

# Basic Vector Space Search Engine Theory

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## Overview:

A Vector Space Search Engine uses very simple techniques from matrix algebra to compare documents based on word frequency.

The first major component of a vector space search engine is the concept of a ***term space***. Simply put, a *term space* consists of every unique word that appears in a collection of documents.

The second major component of a vector space search engine is ***term counts***. *Term counts* are simply records of how many times each term occurs in an individual document. This is usually represented as a table, as in the illustration below.

By using the *term space* as a coordinate space, and the *term counts* as coordinates within that space, we can create a *vector* for each document. In order to understand how we create these vectors, let's look at a simple example. You're probably familiar with Cartesian Coordinates; plotting points along X, Y, and Z axes. Similarly, in the case of a *term space* containing three unique terms we would refer to these axes as the *term1*, *term2*, and *term3* axes. (In vector space search theory these axes are usually referred to as ***dimensions***.) By counting how many times each term appears in a document, and plotting the coordinates along each term *dimension*, we can determine a point in the *term space* that corresponds to the document. Using this point we can then create a vector for the document back to the origin.

Once we have plotted the vector of a document through the term space, we can then calculate the ***magnitude*** of the vector. Think of the *magnitude* as the length of the line between the documents point in the term space and the origin of the term space (at coordinates (0,0,0) in our example). These vector magnitudes will allow us to compare documents by calculating the cosign of the angle between them. For example, identical documents will have a cosign of 1, documents containing similar terms will have positive decimal cosigns, and documents with nothing in common will have cosigns of zero.

## A Simple Example:

In this tutorial we'll go through the entire indexing and search process using a simple three dimensional example that is easy to envision.

To begin, let's assume we have a collection of three documents. Each document contains combinations of the words *cat*, *dog*, and *mouse*. The words *cat*, *dog*, and *mouse* are the *term space*. Thus we can say that each document has coordinates along the *cat*, *dog*, and *mouse* dimensions. These coordinates are

determined by how many times each term appears in the document. For example, document 1 below would have a “cat-dog-mouse vector” of (3,1,4).

**Term Counts**

Term Space	Document 1	Document 2	Document 3
cat	3	1	2
dog	1	2	3
mouse	4	5	0

We calculate the magnitude of the vector for each document using the Pythagorean Theorem, but in this case we have more than two dimensions, so the formula would be:

$$a^2+b^2+c^2=d^2$$

$$\|V_1\| = \sqrt{(3^2)+(1^2)+(4^2)} = \sqrt{9+1+16} = \sqrt{26} = 5.09901$$

$$\|V_2\| = \sqrt{(1^2)+(2^2)+(5^2)} = \sqrt{1+4+25} = \sqrt{30} = 5.47722$$

$$\|V_3\| = \sqrt{(2^2)+(3^2)+(0^2)} = \sqrt{4+9+0} = \sqrt{15} = 3.87298$$

NOTE: The two vertical bars on each side of the vector variable mean “the magnitude of”.

Note that the Pythagorean Theorem formula will continue to apply no matter how many dimensions we are working with. For example, if we had a term space with 1,000 unique words, and thus 1,000 dimensions, the formula would continue to be  $a^2+b^2+c^2+d^2+e^2\dots$ etc., 995 more times until we reach our answer.

Also, the astute observer may have noticed that different documents can have the exact same vector magnitudes. For example, two different documents with vectors of (1,2,3) and (3,2,1) would both have a vector magnitude of 3.74165. This is not a contradiction. As we will see, relevancy scores of documents are based on the dimensionality of the query term that is searched for, thus documents with identical vector magnitudes can return very different query results. In other words, just because two lines are the same length, it doesn’t necessarily mean that they are pointing to the same angle within the term space.

### Querying:

To query the document collection index, we project the vector of our query into the vector space, and then calculate the cosign of the angle between the query and each of the other documents in the collection. In English, this means we project the query vector into the vector space, and then see what other document vectors are nearby.

For example, if the query term is “mouse”, then the “cat-dog-mouse vector” would be (0,0,1). The magnitude of our query vector would then be:

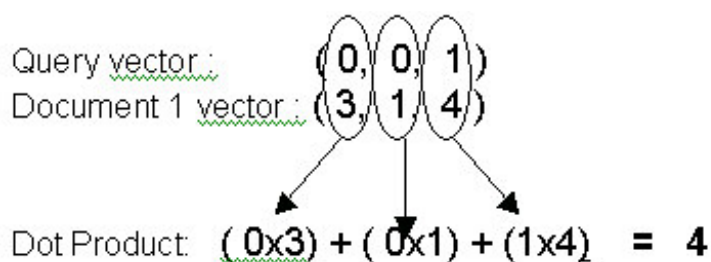
$$\|Q\| = \sqrt{(0^2)+(0^2)+(1^2)} = \sqrt{0+0+1} = \sqrt{1} = 1$$

NOTE: A simple optimization while coding is to check if the query term is in the term space, and if so, then  $\|Q\|$  will always = 1, but this only works with a single search term. For multiple search terms, count how many are within the term space, and take the square root of the count. Because query terms are not represented as values less than 1, the solution to  $\|Q\|$  will always be the square root of a whole number. But this assumes that each term appears only once in each query, which is not necessarily a good assumption because of word stemming which I will discuss shortly.

To calculate the cosign between the query and a document vector, we divide the **Dot Product** of the query vector and the document vector, by the magnitude of the query vector multiplied by the magnitude of the document vector.

$$\frac{Q \cdot V_1}{\|Q\| \times \|V_1\|}$$

The *Dot Product* is the sum of the term counts for each document and the corresponding query term counts multiplied together. For example, if we were to search for the term “mouse”, the coordinates for the Query would be (0,0,1) because the words cat and dog do not appear and the word mouse appears once in the third dimension of the term space. Document 1 in our example collection would have a vector of (3,1,4) based on the term counts listed in the table above. If we wanted to calculate the *Dot Product* between the Query and Document 1 we would make the following calculation:



Query vector : (0, 0, 1)  
 Document 1 vector : (3, 1, 4)  
 Dot Product:  $(0 \times 3) + (0 \times 1) + (1 \times 4) = 4$

Now we divide the Dot Product of 4, by the product of the query and document magnitudes, to get the cosine value. As we saw earlier the magnitude of the vector of Document 1 is 5.09901, and the magnitude of the Query vector = 1. Thus the cosign value is 4 divided by 5.09901.

Let's try it out. The cosign of the angle between the query for “mouse”, and Document 1 would be calculated by:

$$\frac{Q \cdot V_1}{\|Q\| \times \|V_1\|} = \frac{(0 \times 3) + (0 \times 1) + (1 \times 4)}{1 \times 5.09901} = \frac{4}{5.09901} = 0.78446$$

NOTE: If a document doesn't contain any relevant search terms from the query, the *Dot Product* will be zero, because zero divided by any value is still zero, thus the cosign will also be zero. This is good to remember while writing efficient code.

If we performed this calculation for the other two documents we would get the following cosigns:

Doc 1 = 0.78446  
Doc 2 = 0.91287  
Doc 3 = 0.00000

By arranging the documents in descending order according to the cosigns, as so:

Doc 2 = 0.91287  
Doc 1 = 0.78446  
Doc 3 = 0.00000

...we can see that document 2 is the most relevant to the query of “mouse”, and a quick glance at our term counts table above will confirm this. Document 1 is slightly less relevant, and Document 3 is completely irrelevant, because it doesn't contain any instances of the word “mouse”.

An easy way to think of this is that the closer the cosign value is to 1, the more relevant the document is. If the cosign is zero, then the documents are orthogonal in the term space and are not related.

## Collection Indexing Process

The process of indexing a collection is specific to the type of documents being indexed. Vector Space search technology can be used on any type information that can be represented in a structured fashion, so it will work equally well on text, images, cryptographic keys, or even DNA. However, custom parsers must be constructed to handle the information in a regulated fashion, and can often be optimized to make the indexing process more efficient.

As an example, let's assume we want to index a small website.

First each HTML document must be pre-processed, and then indexed as part of a collection. (Collections can only be indexed as a whole. Adding additional files to a collection after it has been indexed changes the dimensionality of the term space and negates the stored document vector magnitudes.)

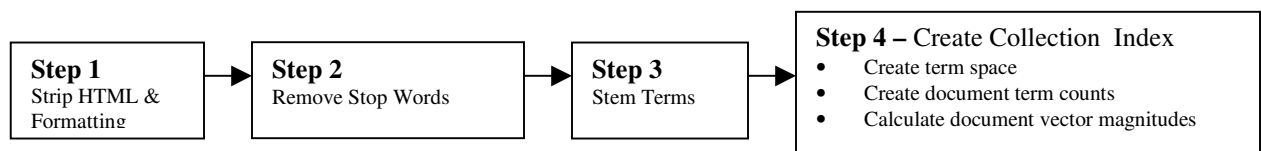
We begin by stripping out all the HTML content because it contains no semantic content. We can also strip out any formatting such as line breaks and carriage returns so that we are left with a simple block of text.

Next, we remove **stop words** from the text. *Stop words* are words that occur commonly in the English language, but don't add any semantic value to the text as a whole. For example, words like “the”, “and”, “of”, and “or” are irrelevant to the actual semantic meaning of the document but would artificially enlarge the term space, and thus the processing time, if they were left in. Also adverbs such as “quickly” (or anything typically ending in “ly”) can be removed because they don't add any additional semantic value.

Next, we **stem** the remaining terms in the document. *Stemming* consists of reducing English word to their root word forms. For example, the words “runner”, “running”, and “runs” would all be stemmed to the word “run.” The *Porter Stemming Algorithm* is typically used for this purpose. This further reduces the term space while maintaining semantic content.

After we have finished with these three steps we are left with (hopefully) the minimal number of terms required to contain the semantic meaning of the original document.

No we can begin indexing the collection by building the term space and calculating the vector magnitude of each document.



Step 1 – Strip out HTML, punctuation, and line breaks to leave only blocked textual content.

Step 2 – Remove stop words (such as “the”) to reduce the size of the term space.

Step 3 – Stem the remaining terms to further reduce the size of the term space while maintaining semantic content. For example, “runner” and “running” will both be stemmed to “run”. The Porter stemming algorithm is commonly used for this purpose.

Step 4A – Populate the term space with one unique instance of each term, from every document, that spans across the entire collection so that all possible terms are included. Store the results.

Step 4B – Count and record how many times each relevant term appears in each document.

Step 4C – Calculate and record the vector magnitude,  $\|V_n\|$ , for each document.

**NOTE:** It is important to keep in mind that how the parser divides up information will affect the search results. For example, if you were indexing the contents of a book, the indexing time and search results would differ greatly depending on whether you broke the text up by chapter, page, or paragraph. You will need to experiment to find the optimal partitions within particular data.

### Vector Space Search Engine Limitations:

Despite how cool Vector Space search technology is, it does have some serious limitations.

First, it is VERY calculation intensive, and therefore quite slow. Because of all the floating-point mathematics, it requires lots and lots of processor time, which kills performance. High performance requires large systems with code optimized to run calculations exclusively in RAM. Hopefully, this will become less of a barrier as processor speeds continue to increase.

Second, dynamic collections will (usually) require re-indexing each time a new document is added. This is because every time you introduce a new term into the term space, you are adding another dimension to the matrix, and all existing documents must be re-indexed so that their vectors are relevant to the new dimensionality. This is perhaps the most serious barrier to the widespread adoption of this technology because it makes real time availability of search results next to impossible.

Third, it requires additional mathematical transformation of the collection matrix in order to detect additional connections between documents with Latent Semantic Indexing. LSI allows us to find additional connections between documents on a semantic level. It is outside the scope of this document, but it is an important next-step in Vector Space search technology, and another barrier to real time usability.

### **Resources and Additional Reading**

- <http://www.perl.com/pub/a/2003/02/19/engine.html> - Excellent article about building a vector space search engine including open source PERL code.
- <http://www.chuggnutt.com/stemmer.php> - Open source implementation of the Porter stemming algorithm in PHP
- [http://www.nittle.org/semantic\\_search.php](http://www.nittle.org/semantic_search.php) - Open source Latent Semantic Indexing package written in Perl. Very much in Beta, not yet suitable for production.
- <http://lsi.arggreenhouse.com/> - Closed source online Latent Semantic Indexing demo by Telecordia Technologies
- "Using Linear Algebra for Information Retrieval" - Berry, M. W.; Dumais, S. T.; and O'Brien, G. W. 1995.
- "Indexing by latent semantic analysis." Journal of the Society for Information Science, 41(6), 391-407. --- first technical Latent Semantic Indexing paper; good background.
- "Enhancing Performance in Latent Semantic Indexing Retrieval" - Susan Dumais, TM-ARH-017527 Technical Report, Bellcore, 1990