Problem Description

Given a set of variables $x_1, \dots, x_n$ and their primed versions $x'_1, \dots, x'_n$, it is possbile to describe a loop which uses these variables by 3 boolean functions:

$$\begin{gathered}
\mathrm{Pref}(x_1, \dots, x_n) \\
\mathrm{Transf}(x_1, \dots, x_n, x'_1, \dots, x'_n) \\
\mathrm{Postf}(x_1, \dots, x_n)
\end{gathered}$$

With:

$\mathrm{Pref}(x_1, \dots, x_n) = 1$ denoting that the set of variables are a valid set of input variables to the loop;

$\mathrm{Transf}(x_1, \dots, x_n, x'_1, \dots, x'_n) = 1$ denoting that the unprimed set of input variables become its corresponding primed versions after one interation of loop

$\mathrm{Postf}(x_1, \dots, x_n) = 1$ denoting that the set of input variables are a valid set of ending state of the loop

The problem is to find the invariant of the loop, a boolean function

$$\mathrm{Invf}(x_1, \dots, x_n)$$

such that

$$\begin{aligned}
\mathrm{Pref}(x_1, \dots, x_n) &\Rightarrow \mathrm{Invf}(x_1, \dots, x_n) \\
\mathrm{Invf}(x_1, \dots, x_n) \wedge \mathrm{Transf}(x_1, \dots, x_n, x'_1, \dots, x'_n) &\Rightarrow \mathrm{Invf}(x'_1, \dots, x'_n) \\
\mathrm{Invf}(x_1, \dots, x_n) &\Rightarrow \mathrm{Postf}(x_1, \dots, x_n)
\end{aligned}$$

Assumption and argument

Assume that the form of Transf is:

$$\begin{aligned}
\mathrm{Transf} \Leftrightarrow\ & (C_1^{(1)} \wedge C_2^{(1)} \wedge \dots \\
& T_1^{(1)} \wedge T_2^{(1)} \wedge \dots \wedge T_n^{(1)}) \vee \\
& (C_1^{(2)} \wedge C_2^{(2)} \wedge \dots \\
& T_1^{(2)} \wedge T_2^{(2)} \wedge \dots \wedge T_n^{(2)}) \vee \\
& \dots
\end{aligned}$$

with elements, or atoms in that form having the form

$$C_j^{(i)} \Leftrightarrow \begin{cases} x_k \geq c \\ x_k > c \\ x_k < c \\ x_k \leq c \\ x_l - x_k \geq c \\ x_l - x_k > c \\ x_l - x_k \leq c \\ x_l - x_k < c \end{cases}$$

$$T_j^{(i)} \Leftrightarrow (x'_k = x_k + c)$$

We would like to prove the argument that given this assumption as a constraint, if there exists an invariant, there must exist another invariant that have the form

$$\mathrm{Invf} \Leftrightarrow C'_1 \wedge C'_2 \dots$$

with C_i being in the same form as $C_j^{(i)}$