In[∘]:=
```
ClearAll[f0, fl, fr, a, b, c, d, hl, hr, hm1, hm2, hp1, hp2, fm1, fm2, fp1, fp2]
```

Second order approximation:

In[∘]:=
```
ford2[h_] := f0 + a h + b h^2
```

In[∘]:=
```
sol2 = FullSimplify[Solve[{ford2[-hl] == fl, ford2[hr] == fr}, {a, b}]]
```

Out[∘]= $\left\{\left\{a \to \frac{(-f0 + fr)\, hl^2 + (f0 - fl)\, hr^2}{hl\, hr\, (hl + hr)},\ b \to \frac{fr\, hl + fl\, hr - f0\, (hl + hr)}{hl\, hr\, (hl + hr)}\right\}\right\}$

In[∘]:=
```
D1ord2 = FullSimplify[ford2'[0] /. sol2]
```

Out[∘]= $\left\{\frac{(-f0 + fr)\, hl^2 + (f0 - fl)\, hr^2}{hl\, hr\, (hl + hr)}\right\}$

In[∘]:=
```
D2ord2 = FullSimplify[ford2''[0] /. sol2]
```

Out[∘]= $\left\{\frac{2\,(fr\, hl + fl\, hr - f0\, (hl + hr))}{hl\, hr\, (hl + hr)}\right\}$

---

a) Coefficients for first derivative at centre point

In[∘]:=
```
c1ord2 = {Row[FullSimplify[Coefficient[D1ord2, fl]]],
  Row[FullSimplify[Coefficient[D1ord2, f0]]],
  Row[FullSimplify[Coefficient[D1ord2, fr]]]}
```

Out[∘]= $\left\{-\frac{hr}{hl\,(hl + hr)},\ \frac{1}{hl} - \frac{1}{hr},\ \frac{hl}{hr\,(hl + hr)}\right\}$

In[∘]:=
```
c1ord2 /. {hl → h, hr → h}
```

Out[∘]= $\left\{-\frac{1}{2\,h},\ 0,\ \frac{1}{2\,h}\right\}$

---

b) Coefficients for second derivative at centre point

In[∘]:=
```
c2ord2 = {Row[FullSimplify[Coefficient[D2ord2, fl]]],
  Row[FullSimplify[Coefficient[D2ord2, f0]]],
  Row[FullSimplify[Coefficient[D2ord2, fr]]]}
```

Out[∘]= $\left\{\frac{2}{hl\,(hl + hr)},\ -\frac{2}{hl\, hr},\ \frac{2}{hr\,(hl + hr)}\right\}$

In[∘]:=
```
c2ord2 /. {hl → h, hr → h}
```

Out[∘]= $\left\{\frac{1}{h^2},\ -\frac{2}{h^2},\ \frac{1}{h^2}\right\}$

---

c) Coefficients for first derivative at left point

In[∘]:=
```
D1ord2l = FullSimplify[ford2'[-hl] /. sol2];
```

In[∘]:=
```
c1ord2l = {Row[FullSimplify[Coefficient[D1ord2l, fl]]],
  Row[FullSimplify[Coefficient[D1ord2l, f0]]],
  Row[FullSimplify[Coefficient[D1ord2l, fr]]]}
```

Out[∘]= $\left\{-\frac{1}{hl}-\frac{1}{hl+hr}, \frac{1}{hl}+\frac{1}{hr}, -\frac{hl}{hl\,hr+hr^2}\right\}$

In[∘]:=
```
c1ord2l /. {hl → h, hr → h}
```

Out[∘]= $\left\{-\frac{3}{2\,h}, \frac{2}{h}, -\frac{1}{2\,h}\right\}$

In[∘]:=
```
sol2l = FullSimplify[Solve[{ford2[h2] == f2, ford2[h3] == f3}, {a, b}]];
(*** as in pdf ***)
```

In[∘]:=
```
D1ord2ll = FullSimplify[ford2'[0] /. sol2l];
```

Out[∘]= ^{3}

In[∘]:=
```
c1ord2ll = {Row[FullSimplify[Coefficient[D1ord2ll,f0]]],
   Row[FullSimplify[Coefficient[D1ord2ll,f2]]],
   Row[FullSimplify[Coefficient[D1ord2ll,f3]]]}
```

Out[∘]= $\left\{-\frac{h2+h3}{h2\ h3}, \frac{1}{h2}+\frac{1}{-h2+h3}, \frac{h2}{(h2-h3)\ h3}\right\}$

In[∘]:=
```
c1ord2ll /. {h2 → h, h3 → 2 h}
 (*** should be same thing ***)
```

Out[∘]= $\left\{-\frac{3}{2\,h}, \frac{2}{h}, -\frac{1}{2\,h}\right\}$

---

d) Coefficients for first derivative at right point

In[∘]:=
```
D1ord2r = FullSimplify[ford2'[hr] /. sol2];
```

In[∘]:=
```
c1ord2r = {Row[FullSimplify[Coefficient[D1ord2r, fl]]],
  Row[FullSimplify[Coefficient[D1ord2r, f0]]],
  Row[FullSimplify[Coefficient[D1ord2r, fr]]]}
```

Out[∘]= $\left\{\frac{hr}{hl^2+hl\,hr}, -\frac{hl+hr}{hl\,hr}, \frac{1}{hr}+\frac{1}{hl+hr}\right\}$

In[∘]:=
```
c1ord2r /. {hl → h, hr → h}
```

Out[∘]= $\left\{\frac{1}{2\,h}, -\frac{2}{h}, \frac{3}{2\,h}\right\}$

In[∘]:=
```
sol2r = FullSimplify[Solve[{ford2[-hm1] == fm1, ford2[-hm2] == fm2}, {a, b}]];
(*** as in pdf ***)
```

In[∘]:=
```
D1ord2rr = FullSimplify[ford2'[0] /. sol2r];
```

In[∘]:=
```
c1ord2rr = {Row[FullSimplify[Coefficient[D1ord2rr, fm2]]],
  Row[FullSimplify[Coefficient[D1ord2rr, fm1]]],
  Row[FullSimplify[Coefficient[D1ord2rr, f0]]]}
```

Out[∘]=
$$\left\{\frac{\text{hm1}}{\text{hm2}\,(-\text{hm1}+\text{hm2})},\ \frac{\text{hm2}}{\text{hm1}^2-\text{hm1}\,\text{hm2}},\ \frac{1}{\text{hm1}}+\frac{1}{\text{hm2}}\right\}$$

In[∘]:=
```
c1ord2rr /. {hm1 → h, hm2 → 2 h}
 (*** should be same thing ***)
```

Out[∘]=
$$\left\{\frac{1}{2\,h},\ -\frac{2}{h},\ \frac{3}{2\,h}\right\}$$

Forth order approximation:

In[1]:=
```
ClearAll[f0, a, b, c, d, hm1, hm2, hp1, hp2, fm1,
 fm2, fp1, fp2, hl2, hl1, hr1, hr2, hr3, f1, f2, f3, f4]
```

In[2]:=
```
ford4[h_] := f0 + a h + b h^2 + c h^3 + d h^4
```

In[3]:=
```
sol4 = FullSimplify[Solve[{ford4[-hm2] == fm2, ford4[-hm1] == fm1,
      ford4[hp1] == fp1, ford4[hp2] == fp2}, {a, b, c, d}, Reals]];
```

In[4]:=
```
D1ord4 = FullSimplify[ford4'[0] /. sol4];
```

In[5]:=
```
D2ord4 = FullSimplify[ford4''[0] /. sol4];
```

a) Coefficients for first derivative at centre point

In[6]:=
```
c1ord4 = {Row[FullSimplify[Coefficient[D1ord4, fm2]]],
  Row[FullSimplify[Coefficient[D1ord4, fm1]]],
  Row[FullSimplify[Coefficient[D1ord4, f0]]],
  Row[FullSimplify[Coefficient[D1ord4, fp1]]],
  Row[FullSimplify[Coefficient[D1ord4, fp2]]]}
```

Out[6]=
$$\left\{-\frac{\text{hm1 hp1 hp2}}{(\text{hm1}-\text{hm2})\,\text{hm2}\,(\text{hm2}+\text{hp1})\,(\text{hm2}+\text{hp2})},\right.$$
$$\frac{\text{hm2 hp1 hp2}}{\text{hm1}\,(\text{hm1}-\text{hm2})\,(\text{hm1}+\text{hp1})\,(\text{hm1}+\text{hp2})},\ \frac{1}{\text{hm1}}+\frac{1}{\text{hm2}}-\frac{\text{hp1}+\text{hp2}}{\text{hp1 hp2}},$$
$$\left.\frac{\text{hm1 hm2 hp2}}{\text{hp1}\,(\text{hm1}+\text{hp1})\,(\text{hm2}+\text{hp1})\,(-\text{hp1}+\text{hp2})},\ \frac{\text{hm1 hm2 hp1}}{(\text{hp1}-\text{hp2})\,\text{hp2}\,(\text{hm1}+\text{hp2})\,(\text{hm2}+\text{hp2})}\right\}$$

In[∘]:=
```
c1ord4 /. {hm2 → 2 h, hm1 → h, hp1 → h, hp2 → 2 h}
```

Out[∘]=
$$\left\{\frac{1}{12\,h},\ -\frac{2}{3\,h},\ 0,\ \frac{2}{3\,h},\ -\frac{1}{12\,h}\right\}$$

b) Coefficients for second derivative at centre point

In[8]:=
```
c2ord4 = {Row[FullSimplify[Coefficient[D2ord4, fm2]]],
   Row[FullSimplify[Coefficient[D2ord4, fm1]]],
   Row[FullSimplify[Coefficient[D2ord4, f0]]],
   Row[FullSimplify[Coefficient[D2ord4, fp1]]],
   Row[FullSimplify[Coefficient[D2ord4, fp2]]]}
```

Out[8]=

$$\left\{\frac{-2\,hp1\,hp2+2\,hm1\,(hp1+hp2)}{(hm1-hm2)\,hm2\,(hm2+hp1)\,(hm2+hp2)},\ \frac{2\,hp1\,hp2-2\,hm2\,(hp1+hp2)}{hm1\,(hm1-hm2)\,(hm1+hp1)\,(hm1+hp2)},\ \frac{2\,(hm1\,(hm2-hp1-hp2)+hp1\,hp2-hm2\,(hp1+hp2))}{hm1\,hm2\,hp1\,hp2},\ \frac{2\,hm1\,hm2-2\,(hm1+hm2)\,hp2}{hp1\,(hm1+hp1)\,(hm2+hp1)\,(hp1-hp2)},\ \frac{-2\,hm1\,hm2+2\,(hm1+hm2)\,hp1}{(hp1-hp2)\,hp2\,(hm1+hp2)\,(hm2+hp2)}\right\}$$

In[9]:=
```
c2ord4 /. {hm2 → 2 h, hm1 → h, hp1 → h, hp2 → 2 h}
```

Out[9]=

$$\left\{-\frac{1}{12\,h^2},\ \frac{4}{3\,h^2},\ -\frac{5}{2\,h^2},\ \frac{4}{3\,h^2},\ -\frac{1}{12\,h^2}\right\}$$

In[13]:=
```
FortranForm[c1ord4]
```

Out[13]//FortranForm=

```
        List(Row(List(-((hm1*hp1*hp2)/((hm1 - hm2)*hm2*(hm2 + hp1)*(hm2 + hp2))))),
     -  Row(List((hm2*hp1*hp2)/(hm1*(hm1 - hm2)*(hm1 + hp1)*(hm1 + hp2)))),
     -  Row(List(1/hm1 + 1/hm2 - (hp1 + hp2)/(hp1*hp2))),
     -  Row(List((hm1*hm2*hp2)/(hp1*(hm1 + hp1)*(hm2 + hp1)*(-hp1 + hp2)))),
     -  Row(List((hm1*hm2*hp1)/((hp1 - hp2)*hp2*(hm1 + hp2)*(hm2 + hp2)))))
```

In[15]:=
```
FortranForm[c2ord4]
```

Out[15]//FortranForm=

```
        List(Row(List((-2*hp1*hp2 + 2*hm1*(hp1 + hp2))/
     -      ((hm1 - hm2)*hm2*(hm2 + hp1)*(hm2 + hp2)))),
     -  Row(List((2*hp1*hp2 - 2*hm2*(hp1 + hp2))/
     -      (hm1*(hm1 - hm2)*(hm1 + hp1)*(hm1 + hp2)))),
     -  Row(List((2*(hm1*(hm2 - hp1 - hp2) + hp1*hp2 - hm2*(hp1 + hp2)))/
     -      (hm1*hm2*hp1*hp2))),Row(List((2*hm1*hm2 - 2*(hm1 + hm2)*hp2)/
     -      (hp1*(hm1 + hp1)*(hm2 + hp1)*(hp1 - hp2)))),
     -  Row(List((-2*hm1*hm2 + 2*(hm1 + hm2)*hp1)/
     -      ((hp1 - hp2)*hp2*(hm1 + hp2)*(hm2 + hp2)))))
```

c) Coefficients for first derivative at first point left

In[∘]:=
```
D1ord4l1 = FullSimplify[ford4'[-hm1] /. sol4];
```

In[∘]:=
```
D2ord4l1 = FullSimplify[ford4''[-hm1] /. sol4];
```

In[•]:=
```
c1ord4l1 = {Row[FullSimplify[Coefficient[D1ord4l1, fm2]]],
  Row[FullSimplify[Coefficient[D1ord4l1, fm1]]],
  Row[FullSimplify[Coefficient[D1ord4l1, f0]]],
  Row[FullSimplify[Coefficient[D1ord4l1, fp1]]],
  Row[FullSimplify[Coefficient[D1ord4l1, fp2]]]}
```

Out[•]=
$$\left\{\frac{\text{hm1}\,(\text{hm1}+\text{hp1})\,(\text{hm1}+\text{hp2})}{(\text{hm1}-\text{hm2})\,\text{hm2}\,(\text{hm2}+\text{hp1})\,(\text{hm2}+\text{hp2})},\right.$$
$$-\frac{1}{\text{hm1}}+\frac{1}{-\text{hm1}+\text{hm2}}-\frac{1}{\text{hm1}+\text{hp1}}-\frac{1}{\text{hm1}+\text{hp2}},\ -\frac{(\text{hm1}-\text{hm2})\,(\text{hm1}+\text{hp1})\,(\text{hm1}+\text{hp2})}{\text{hm1}\,\text{hm2}\,\text{hp1}\,\text{hp2}},$$
$$\left.-\frac{\text{hm1}\,(\text{hm1}-\text{hm2})\,(\text{hm1}+\text{hp2})}{\text{hp1}\,(\text{hm1}+\text{hp1})\,(\text{hm2}+\text{hp1})\,(\text{hp1}-\text{hp2})},\ \frac{\text{hm1}\,(\text{hm1}-\text{hm2})\,(\text{hm1}+\text{hp1})}{(\text{hp1}-\text{hp2})\,\text{hp2}\,(\text{hm1}+\text{hp2})\,(\text{hm2}+\text{hp2})}\right\}$$

In[•]:=
```
FullSimplify[c1ord4l1 /. {hm2 → 2 h, hm1 → h, hp1 → h, hp2 → 2 h}]
```

Out[•]=
$$\left\{-\frac{1}{4\,h},\ -\frac{5}{6\,h},\ \frac{3}{2\,h},\ -\frac{1}{2\,h},\ \frac{1}{12\,h}\right\}$$

In[•]:=
```
sol4l1 = FullSimplify[Solve[
    {ford4[hm1] == f1, ford4[h1] == f2, ford4[h2] == f3, ford4[h3] == f4}, {a, b, c, d}]];
```

In[•]:=
```
D1ord4l1 = FullSimplify[ford4'[0] /. sol4l1];
c1ord4l1 = {Row[FullSimplify[Coefficient[D1ord4l1, f1]]],
  Row[FullSimplify[Coefficient[D1ord4l1, f0]]],
  Row[FullSimplify[Coefficient[D1ord4l1, f2]]],
  Row[FullSimplify[Coefficient[D1ord4l1, f3]]],
  Row[FullSimplify[Coefficient[D1ord4l1, f4]]]}
```

Out[•]=
$$\left\{\frac{\text{h1}\,\text{h2}\,\text{h3}}{(\text{h1}-\text{hm1})\,\text{hm1}\,(-\text{h2}+\text{hm1})\,(-\text{h3}+\text{hm1})},\right.$$
$$-\frac{1}{\text{h1}}-\frac{1}{\text{h2}}-\frac{\text{h3}+\text{hm1}}{\text{h3}\,\text{hm1}},\ \frac{\text{h2}\,\text{h3}\,\text{hm1}}{\text{h1}\,(-\text{h1}+\text{h2})\,(-\text{h1}+\text{h3})\,(-\text{h1}+\text{hm1})},$$
$$\left.\frac{\text{h1}\,\text{h3}\,\text{hm1}}{(\text{h1}-\text{h2})\,\text{h2}\,(\text{h2}-\text{h3})\,(\text{h2}-\text{hm1})},\ \frac{\text{h1}\,\text{h2}\,\text{hm1}}{(\text{h1}-\text{h3})\,\text{h3}\,(-\text{h2}+\text{h3})\,(\text{h3}-\text{hm1})}\right\}$$

implement these

Out[•]=
$$\left\{\frac{\text{h1}\,\text{h2}\,\text{h3}}{(\text{h1}-\text{hm1})\,\text{hm1}\,(-\text{h2}+\text{hm1})\,(-\text{h3}+\text{hm1})},\right.$$
$$-\frac{1}{\text{h1}}-\frac{1}{\text{h2}}-\frac{\text{h3}+\text{hm1}}{\text{h3}\,\text{hm1}},\ \frac{\text{h2}\,\text{h3}\,\text{hm1}}{\text{h1}\,(-\text{h1}+\text{h2})\,(-\text{h1}+\text{h3})\,(-\text{h1}+\text{hm1})},$$
$$\left.\frac{\text{h1}\,\text{h3}\,\text{hm1}}{(\text{h1}-\text{h2})\,\text{h2}\,(\text{h2}-\text{h3})\,(\text{h2}-\text{hm1})},\ \frac{\text{h1}\,\text{h2}\,\text{hm1}}{(\text{h1}-\text{h3})\,\text{h3}\,(-\text{h2}+\text{h3})\,(\text{h3}-\text{hm1})}\right\}$$

In[•]:=
```
FullSimplify[c1ord4l1 /. {hm1 → -h, h1 → h, h2 → 2 h, h3 → 3 h}]
```

Out[•]=
$$\left\{-\frac{1}{4\,h},\ -\frac{5}{6\,h},\ \frac{3}{2\,h},\ -\frac{1}{2\,h},\ \frac{1}{12\,h}\right\}$$

second derivative at first point left

In[∘]:=
```
c2ord4l1 = {Row[FullSimplify[Coefficient[D2ord4l1, fm2]]],
  Row[FullSimplify[Coefficient[D2ord4l1, fm1]]],
  Row[FullSimplify[Coefficient[D2ord4l1, f0]]],
  Row[FullSimplify[Coefficient[D2ord4l1, fp1]]],
  Row[FullSimplify[Coefficient[D2ord4l1, fp2]]]}
```

Out[∘]=
$$\left\{-\frac{2\left(3\,\mathrm{hm1}^2+\mathrm{hp1}\,\mathrm{hp2}+2\,\mathrm{hm1}\left(\mathrm{hp1}+\mathrm{hp2}\right)\right)}{\left(\mathrm{hm1}-\mathrm{hm2}\right)\mathrm{hm2}\left(\mathrm{hm2}+\mathrm{hp1}\right)\left(\mathrm{hm2}+\mathrm{hp2}\right)},\right.$$
$$\left(2\left(6\,\mathrm{hm1}^2+\mathrm{hp1}\,\mathrm{hp2}-\mathrm{hm2}\left(\mathrm{hp1}+\mathrm{hp2}\right)+3\,\mathrm{hm1}\left(-\mathrm{hm2}+\mathrm{hp1}+\mathrm{hp2}\right)\right)\right)/$$
$$\left(\mathrm{hm1}\left(\mathrm{hm1}-\mathrm{hm2}\right)\left(\mathrm{hm1}+\mathrm{hp1}\right)\left(\mathrm{hm1}+\mathrm{hp2}\right)\right),$$
$$\left(6\,\mathrm{hm1}^2+2\,\mathrm{hp1}\,\mathrm{hp2}-2\,\mathrm{hm2}\left(\mathrm{hp1}+\mathrm{hp2}\right)+4\,\mathrm{hm1}\left(-\mathrm{hm2}+\mathrm{hp1}+\mathrm{hp2}\right)\right)/\left(\mathrm{hm1}\,\mathrm{hm2}\,\mathrm{hp1}\,\mathrm{hp2}\right),$$
$$\left.\frac{6\,\mathrm{hm1}^2-2\,\mathrm{hm2}\,\mathrm{hp2}+4\,\mathrm{hm1}\left(-\mathrm{hm2}+\mathrm{hp2}\right)}{\mathrm{hp1}\left(\mathrm{hm1}+\mathrm{hp1}\right)\left(\mathrm{hm2}+\mathrm{hp1}\right)\left(\mathrm{hp1}-\mathrm{hp2}\right)},\frac{-6\,\mathrm{hm1}^2+4\,\mathrm{hm1}\left(\mathrm{hm2}-\mathrm{hp1}\right)+2\,\mathrm{hm2}\,\mathrm{hp1}}{\left(\mathrm{hp1}-\mathrm{hp2}\right)\mathrm{hp2}\left(\mathrm{hm1}+\mathrm{hp2}\right)\left(\mathrm{hm2}+\mathrm{hp2}\right)}\right\}$$

In[∘]:=
```
FullSimplify[c2ord4l1 /. {hm2 → 2 h, hm1 → h, hp1 → h, hp2 → 2 h}]
```

Out[∘]=
$$\left\{\frac{11}{12\,h^2},-\frac{5}{3\,h^2},\frac{1}{2\,h^2},\frac{1}{3\,h^2},-\frac{1}{12\,h^2}\right\}$$

In[∘]:=
```
D2ord4l1 = FullSimplify[ford4''[0] /. sol4l1];
c2ord4l1 = {Row[FullSimplify[Coefficient[D2ord4l1, f1]]],
  Row[FullSimplify[Coefficient[D2ord4l1, f0]]],
  Row[FullSimplify[Coefficient[D2ord4l1, f2]]],
  Row[FullSimplify[Coefficient[D2ord4l1, f3]]],
  Row[FullSimplify[Coefficient[D2ord4l1, f4]]]}
```

Out[∘]=
$$\left\{-\frac{2\left(\mathrm{h2}\,\mathrm{h3}+\mathrm{h1}\left(\mathrm{h2}+\mathrm{h3}\right)\right)}{\left(\mathrm{h1}-\mathrm{hm1}\right)\mathrm{hm1}\left(-\mathrm{h2}+\mathrm{hm1}\right)\left(-\mathrm{h3}+\mathrm{hm1}\right)},\right.$$
$$\frac{2\left(\mathrm{h3}\,\mathrm{hm1}+\mathrm{h2}\left(\mathrm{h3}+\mathrm{hm1}\right)+\mathrm{h1}\left(\mathrm{h2}+\mathrm{h3}+\mathrm{hm1}\right)\right)}{\mathrm{h1}\,\mathrm{h2}\,\mathrm{h3}\,\mathrm{hm1}},\frac{2\left(\mathrm{h3}\,\mathrm{hm1}+\mathrm{h2}\left(\mathrm{h3}+\mathrm{hm1}\right)\right)}{\mathrm{h1}\left(\mathrm{h1}-\mathrm{h2}\right)\left(\mathrm{h1}-\mathrm{h3}\right)\left(\mathrm{h1}-\mathrm{hm1}\right)},$$
$$\left.-\frac{2\left(\mathrm{h3}\,\mathrm{hm1}+\mathrm{h1}\left(\mathrm{h3}+\mathrm{hm1}\right)\right)}{\left(\mathrm{h1}-\mathrm{h2}\right)\mathrm{h2}\left(\mathrm{h2}-\mathrm{h3}\right)\left(\mathrm{h2}-\mathrm{hm1}\right)},-\frac{2\left(\mathrm{h2}\,\mathrm{hm1}+\mathrm{h1}\left(\mathrm{h2}+\mathrm{hm1}\right)\right)}{\left(\mathrm{h1}-\mathrm{h3}\right)\mathrm{h3}\left(-\mathrm{h2}+\mathrm{h3}\right)\left(\mathrm{h3}-\mathrm{hm1}\right)}\right\}$$

implement these

Out[∘]=
$$\left\{-\frac{2\left(\mathrm{h2}\,\mathrm{h3}+\mathrm{h1}\left(\mathrm{h2}+\mathrm{h3}\right)\right)}{\left(\mathrm{h1}-\mathrm{hm1}\right)\mathrm{hm1}\left(-\mathrm{h2}+\mathrm{hm1}\right)\left(-\mathrm{h3}+\mathrm{hm1}\right)},\right.$$
$$\frac{2\left(\mathrm{h3}\,\mathrm{hm1}+\mathrm{h2}\left(\mathrm{h3}+\mathrm{hm1}\right)+\mathrm{h1}\left(\mathrm{h2}+\mathrm{h3}+\mathrm{hm1}\right)\right)}{\mathrm{h1}\,\mathrm{h2}\,\mathrm{h3}\,\mathrm{hm1}},\frac{2\left(\mathrm{h3}\,\mathrm{hm1}+\mathrm{h2}\left(\mathrm{h3}+\mathrm{hm1}\right)\right)}{\mathrm{h1}\left(\mathrm{h1}-\mathrm{h2}\right)\left(\mathrm{h1}-\mathrm{h3}\right)\left(\mathrm{h1}-\mathrm{hm1}\right)},$$
$$\left.-\frac{2\left(\mathrm{h3}\,\mathrm{hm1}+\mathrm{h1}\left(\mathrm{h3}+\mathrm{hm1}\right)\right)}{\left(\mathrm{h1}-\mathrm{h2}\right)\mathrm{h2}\left(\mathrm{h2}-\mathrm{h3}\right)\left(\mathrm{h2}-\mathrm{hm1}\right)},-\frac{2\left(\mathrm{h2}\,\mathrm{hm1}+\mathrm{h1}\left(\mathrm{h2}+\mathrm{hm1}\right)\right)}{\left(\mathrm{h1}-\mathrm{h3}\right)\mathrm{h3}\left(-\mathrm{h2}+\mathrm{h3}\right)\left(\mathrm{h3}-\mathrm{hm1}\right)}\right\}$$

In[∘]:=
```
FullSimplify[c2ord4l1 /. {hm1 → -h, h1 → h, h2 → 2 h, h3 → 3 h}]
```

Out[∘]=
$$\left\{\frac{11}{12\,h^2},-\frac{5}{3\,h^2},\frac{1}{2\,h^2},\frac{1}{3\,h^2},-\frac{1}{12\,h^2}\right\}$$

In[•]:=

d) Coefficients for first derivative at second point left

```
In[•]:= D1ord4l2 = FullSimplify[ford4'[-hm2] /. sol4];
```

```
In[•]:= c1ord4l2 = {Row[FullSimplify[Coefficient[D1ord4l2, fm2]]],
   Row[FullSimplify[Coefficient[D1ord4l2, fm1]]],
   Row[FullSimplify[Coefficient[D1ord4l2, f0]]],
   Row[FullSimplify[Coefficient[D1ord4l2, fp1]]],
   Row[FullSimplify[Coefficient[D1ord4l2, fp2]]]}
```

Out[•]=
$$\left\{\frac{hm1-2\,hm2}{hm2\,(-hm1+hm2)}-\frac{1}{hm2+hp1}-\frac{1}{hm2+hp2},\ -\frac{hm2\,(hm2+hp1)\,(hm2+hp2)}{hm1\,(hm1-hm2)\,(hm1+hp1)\,(hm1+hp2)},\ \frac{(hm1-hm2)\,(hm2+hp1)\,(hm2+hp2)}{hm1\,hm2\,hp1\,hp2},\ \frac{(hm1-hm2)\,hm2\,(hm2+hp2)}{hp1\,(hm1+hp1)\,(hm2+hp1)\,(hp1-hp2)},\ -\frac{(hm1-hm2)\,hm2\,(hm2+hp1)}{(hp1-hp2)\,hp2\,(hm1+hp2)\,(hm2+hp2)}\right\}$$

```
In[•]:= FullSimplify[c1ord4l2 /. {hm2 → 2 h, hm1 → h, hp1 → h, hp2 → 2 h}]
```

Out[•]=
$$\left\{-\frac{25}{12\,h},\ \frac{4}{h},\ -\frac{3}{h},\ \frac{4}{3\,h},\ -\frac{1}{4\,h}\right\}$$

```
In[•]:= sol4l2 = FullSimplify[Solve[
     {ford4[h1] == f1, ford4[h2] == f2, ford4[h3] == f3, ford4[h4] == f4}, {a, b, c, d}]];
```

```
In[•]:= D1ord4l2 = FullSimplify[ford4'[0] /. sol4l2]
```

Out[•]=
$$\left\{f0\left(-\frac{1}{h1}-\frac{1}{h2}-\frac{1}{h3}-\frac{1}{h4}\right)+\frac{f2\,h1\,h3\,h4}{(h1-h2)\,h2\,(h2-h3)\,(h2-h4)}+\frac{f3\,h1\,h2\,h4}{(h1-h3)\,h3\,(-h2+h3)\,(h3-h4)}+\frac{f1\,h2\,h3\,h4}{h1\,(-h1+h2)\,(-h1+h3)\,(-h1+h4)}+\frac{f4\,h1\,h2\,h3}{(h1-h4)\,h4\,(-h2+h4)\,(-h3+h4)}\right\}$$

```
In[•]:= c1ord4l2 = {Row[FullSimplify[Coefficient[D1ord4l2, f0]]],
   Row[FullSimplify[Coefficient[D1ord4l2, f1]]],
   Row[FullSimplify[Coefficient[D1ord4l2, f2]]],
   Row[FullSimplify[Coefficient[D1ord4l2, f3]]],
   Row[FullSimplify[Coefficient[D1ord4l2, f4]]]}
```

Out[•]=
$$\left\{-\frac{1}{h1}-\frac{1}{h2}-\frac{h3+h4}{h3\,h4},\ \frac{h2\,h3\,h4}{h1\,(-h1+h2)\,(-h1+h3)\,(-h1+h4)},\ \frac{h1\,h3\,h4}{(h1-h2)\,h2\,(h2-h3)\,(h2-h4)},\ \frac{h1\,h2\,h4}{(h1-h3)\,h3\,(-h2+h3)\,(h3-h4)},\ \frac{h1\,h2\,h3}{(h1-h4)\,h4\,(-h2+h4)\,(-h3+h4)}\right\}$$

implement these

Out[•]= $\left\{-\frac{1}{h1}-\frac{1}{h2}-\frac{h3+h4}{h3\,h4},\ \frac{h2\,h3\,h4}{h1\,(-h1+h2)\,(-h1+h3)\,(-h1+h4)},\ \frac{h1\,h3\,h4}{(h1-h2)\,h2\,(h2-h3)\,(h2-h4)},\ \frac{h1\,h2\,h4}{(h1-h3)\,h3\,(-h2+h3)\,(h3-h4)},\ \frac{h1\,h2\,h3}{(h1-h4)\,h4\,(-h2+h4)\,(-h3+h4)}\right\}$

In[•]:=
```
c1ord4l2 /. {h1 → h, h2 → 2 h, h3 → 3 h, h4 → 4 h}
```

Out[•]= $\left\{-\frac{25}{12\,h},\ \frac{4}{h},\ -\frac{3}{h},\ \frac{4}{3\,h},\ -\frac{1}{4\,h}\right\}$

e) Coefficients for first and second derivative at first point right

In[•]:=
```
D1ord4r1 = FullSimplify[ford4'[hp1] /. sol4];
```

In[•]:=
```
D2ord4r1 = FullSimplify[ford4''[hp1] /. sol4];
```

In[•]:=
```
c1ord4r1 = {Row[FullSimplify[Coefficient[D1ord4r1, fm2]]],
  Row[FullSimplify[Coefficient[D1ord4r1, fm1]]],
  Row[FullSimplify[Coefficient[D1ord4r1, f0]]],
  Row[FullSimplify[Coefficient[D1ord4r1, fp1]]],
  Row[FullSimplify[Coefficient[D1ord4r1, fp2]]]}
```

Out[•]= $\left\{-\frac{hp1\,(hm1+hp1)\,(hp1-hp2)}{(hm1-hm2)\,hm2\,(hm2+hp1)\,(hm2+hp2)},\ \frac{hp1\,(hm2+hp1)\,(hp1-hp2)}{hm1\,(hm1-hm2)\,(hm1+hp1)\,(hm1+hp2)},\ \frac{(hm1+hp1)\,(hm2+hp1)\,(hp1-hp2)}{hm1\,hm2\,hp1\,hp2},\ \frac{1}{hp1}+\frac{1}{hm1+hp1}+\frac{1}{hm2+hp1}+\frac{1}{hp1-hp2},\ -\frac{hp1\,(hm1+hp1)\,(hm2+hp1)}{(hp1-hp2)\,hp2\,(hm1+hp2)\,(hm2+hp2)}\right\}$

In[•]:=
```
FullSimplify[c1ord4r1 /. {hm2 → 2 h, hm1 → h, hp1 → h, hp2 → 2 h}]
```

Out[•]= $\left\{-\frac{1}{12\,h},\ \frac{1}{2\,h},\ -\frac{3}{2\,h},\ \frac{5}{6\,h},\ \frac{1}{4\,h}\right\}$

In[•]:=
```
sol4r1 = FullSimplify[Solve[
    {ford4[h3] == f1, ford4[h2] == f2, ford4[h1] == f3, ford4[hp1] == f4}, {a, b, c, d}]];
D1ord4r1 = FullSimplify[ford4'[0] /. sol4r1];
```

In[•]:=
```
c1ord4r1 = {Row[FullSimplify[Coefficient[D1ord4r1, f1]]],
  Row[FullSimplify[Coefficient[D1ord4r1, f2]]],
  Row[FullSimplify[Coefficient[D1ord4r1, f3]]],
  Row[FullSimplify[Coefficient[D1ord4r1, f0]]],
  Row[FullSimplify[Coefficient[D1ord4r1, f4]]]}
```

Out[•]=
$$\left\{\frac{h1\ h2\ hp1}{(h1-h3)\ h3\ (-h2+h3)\ (h3-hp1)},\ \frac{h1\ h3\ hp1}{(h1-h2)\ h2\ (h2-h3)\ (h2-hp1)},\ \frac{h2\ h3\ hp1}{h1\ (-h1+h2)\ (-h1+h3)\ (-h1+hp1)},\ -\frac{1}{h1}-\frac{1}{h2}-\frac{h3+hp1}{h3\ hp1},\ \frac{h1\ h2\ h3}{(h1-hp1)\ hp1\ (-h2+hp1)\ (-h3+hp1)}\right\}$$

implement these

Out[•]=
$$\left\{\frac{h1\ h2\ hp1}{(h1-h3)\ h3\ (-h2+h3)\ (h3-hp1)},\ \frac{h1\ h3\ hp1}{(h1-h2)\ h2\ (h2-h3)\ (h2-hp1)},\ \frac{h2\ h3\ hp1}{h1\ (-h1+h2)\ (-h1+h3)\ (-h1+hp1)},\ -\frac{1}{h1}-\frac{1}{h2}-\frac{h3+hp1}{h3\ hp1},\ \frac{h1\ h2\ h3}{(h1-hp1)\ hp1\ (-h2+hp1)\ (-h3+hp1)}\right\}$$

In[•]:=
```
FullSimplify[c1ord4r1 /. {h3 → -3 h, h2 → -2 h, h1 → -h, hp1 → h}]
```

Out[•]=
$$\left\{-\frac{1}{12\ h},\ \frac{1}{2\ h},\ -\frac{3}{2\ h},\ \frac{5}{6\ h},\ \frac{1}{4\ h}\right\}$$

In[•]:=
$$\left\{-\frac{1}{12\ h},\ \frac{1}{2\ h},\ -\frac{3}{2\ h},\ \frac{5}{6\ h},\ \frac{1}{4\ h}\right\}$$

Out[•]=
$$\left\{-\frac{1}{12\ h},\ \frac{1}{2\ h},\ -\frac{3}{2\ h},\ \frac{5}{6\ h},\ \frac{1}{4\ h}\right\}$$

second derivative

In[•]:=
```
c2ord4r1 = {Row[FullSimplify[Coefficient[D2ord4r1, fm2]]],
  Row[FullSimplify[Coefficient[D2ord4r1, fm1]]],
  Row[FullSimplify[Coefficient[D2ord4r1, f0]]],
  Row[FullSimplify[Coefficient[D2ord4r1, fp1]]],
  Row[FullSimplify[Coefficient[D2ord4r1, fp2]]]}
```

Out[•]=
$$\left\{\frac{-6\ hp1^2+4\ hp1\ hp2+2\ hm1\ (-2\ hp1+hp2)}{(hm1-hm2)\ hm2\ (hm2+hp1)\ (hm2+hp2)},\ \frac{4\ hm2\ hp1+6\ hp1^2-2\ hm2\ hp2-4\ hp1\ hp2}{hm1\ (hm1-hm2)\ (hm1+hp1)\ (hm1+hp2)},\right.$$
$$(2\ hm1\ hm2+4\ hm1\ hp1+4\ hm2\ hp1+6\ hp1^2-2\ (hm1+hm2+2\ hp1)\ hp2)\ /\ (hm1\ hm2\ hp1\ hp2),$$
$$(2\ (hm1\ hm2+3\ hm1\ hp1+3\ hm2\ hp1+6\ hp1^2-(hm1+hm2+3\ hp1)\ hp2))\ /$$
$$\left.(hp1\ (hm1+hp1)\ (hm2+hp1)\ (hp1-hp2)),\ -\frac{2\ (hm1\ hm2+2\ (hm1+hm2)\ hp1+3\ hp1^2)}{(hp1-hp2)\ hp2\ (hm1+hp2)\ (hm2+hp2)}\right\}$$

In[∘]:=
```
FullSimplify[c2ord4r1 /. {hm2 → 2 h, hm1 → h, hp1 → h, hp2 → 2 h}]
```

Out[∘]= $\left\{-\frac{1}{12\,h^2}, \frac{1}{3\,h^2}, \frac{1}{2\,h^2}, -\frac{5}{3\,h^2}, \frac{11}{12\,h^2}\right\}$

```
D2ord4r1 = FullSimplify[ford4''[0] /. sol4r1];
c2ord4r1 = {Row[FullSimplify[Coefficient[D2ord4r1, f1]]],
   Row[FullSimplify[Coefficient[D2ord4r1, f2]]],
   Row[FullSimplify[Coefficient[D2ord4r1, f3]]],
   Row[FullSimplify[Coefficient[D2ord4r1, f0]]],
   Row[FullSimplify[Coefficient[D2ord4r1, f4]]]}
```

implement these

Out[∘]= $$\left\{-\frac{2\,(h2\,hp1 + h1\,(h2 + hp1))}{(h1 - h3)\,h3\,(-h2 + h3)\,(h3 - hp1)}, -\frac{2\,(h3\,hp1 + h1\,(h3 + hp1))}{(h1 - h2)\,h2\,(h2 - h3)\,(h2 - hp1)}, \frac{2\,(h3\,hp1 + h2\,(h3 + hp1))}{h1\,(h1 - h2)\,(h1 - h3)\,(h1 - hp1)}, \frac{2\,(h3\,hp1 + h2\,(h3 + hp1) + h1\,(h2 + h3 + hp1))}{h1\,h2\,h3\,hp1}, -\frac{2\,(h2\,h3 + h1\,(h2 + h3))}{(h1 - hp1)\,hp1\,(-h2 + hp1)\,(-h3 + hp1)}\right\}$$

In[∘]:=
```
FullSimplify[c2ord4r1 /. {h3 → -3 h, h2 → -2 h, h1 → -h, hp1 → h}]
```

Out[∘]= $\left\{-\frac{1}{12\,h^2}, \frac{1}{3\,h^2}, \frac{1}{2\,h^2}, -\frac{5}{3\,h^2}, \frac{11}{12\,h^2}\right\}$

---

f) Coefficients for first derivative at second point right

In[∘]:=
```
D1ord4r2 = FullSimplify[ford4'[hp2] /. sol4];
```

In[∘]:=
```
c1ord4r2 = {Row[FullSimplify[Coefficient[D1ord4r2, fm2]]],
   Row[FullSimplify[Coefficient[D1ord4r2, fm1]]],
   Row[FullSimplify[Coefficient[D1ord4r2, f0]]],
   Row[FullSimplify[Coefficient[D1ord4r2, fp1]]],
   Row[FullSimplify[Coefficient[D1ord4r2, fp2]]]}
```

Out[∘]= $$\left\{\frac{(hp1 - hp2)\,hp2\,(hm1 + hp2)}{(hm1 - hm2)\,hm2\,(hm2 + hp1)\,(hm2 + hp2)}, -\frac{(hp1 - hp2)\,hp2\,(hm2 + hp2)}{hm1\,(hm1 - hm2)\,(hm1 + hp1)\,(hm1 + hp2)}, -\frac{(hp1 - hp2)\,(hm1 + hp2)\,(hm2 + hp2)}{hm1\,hm2\,hp1\,hp2}, \frac{hp2\,(hm1 + hp2)\,(hm2 + hp2)}{hp1\,(hm1 + hp1)\,(hm2 + hp1)\,(hp1 - hp2)}, \frac{1}{hp2} + \frac{1}{hm1 + hp2} + \frac{1}{hm2 + hp2} + \frac{1}{-hp1 + hp2}\right\}$$

In[∘]:=
```
FullSimplify[c1ord4r2 /. {hm2 → 2 h, hm1 → h, hp1 → h, hp2 → 2 h}]
```

Out[∘]= $\left\{\frac{1}{4\,h}, -\frac{4}{3\,h}, \frac{3}{h}, -\frac{4}{h}, \frac{25}{12\,h}\right\}$

In[∘]:=
```
sol4r2 = FullSimplify[Solve[
    {ford4[h4] == f1, ford4[h3] == f2, ford4[h2] == f3, ford4[h1] == f4}, {a, b, c, d}]];
D1ord4r2 = FullSimplify[ford4'[0] /. sol4r2];
```

In[∘]:=
```
c1ord4r2 = {Row[FullSimplify[Coefficient[D1ord4r2, f1]]],
  Row[FullSimplify[Coefficient[D1ord4r2, f2]]],
  Row[FullSimplify[Coefficient[D1ord4r2, f3]]],
  Row[FullSimplify[Coefficient[D1ord4r2, f4]]],
  Row[FullSimplify[Coefficient[D1ord4r2, f0]]]}
```

Out[∘]=
$$\left\{\frac{h1\ h2\ h3}{(h1-h4)\ h4\ (-h2+h4)\ (-h3+h4)},\ \frac{h1\ h2\ h4}{(h1-h3)\ h3\ (-h2+h3)\ (h3-h4)},\ \frac{h1\ h3\ h4}{(h1-h2)\ h2\ (h2-h3)\ (h2-h4)},\ \frac{h2\ h3\ h4}{h1\ (-h1+h2)\ (-h1+h3)\ (-h1+h4)},\ -\frac{1}{h1}-\frac{1}{h2}-\frac{h3+h4}{h3\ h4}\right\}$$

implement these

Out[∘]=
$$\left\{\frac{h1\ h2\ h3}{(h1-h4)\ h4\ (-h2+h4)\ (-h3+h4)},\ \frac{h1\ h2\ h4}{(h1-h3)\ h3\ (-h2+h3)\ (h3-h4)},\ \frac{h1\ h3\ h4}{(h1-h2)\ h2\ (h2-h3)\ (h2-h4)},\ \frac{h2\ h3\ h4}{h1\ (-h1+h2)\ (-h1+h3)\ (-h1+h4)},\ -\frac{1}{h1}-\frac{1}{h2}-\frac{h3+h4}{h3\ h4}\right\}$$

In[∘]:=
```
FullSimplify[c1ord4r2 /. {h4 → -4 h, h3 → -3 h, h2 → -2 h, h1 → -h}]
```

Out[∘]=
$$\left\{\frac{1}{4\ h},\ -\frac{4}{3\ h},\ \frac{3}{h},\ -\frac{4}{h},\ \frac{25}{12\ h}\right\}$$