

## Slides

If the two outcomes are *not* mutually exclusive:

$$P(\{2 \times H\} \cup \{3 \times H\}) = \frac{4}{8} + \frac{1}{8} - \frac{1}{8} = \frac{4}{8} = 0.5$$

$$P(\{2 \times H\} \cap \{1 \times T\}) + P(3 \times H)$$

- the mean of  $x$  is 9
- the variance of  $x$  is 11
- the mean of  $y$  is 7.50
- the variance of  $y$  is 4.125
- the correlation coefficient of  $x$  and  $y$  is 0.816
- the best fit line is given by  $y = 3.00 + 0.500x$

urn:	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
10 samples selected with replacement:	5	2	12	19	10	5	3	19	11	4										
10 samples selected with replacement:	<span style="border: 1px solid black;">5</span>	2	12	<span style="border: 1px solid black;">19</span>	10	<span style="border: 1px solid black;">5</span>	3	<span style="border: 1px solid black;">19</span>	11	4										
10 samples selected without replacement:	15	8	13	5	4	19	7	1	20	10										

$$P(H, H, T) + P(\begin{smallmatrix} \square & \bullet \\ \bullet & \bullet \end{smallmatrix}, \begin{smallmatrix} \bullet & \bullet \\ \bullet & \bullet \end{smallmatrix}) - P(H, H, H \cap \begin{smallmatrix} \square & \bullet \\ \bullet & \bullet \end{smallmatrix}, \begin{smallmatrix} \bullet & \bullet \\ \bullet & \bullet \end{smallmatrix})$$

$$p = 2/3$$

We failed to reject  $H_0$  for  $\hat{p} = \frac{2}{5} = 0.4$ . How about  $\hat{p} = \frac{6}{15} = 0.4$ ?

$$k \in R$$

Next, how about  $\hat{p} = \frac{12}{30} = 0.4$ ?

Which values of  $p$  are compatible with the observation of  $k = 2$  successes out of  $n = 5$  claims?

$$P(k \leq 2 | p = 2/3, n = 5) = \sum_{i=0}^2 \binom{5}{i} (2/3)^i (1/3)^{n-i} = 0.21 \geq \alpha/2$$

Assuming that  $H_0$  is true, the probability of incurring a type-I error is:

1 experiment:  $\alpha = 5\%$

2 experiments:  $1 - (1 - \alpha)^2 = 9.75\%$

3 experiments:  $1 - (1 - \alpha)^3 = 14\%$

48 experiments:  $1 - (1 - \alpha)^{48} = 91.5\%$

$N$  experiments:  $1 - (1 - \alpha)^N\%$

Bonferroni correction:  $\alpha/N$

What is  $\lambda$ ?

**the binomial distribution**

**the Poisson distribution**

What is so normal about the Gaussian distribution?

What is  $s[z]$ ?

$$s[d] = \sqrt{s[x]^2 + s[y]^2}$$

$$z = x + y$$

$$z = x - y$$

$$s[z]^2 = s[x]^2 + s[y]^2$$

$$z = xy$$

$$\left(\frac{s[z]}{z}\right)^2 = \left(\frac{s[x]}{x}\right)^2 + \left(\frac{s[y]}{y}\right)^2$$

$$s[y]^2 = y^2 \left(2\frac{s[t]}{t}\right)^2$$

$$2 \times 0.0000042 = 0.0000084 < \alpha$$

$$\mathrm{P}(\geq 9.0 \text{ earthquake})?$$

$$\frac{44 \ + \ 51 \ + \ 79 \ + \ 65 \ + \ 27 \ + \ 31 \ + \ 4 \ + \ 355 \ + \ 22 \ + \ 352 \ + \ 287 \ + \ 7 \ + \ 287 \ + \ 339 \ + \ 0}{276 \ + \ 342 \ + \ 355 \ + \ 334 \ + \ 296 \ + \ 7 \ + \ 17 \ + \ 351 \ + \ 349 \ + \ 37 \ + \ 339 \ + \ 40 \ + \ 324 \ + \ 325 \ + \ 334}$$

$$=189.2^{\circ}$$

$$\sin[\theta\pm\phi]=\sin[\theta]\cos[\phi]\pm\cos[\theta]\sin[\phi]$$

$$359^{\circ}-1^{\circ}=2^{\circ}$$

$$\bar{R}=0.88$$

$$s_c=0.51$$

$$s_c=2.04$$

x	0.85	0.84	0.83	0.8	0.77	0.75	0.74	0.73	0.71	0.6
y	0.53	0.54	0.56	0.6	0.64	0.66	0.67	0.68	0.71	0.8
x	-0.87	-0.85	-0.83	-0.8	-0.77	-0.75	-0.74	-0.73	-0.69	-0.64
y	-0.50	-0.53	-0.56	-0.6	-0.64	-0.66	-0.67	-0.68	-0.72	-0.77

## Quizzes

Given  $k = 9$  successes out of  $n=10$  binomial trials, carry out a hypothesis test for

$$H_0 : p = 0.6$$

versus

$$H_a : p > 0.6$$

at a 95% confidence level.

$X$	0	1	2	3	4	5	6	7	8	9	10
$P(X)$	0.0001	0.0016	0.01062	0.0425	0.1115	0.2007	0.2508	0.2150	0.1209	0.0403	0.0060
$P(x \leq X)$	0.0001	0.0017	0.01230	0.0548	0.1662	0.3669	0.6177	0.8327	0.9536	0.9940	1
$P(x \geq X)$	1	0.9999	0.9983	0.9877	0.9452	0.8338	0.6331	0.3823	0.1673	0.0464	0.0060

Can the null hypothesis be rejected?

Given  $k = 9$  successes out of  $n=10$  binomial trials, carry out hypothesis test of

$$H_o : p = 0.6$$

versus

$$H_a : p \neq 0.6$$

at a 95% confidence level.

$X$	0	1	2	3	4	5	6	7	8	9	10
$P(X)$	0.0001	0.0016	0.01062	0.0425	0.1115	0.2007	0.2508	0.2150	0.1209	0.0403	0.0060
$P(x \leq X)$	0.0001	0.0017	0.01230	0.0548	0.1662	0.3669	0.6177	0.8327	0.9536	0.9940	1
$P(x \geq X)$	1	0.9999	0.9983	0.9877	0.9452	0.8338	0.6331	0.3823	0.1673	0.0464	0.0060

Can the null hypothesis be rejected?

Given  $k = 9$  successes out of  $n=10$  trials, carry out a hypothesis test for

$$H_o : p = 0.6$$

versus

$$H_a : p < 0.6$$

at a 95% confidence level?

$X$	0	1	2	3	4	5	6	7	8	9	10
$P(X)$	0.0001	0.0016	0.01062	0.0425	0.1115	0.2007	0.2508	0.2150	0.1209	0.0403	0.0060
$P(x \leq X)$	0.0001	0.0017	0.01230	0.0548	0.1662	0.3669	0.6177	0.8327	0.9536	0.9940	1
$P(x \geq X)$	1	0.9999	0.9983	0.9877	0.9452	0.8338	0.6331	0.3823	0.1673	0.0464	0.0060

Can the null hypothesis be rejected?

Here are some key quantiles (column names) of a binomial distribution with  $n = 100$  and parameter values  $p$  given as row names:

	0.01	0.025	0.05	0.1	0.5	0.9	0.95	0.975	0.99
0.466	35	37	38	40	47	53	55	56	58
0.497	38	40	42	43	50	56	58	60	61
0.513	40	42	43	45	51	58	59	61	63
0.682	57	59	60	62	68	74	76	77	79
0.697	59	60	62	64	70	76	77	78	80
0.724	62	63	65	67	72	78	80	81	82

What is the 95% confidence interval for  $p$  if the number of successes is  $k = 60$  out of  $n = 100$ ?

$$\begin{aligned}
P(\text{wrong decision}) &= 1 - \left(1 - \frac{1}{7 \times 10^7}\right)^{1 \times 10^7} \\
&= 1 - \exp\left(1 \times 10^7 \ln\left(1 - \frac{1}{7 \times 10^7}\right)\right) \\
&= 1 - \exp\left(-\frac{1 \times 10^7}{7 \times 10^7}\right) \\
&= 1 - \exp(-1/7) = 0.13
\end{aligned}$$

$$\frac{P(\text{double SIDS})}{P(\text{double homicide})} = \frac{21700 \times 123}{1300 \times 228} = \frac{9}{1}$$