In[170]:=
```
ClearAll[f0, fl, fr, a, b, c, d, hl, hr, hm1, hm2, hp1, hp2, fm1, fm2, fp1, fp2]
```

Second order approximation:

In[229]:=
```
ford2[h_] := f0 + a h + b h^2
```

In[230]:=
```
sol2 = FullSimplify[Solve[{ford2[-hl] == fl, ford2[hr] == fr}, {a, b}]]
```

Out[230]= $\left\{\left\{a \to \frac{(-f0+fr)\,hl^2+(f0-fl)\,hr^2}{hl\,hr\,(hl+hr)},\ b \to \frac{fr\,hl+fl\,hr-f0\,(hl+hr)}{hl\,hr\,(hl+hr)}\right\}\right\}$

In[231]:=
```
D1ord2 = FullSimplify[ford2'[0] /. sol2]
```

Out[231]= $\left\{\frac{(-f0+fr)\,hl^2+(f0-fl)\,hr^2}{hl\,hr\,(hl+hr)}\right\}$

In[232]:=
```
D2ord2 = FullSimplify[ford2''[0] /. sol2]
```

Out[232]= $\left\{\frac{2\,(fr\,hl+fl\,hr-f0\,(hl+hr))}{hl\,hr\,(hl+hr)}\right\}$

---

a) Coefficients for first derivative at centre point

In[175]:=
```
c1ord2 = {Row[FullSimplify[Coefficient[D1ord2, fl]]],
  Row[FullSimplify[Coefficient[D1ord2, f0]]],
  Row[FullSimplify[Coefficient[D1ord2, fr]]]}
```

Out[175]= $\left\{-\frac{hr}{hl\,(hl+hr)},\ \frac{1}{hl}-\frac{1}{hr},\ \frac{hl}{hr\,(hl+hr)}\right\}$

In[176]:=
```
c1ord2 /. {hl → h, hr → h}
```

Out[176]= $\left\{-\frac{1}{2\,h},\ 0,\ \frac{1}{2\,h}\right\}$

---

b) Coefficients for second derivative at centre point

In[177]:=
```
c2ord2 = {Row[FullSimplify[Coefficient[D2ord2, fl]]],
  Row[FullSimplify[Coefficient[D2ord2, f0]]],
  Row[FullSimplify[Coefficient[D2ord2, fr]]]}
```

Out[177]= $\left\{\frac{2}{hl\,(hl+hr)},\ -\frac{2}{hl\,hr},\ \frac{2}{hr\,(hl+hr)}\right\}$

In[178]:=
```
c2ord2 /. {hl → h, hr → h}
```

Out[178]= $\left\{\frac{1}{h^2},\ -\frac{2}{h^2},\ \frac{1}{h^2}\right\}$

---

c) Coefficients for first derivative at left point

In[214]:=
```
D1ord2l = FullSimplify[ford2'[-hl] /. sol2];
```

In[215]:=
```
c1ord2l = {Row[FullSimplify[Coefficient[D1ord2l, fl]]],
  Row[FullSimplify[Coefficient[D1ord2l, f0]]],
  Row[FullSimplify[Coefficient[D1ord2l, fr]]]}
```

Out[215]= $\left\{-\frac{1}{hl}-\frac{1}{hl+hr}, \frac{1}{hl}+\frac{1}{hr}, -\frac{hl}{hl\,hr+hr^2}\right\}$

In[216]:=
```
c1ord2l /. {hl → h, hr → h}
```

Out[216]= $\left\{-\frac{3}{2\,h}, \frac{2}{h}, -\frac{1}{2\,h}\right\}$

In[217]:=
```
sol2l = FullSimplify[Solve[{ford2[h2] == f2, ford2[h3] == f3}, {a, b}]];
(*** as in pdf ***)
```

In[218]:=
```
D1ord2ll = FullSimplify[ford2'[0] /. sol2l];
```

In[219]:=
```
c1ord2ll = {Row[FullSimplify[Coefficient[D1ord2ll, f0]]],
  Row[FullSimplify[Coefficient[D1ord2ll, f2]]],
  Row[FullSimplify[Coefficient[D1ord2ll, f3]]]}
```

Out[219]= $\left\{-\frac{h2+h3}{h2\,h3}, \frac{1}{h2}+\frac{1}{-h2+h3}, \frac{h2}{(h2-h3)\,h3}\right\}$

In[220]:=
```
c1ord2ll /. {h2 → h, h3 → 2 h}
 (*** should be same thing ***)
```

Out[220]= $\left\{-\frac{3}{2\,h}, \frac{2}{h}, -\frac{1}{2\,h}\right\}$

---

d) Coefficients for first derivative at right point

In[233]:=
```
D1ord2r = FullSimplify[ford2'[hr] /. sol2];
```

In[234]:=
```
c1ord2r = {Row[FullSimplify[Coefficient[D1ord2r, fl]]],
  Row[FullSimplify[Coefficient[D1ord2r, f0]]],
  Row[FullSimplify[Coefficient[D1ord2r, fr]]]}
```

Out[234]= $\left\{\frac{hr}{hl^2+hl\,hr}, -\frac{hl+hr}{hl\,hr}, \frac{1}{hr}+\frac{1}{hl+hr}\right\}$

In[235]:=
```
c1ord2r /. {hl → h, hr → h}
```

Out[235]= $\left\{\frac{1}{2\,h}, -\frac{2}{h}, \frac{3}{2\,h}\right\}$

In[240]:=
```
sol2r = FullSimplify[Solve[{ford2[-hm1] == fm1, ford2[-hm2] == fm2}, {a, b}]];
(*** as in pdf ***)
```

In[241]:=
```
D1ord2rr = FullSimplify[ford2'[0] /. sol2r];
```

In[250]:=
```
c1ord2rr = {Row[FullSimplify[Coefficient[D1ord2rr, fm2]]],
  Row[FullSimplify[Coefficient[D1ord2rr, fm1]]],
  Row[FullSimplify[Coefficient[D1ord2rr, f0]]]}
```

Out[250]= $\left\{\frac{hm1}{hm2\,(-hm1+hm2)},\ \frac{hm2}{hm1^2-hm1\,hm2},\ \frac{1}{hm1}+\frac{1}{hm2}\right\}$

In[251]:=
```
c1ord2rr /. {hm1 → h, hm2 → 2 h}
 (*** should be same thing ***)
```

Out[251]= $\left\{\frac{1}{2\,h},\ -\frac{2}{h},\ \frac{3}{2\,h}\right\}$

Forth order approximation:

In[193]:=
```
ClearAll[f0, a, b, c, d, hm1, hm2, hp1, hp2, fm1, fm2, fp1, fp2]
```

In[194]:=
```
ford4[h_] := f0 + a h + b h^2 + c h^3 + d h^4
```

In[195]:=
```
sol4 = FullSimplify[Solve[{ford4[-hm2] == fm2, ford4[-hm1] == fm1,
     ford4[hp1] == fp1, ford4[hp2] == fp2}, {a, b, c, d}, Reals]];
```

In[196]:=
```
D1ord4 = FullSimplify[ford4'[0] /. sol4];
```

In[197]:=
```
D2ord4 = FullSimplify[ford4''[0] /. sol4];
```

a) Coefficients for first derivative at centre point

In[198]:=
```
c1ord4 = {Row[FullSimplify[Coefficient[D1ord4, fm2]]],
  Row[FullSimplify[Coefficient[D1ord4, fm1]]],
  Row[FullSimplify[Coefficient[D1ord4, f0]]],
  Row[FullSimplify[Coefficient[D1ord4, fp1]]],
  Row[FullSimplify[Coefficient[D1ord4, fp2]]]}
```

Out[198]= $\left\{-\frac{hm1\,hp1\,hp2}{(hm1-hm2)\,hm2\,(hm2+hp1)\,(hm2+hp2)},\right.$
$\frac{hm2\,hp1\,hp2}{hm1\,(hm1-hm2)\,(hm1+hp1)\,(hm1+hp2)},\ \frac{1}{hm1}+\frac{1}{hm2}-\frac{hp1+hp2}{hp1\,hp2},$
$\left.\frac{hm1\,hm2\,hp2}{hp1\,(hm1+hp1)\,(hm2+hp1)\,(-hp1+hp2)},\ \frac{hm1\,hm2\,hp1}{(hp1-hp2)\,hp2\,(hm1+hp2)\,(hm2+hp2)}\right\}$

In[199]:=
```
c1ord4 /. {hm2 → 2 h, hm1 → h, hp1 → h, hp2 → 2 h}
```

Out[199]= $\left\{\frac{1}{12\,h},\ -\frac{2}{3\,h},\ 0,\ \frac{2}{3\,h},\ -\frac{1}{12\,h}\right\}$

b) Coefficients for second derivative at centre point

In[200]:=
```
c2ord4 = {Row[FullSimplify[Coefficient[D2ord4, fm2]]],
  Row[FullSimplify[Coefficient[D2ord4, fm1]]],
  Row[FullSimplify[Coefficient[D2ord4, f0]]],
  Row[FullSimplify[Coefficient[D2ord4, fp1]]],
  Row[FullSimplify[Coefficient[D2ord4, fp2]]]}
```

Out[200]=
$$\left\{\frac{-2\,\text{hp1}\,\text{hp2}+2\,\text{hm1}\,(\text{hp1}+\text{hp2})}{(\text{hm1}-\text{hm2})\,\text{hm2}\,(\text{hm2}+\text{hp1})\,(\text{hm2}+\text{hp2})},\ \frac{2\,\text{hp1}\,\text{hp2}-2\,\text{hm2}\,(\text{hp1}+\text{hp2})}{\text{hm1}\,(\text{hm1}-\text{hm2})\,(\text{hm1}+\text{hp1})\,(\text{hm1}+\text{hp2})},\right.$$
$$\frac{2\,(\text{hm1}\,(\text{hm2}-\text{hp1}-\text{hp2})+\text{hp1}\,\text{hp2}-\text{hm2}\,(\text{hp1}+\text{hp2}))}{\text{hm1}\,\text{hm2}\,\text{hp1}\,\text{hp2}},$$
$$\left.\frac{2\,\text{hm1}\,\text{hm2}-2\,(\text{hm1}+\text{hm2})\,\text{hp2}}{\text{hp1}\,(\text{hm1}+\text{hp1})\,(\text{hm2}+\text{hp1})\,(\text{hp1}-\text{hp2})},\ \frac{-2\,\text{hm1}\,\text{hm2}+2\,(\text{hm1}+\text{hm2})\,\text{hp1}}{(\text{hp1}-\text{hp2})\,\text{hp2}\,(\text{hm1}+\text{hp2})\,(\text{hm2}+\text{hp2})}\right\}$$

In[201]:=
```
c2ord4 /. {hm2 → 2 h, hm1 → h, hp1 → h, hp2 → 2 h}
```

Out[201]=
$$\left\{-\frac{1}{12\,h^2},\ \frac{4}{3\,h^2},\ -\frac{5}{2\,h^2},\ \frac{4}{3\,h^2},\ -\frac{1}{12\,h^2}\right\}$$

---

c) Coefficients for first derivative at first point left

In[202]:=
```
D1ord4l1 = FullSimplify[ford4'[-hm1] /. sol4];
```

In[203]:=
```
c1ord4l1 = {Row[FullSimplify[Coefficient[D1ord4l1, fm2]]],
  Row[FullSimplify[Coefficient[D1ord4l1, fm1]]],
  Row[FullSimplify[Coefficient[D1ord4l1, f0]]],
  Row[FullSimplify[Coefficient[D1ord4l1, fp1]]],
  Row[FullSimplify[Coefficient[D1ord4l1, fp2]]]}
```

Out[203]=
$$\left\{\frac{\text{hm1}\,(\text{hm1}+\text{hp1})\,(\text{hm1}+\text{hp2})}{(\text{hm1}-\text{hm2})\,\text{hm2}\,(\text{hm2}+\text{hp1})\,(\text{hm2}+\text{hp2})},\right.$$
$$-\frac{1}{\text{hm1}}+\frac{1}{-\text{hm1}+\text{hm2}}-\frac{1}{\text{hm1}+\text{hp1}}-\frac{1}{\text{hm1}+\text{hp2}},\ -\frac{(\text{hm1}-\text{hm2})\,(\text{hm1}+\text{hp1})\,(\text{hm1}+\text{hp2})}{\text{hm1}\,\text{hm2}\,\text{hp1}\,\text{hp2}},$$
$$\left.-\frac{\text{hm1}\,(\text{hm1}-\text{hm2})\,(\text{hm1}+\text{hp2})}{\text{hp1}\,(\text{hm1}+\text{hp1})\,(\text{hm2}+\text{hp1})\,(\text{hp1}-\text{hp2})},\ \frac{\text{hm1}\,(\text{hm1}-\text{hm2})\,(\text{hm1}+\text{hp1})}{(\text{hp1}-\text{hp2})\,\text{hp2}\,(\text{hm1}+\text{hp2})\,(\text{hm2}+\text{hp2})}\right\}$$

In[204]:=
```
FullSimplify[c1ord4l1 /. {hm2 → 2 h, hm1 → h, hp1 → h, hp2 → 2 h}]
```

Out[204]=
$$\left\{-\frac{1}{4\,h},\ -\frac{5}{6\,h},\ \frac{3}{2\,h},\ -\frac{1}{2\,h},\ \frac{1}{12\,h}\right\}$$

---

d) Coefficients for first derivative at second point left

In[205]:=
```
D1ord4l2 = FullSimplify[ford4'[-hm2] /. sol4];
```

In[206]:=
```
c1ord4l2 = {Row[FullSimplify[Coefficient[D1ord4l2, fm2]]],
  Row[FullSimplify[Coefficient[D1ord4l2, fm1]]],
  Row[FullSimplify[Coefficient[D1ord4l2, f0]]],
  Row[FullSimplify[Coefficient[D1ord4l2, fp1]]],
  Row[FullSimplify[Coefficient[D1ord4l2, fp2]]]}
```

Out[206]=
$$\left\{\frac{hm1-2\,hm2}{hm2\,(-hm1+hm2)}-\frac{1}{hm2+hp1}-\frac{1}{hm2+hp2},\ -\frac{hm2\,(hm2+hp1)\,(hm2+hp2)}{hm1\,(hm1-hm2)\,(hm1+hp1)\,(hm1+hp2)},\ \frac{(hm1-hm2)\,(hm2+hp1)\,(hm2+hp2)}{hm1\,hm2\,hp1\,hp2},\ \frac{(hm1-hm2)\,hm2\,(hm2+hp2)}{hp1\,(hm1+hp1)\,(hm2+hp1)\,(hp1-hp2)},\ -\frac{(hm1-hm2)\,hm2\,(hm2+hp1)}{(hp1-hp2)\,hp2\,(hm1+hp2)\,(hm2+hp2)}\right\}$$

In[207]:=
```
FullSimplify[c1ord4l2 /. {hm2 → 2 h, hm1 → h, hp1 → h, hp2 → 2 h}]
```

Out[207]=
$$\left\{-\frac{25}{12\,h},\ \frac{4}{h},\ -\frac{3}{h},\ \frac{4}{3\,h},\ -\frac{1}{4\,h}\right\}$$

e) Coefficients for first derivative at first point right

In[208]:=
```
D1ord4r1 = FullSimplify[ford4'[hp1] /. sol4];
```

In[209]:=
```
c1ord4r1 = {Row[FullSimplify[Coefficient[D1ord4r1, fm2]]],
  Row[FullSimplify[Coefficient[D1ord4r1, fm1]]],
  Row[FullSimplify[Coefficient[D1ord4r1, f0]]],
  Row[FullSimplify[Coefficient[D1ord4r1, fp1]]],
  Row[FullSimplify[Coefficient[D1ord4r1, fp2]]]}
```

Out[209]=
$$\left\{-\frac{hp1\,(hm1+hp1)\,(hp1-hp2)}{(hm1-hm2)\,hm2\,(hm2+hp1)\,(hm2+hp2)},\ \frac{hp1\,(hm2+hp1)\,(hp1-hp2)}{hm1\,(hm1-hm2)\,(hm1+hp1)\,(hm1+hp2)},\ \frac{(hm1+hp1)\,(hm2+hp1)\,(hp1-hp2)}{hm1\,hm2\,hp1\,hp2},\ \frac{1}{hp1}+\frac{1}{hm1+hp1}+\frac{1}{hm2+hp1}+\frac{1}{hp1-hp2},\ -\frac{hp1\,(hm1+hp1)\,(hm2+hp1)}{(hp1-hp2)\,hp2\,(hm1+hp2)\,(hm2+hp2)}\right\}$$

In[210]:=
```
FullSimplify[c1ord4r1 /. {hm2 → 2 h, hm1 → h, hp1 → h, hp2 → 2 h}]
```

Out[210]=
$$\left\{-\frac{1}{12\,h},\ \frac{1}{2\,h},\ -\frac{3}{2\,h},\ \frac{5}{6\,h},\ \frac{1}{4\,h}\right\}$$

f) Coefficients for first derivative at second point right

In[211]:=
```
D1ord4r2 = FullSimplify[ford4'[hp2] /. sol4];
```

In[212]:=
```
c1ord4r2 = {Row[FullSimplify[Coefficient[D1ord4r2, fm2]]],
  Row[FullSimplify[Coefficient[D1ord4r2, fm1]]],
  Row[FullSimplify[Coefficient[D1ord4r2, f0]]],
  Row[FullSimplify[Coefficient[D1ord4r2, fp1]]],
  Row[FullSimplify[Coefficient[D1ord4r2, fp2]]]}
```

Out[212]=

$$\left\{\frac{(hp1-hp2)\ hp2\ (hm1+hp2)}{(hm1-hm2)\ hm2\ (hm2+hp1)\ (hm2+hp2)},\right.$$
$$-\frac{(hp1-hp2)\ hp2\ (hm2+hp2)}{hm1\ (hm1-hm2)\ (hm1+hp1)\ (hm1+hp2)},\ -\frac{(hp1-hp2)\ (hm1+hp2)\ (hm2+hp2)}{hm1\ hm2\ hp1\ hp2},$$
$$\left.\frac{hp2\ (hm1+hp2)\ (hm2+hp2)}{hp1\ (hm1+hp1)\ (hm2+hp1)\ (hp1-hp2)},\ \frac{1}{hp2}+\frac{1}{hm1+hp2}+\frac{1}{hm2+hp2}+\frac{1}{-hp1+hp2}\right\}$$

In[213]:=
```
FullSimplify[c1ord4r2 /. {hm2 → 2 h, hm1 → h, hp1 → h, hp2 → 2 h}]
```

Out[213]=

$$\left\{\frac{1}{4\ h},\ -\frac{4}{3\ h},\ \frac{3}{h},\ -\frac{4}{h},\ \frac{25}{12\ h}\right\}$$