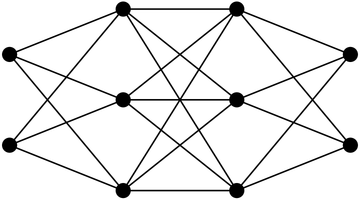
Distributivity, size, homomorphism

Simon Burton

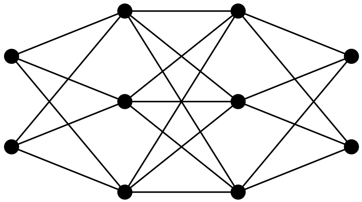
School of Physics, The University of Sydney

Path counting



$$|\text{paths}| = 2.3.3.2 = 36$$

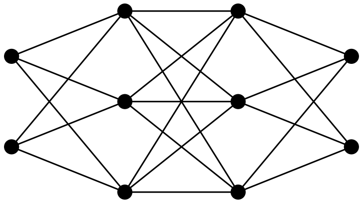
Path counting



$$\begin{pmatrix}1 & 1 & 1\\1 & 1 & 1\end{pmatrix}\begin{pmatrix}1 & 1 & 1\\1 & 1 & 1\\1 & 1 & 1\end{pmatrix}\begin{pmatrix}1 & 1\\1 & 1\\1 & 1\end{pmatrix} = \begin{pmatrix}9 & 9\\9 & 9\end{pmatrix}$$

$$\sum_{jk} A_{ij} B_{jk} C_{kl} = \sum_{j} A_{ij} \sum_{k} B_{jk} C_{kl}$$

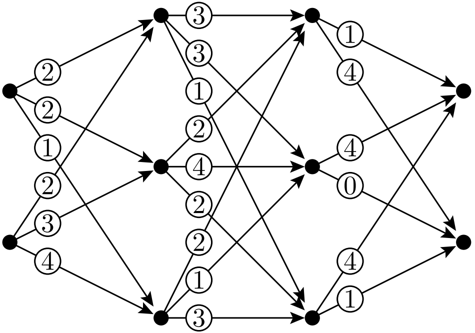
Path counting



Distributivity:

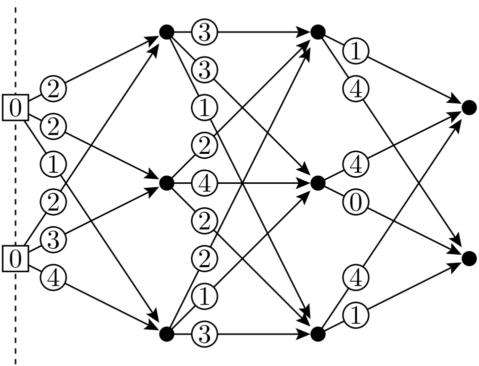
$$a(b+c) = ab + ac$$

Minimum weight path

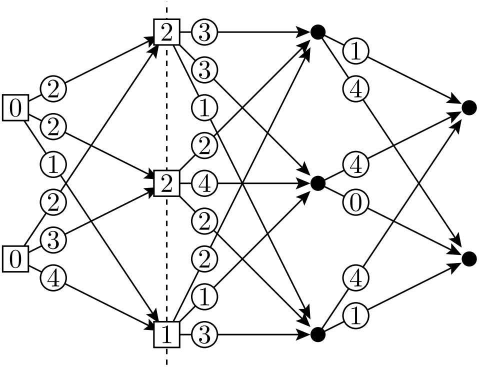


$$\min_{ijk}(A_{ij} + B_{jk} + C_{kl})$$

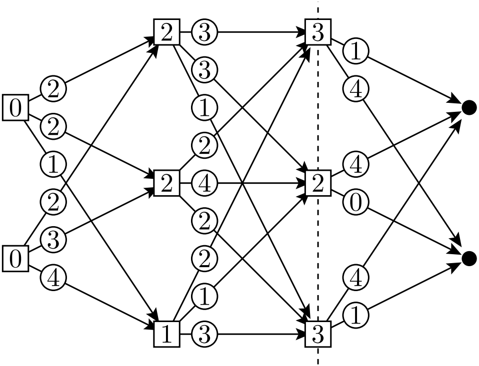
Minimum weight path



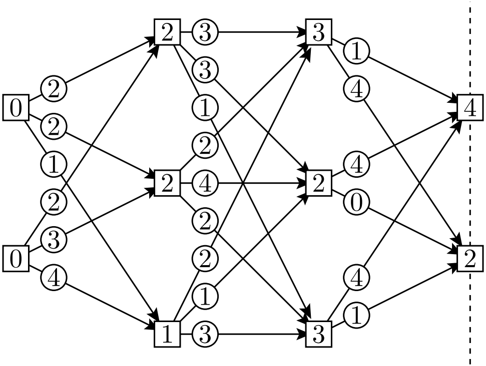
Minimum weight path



Minimum weight path

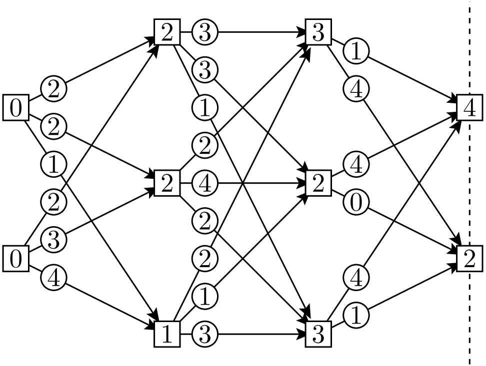


Minimum weight path



$$\min_{ijk}(A_{ij} + B_{jk} + C_{kl}) = \min_{k}(\min_{j}((\min_{i} A_{ij}) + B_{jk}) + C_{kl})$$

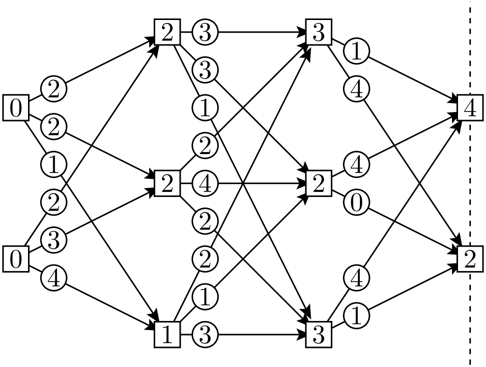
Minimum weight path



Distributivity:

$$a + \min(b, c) = \min(a + b, a + c)$$

Minimum weight path



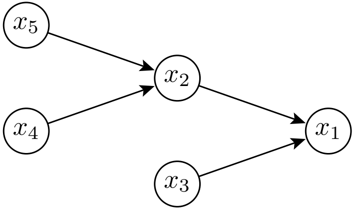
... Bellman equation
... Hamilton-Jacobi equation

Belief propagation

Joint probabilities factor as:

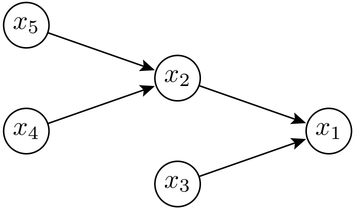
$$p(x_1, x_2, x_3, x_4, x_5) = p(x_1|x_2, x_3, x_4, x_5)p(x_2|x_3, x_4, x_5)$$
$$p(x_3|x_4, x_5)p(x_4|x_5)p(x_5)$$

Belief propagation



$$
\begin{aligned}
p(x_1|x_2, x_3, x_4, x_5) &= p(x_1|x_2, x_3) \\
p(x_2|x_3, x_4, x_5) &= p(x_2|x_4, x_5) \\
p(x_3|x_4, x_5) &= p(x_3) \\
p(x_4|x_5) &= p(x_4)
\end{aligned}
$$

Belief propagation



$$
\begin{aligned}
p(x_1) &= \sum_{x_1 \ldots x_5} p(x_1, x_2, x_3, x_4, x_5) \\
&= \sum_{x_1 \ldots x_5} p(x_1|x_2, x_3)p(x_2|x_4, x_5)p(x_3)p(x_4)p(x_5) \\
&= \sum_{x_1 x_2 x_3} p(x_1|x_2, x_3)p(x_3) \sum_{x_4 x_5} p(x_2|x_4, x_5)p(x_4)p(x_5)
\end{aligned}
$$

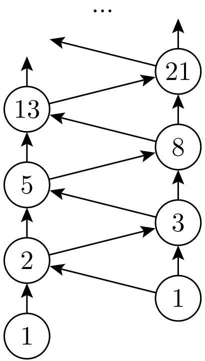
Linear recurrence relations

For $n = 1, 2, \ldots$:

$$a_{n+2} = a_{n+1} + a_n \text{ with } a_1 = 1, a_2 = 1.$$

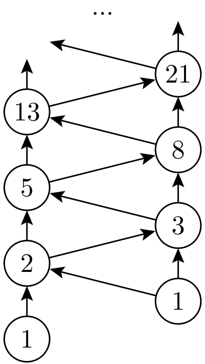
```
def fibonacci(n):
    if n==1 or n==2:
        return 1
    return fibonacci(n-1) + fibonacci(n-2)
```

Linear recurrence relations



$$a_{n+2} = a_{n+1} + a_n \text{ with } a_1 = 1, a_2 = 1.$$

Linear recurrence relations



$$Sn := n + 1$$

$$f(SSn) = f(Sn) + f(n).$$

Perturbation theory

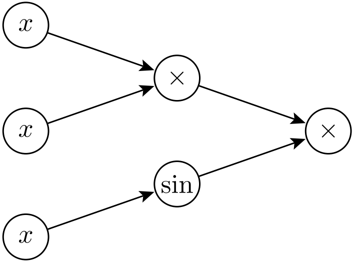
$$f(x) = x^2 \sin(x)$$
$$f'(x) = 2x \cos(x) + x^2 \cos(x)$$

Compute to first order:

$$f(x+\varepsilon) = (x+\varepsilon)^2 \sin(x+\varepsilon)$$

with $\varepsilon^2 = 0$.

Automatic differentiation



Chain rule:

$$D(f \circ g)(x) = Df(g(x)) \cdot Dg(x)$$

... neural networks

Renormalization

Classical spin chain: $H = \sum_{i=1}^{N} \sigma_i \sigma_{i+1}$

$$Z = \sum_{\sigma_1 \ldots \sigma_N} \exp\Big(\sum_{i=1}^{N} \sigma_i \sigma_{i+1}\Big) = \sum_{\sigma_1 \ldots \sigma_N} \prod_{i=1}^{N} \exp\big(\sigma_i \sigma_{i+1}\big)$$

$$= \sum_{\sigma_1 \sigma_3 \ldots} \prod_{i=2,4\ldots} \Big(\sum_{\sigma_i} \exp\big(\sigma_i \sigma_{i+1}\big)\Big)$$

Sum over paths

$$\frac{d}{dt}\psi(t) = Hf(t)$$

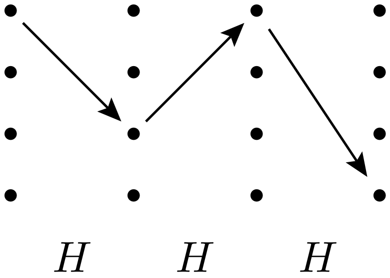
has solution:

$$\begin{aligned}\psi(t) &= \exp(tH) \\ &= \sum_n \frac{1}{n!} t^n H^n\end{aligned}$$

Sum over paths

$$\exp(H) = 1 + H + \frac{1}{2}H^2 + ... + \frac{1}{n!}H^n + ...$$

$$\frac{1}{3!}H^3 = \frac{1}{3!}\sum$$



Quantum

prepare   measure

$$
\begin{tikzcd}
 & |0\rangle \arrow[r] & p|\langle u|0\rangle|^2 \\
{} \arrow[ur, "p"] \arrow[dr, "1-p"'] & & \\
 & |1\rangle \arrow[r] & (1-p)|\langle u|1\rangle|^2
\end{tikzcd}
$$

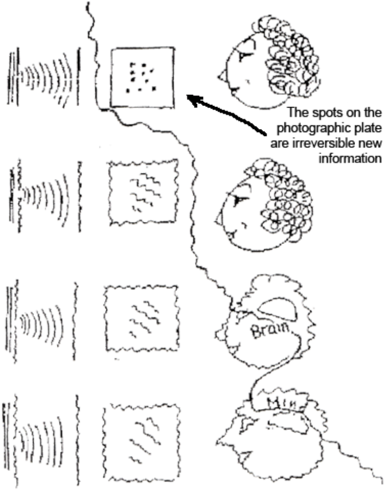
Quantum

prepare   measure

$$\begin{array}{lcl} \xrightarrow{\;p\;} |0\rangle & \longrightarrow & p|\langle u|0\rangle|^2 \\[2em] \xrightarrow{\;1-p\;} |1\rangle & \longrightarrow & (1-p)|\langle u|1\rangle|^2 \end{array}$$

$$\begin{aligned} &p|\langle u|0\rangle|^2 + (1-p)|\langle u|1\rangle|^2 \\ =&\langle u|\Big(p|0\rangle\langle 0| + (1-p)|1\rangle\langle 1|\Big)|u\rangle \end{aligned}$$

Quantum


The spots on the photographic plate are irreversible new information
Brain

Size

$$\mu(A \cup B) = \mu(A) + \mu(B)$$

Size

$$\mu(A \cup B) = \mu(A) + \mu(B) - \mu(A \cap B)$$

Size

$$\mu(A \cup B) = \mu(A) + \mu(B) - \mu(A \cap B)$$
$$\chi(X \cup Y) = \chi(X) + \chi(Y) - \chi(X \cap Y)$$
$$H(p, q) = H(p) + H(q) - I(p : q)$$

Size

$$\begin{aligned}\mu(A \cup B \cup C) = \mu(A) + \mu(B) + \mu(C) \\ - \mu(A \cap B) - \mu(A \cap C) - \mu(B \cap C) \\ + \mu(A \cap B \cap C)\end{aligned}$$

Convexity

Consider a convex combination

$$\alpha a + (1-\alpha)b$$

for $a, b \in \mathbb{R}^n$. We have:

$$x + \big(\alpha a + (1-\alpha)b\big) = \alpha(x+a) + (1-\alpha)(x+b)$$

Convexity

Define:

$$a +_\alpha b := \alpha a + (1 - \alpha) b$$

then

$$x + (a +_\alpha b) = (x + a) +_\alpha (x + b).$$

Convexity

Define:

$$a +_\alpha b := \alpha a + (1-\alpha) b$$

then

$$x + (a +_\alpha b) = (x + a) +_\alpha (x + b).$$

And more:

$$x +_\alpha (a +_\beta b) = (x +_\alpha a) +_\beta (x +_\alpha b).$$

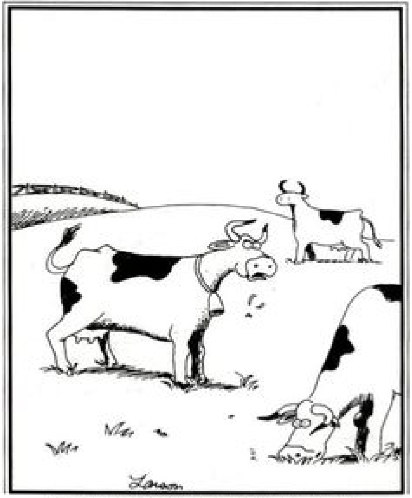
Conjugation

Given a group G with elements g, h, k define

$$g \triangleright h := g^{-1} h g.$$

Then we have

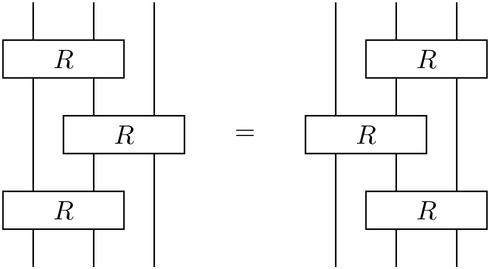
$$g \triangleright (h \triangleright k) = (g \triangleright h) \triangleright (g \triangleright k).$$



"Hey, wait a minute! This is grass! We've been eating grass!"

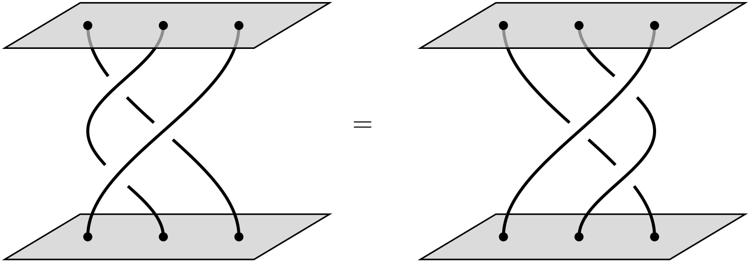
Yang-Baxter equation

$$(R \otimes I)(I \otimes R)(R \otimes I) = (I \otimes R)(R \otimes I)(I \otimes R)$$

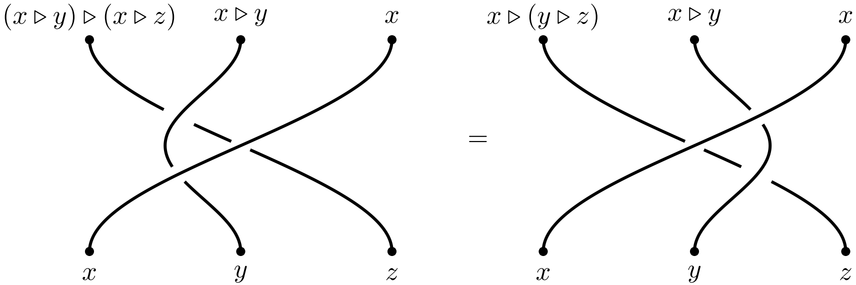


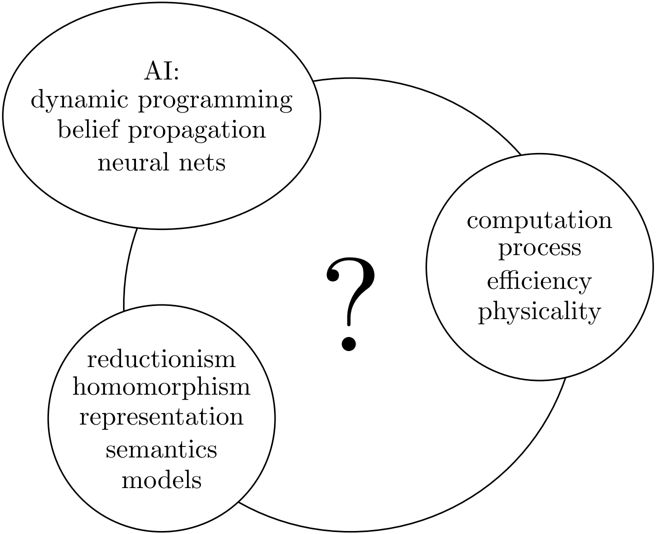
Yang-Baxter equation

$$\sigma_1\sigma_2\sigma_1 = \sigma_2\sigma_1\sigma_2.$$



Yang-Baxter equation




AI:
dynamic programming
belief propagation
neural nets
?
computation
process
efficiency
physicality
reductionism
homomorphism
representation
semantics
models