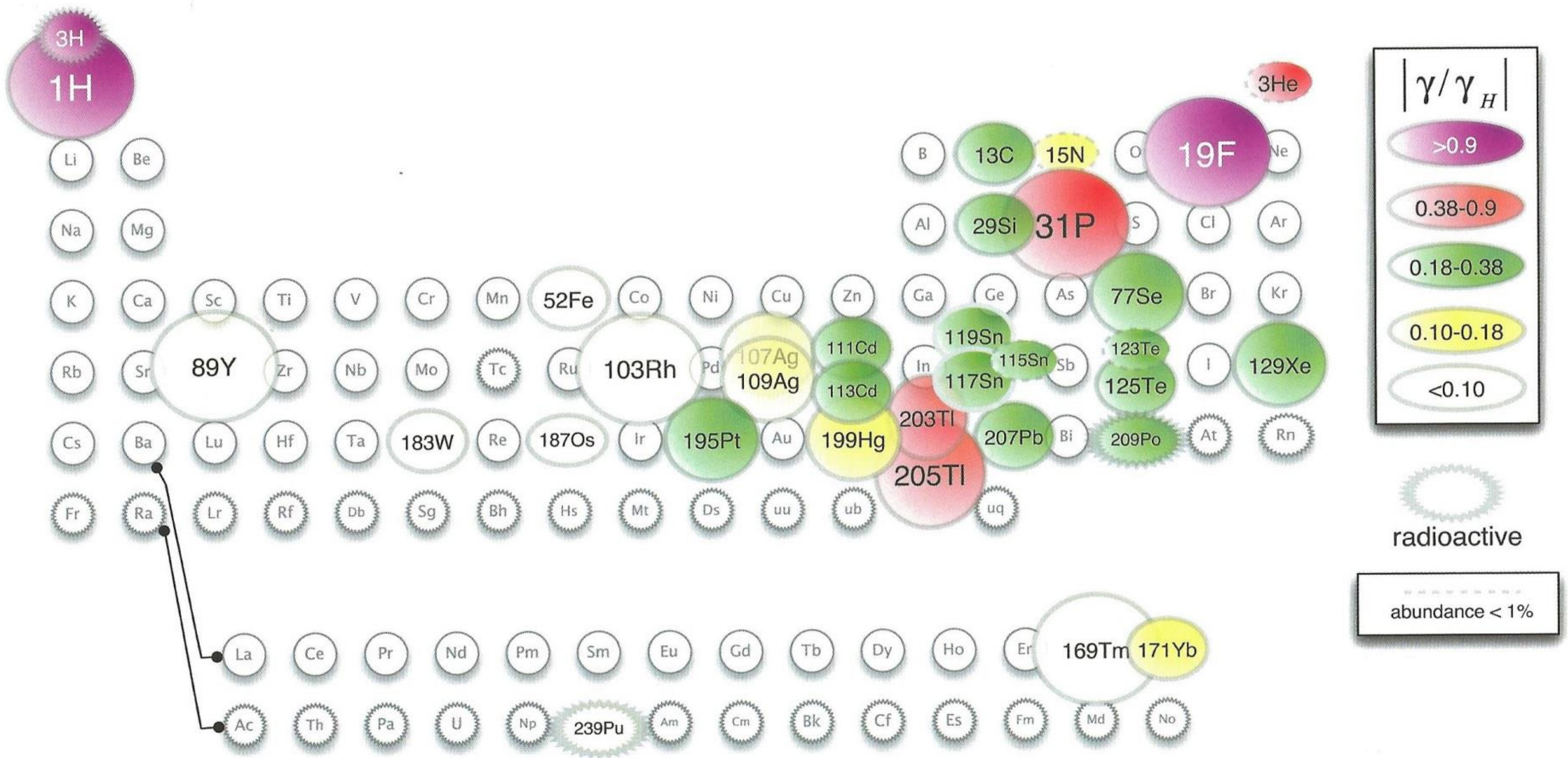
- $\mu = \gamma\, I$
- γ : gyromagnetic ratio
- γ = 42.577 MHz/T for ^{1}H



$$\Delta E = \hbar\gamma B_0 \\ = \hbar\omega$$



Nuclear Spin: *I* = 1/2



Levitt MH. *Spin dynamics*. Wiley

Spin-1/2 Nuclei

Table 1 The spin properties of spin-$\frac{1}{2}$ nuclei[a].

Isotope[b]	Natural abundance,[c] x/%	Magnetic moment,[d] μ/μ_N	Magnetogyric ratio,[d] $\gamma/10^7$ rad s^{-1} T^{-1}	Frequency ratio,[e] Ξ/%	Relative receptivity[g]	
					D^p	D^C
^{1}H	99.9885	4.837 353 570	26.752 2128	100.000 000[h]	1.000	5.87×10^{3}
^{3}H[i]	–	5.159 714 367	28.534 9779	106.663 974	–	–
^{3}He	1.37×10^{-4}	–3.685 154 336	–20.380 1587	76.179 437	6.06×10^{-7}	3.56×10^{-3}
^{13}C	1.07	1.216 613	6.728 284	25.145 020	1.70×10^{-4}	1.00
^{15}N	0.368	–0.490 497 46	–2.712 618 04	10.136 767	3.84×10^{-6}	2.25×10^{-2}
^{19}F	100	4.553 333	25.181 48	94.094 011	0.834	4.90×10^{3}
^{29}Si	4.6832	–0.961 79	–5.3190	19.867 187	3.68×10^{-4}	2.16
^{31}P	100	1.959 99	10.8394	40.480 742	6.65×10^{-2}	3.91 [illegible]0^{2}

Johannes Fischer
X-nuclear Imaging
Day 2, 13:50

Harris AK. *Pure Appl. Chem.*, 73(11): 1795–1818 (2001)

Quadrupolar Nuclei

Isotope[b]	Spin[c]	Natural abundance,[c] $x/\%$	Magnetic moment,[d] μ/μ_N	Magnetogyric ratio,[d] $\gamma/10^7$ rad s^{-1} T^{-1}	Quadrupole moment[e] Q/fm^2	Frequency ratio,[e] Ξ /%	Line-width factor,[h] ℓ/fm^4	Relative receptivity[i] D^p	D^C
^{2}H[j]	1	0.0115	1.212 600 77	4.106 627 91	0.2860	15.350 609	0.41	1.11×10^{-6}	6.52×10^{-3}
^{6}Li	1	7.59	1.162 5637	3.937 1709	–0.0808	14.716 086	0.033	6.45×10^{-4}	3.79
^{7}Li	3/2	92.41	4.204 075 05	10.397 7013	–4.01	38.863 797	21	0.271	1.59×10^{3}
^{9}Be	3/2	100	–1.520 136	–3.759 666	5.288	14.051 813	37	1.39×10^{-2}	81.5
^{10}B	3	19.9	2.079 2055	2.874 6786	8.459	10.743 658	14	3.95×10^{-3}	23.2
^{11}B	3/2	80.1	3.471 0308	8.584 7044	4.059	32.083 974	22	0.132	7.77×10^{2}
^{14}N[j]	1	99.632	0.571 004 28	1.933 7792	2.044	7.226 317	21	1.00×10^{-3}	5.90
^{17}O	5/2	0.038	–2.240 77	–3.628 08	–2.558	13.556 457	2.1	1.11×10^{-5}	6.50×10^{-2}
^{21}Ne	3/2	0.27	–0.854 376	–2.113 08	10.155	7.894 296[m]	140	6.65×10^{-6}	3.91×10^{-2}
^{23}Na	3/2	100	2.862 9811	7.080 8493	10.4	26.451 900	140	9.27×10^{-2}	5.45×10^{2}
^{25}Mg	5/2	10.00	–1.012 20	–1.638 87	19.94	6.121 635	130	2.68×10^{-4}	1.58
^{27}Al	5/2	100	4.308 6865	6.976 2715	14.66	26.056 859	69	0.207	1.22×10^{3}
^{33}S	3/2	0.76	0.831 1696	2.055 685	–6.78	7.676 000	61	1.72×10^{-5}	0.101
^{35}Cl	3/2	75.78	1.061 035	2.624 198	–8.165	9.797 909	89	3.58×10^{-3}	21.0
^{37}Cl	3/2	24.22	0.883 1998	2.184 368	–6.435	8.155 725	55	6.59×10^{-4}	3.87
^{39}K	3/2	93.2581	0.505 433 76	1.250 0608	5.85	4.666 373	46	4.76×10^{-4}	2.79
(^{40}K)	4	0.0117	–1.451 3203	–1.554 2854	–7.3	5.802 018	5.2	6.12×10^{-7}	3.59×10^{-3}
(^{41}K)	3/2	6.7302	0.277 396 09	0.686 068 08	7.11	2.561 305[n]	67	5.68×10^{-6}	3.33×10^{-2}
^{43}Ca	7/2	0.135	–1.494 067	–1.803 069	–4.08	6.730 029[o]	2.3	8.68×10^{-6}	5.10×10^{-2}

Harris AK. *Pure Appl. Chem.*, 73(11): 1795–1818 (2001)

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Nuclei for in vivo MR

Non-vanishing Spin

Nucleus	Spin	γ [MHz/T]
^{1}H	1/2	42.58
^{31}P	1/2	17.25
^{23}Na	3/2	11.27
^{14}N	1	3.08
^{13}C	1/2	10.71
^{19}F	1/2	40.08
^{3}He	-1/2	32.43
^{129}Xe	-1/2	11.78

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Bloch Equations

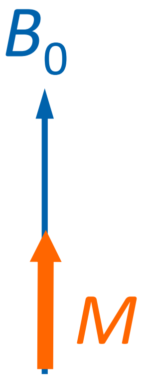
Thermal Equilibrium

$$\vec{M} || \vec{B}$$

$$\frac{dM_x}{dt} = \gamma\left(\vec{M} \times \vec{B}\right)_x = 0$$

$$\frac{dM_y}{dt} = \gamma\left(\vec{M} \times \vec{B}\right)_y = 0$$

$$\frac{dM_z}{dt} = \gamma\left(\vec{M} \times \vec{B}\right)_z = 0$$



Bloch Equations

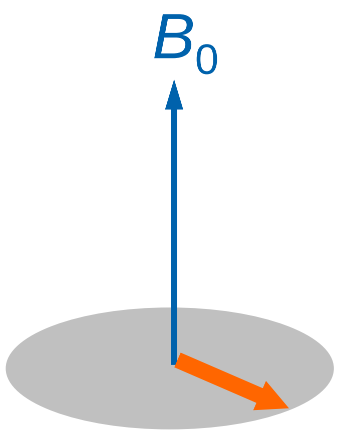
Precession

$$\vec{M} \perp \vec{B}$$

$$\frac{dM_x}{dt} = \gamma\left(\vec{M} \times \vec{B}\right)_x = \gamma\ B_z\, M_y$$

$$\frac{dM_y}{dt} = \gamma\left(\vec{M} \times \vec{B}\right)_y = -\gamma\ B_z\, M_x$$

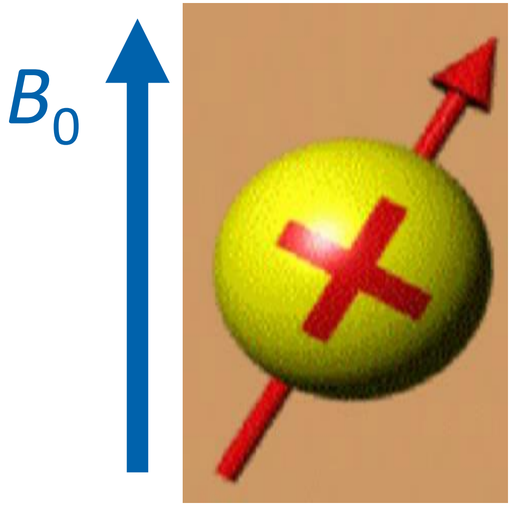
$$\frac{dM_z}{dt} = \gamma\left(\vec{M} \times \vec{B}\right)_z = 0$$



Nuclear Precession

Rotation of Magnetisation

$$\omega_0 = \gamma \cdot B_0$$



Bloch Equation

From Laboratory to Rotating Frame

Coordinate Transformation

- Bloch equation in lab frame:

$$\frac{dM_x}{dt} = \gamma\left(\vec{M} \times \vec{B}\right)_x$$

$$\frac{dM_y}{dt} = \gamma\left(\vec{M} \times \vec{B}\right)_y$$

$$\frac{dM_z}{dt} = \gamma\left(\vec{M} \times \vec{B}\right)_z$$

- Bloch equation in rotating frame:

$$\frac{dM'_x}{dt} = \gamma\left(\vec{M}' \times (\frac{\vec{\omega}}{\gamma} - \vec{B})\right)_x$$

$$\frac{dM'_y}{dt} = \gamma\left(\vec{M}' \times (\frac{\vec{\omega}}{\gamma} - \vec{B})\right)_y$$

$$\frac{dM'_z}{dt} = \gamma\left(\vec{M}' \times (\frac{\vec{\omega}}{\gamma} - \vec{B})\right)_z$$

with $\vec{\omega} = \omega \cdot \vec{e}_z$

- Rotating frame:

$$\begin{pmatrix} M'_x \\ M'_y \\ M'_z \end{pmatrix} = \begin{pmatrix} \cos(\omega t) & -\sin(\omega t) & 0 \\ \sin(\omega t) & \cos(\omega t) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}$$

Bloch Equation

Resonant Excitation

$$\frac{dM_x}{dt} = \gamma\left(\vec{M} \times (\frac{\vec{\omega}}{\gamma} - \vec{B})\right)_x$$

$$\frac{dM_y}{dt} = \gamma\left(\vec{M} \times (\frac{\vec{\omega}}{\gamma} - \vec{B})\right)_y$$

$$\frac{dM_z}{dt} = \gamma\left(\vec{M} \times (\frac{\vec{\omega}}{\gamma} - \vec{B})\right)_z$$

$$\frac{\vec{\omega}}{\gamma} - \vec{B} = \frac{\overrightarrow{\omega_0}}{\gamma} - \vec{B}_0 + \vec{B}_1 = \vec{B}_1$$

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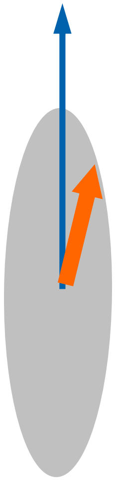
Bloch Equation

Resonant Excitation

$$\frac{dM_x}{dt} = \gamma\left(\vec{M} \times \vec{B}_1\right)_x = 0$$

$$\frac{dM_y}{dt} = \gamma\left(\vec{M} \times \vec{B}_1\right)_y = -\gamma\, B_1\, M_z$$

$$\frac{dM_z}{dt} = \gamma\left(\vec{M} \times \vec{B}_1\right)_z = \gamma\, B_1\, M_y$$



$$\vec{B}_1 = B_1(t)\begin{pmatrix}1\\0\\0\end{pmatrix}$$

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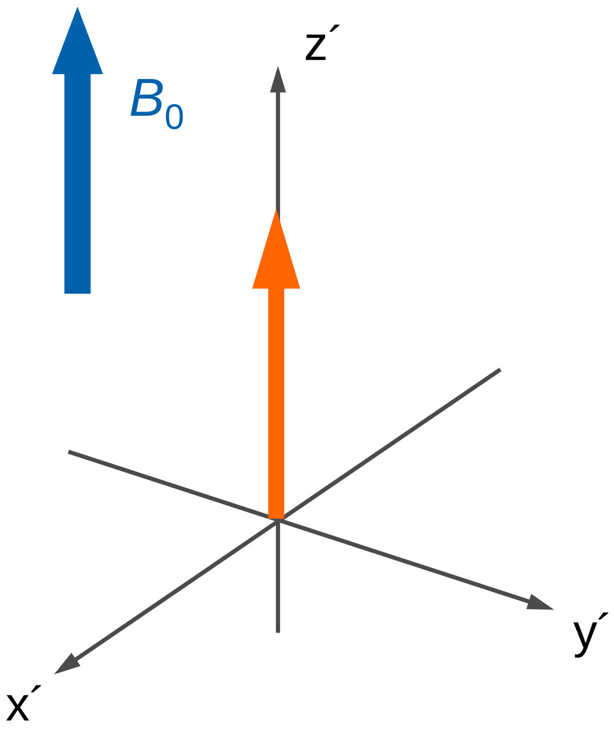
Resonant Excitation

RF Pulse

Larmor Frequency

- Eigenfrequency of magnetisation: ω



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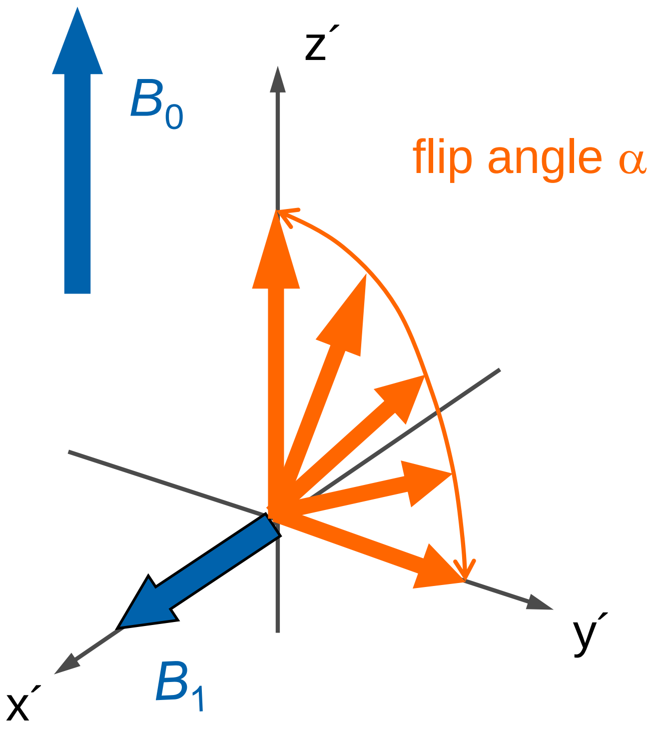
Resonant Excitation

RF Pulse

Larmor Frequency

- Eigenfrequency of magnetisation: ω
- external radio-frequency field (B_1-field)
- Rotation (precession) about B_1



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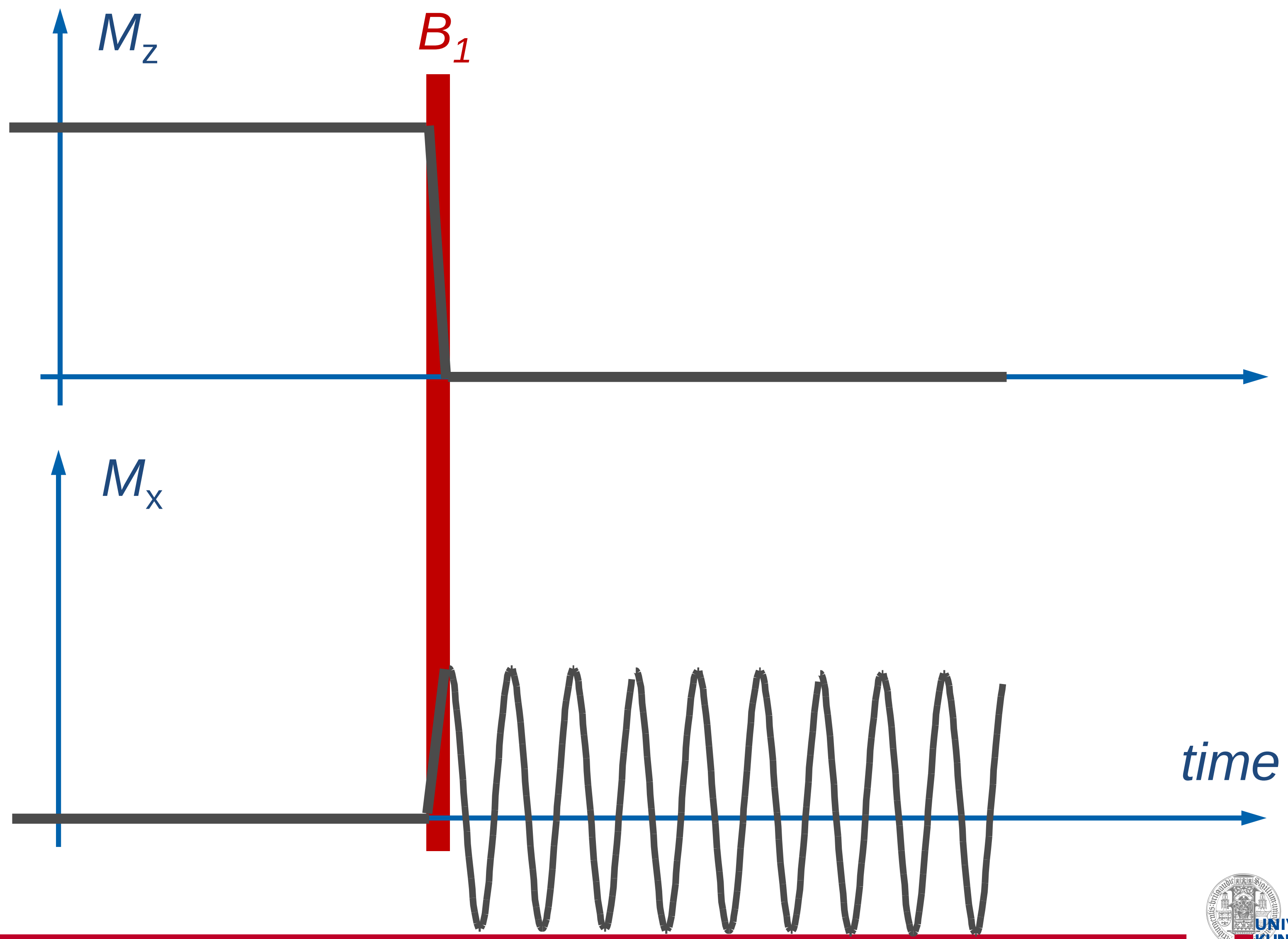
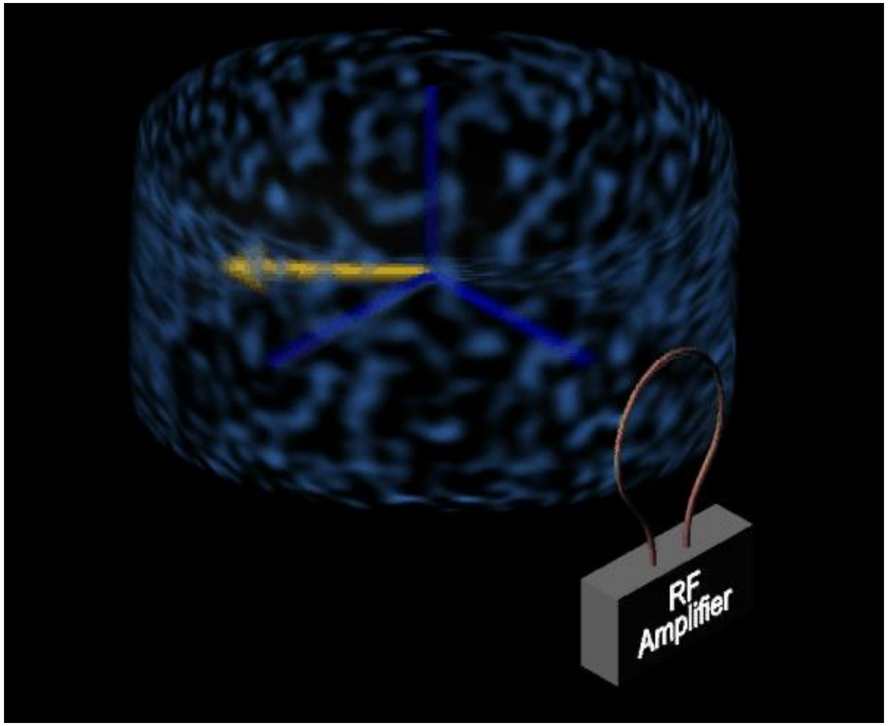
A Simple MR Experiment

RF Excitation + Precession



Signal Reception

Free Induction Decay



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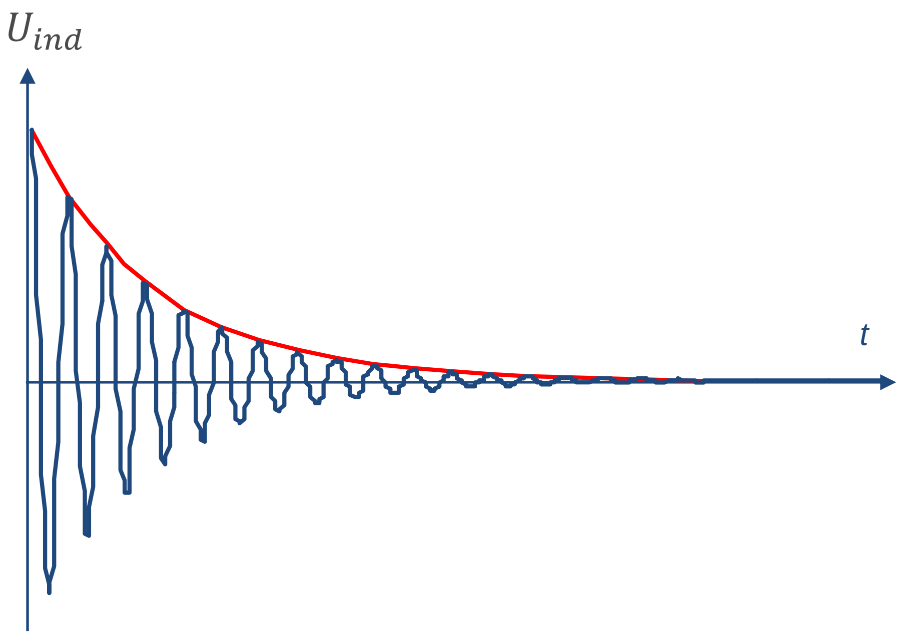
The Animated Physics of MRI, University Toronto

Free Induction Decay: FID



$$U_{ind} = -\frac{d\Phi}{dt} \sim -\frac{dM}{dt}$$

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Relaxation

Return to Thermal Equilibrium

Thermal Equilibrium

- Magnetisation parallel to B_0

Relaxation

- Return to thermal equilibrium
- T1 relaxation: parallel (longitudinal) to B_0
- T2 relaxation: orthogonal (transversal) to B_0

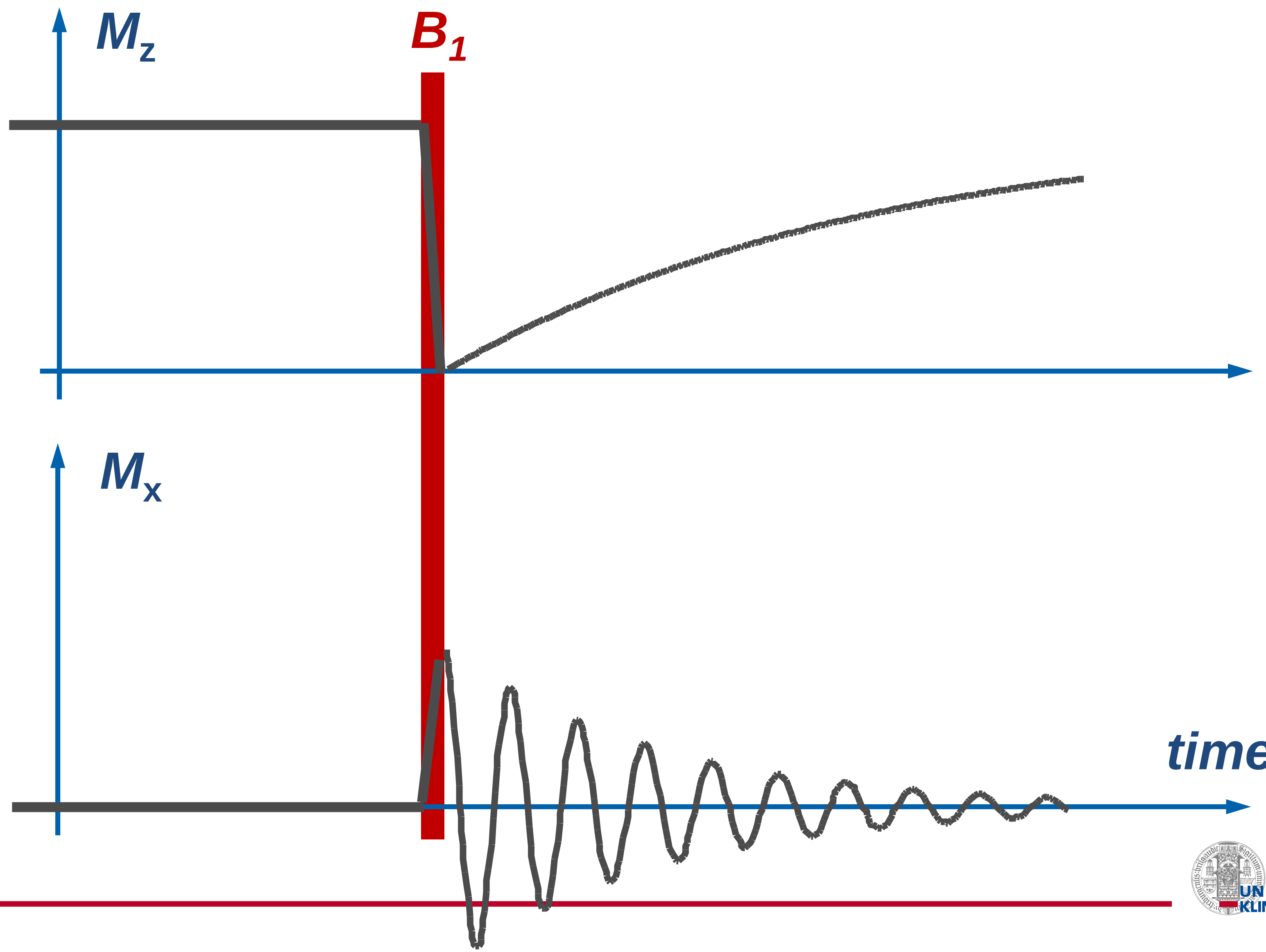
$$\frac{dM_x}{dt} = \gamma\left(\vec{M} \times \vec{B}\right)_x - \frac{M_x}{T_2}$$

$$\frac{dM_y}{dt} = \gamma\left(\vec{M} \times \vec{B}\right)_y - \frac{M_y}{T_2}$$

$$\frac{dM_z}{dt} = \gamma\left(\vec{M} \times \vec{B}\right)_z - \frac{M_z - M_0}{T_1}$$

A Realistic MR Experiment

T1 and T2-Relaxation

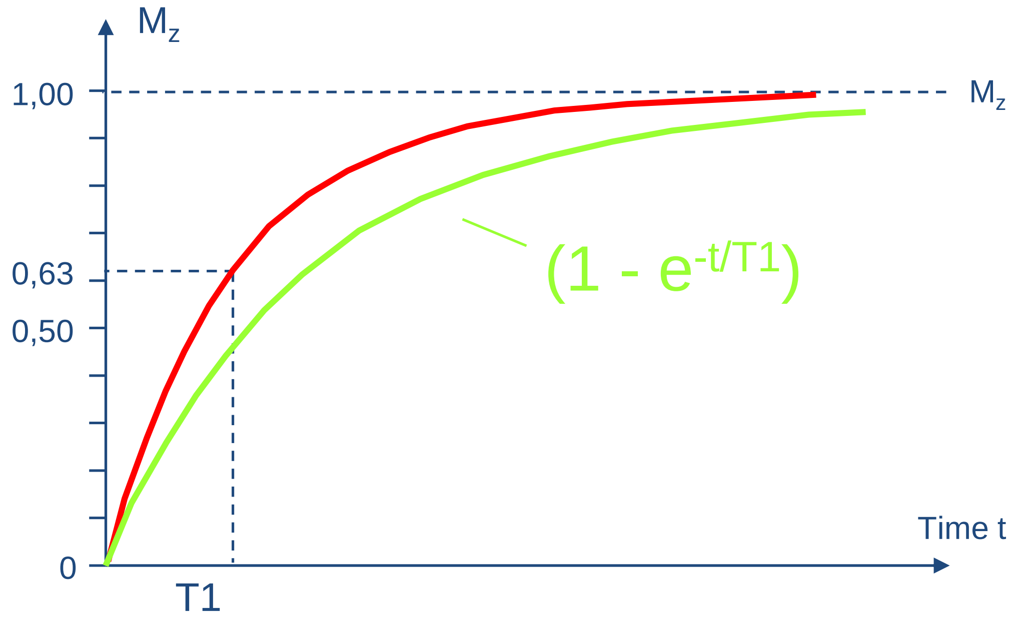


T1 Relaxation

Properties

Spin Lattice Relaxation

- Energy exchange between spin and magnetic surroundings (lattice)
- Longitudinal Magnetisation
- T1 depends on
  - tissue type
  - B_0
  - temperature
- T1 can be shortened by contrast agents



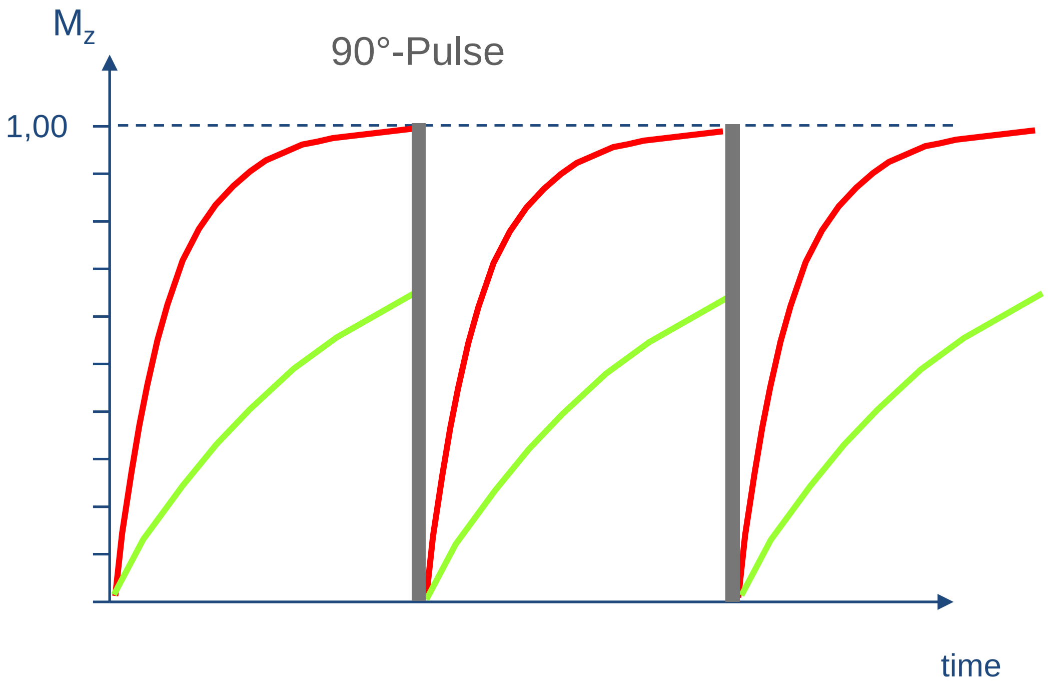
Tissue	T1 [ms] (@ 1.5 T)
Fat	300
WBM	600
GBM	800
CSF	4500

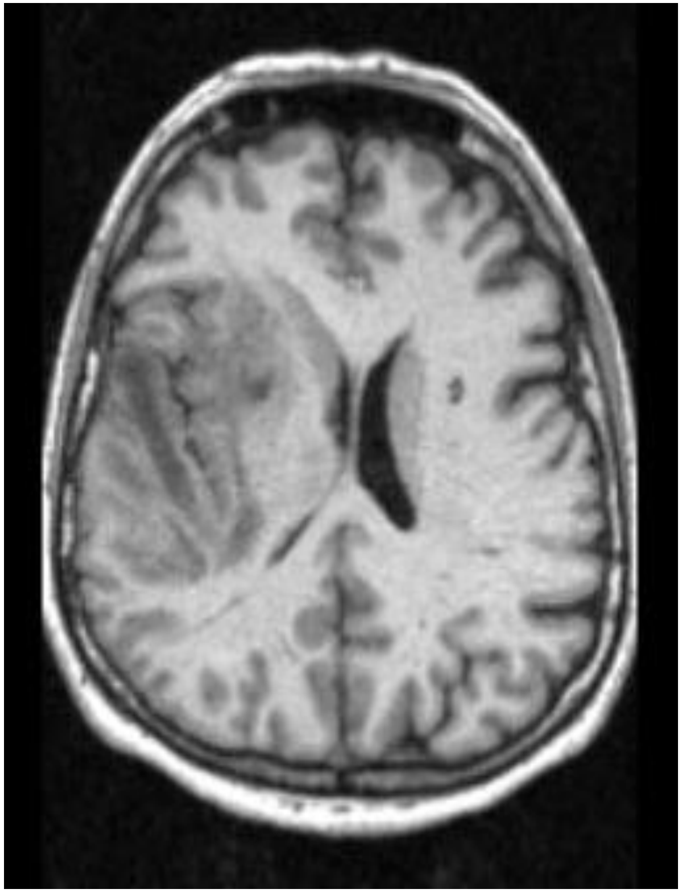
T1 Weighting

Image Contrast

Stopped Pulse Experiment

- 90° rf pulse
- Repetition time: TR



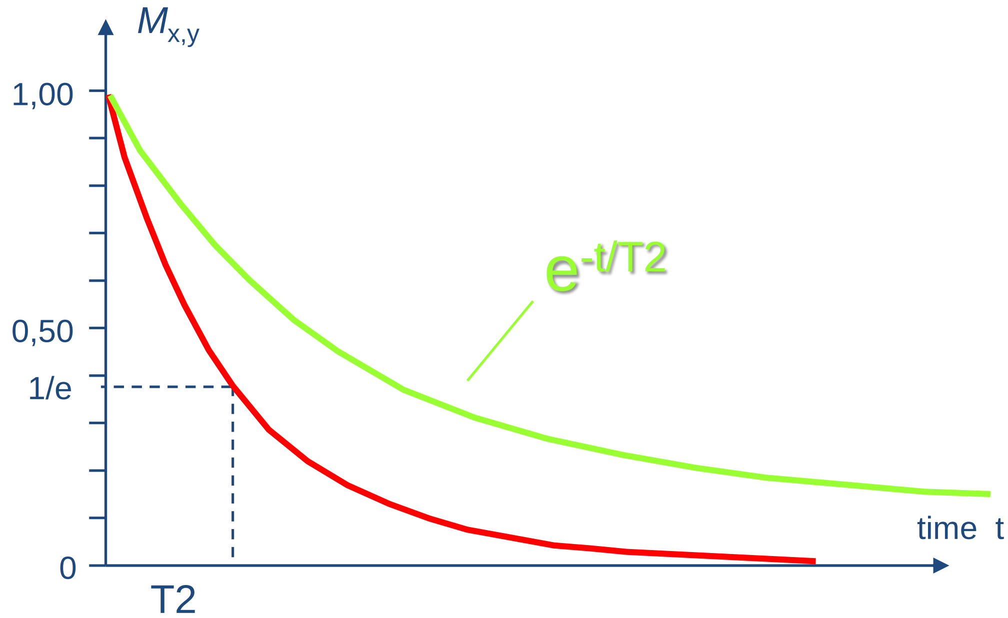


T2 Relaxation

Properties

Spin Spin Relaxation

- Loss of phase coherence between spins
- Transverse Magnetisation
- T2 depends on
  - tissue type
  - B_0
- T2 can be shortened by contrast agents



Tissue	T2 [ms] (@ 1.5 T)
Fat	60
WBM	80
GBM	100
CSF	2200

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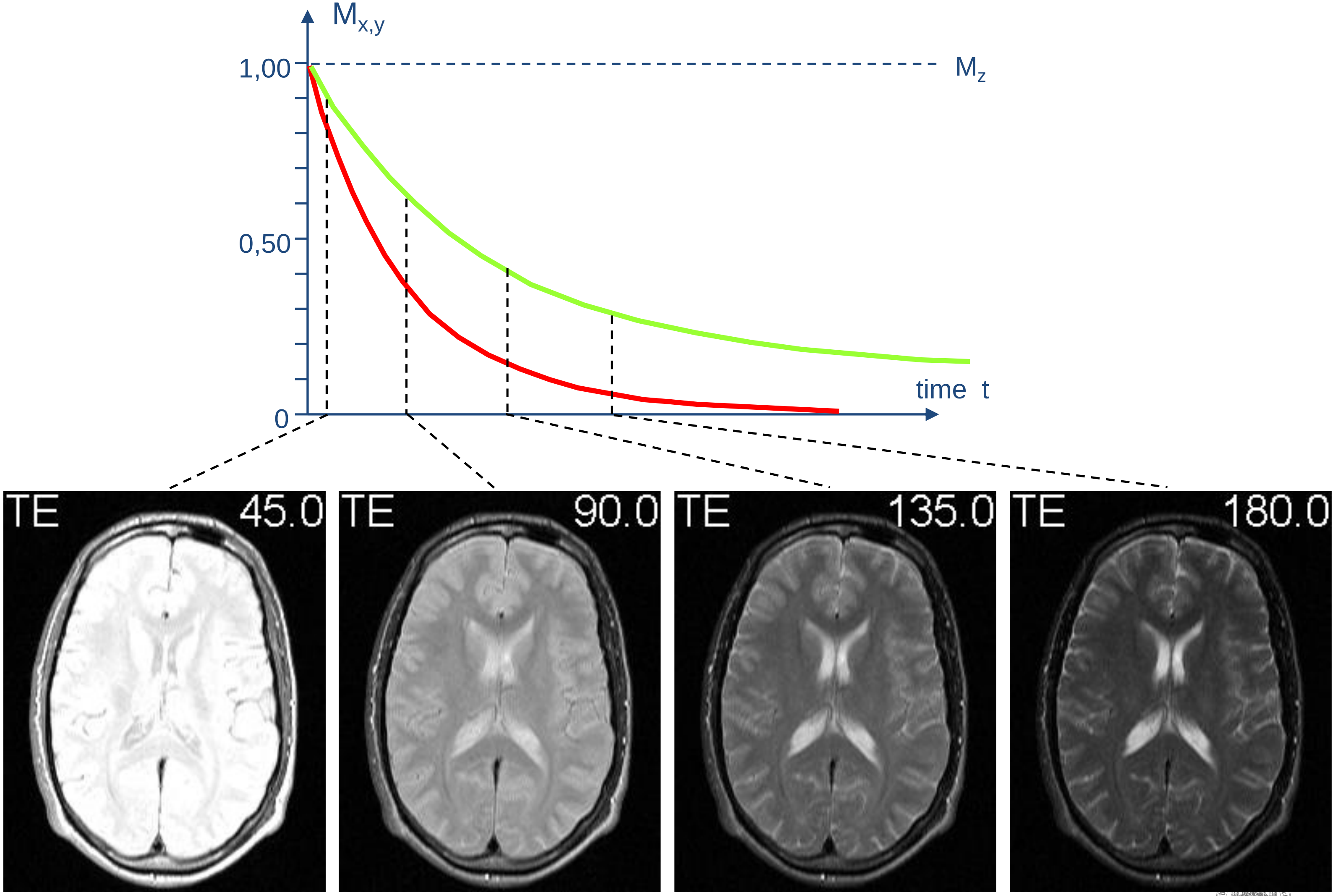
T2 Weighting
Image Contrast



Spin Gymnastics

Spin Gymnastics

Solutions for Special Cases of the Bloch Equation

Concept

- Describe time evolution of magnetization
- Matrix multiplications and vector addition

Matrix Notation for

- RF excitation (on-resonant)
- Precession (off-resonance)
- Relaxation
- Spoiling

Magnetization

- Vector: $\vec{M} = \begin{pmatrix} M_x \\ M_y \\ M_z \end{pmatrix}$
- Equilibrium magnetization:

$$\vec{M}_0 = \begin{pmatrix} 0 \\ 0 \\ M_0 \end{pmatrix}$$

Bloch Equation

Relaxation

On-resonant Equation

$$\frac{dM'_x}{dt} = -\frac{M'_x}{T_2}$$

$$\frac{dM'_y}{dt} = -\frac{M'_y}{T_2}$$

$$\frac{dM'_z}{dt} = -\frac{M'_z - M_0}{T_1}$$

Matrix/Vector Notation

$$\vec{M}(t) = \vec{M}_0 - \left(\vec{M}_0 - \vec{M}(0)\right) \cdot \boldsymbol{R_{relax}}$$

with

$$\boldsymbol{R_{relax}}(T_1, T_2, t) = \begin{pmatrix} e^{-t/T_2} & 0 & 0 \\ 0 & e^{-t/T_2} & 0 \\ 0 & 0 & e^{-t/T_1} \end{pmatrix}$$

Solution

$$M_{x,y}(t) = M_{x,y}(0) \cdot e^{-\frac{t}{T_2}}$$

$$M_z(t) = M_0 - (M_0 - M_z(0)) \cdot e^{-\frac{t}{T_1}}$$

Bloch Equation

Radio-frequency Excitation

RF Excitation

- On-resonant B_1-Pulse

$$\frac{dM_x}{dt} = 0$$
$$\frac{dM_y}{dt} = \gamma B_1 \cdot M_z$$
$$\frac{dM_z}{dt} = -\gamma B_1 \cdot M_y$$

- Solution

$$\vec{M}_+ = \boldsymbol{R_x}(\gamma B_1 \cdot t) \cdot \vec{M}_-$$

with

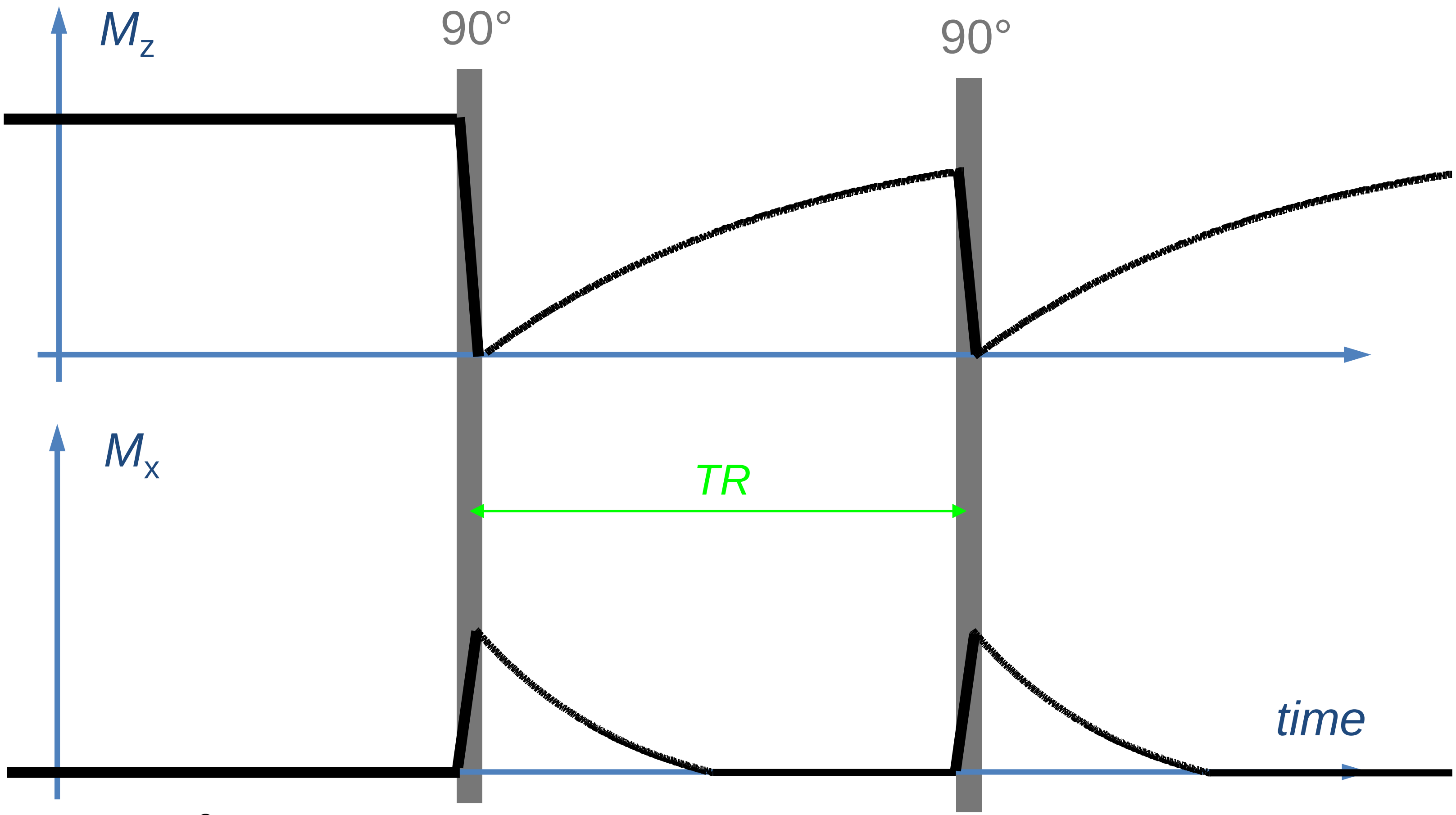
$$\boldsymbol{R_x}(\gamma B_1 \cdot t) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\gamma B_1 \cdot t) & \sin(\gamma B_1 \cdot t) \\ 0 & -\sin(\gamma B_1 \cdot t) & \cos(\gamma B_1 \cdot t) \end{pmatrix}$$

- Flip angle

$$\alpha_x = \gamma B_1 t = \gamma \int_0^\tau B_1(t)dt$$

MR-Experiment: (90°y-TR)n

RF-Pulse and Delay TR

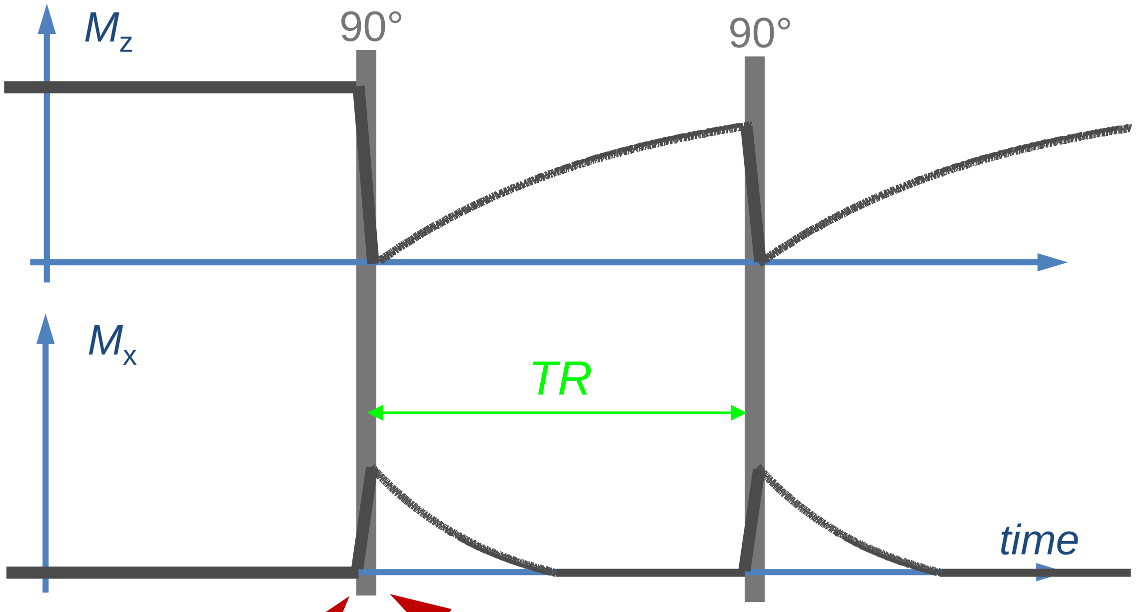


$\vec{M}_0 = \begin{pmatrix} 0 \\ 0 \\ M_0 \end{pmatrix}$ $\boldsymbol{R_y}(90°)$ $\boldsymbol{R_y}(90°)$

$\widehat{\boldsymbol{R}}_{\boldsymbol{relax}}(T_1, T_2, TR)$ $\widehat{\boldsymbol{R}}_{\boldsymbol{relax}}(T_1, T_2, TR)$

MR-Experiment: $(90°_y\text{-TR})_n$

Step 1



$$\vec{M}(0_-) = \vec{M}_0 = \begin{pmatrix} 0 \\ 0 \\ M_0 \end{pmatrix}$$

$$\vec{M}(0_+) = \boldsymbol{R_y}(90°) \cdot \vec{M}(0_-) = \begin{pmatrix} M_0 \\ 0 \\ 0 \end{pmatrix}$$

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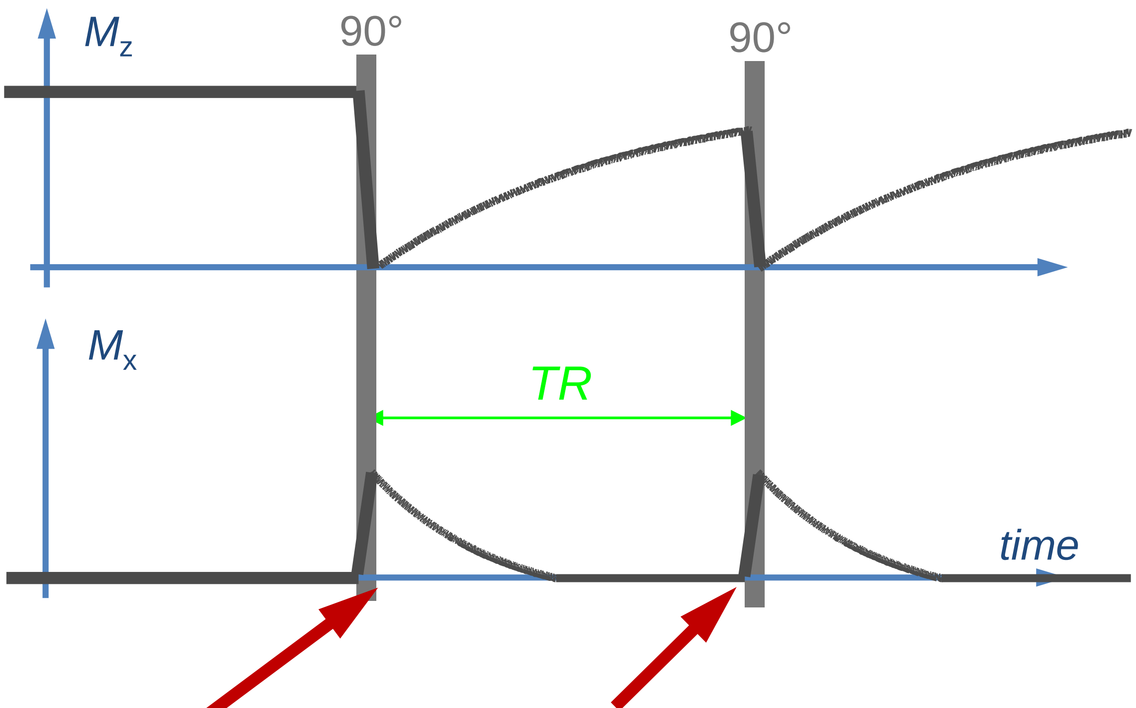
MR-Experiment: $(90°_y\text{-}TR)_n$

Step 2

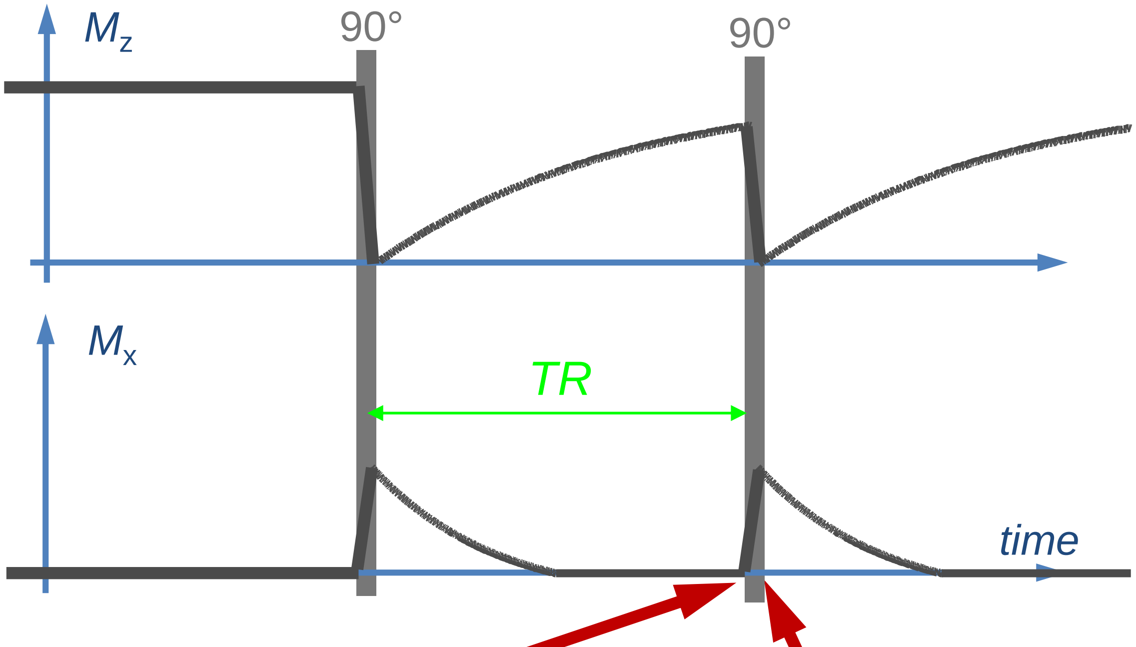


$$\vec{M}(0_+) = \begin{pmatrix} M_0 \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{M}(TR) = \vec{M}_0 - \left(\vec{M}_0 - \vec{M}(0_+)\right) \cdot \boldsymbol{R_{relax}}(T_1, T_2, TR)$$
$$= \begin{pmatrix} M_0 \cdot e^{-TR/T2} \\ 0 \\ M_0 \cdot \left(1 - e^{-TR/T_1}\right) \end{pmatrix} \approx \begin{pmatrix} 0 \\ 0 \\ M_0 \cdot \left(1 - e^{-TR/T_1}\right) \end{pmatrix}$$

MR-Experiment: $(90°_y\text{-}TR)_n$

Step 3



$$\vec{M}(0_-) = \begin{pmatrix} 0 \\ 0 \\ M_0 \cdot \left(1 - e^{-TR/T_1}\right) \end{pmatrix}$$

$$\vec{M}(0_+) = \hat{R}_{RF}(90°_y) \cdot \vec{M}(0_-) = \begin{pmatrix} M_0\left(1 - e^{-TR/T_1}\right) \\ 0 \\ 0 \end{pmatrix}$$

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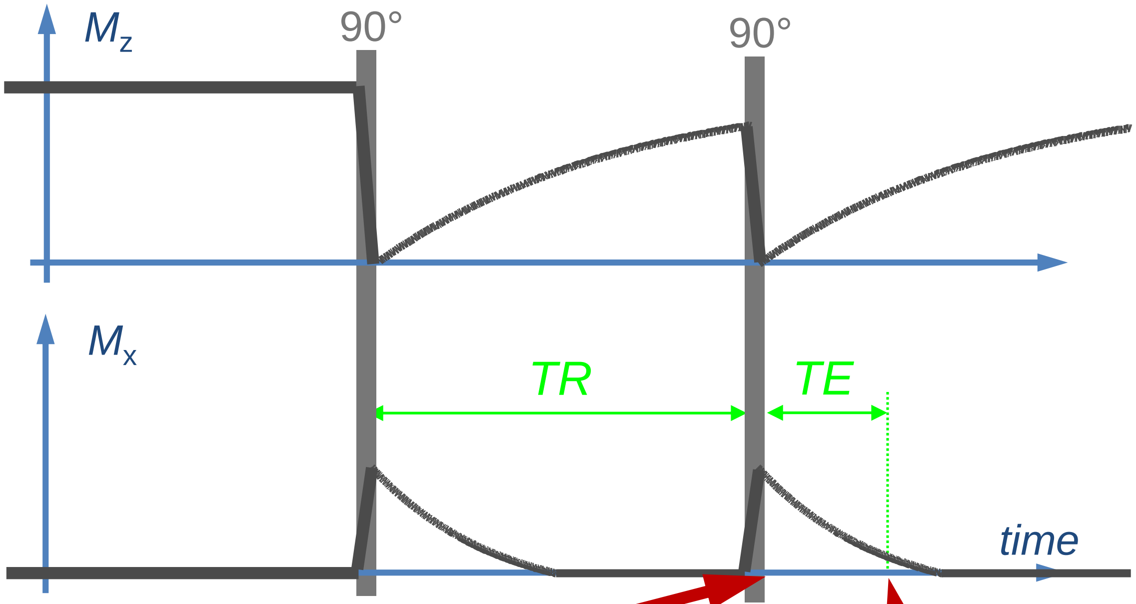
MR-Experiment: $(90°_y\text{-TR})_n$

Step 4



$$\vec{M}(0_+) = \begin{pmatrix} M_0\left(1 - e^{-TR/T_1}\right) \\ 0 \\ 0 \end{pmatrix}$$

$$\vec{M}(TE) = \vec{M}_0 - \left(\vec{M}_0 - \vec{M}(0_+)\right) \cdot R_{relax}(T_1, T_2, TE)$$
$$= \begin{pmatrix} M_0 \cdot \left(1 - e^{-TR/T_1}\right) \cdot e^{-TE/T2} \\ 0 \\ M_0 \cdot \left(1 - e^{-TE/T_1}\right) \end{pmatrix}$$

Spin Gymnastics

Propagation in Time

Vector/Matrix Notation

- iteration from time t_n to t_{n+1}
- note: $t_n \approx t_{n+1}$ is possible
- every interaction described by

$$\vec{M}(t_{n+1}) = \vec{A}_n + \boldsymbol{B_n} \cdot \vec{M}(t_n)$$

Iteration

- $$\begin{aligned}\vec{M}(t_{n+1}) &= \vec{A}_n + \boldsymbol{B_n} \cdot \vec{M}(t_n) \\ &= \vec{A}_n + \boldsymbol{B_n} \cdot \left(\vec{A}_{n-1} + \boldsymbol{B_{n-1}} \cdot \vec{M}(t_{n-1})\right) \\ &= \vec{A}_n + \boldsymbol{B_n} \cdot \vec{A}_{n-1} + \boldsymbol{B_n} \cdot \boldsymbol{B_{n-1}} \cdot \vec{M}(t_{n-1})\end{aligned}$$
- repeated insertion yields

$$\vec{M}(t_{n+1}) = \sum_{i=1}^{n} \left[\prod_{j=i+1}^{n} \boldsymbol{B_j}\right] \vec{A}_i + \left[\prod_{j=1}^{n} \boldsymbol{B_j}\right] \cdot \vec{M}(t_1) \quad = \vec{A} + \boldsymbol{B} \cdot \vec{M}(t_1)$$

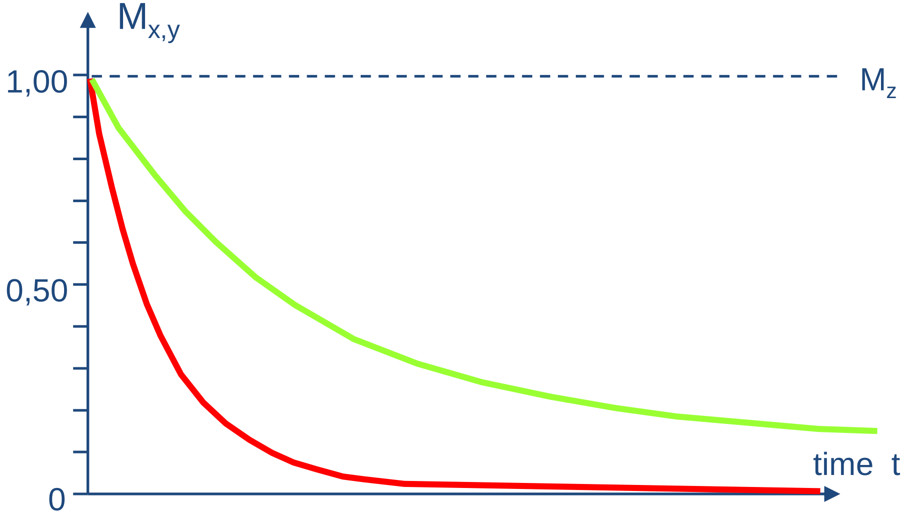
Spin Echo

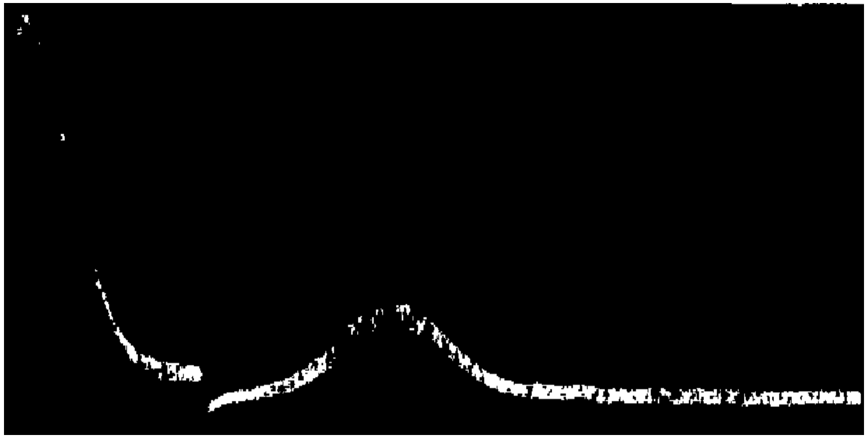
Magnetic Field Inhomogeneities

T2* Relaxation

Problem

- Apparent signal decay faster than T2
- effective relaxation time: T2*
- T2* depends on
  - T2
  - Magnetic surroundings





EL Hahn: *Spin Echoes*. Phys Rev (1950)

Solution

- refocussing

Jürgen Hennig
Spin Echoes
Day 2, 10:00

Spin Echo Formation

A Summary

Initial 90° RF Excitation

- Generates transverse magnetisation
- Coherent superposition of individual magnetisation vectors at $t = 0$

Dephasing

- Loss of phase coherence over time TE/2
- Caused by off-resonances, Brownian motion (T2), field imhomogeneities

180° Refocusing RF Pulse

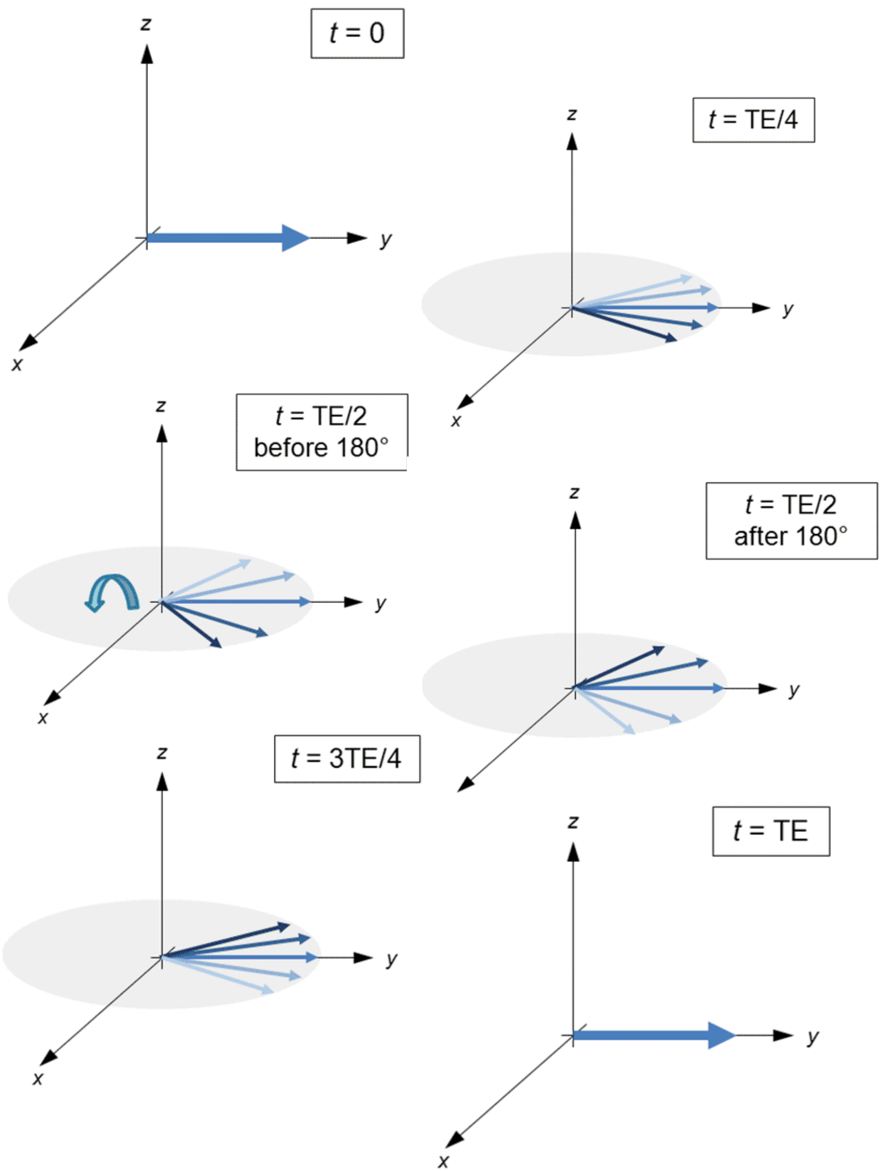
- Change sign of signal phase ($\phi \rightarrow -\phi$) at t = TE/2

Rephasing

- Built-up of phase coherence
- Compensation of reversible phase losses

Echo

- Constructive interference of phases at t = TE



Spin Echo

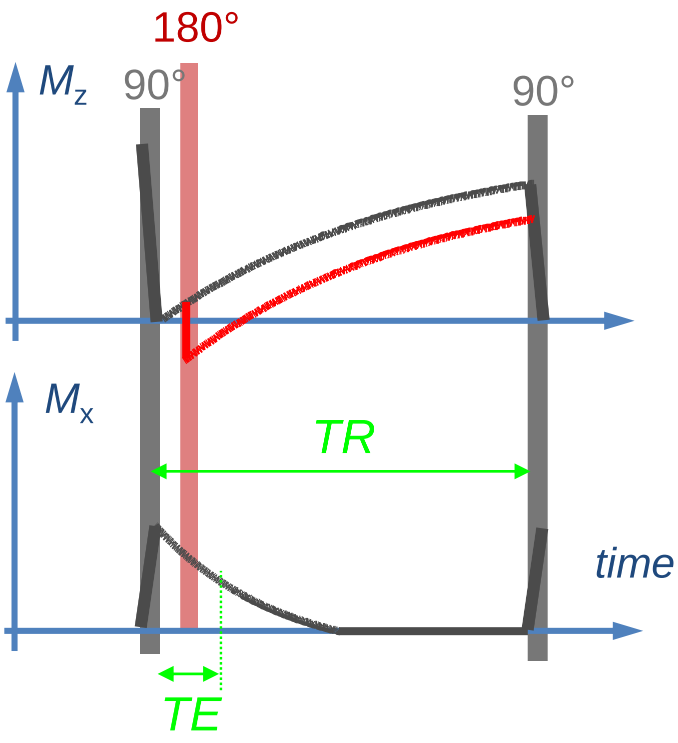
Signal Equation

90°-TE/2-180°-TE/2-ACQ-(TR-TE)

- SE-MRI: series of 90° pulses (and 180°)
- signal proportional to transverse magnetization

$$S \sim M_{xy} = M_0 \cdot \left(1 - e^{-TR/T_1}\right) \cdot e^{-TE/T_2}$$

- additional effect of 180° pulse on magnetization

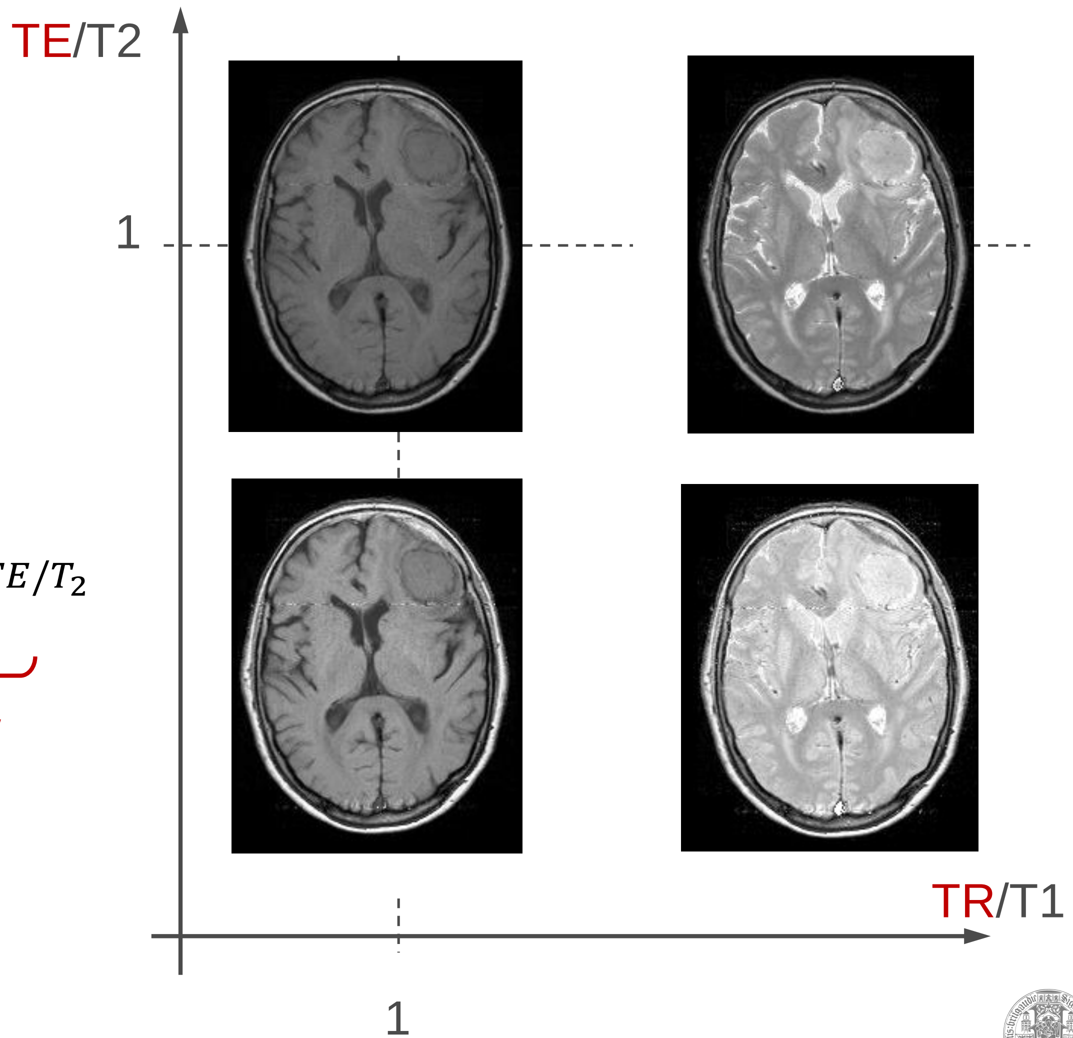


$$S \sim M_{xy} = M_{z,0} \cdot \left(1 + e^{-TR/T_1} - 2e^{-(TR-TE/2)/T_1}\right) \cdot e^{-TE/T_2}$$

T1 and T2 Contrast

How to generate a specific weighting?

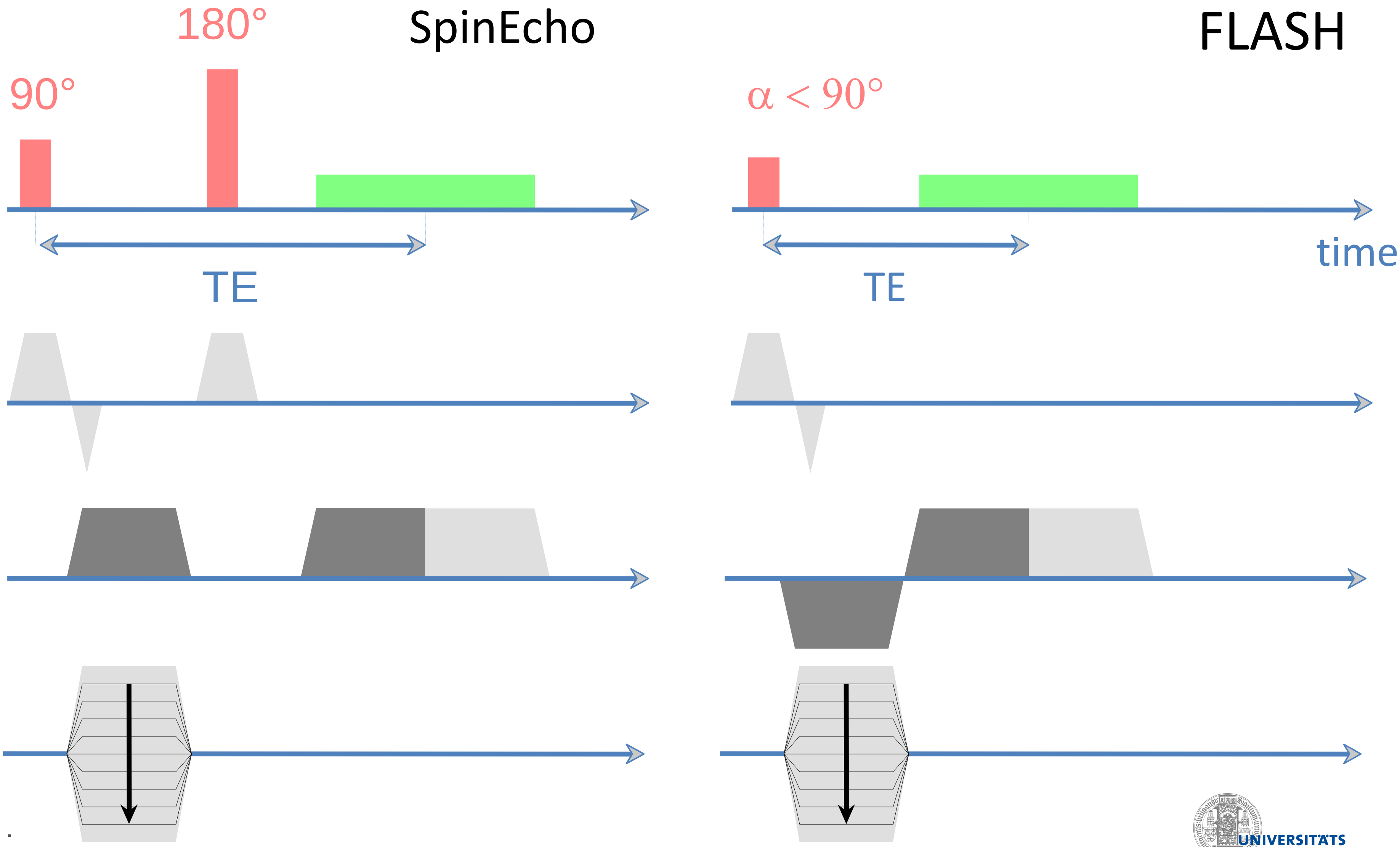
$$\underbrace{M_0}_{\rho} \cdot \underbrace{\left(1 - e^{-TR/T_1}\right)}_{\text{T1-w}} \cdot \underbrace{e^{-TE/T_2}}_{\text{T2-w}}$$



Spoiled Gradient Echo
(Fast Low Angle Shot, FLASH)

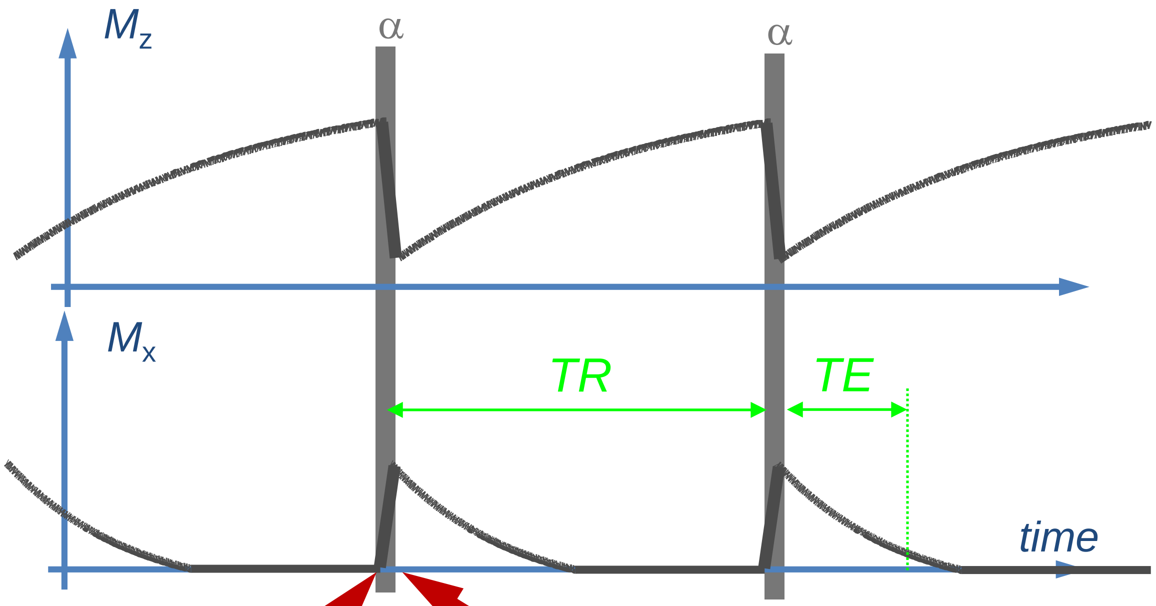
Spin Echo

Long Repetition Times



FLASH Signal Equation

Step 1



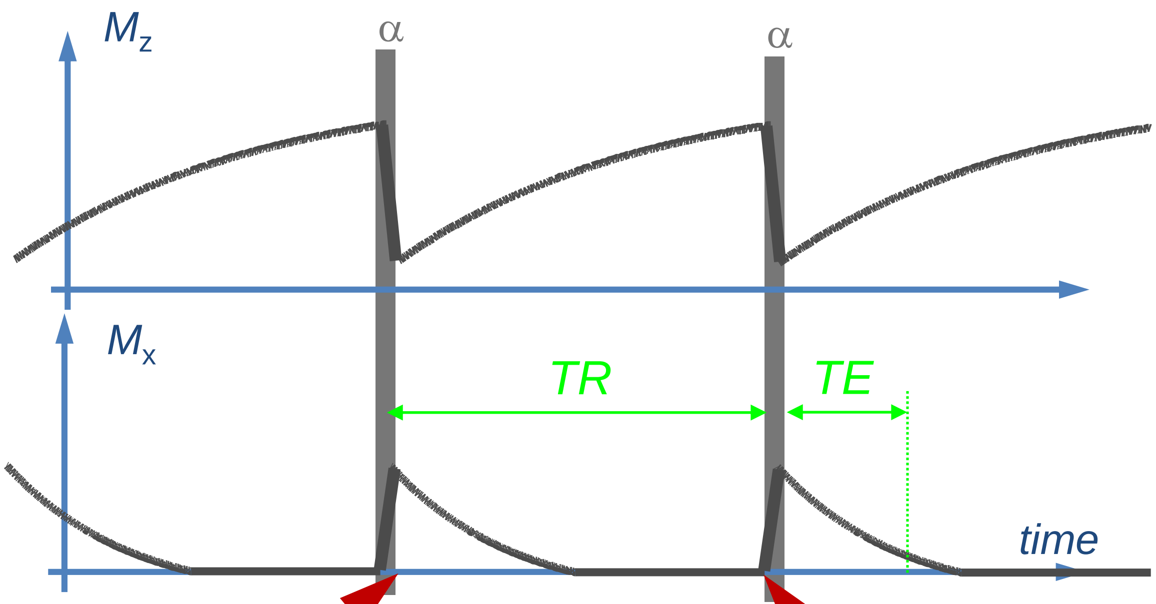
$$\vec{M}(0_-) = \begin{pmatrix} 0 \\ 0 \\ M_{z-} \end{pmatrix}$$

$$\vec{M}(0_+) = \hat{R}_{RF}(\alpha_y)\cdot\vec{M}(0_-) = \begin{pmatrix} \sin\alpha_y \cdot M_{z-} \\ 0 \\ \cos\alpha_y \cdot M_{z-} \end{pmatrix}$$

.

FLASH Signal Equation

Step 2



$$\vec{M}(TR) = \vec{M}_0 - \left(\vec{M}_0 - \vec{M}(0_+)\right) \cdot R_{relax}(T_1, T_2^*, TR)$$

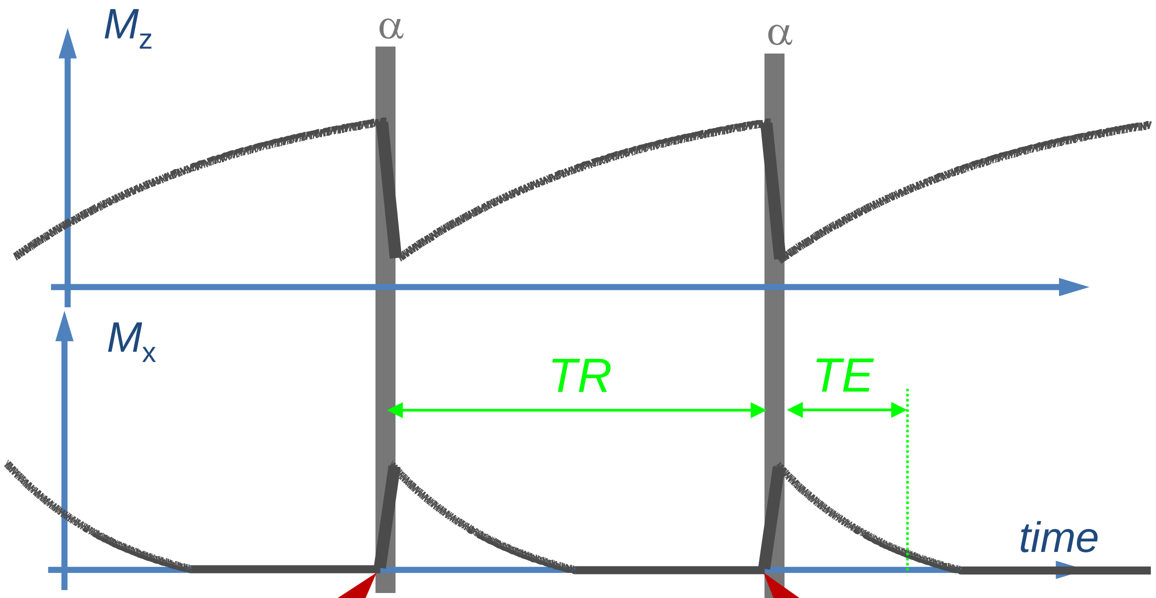
$$= \begin{pmatrix} \sin\alpha_y M_{z-} \cdot e^{-TR/T_2^*} \approx 0 \\ 0 \\ M_0 - \left(M_0 - \cos\alpha_y M_{z-}\right) e^{-TR/T_1} \end{pmatrix}$$

$$\vec{M}(0_+) = \begin{pmatrix} \sin\alpha_y \cdot M_{z-} \\ 0 \\ \cos\alpha_y \cdot M_{z-} \end{pmatrix}$$

.

FLASH Signal Equation

Step 3



$$\vec{M}(TR) = \vec{M}_0 - \left(\vec{M}_0 - \vec{M}(0_+)\right) \cdot R_{relax}(T_1, T_2^*, TR)$$

$$= \begin{pmatrix} 0 \\ 0 \\ M_0 - \left(M_0 - \cos\alpha_y M_{z-}\right) e^{-TR/T_1} \end{pmatrix}$$

$$\vec{M}(0_-) = \begin{pmatrix} 0 \\ 0 \\ M_{z-} \end{pmatrix}$$

.

Dynamic Steady State

Repetitive Signal

Longitudinal Magnetization

- Steady state condition

$$M_{z-} = M_0 - \left(M_0 - \cos\alpha_y \cdot M_{z-}\right)e^{-TR/T_1}$$

- z magnetization

$$M_{z-} = M_0 \frac{1 - e^{-TR/T_1}}{1 - \cos\alpha_y \cdot e^{-TR/T_1}}$$

FLASH Signal

- Steady state condition

$$S \sim M_{z-} \cdot \sin\alpha_y \cdot e^{-TE/T_2^*}$$

$$= M_0 \sin\alpha_y \cdot \frac{1 - e^{-TR/T_1}}{1 - \cos\alpha_y \cdot e^{-TR/T_1}} \cdot e^{-TE/T_2^*}$$

Jochen Leupold
Gradient Echo and
Steady State
Day 2, 9:00

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Ernst Angle

Maximum Signal

Signal Equation

$$S = M_0 \sin\alpha_y \cdot \frac{1-e^{-TR/T_1}}{1-\cos\alpha_y \cdot e^{-TR/T_1}} \cdot e^{-TE/T_2*}$$

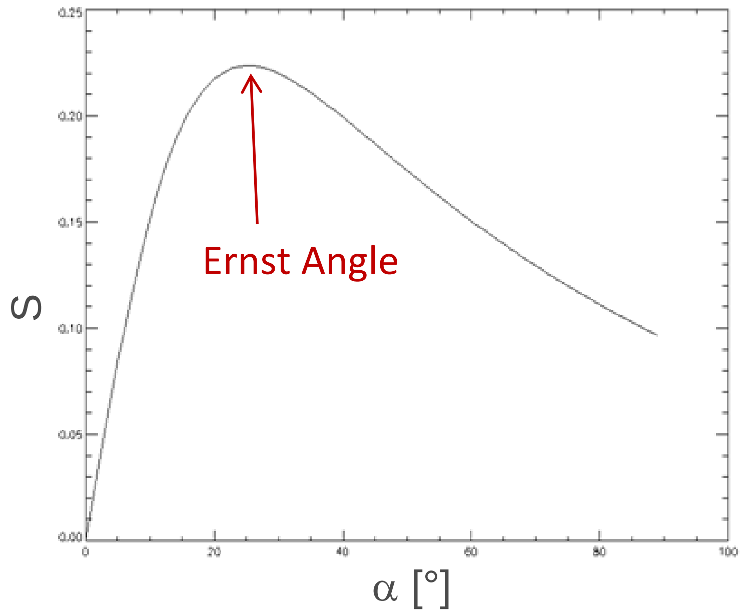
$$= M_0 \cdot \frac{\sin\alpha_y}{1-\cos\alpha_y \cdot e^{-TR/T_1}} \left(1-e^{-TR/T_1}\right) \cdot e^{-TE/T_2*}$$

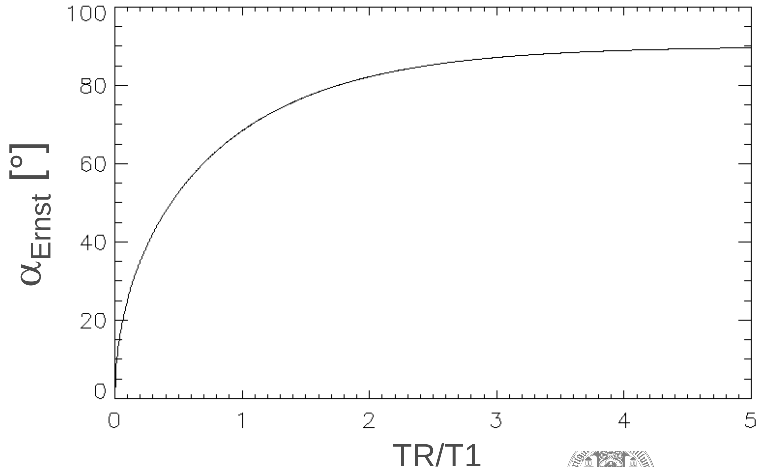
- Optimal flip angle

$$\frac{dS}{d\alpha} = 0$$

- Ernst angle

$$\alpha_{Ernst} = \arccos\left(e^{-TR/T_1}\right)$$





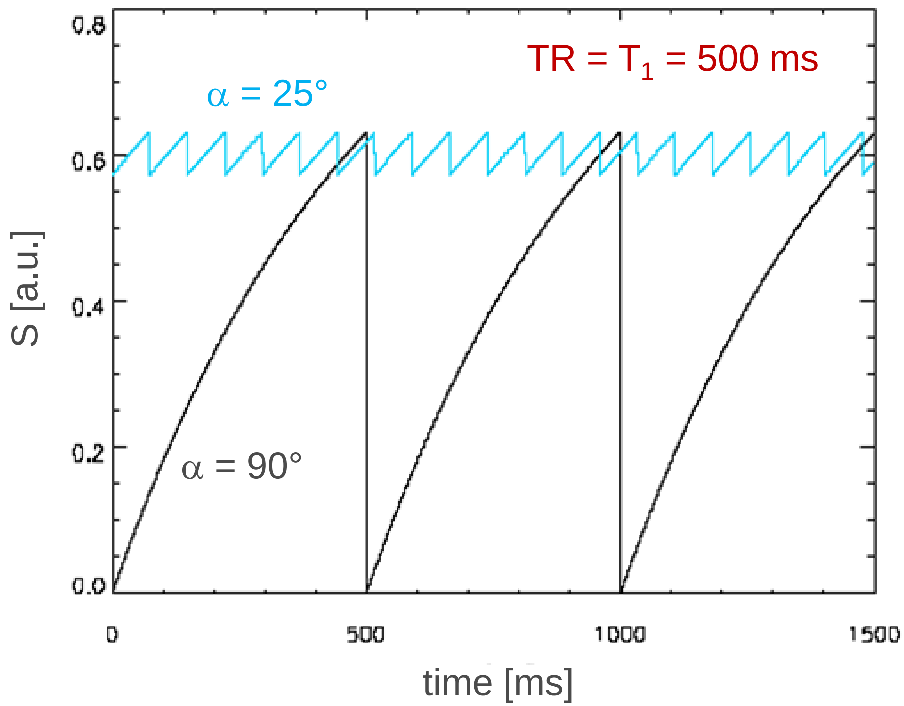
Ernst RR. Review of Scientific Instruments. **37**: 93 (1966).

Fast Low Angle Shot (FLASH)

Advantages and Disadvantages

Advantages

- low flip-angles
- low rf power deposition
- short TR
- short TE
- fast acquisition



Disadvantages

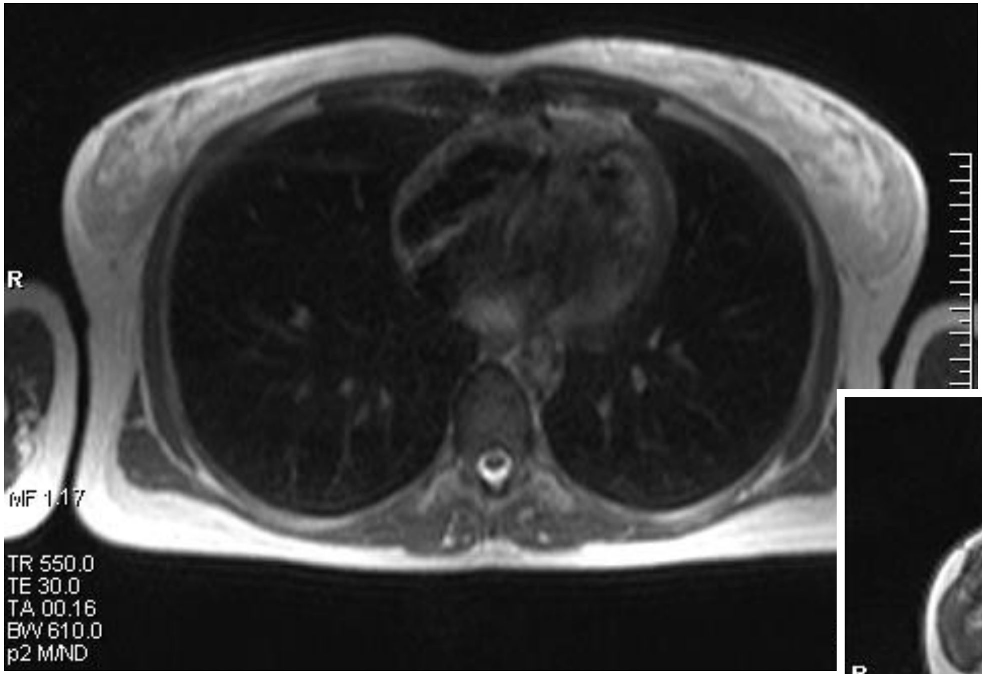
- Signal reduced over SE by a factor

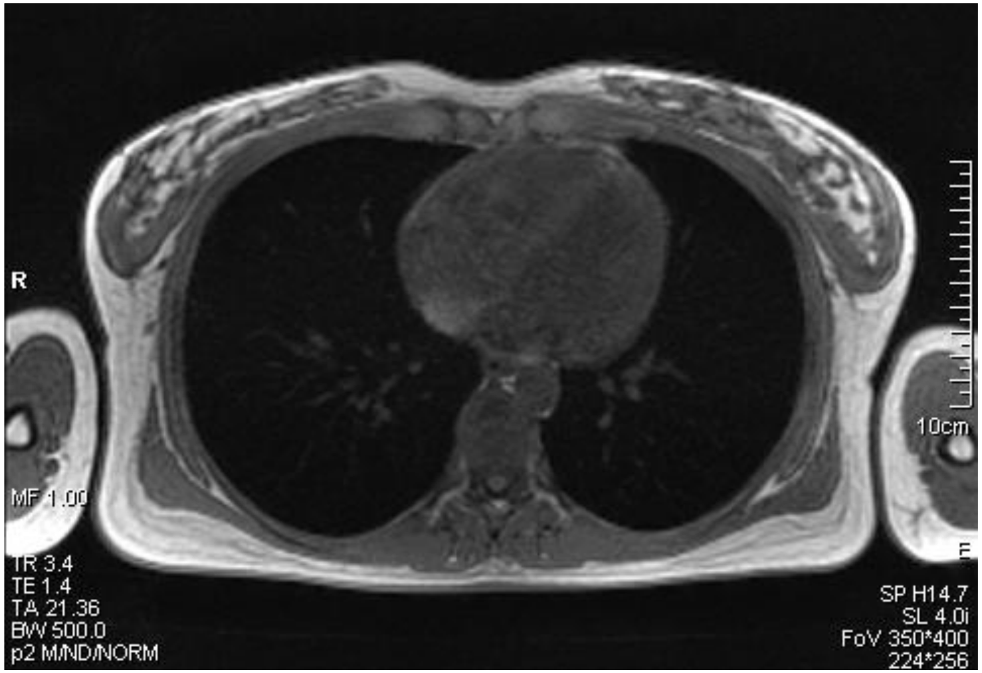
$$\frac{\sin\alpha}{1-\cos\alpha\cdot e^{-TR/T_1}}$$

- Sensitive to susceptibility changes (T2*)

Example: SE vs. FLASH

Lung MRI





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Spatial Encoding

Precession Frequency

Homogeneous Magnetic Field

$$\omega_0 = \gamma \cdot B_0$$

Precession Frequency

Inhomogeneous Magnetic Field: Gradient

$$\omega_0(x) = \gamma(B_0 + x \cdot G_x)$$

Magnetic Field Gradients

Switchable Field Inhomogeneities

Sebastian Littin
MR hardware and
imperfections
Day 1, 14:20

Spatial Localization in MRI

Use of Gradients

Slice Selection

- Excite magnetization only within a (thin) slice

Frequency Encoding

- Acquire MR signal in the presence of a gradient

Phase Encoding

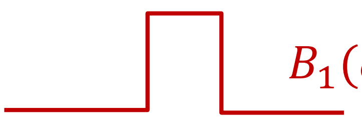
- Prepare phase of transverse magnetization with an encoding gradient

Slice Selection

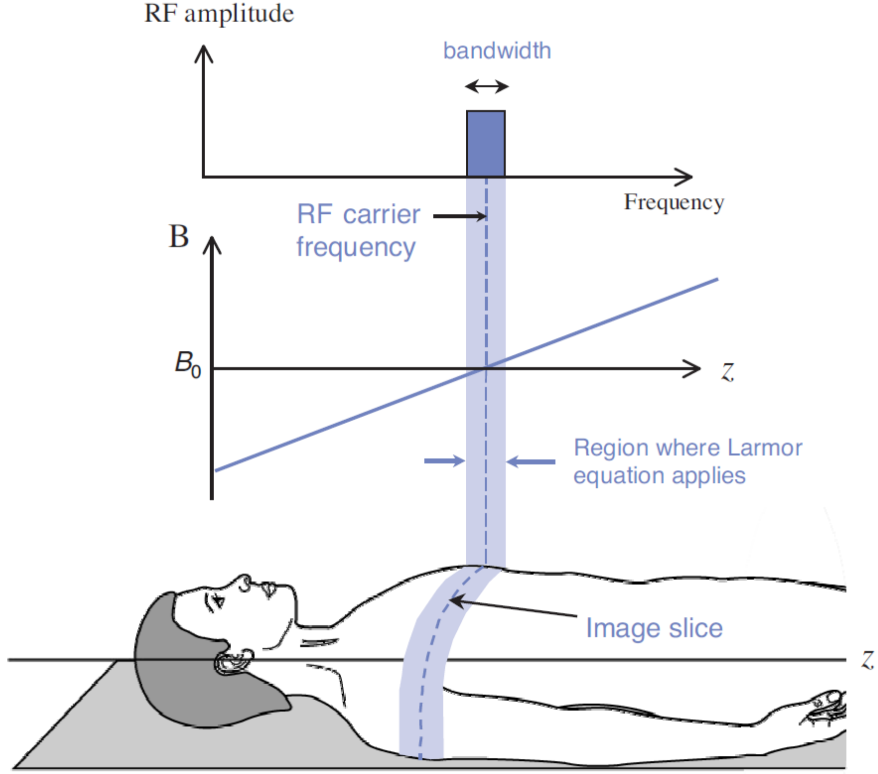
Reduction of 3D to 2D Encoding Problem

Slice-selective Excitation

- simultaneous gradient (G_z) and RF pulse ($B_1(t)$)
- rectangular slice profile
- band-limited RF pulse



$B_1(\omega)$



D. McRobbie: *MRI: From Picture to Proton*

Bloch Equation

Slice-Selective Excitation

$$\vec{B} = \vec{B}_0 + \vec{B}_1(t) + \vec{e}_z \cdot zG_z(t) = \begin{pmatrix} B_1(t) \\ 0 \\ B_0 + zG_z(t) \end{pmatrix}$$

$$\frac{dM_x}{dt} = \gamma\left(\vec{M} \times (\frac{\vec{\omega}}{\gamma} - \vec{B})\right)_x = \gamma zG_z(t)\, M_y$$

$$\frac{dM_y}{dt} = \gamma\left(\vec{M} \times (\frac{\vec{\omega}}{\gamma} - \vec{B})\right)_y = -\gamma zG_z(t)\, M_x + \gamma B_1(t)\, M_z$$

$$\frac{dM_z}{dt} = \gamma\left(\vec{M} \times (\frac{\vec{\omega}}{\gamma} - \vec{B})\right)_z = -\gamma B_1(t)\, M_y$$

$$M_\perp = M_x + iM_y \quad : \quad \frac{dM_\perp}{dt} = -i\gamma zG_z(t) M_\perp + i\gamma B_1(t) M_z$$

Small Tip Angle Approximation

Reduction of 3D to 2D Encoding Problem

$$\frac{dM_\perp}{dt} = -i\gamma z G_z(t) M_\perp + i\gamma B_1(t) M_z$$
$$= \quad -i\Delta\omega(z) M_\perp + i\gamma B_1(t) M_z$$

With $M_\perp(t=0) = 0$ we get:

$$M_\perp(t,z) = i e^{-\Delta\omega(z)t} \int_{t'=0}^{t} \gamma B_1(t') M_z(t')\, e^{i\Delta\omega(z)t'} dt'$$

$$\approx i e^{-i\Delta\omega(z)t} M_0 \int_{t'=0}^{t} \gamma B_1(t')\, e^{i\Delta\omega(z)t'} dt'$$

Small Tip Angle Approximation

Reduction of 3D to 2D Encoding Problem

Toni Stöcker
RF Pulses
Day 1, 13:20

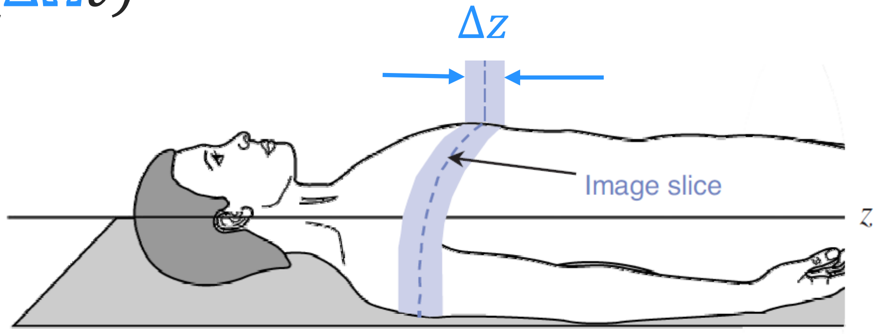
$$M_\perp(t,z) \approx i e^{-i\Delta\omega(z)t} M_0 \int_{t'=0}^{t} \gamma B_1(t')\, e^{i\Delta\omega(z)t'} dt'$$

$$M_\perp(t_{pulse}, z) = FT\left(B_1(t)\right) \rightarrow B_1(t) = FT^{-1}\left(M_\perp(t_{pulse}, z)\right)$$

$$B_1(t) = B_1 \cdot \frac{\sin(\Delta\Omega t)}{\Delta\Omega t} \equiv B_1 \cdot \mathrm{sinc}(\Delta\Omega t)$$

with

$$\Delta\Omega = \gamma \Delta z G_z$$



D. McRobbie: *MRI: From Picture to Proton*

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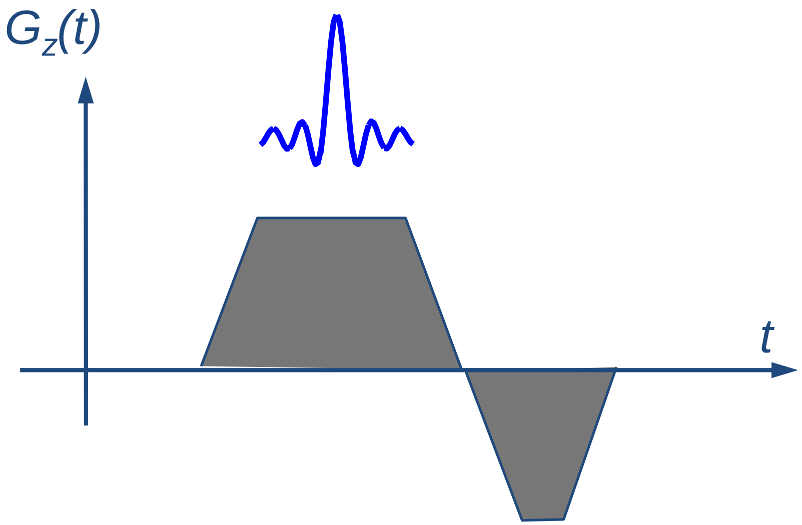
UNIVERSITÄTS KLINIKUM FREIBURG

Slice-selective Excitation

Reduction of 3D to 2D Encoding Problem



$$B_1(t) = B_1 \cdot \frac{\sin(\gamma \Delta z G_z t)}{\gamma \Delta z G_z t}$$

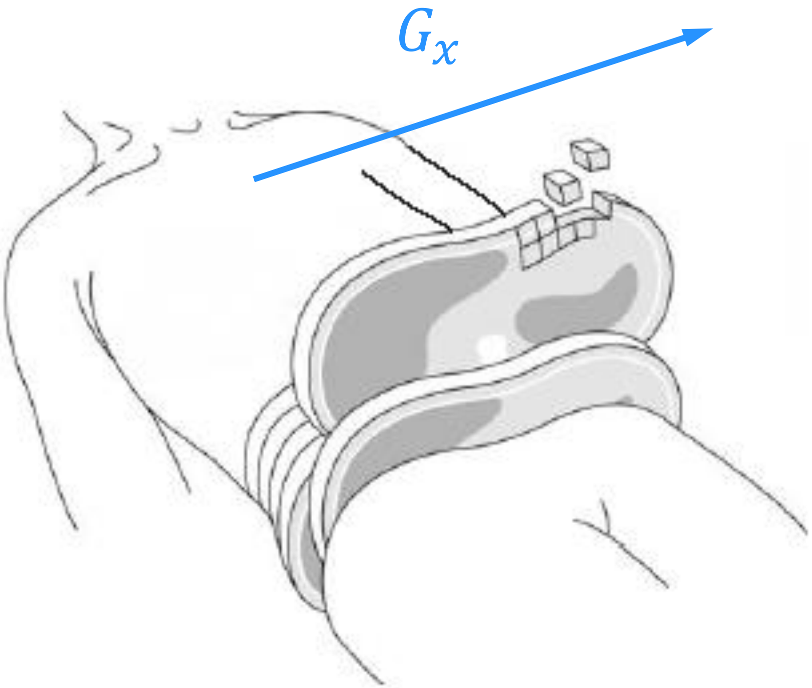
$$M_\perp(t_{pulse}, z) \approx i e^{-i\gamma \Delta z G_z t_{pulse}/2} M_0 \int_{t'=0}^{t_{pulse}} \gamma B_1(t')\, e^{i\gamma \Delta z G_z t'} dt'$$

Frequency Encoding

Encoding of One In-Plane Axis

Precession Frequency

- $$\omega(\vec{x}) = \gamma\left(B_0 + \vec{x} \cdot \vec{G}\right)$$
$$= \omega_0 + \gamma x \cdot G_x$$



Signal

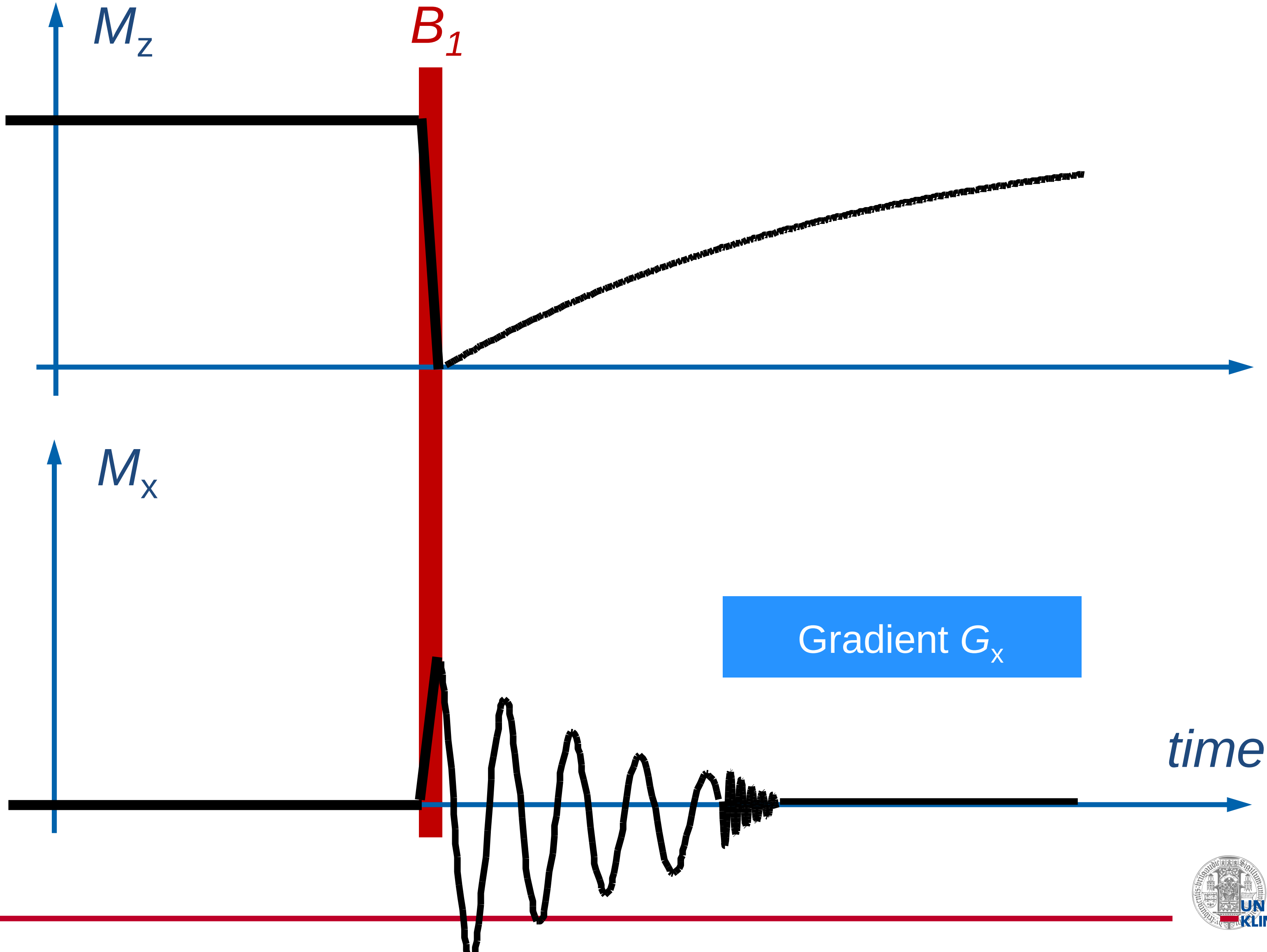
- $$S(t) = \int M_\perp(x,y,z) \cdot e^{i\omega(x)t} dx dy$$
$$= \int P_y(x) \cdot e^{i\gamma x \cdot G_x t} dx$$
$$= \int P_y(x) \cdot e^{i k_x \cdot x} dx$$

Projection

$$P_y(x) = \int M_\perp(x,y,z) dy$$

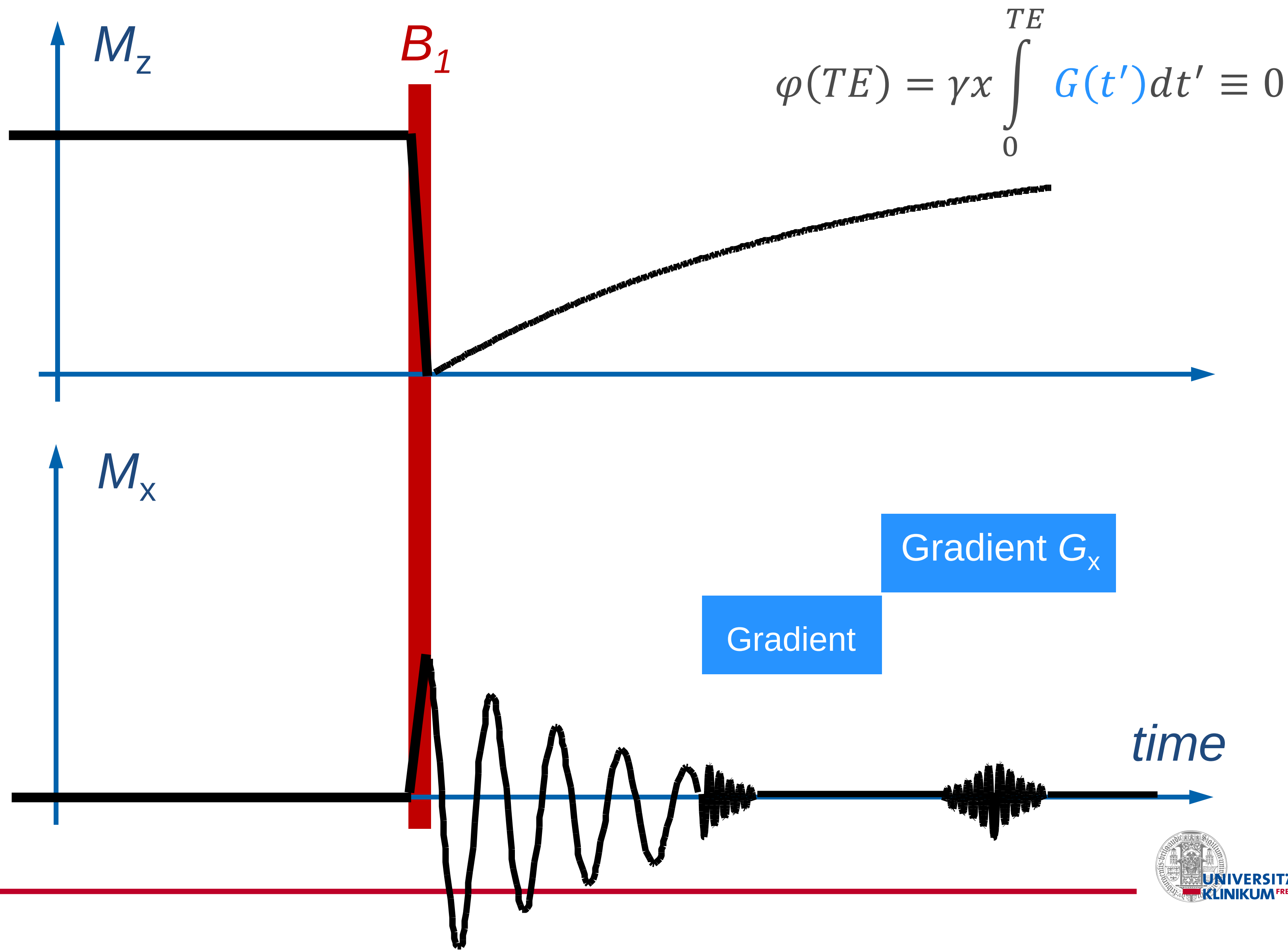
A Simple MR Experiment

Signal Dephasing in a Gradient



A Simple MR Experiment

Gradient Echo

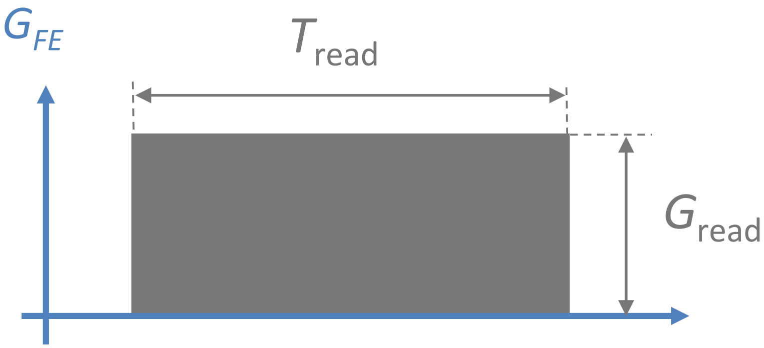


Data Sampling

Nyquist Theorem

Frequency Encoding

- Readout time: T_{read}
- Bandwidth BW = T_{read}^{-1}
- Readout gradient amplitude: G_{read}



Discrete data sampling

- Dwell time: τ
- Sampling points N_{read}

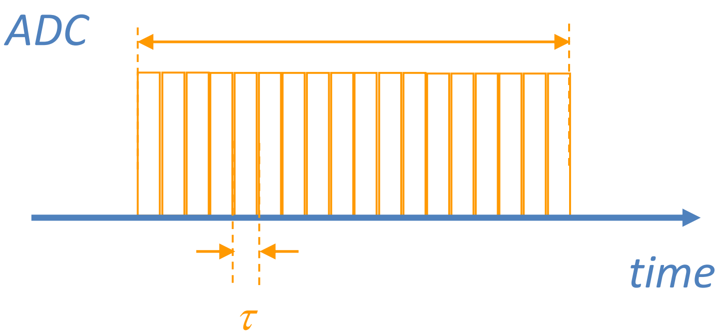
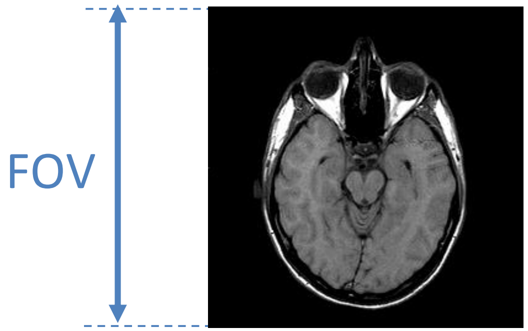


Image Dimensions

- Field of view: FOV
- Spatial resolution: Δx



Data Sampling

Nyquist Theorem

Relations

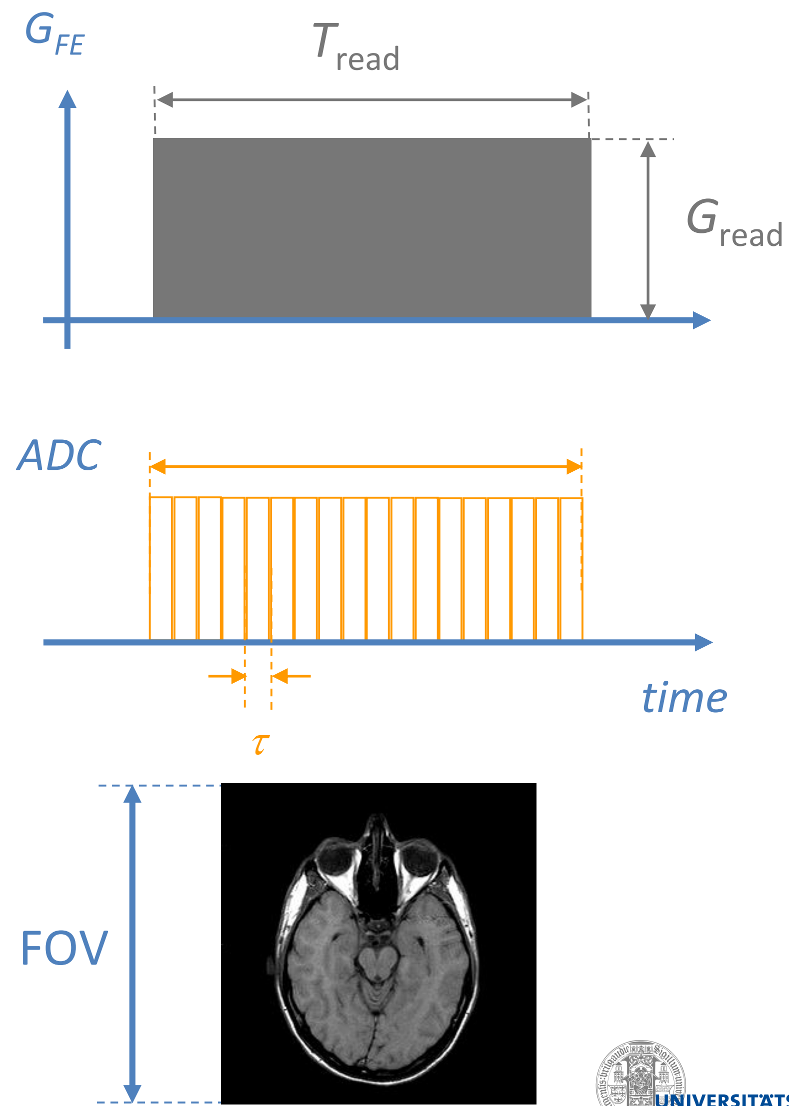
$$T_{read} = \frac{1}{BW} = N_{read} \cdot \tau$$
$$FOV = N_{read} \cdot \Delta x$$

Nyquist Theorem

- 2 frequencies can be distingui-shed, if their phase difference over the data acquisition period is 2π

$$\begin{aligned}\Delta\phi &= \Delta\omega \cdot T_{read} = \gamma \cdot \Delta B \cdot T_{read} \\ &= \gamma \cdot \Delta x \cdot G_{read} \cdot T_{read} \\ &\equiv 2\pi\end{aligned}$$

$$G_{read} = \frac{2\pi}{\gamma \cdot \Delta x \cdot T_{read}} = \frac{2\pi \cdot N_{read} \cdot BW}{\gamma \cdot FOV}$$



Readout Gradient

An Example

Parameters

- FOV = 500 mm
- Matrix: 256
- Δx = FOV/Matrix = 1,95 mm
- T_{read} = 5,12 ms
- BW = 195 Hz/px
- $\gamma = 2\pi \cdot$ 42,577 MHz/T

$$G_{read} = \frac{2\pi}{\gamma \cdot \Delta x \cdot T_{read}} = 2{,}35\ \mathrm{mT/m}$$

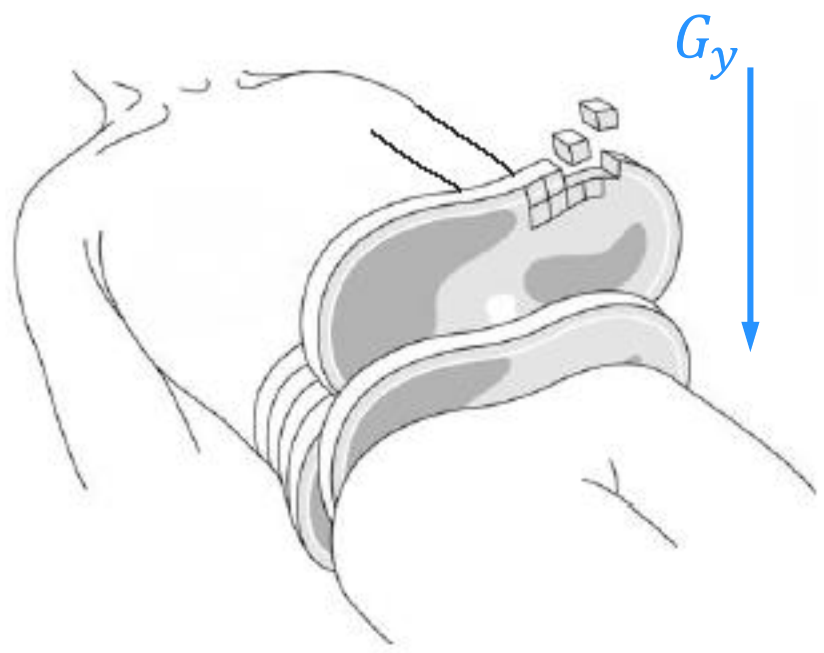
Phase Encoding
Gradient Table

Phase-modulated Projections

$$S(k_x, k_y) = \int M_\perp(x, y, z)\ e^{ik_y \cdot y} \cdot e^{ik_x \cdot x} dxdy$$

$$k_y = \gamma \cdot G_y \cdot T$$

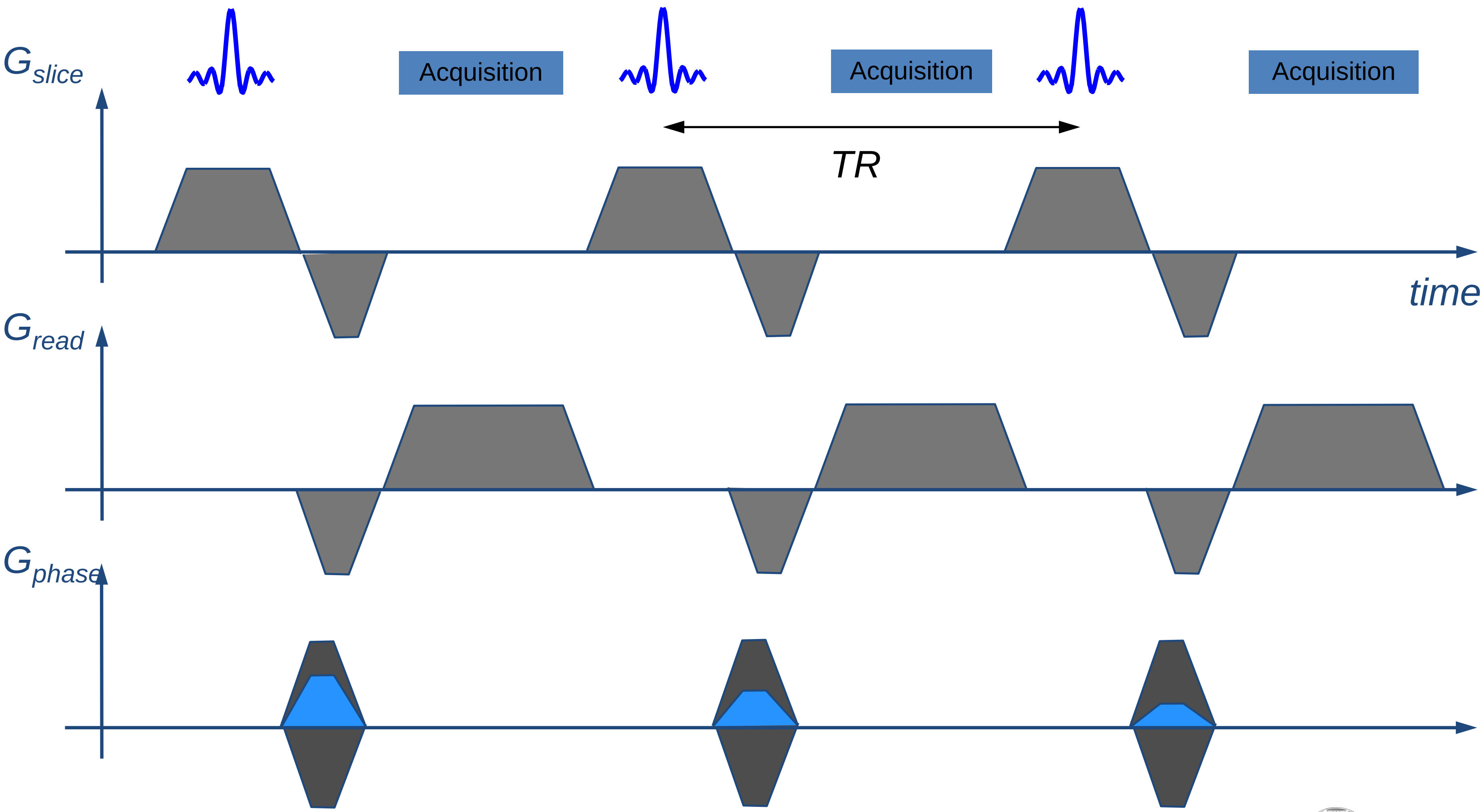
- Cartesian sampling:
  gradient table $G_y = n \cdot \Delta G_y$
- phase encoding between slice selection and readout



Frank Zijlstra
K-Spase sampling schemes
Day 1, 13:50

Gradient Echo Pulse Sequence

Timing of Gradients, RF-Pulses and Acquisitions



k-Space:
The Final Frontier

k-Space

Fourier Transform of the MR Signal

Properties

- Non-locality
- Point symmetry
- Band limited
- Asymmetric acquisition

Measurement Time

- TA = TR x # k-Lines
- TR = 600 ms
- 256 Lines
- TA = 2 min 34 s

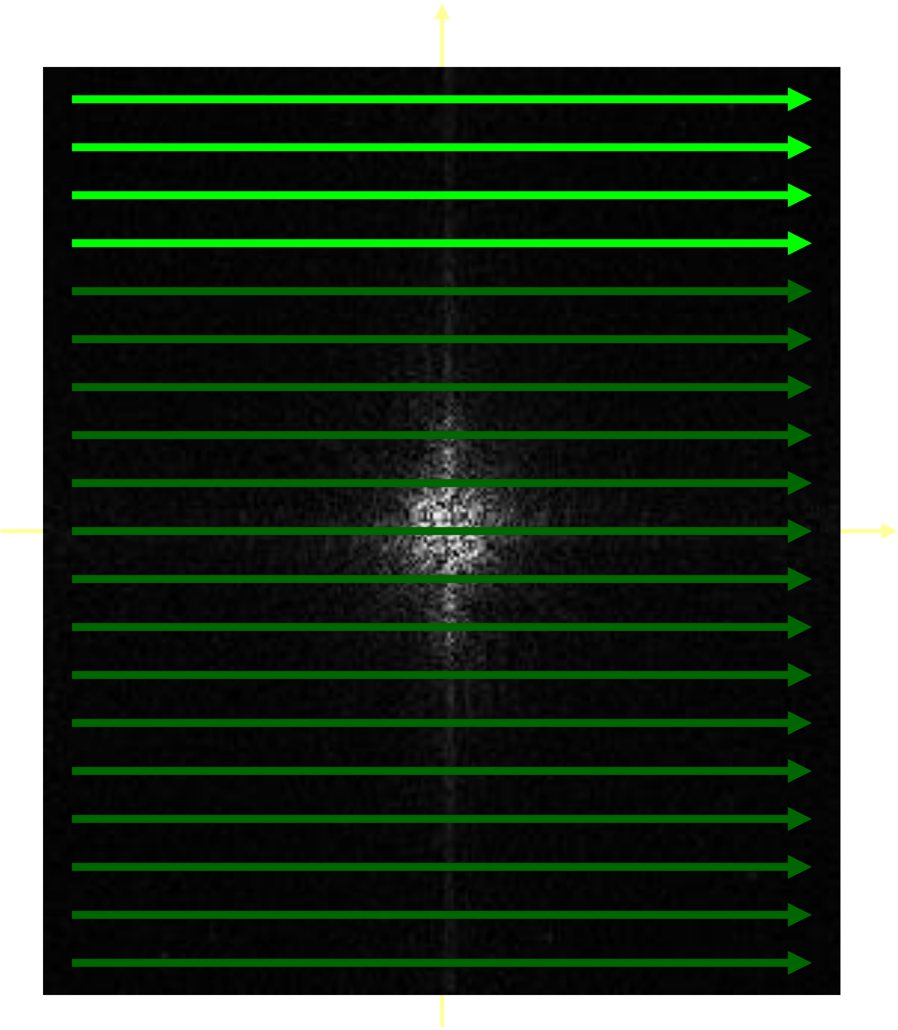


Image and *k*-Space

Reciprocity

Fourier

Transform

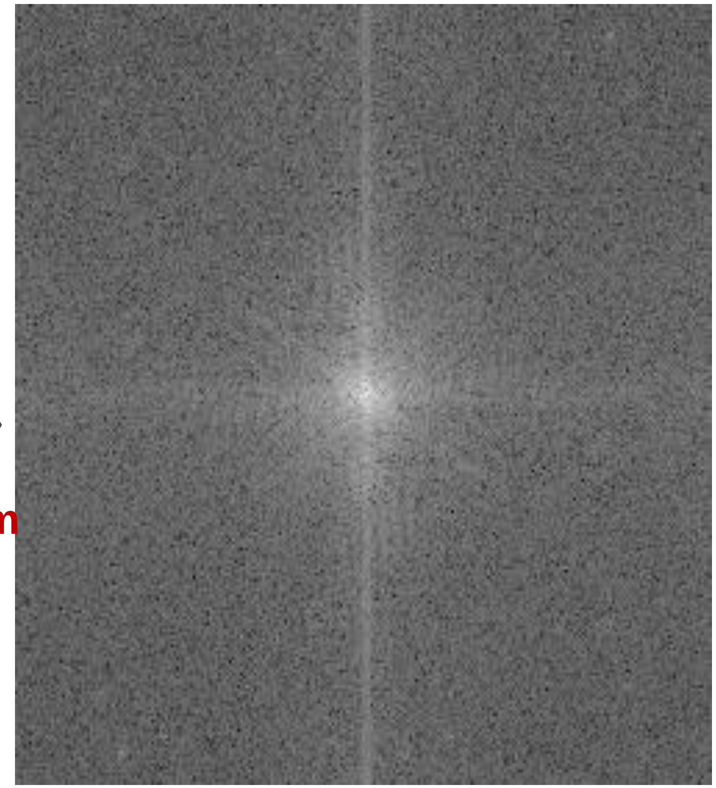


Image and *k*-Space

Low Frequency Components

Fourier

Transform

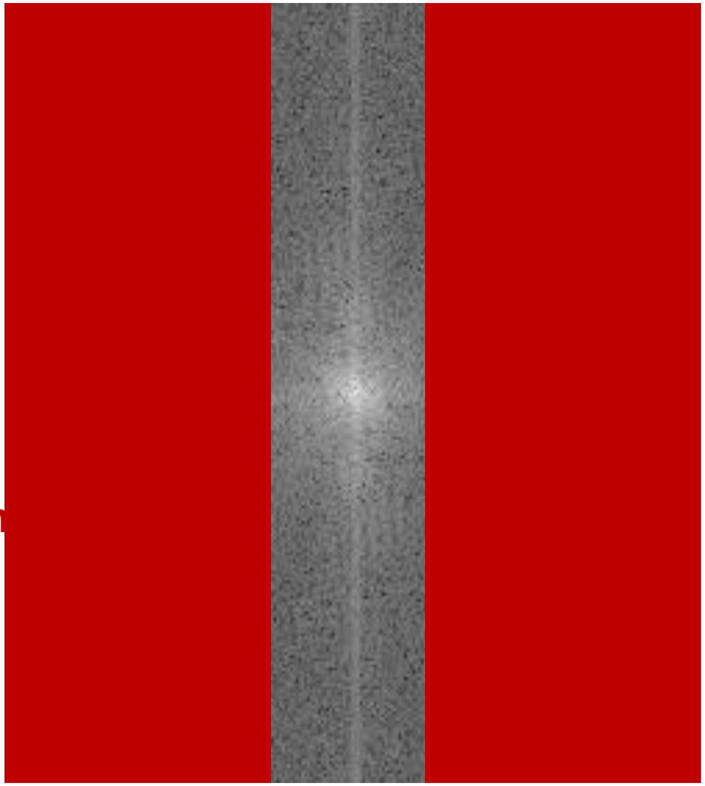


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Image and *k*-Space

High Frequency Components

Fourier

Transform

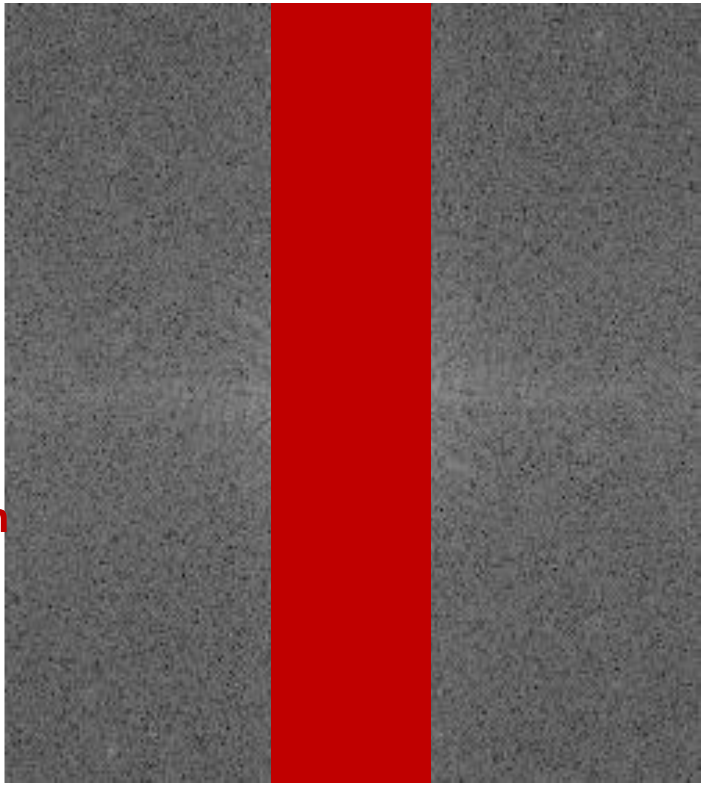
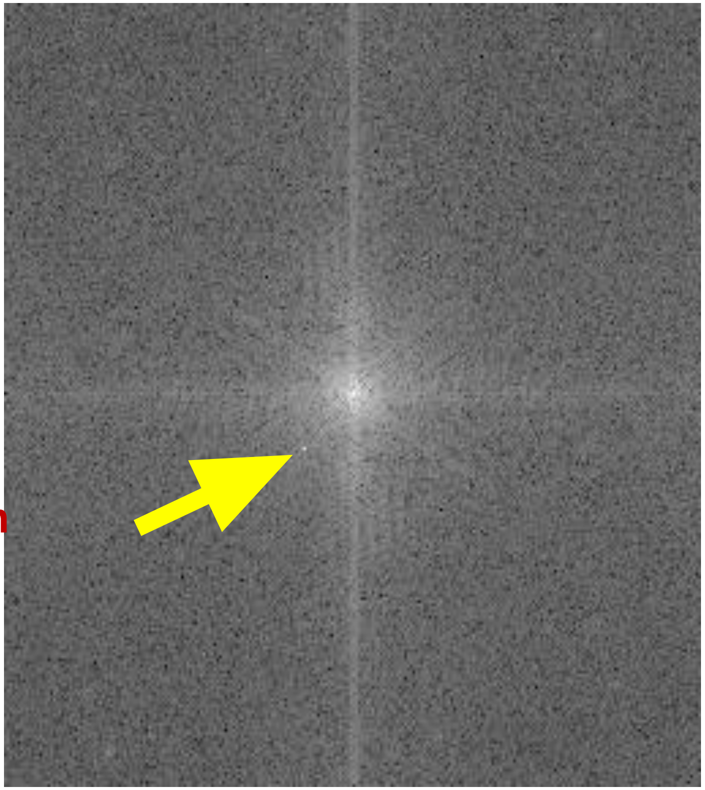


Image and *k*-Space

Non-Locality

Fourier

Transform



Thank you for your attention!