

```
In[ ]:= ClearAll[f0, fl, fr, a, b, c, d, hl, hr, hm1, hm2, hp1, hp2, fm1, fm2, fp1, fp2]
```

Second order approximation:

```
In[ ]:= ford2[h_] := f0 + a h + b h^2
```

```
In[ ]:= sol2 = FullSimplify[Solve[{ford2[-hl] == fl, ford2[hr] == fr}, {a, b}]]
```

```
Out[ ]:= { {a -> ((-f0 + fr) hl^2 + (f0 - fl) hr^2) / (hl hr (hl + hr)), b -> (fr hl + fl hr - f0 (hl + hr)) / (hl hr (hl + hr)) }
```

```
In[ ]:= D1ord2 = FullSimplify[ford2'[0] /. sol2]
```

```
Out[ ]:= { (-f0 + fr) hl^2 + (f0 - fl) hr^2 / (hl hr (hl + hr)) }
```

```
In[ ]:= D2ord2 = FullSimplify[ford2''[0] /. sol2]
```

```
Out[ ]:= { 2 (fr hl + fl hr - f0 (hl + hr)) / (hl hr (hl + hr)) }
```

a) Coefficients for first derivative at centre point

```
In[ ]:= c1ord2 = {Row[FullSimplify[Coefficient[D1ord2, fl]]],  
                Row[FullSimplify[Coefficient[D1ord2, f0]]],  
                Row[FullSimplify[Coefficient[D1ord2, fr]]]}
```

```
Out[ ]:= { -hr / (hl (hl + hr)), 1 / hl - 1 / hr, hl / (hr (hl + hr)) }
```

```
In[ ]:= c1ord2 /. {hl -> h, hr -> h}
```

```
Out[ ]:= { -1 / (2 h), 0, 1 / (2 h) }
```

b) Coefficients for second derivative at centre point

```
In[ ]:= c2ord2 = {Row[FullSimplify[Coefficient[D2ord2, fl]]],  
                Row[FullSimplify[Coefficient[D2ord2, f0]]],  
                Row[FullSimplify[Coefficient[D2ord2, fr]]]}
```

```
Out[ ]:= { 2 / (hl (hl + hr)), -2 / (hl hr), 2 / (hr (hl + hr)) }
```

```
In[ ]:= c2ord2 /. {hl -> h, hr -> h}
```

```
Out[ ]:= { 1 / h^2, -2 / h^2, 1 / h^2 }
```

c) Coefficients for first derivative at left point

```
In[ ]:= D1ord2l = FullSimplify[ford2'[-hl] /. sol2];
```

```
In[ ]:= c1ord2l = {Row[FullSimplify[Coefficient[D1ord2l, fl]]],
  Row[FullSimplify[Coefficient[D1ord2l, f0]]],
  Row[FullSimplify[Coefficient[D1ord2l, fr]]]}
```

$$\text{Out[]} = \left\{ -\frac{1}{hl} - \frac{1}{hl + hr}, \frac{1}{hl} + \frac{1}{hr}, -\frac{hl}{hl \, hr + hr^2} \right\}$$

```
In[ ]:= c1ord2l /. {hl -> h, hr -> h}
```

$$\text{Out[]} = \left\{ -\frac{3}{2 \, h}, \frac{2}{h}, -\frac{1}{2 \, h} \right\}$$

```
In[ ]:= sol2l = FullSimplify[Solve[{ford2[h2] == f2, ford2[h3] == f3}, {a, b}]];
  (** as in pdf **)
```

```
In[ ]:= D1ord2ll = FullSimplify[ford2'[0] /. sol2l];
```

$$\text{Out[]} = \frac{3}{2}$$

```
In[ ]:= c1ord2ll = {Row[FullSimplify[Coefficient[D1ord2ll, f0]]],
  Row[FullSimplify[Coefficient[D1ord2ll, f2]]],
  Row[FullSimplify[Coefficient[D1ord2ll, f3]]]}
```

$$\text{Out[]} = \left\{ -\frac{h2+h3}{h2 \, h3}, \frac{1}{h2} + \frac{1}{-h2+h3}, \frac{h2}{(h2-h3) \, h3} \right\}$$

```
In[ ]:= c1ord2ll /. {h2 -> h, h3 -> 2 h}
  (** should be same thing **)
```

$$\text{Out[]} = \left\{ -\frac{3}{2 \, h}, \frac{2}{h}, -\frac{1}{2 \, h} \right\}$$

d) Coefficients for first derivative at right point

```
In[ ]:= D1ord2r = FullSimplify[ford2'[hr] /. sol2];
```

```
In[ ]:= c1ord2r = {Row[FullSimplify[Coefficient[D1ord2r, fl]]],
  Row[FullSimplify[Coefficient[D1ord2r, f0]]],
  Row[FullSimplify[Coefficient[D1ord2r, fr]]]}
```

$$\text{Out[]} = \left\{ \frac{hr}{hl^2 + hl \, hr}, -\frac{hl + hr}{hl \, hr}, \frac{1}{hr} + \frac{1}{hl + hr} \right\}$$

```
In[ ]:= c1ord2r /. {hl -> h, hr -> h}
```

$$\text{Out[]} = \left\{ \frac{1}{2 \, h}, -\frac{2}{h}, \frac{3}{2 \, h} \right\}$$

```
In[ ]:= sol2r = FullSimplify[Solve[{ford2[-hm1] == fm1, ford2[-hm2] == fm2}, {a, b}]];
  (** as in pdf **)
```

```
In[ ]:= D1ord2rr = FullSimplify[ford2'[0] /. sol2r];
```

```
In[ ]:= c1ord2rr = {Row[FullSimplify[Coefficient[D1ord2rr, fm2]]],
  Row[FullSimplify[Coefficient[D1ord2rr, fm1]]],
  Row[FullSimplify[Coefficient[D1ord2rr, f0]]]}
```

$$\text{Out[]} = \left\{ \frac{hm1}{hm2 (-hm1 + hm2)}, \frac{hm2}{hm1^2 - hm1 hm2}, \frac{1}{hm1} + \frac{1}{hm2} \right\}$$

```
In[ ]:= c1ord2rr /. {hm1 -> h, hm2 -> 2 h}
(***) should be same thing (***)
```

$$\text{Out[]} = \left\{ \frac{1}{2 h}, -\frac{2}{h}, \frac{3}{2 h} \right\}$$

Forth order approximation:

```
In[1]:= ClearAll[f0, a, b, c, d, hm1, hm2, hp1, hp2, fm1,
  fm2, fp1, fp2, hl2, hl1, hr1, hr2, hr3, f1, f2, f3, f4]
```

```
In[2]:= ford4[h_] := f0 + a h + b h^2 + c h^3 + d h^4
```

```
In[3]:= sol4 = FullSimplify[Solve[{ford4[-hm2] == fm2, ford4[-hm1] == fm1,
  ford4[hp1] == fp1, ford4[hp2] == fp2}, {a, b, c, d}, Reals]];
```

```
In[4]:= D1ord4 = FullSimplify[ford4'[0] /. sol4];
```

```
In[5]:= D2ord4 = FullSimplify[ford4''[0] /. sol4];
```

a) Coefficients for first derivative at centre point

```
In[6]:= c1ord4 = {Row[FullSimplify[Coefficient[D1ord4, fm2]]],
  Row[FullSimplify[Coefficient[D1ord4, fm1]]],
  Row[FullSimplify[Coefficient[D1ord4, f0]]],
  Row[FullSimplify[Coefficient[D1ord4, fp1]]],
  Row[FullSimplify[Coefficient[D1ord4, fp2]]]}
```

$$\text{Out[6]} = \left\{ -\frac{hm1 hp1 hp2}{(hm1 - hm2) hm2 (hm2 + hp1) (hm2 + hp2)}, \frac{hm2 hp1 hp2}{hm1 (hm1 - hm2) (hm1 + hp1) (hm1 + hp2)}, \frac{1}{hm1} + \frac{1}{hm2} - \frac{hp1 + hp2}{hp1 hp2}, \frac{hm1 hm2 hp2}{hp1 (hm1 + hp1) (hm2 + hp1) (-hp1 + hp2)}, \frac{hm1 hm2 hp1}{(hp1 - hp2) hp2 (hm1 + hp2) (hm2 + hp2)} \right\}$$

```
In[ ]:= c1ord4 /. {hm2 -> 2 h, hm1 -> h, hp1 -> h, hp2 -> 2 h}
```

$$\text{Out[]} = \left\{ \frac{1}{12 h}, -\frac{2}{3 h}, 0, \frac{2}{3 h}, -\frac{1}{12 h} \right\}$$

b) Coefficients for second derivative at centre point

```
In[8]:= c2ord4 = {Row[FullSimplify[Coefficient[D2ord4, fm2]]],
  Row[FullSimplify[Coefficient[D2ord4, fm1]]],
  Row[FullSimplify[Coefficient[D2ord4, f0]]],
  Row[FullSimplify[Coefficient[D2ord4, fp1]]],
  Row[FullSimplify[Coefficient[D2ord4, fp2]]]}
```

$$\text{Out[8]} = \left\{ \frac{-2 \, hp1 \, hp2 + 2 \, hm1 \, (hp1 + hp2)}{(hm1 - hm2) \, hm2 \, (hm2 + hp1) \, (hm2 + hp2)}, \frac{2 \, hp1 \, hp2 - 2 \, hm2 \, (hp1 + hp2)}{hm1 \, (hm1 - hm2) \, (hm1 + hp1) \, (hm1 + hp2)}, \right. \\ \left. \frac{2 \, (hm1 \, (hm2 - hp1 - hp2) + hp1 \, hp2 - hm2 \, (hp1 + hp2))}{hm1 \, hm2 \, hp1 \, hp2}, \frac{2 \, hm1 \, hm2 - 2 \, (hm1 + hm2) \, hp2}{hp1 \, (hm1 + hp1) \, (hm2 + hp1) \, (hp1 - hp2)}, \frac{-2 \, hm1 \, hm2 + 2 \, (hm1 + hm2) \, hp1}{(hp1 - hp2) \, hp2 \, (hm1 + hp2) \, (hm2 + hp2)} \right\}$$

```
In[9]:= c2ord4 /. {hm2 -> 2 h, hm1 -> h, hp1 -> h, hp2 -> 2 h}
```

$$\text{Out[9]} = \left\{ -\frac{1}{12 \, h^2}, \frac{4}{3 \, h^2}, -\frac{5}{2 \, h^2}, \frac{4}{3 \, h^2}, -\frac{1}{12 \, h^2} \right\}$$

```
In[13]:= FortranForm[c1ord4]
```

```
Out[13]//FortranForm=
```

```
List(Row(List(-( (hm1*hp1*hp2)/( (hm1 - hm2)*hm2*(hm2 + hp1)*(hm2 + hp2)) )),
- Row(List((hm2*hp1*hp2)/(hm1*(hm1 - hm2)*(hm1 + hp1)*(hm1 + hp2))),
- Row(List(1/hm1 + 1/hm2 - (hp1 + hp2)/(hp1*hp2))),
- Row(List((hm1*hm2*hp2)/(hp1*(hm1 + hp1)*(hm2 + hp1)*(-hp1 + hp2))),
- Row(List((hm1*hm2*hp1)/( (hp1 - hp2)*hp2*(hm1 + hp2)*(hm2 + hp2)))))
```

```
In[15]:= FortranForm[c2ord4]
```

```
Out[15]//FortranForm=
```

```
List(Row(List((-2*hp1*hp2 + 2*hm1*(hp1 + hp2))/
- ((hm1 - hm2)*hm2*(hm2 + hp1)*(hm2 + hp2))),
- Row(List((2*hp1*hp2 - 2*hm2*(hp1 + hp2))/
- (hm1*(hm1 - hm2)*(hm1 + hp1)*(hm1 + hp2))),
- Row(List((2*(hm1*(hm2 - hp1 - hp2) + hp1*hp2 - hm2*(hp1 + hp2))/
- (hm1*hm2*hp1*hp2))),Row(List((2*hm1*hm2 - 2*(hm1 + hm2)*hp2)/
- (hp1*(hm1 + hp1)*(hm2 + hp1)*(hp1 - hp2))),
- Row(List((-2*hm1*hm2 + 2*(hm1 + hm2)*hp1)/
- ((hp1 - hp2)*hp2*(hm1 + hp2)*(hm2 + hp2)))))
```

c) Coefficients for first derivative at first point left

```
In[*]:= D1ord4l1 = FullSimplify[ford4'[-hm1] /. sol4];
```

```
In[*]:= D2ord4l1 = FullSimplify[ford4''[-hm1] /. sol4];
```

```
In[ ]:= c1ord4l1 = {Row[FullSimplify[Coefficient[D1ord4l1, fm2]]],
  Row[FullSimplify[Coefficient[D1ord4l1, fm1]]],
  Row[FullSimplify[Coefficient[D1ord4l1, f0]]],
  Row[FullSimplify[Coefficient[D1ord4l1, fp1]]],
  Row[FullSimplify[Coefficient[D1ord4l1, fp2]]]}
```

$$\text{Out[]} = \left\{ \frac{hm1 (hm1 + hp1) (hm1 + hp2)}{(hm1 - hm2) hm2 (hm2 + hp1) (hm2 + hp2)}, \right. \\ \left. -\frac{1}{hm1} + \frac{1}{-hm1 + hm2} - \frac{1}{hm1 + hp1} - \frac{1}{hm1 + hp2}, -\frac{(hm1 - hm2) (hm1 + hp1) (hm1 + hp2)}{hm1 hm2 hp1 hp2}, \right. \\ \left. -\frac{hm1 (hm1 - hm2) (hm1 + hp2)}{hp1 (hm1 + hp1) (hm2 + hp1) (hp1 - hp2)}, \frac{hm1 (hm1 - hm2) (hm1 + hp1)}{(hp1 - hp2) hp2 (hm1 + hp2) (hm2 + hp2)} \right\}$$

```
In[ ]:= FullSimplify[c1ord4l1 /. {hm2 -> 2 h, hm1 -> h, hp1 -> h, hp2 -> 2 h}]
```

$$\text{Out[]} = \left\{ -\frac{1}{4 h}, -\frac{5}{6 h}, \frac{3}{2 h}, -\frac{1}{2 h}, \frac{1}{12 h} \right\}$$

```
In[ ]:= sol4l1 = FullSimplify[Solve[
  {ford4[hm1] == f1, ford4[h1] == f2, ford4[h2] == f3, ford4[h3] == f4}, {a, b, c, d}]];
```

```
In[ ]:= D1ord4l1 = FullSimplify[ford4'[0] /. sol4l1];
c1ord4l1 = {Row[FullSimplify[Coefficient[D1ord4l1, f1]]],
  Row[FullSimplify[Coefficient[D1ord4l1, f0]]],
  Row[FullSimplify[Coefficient[D1ord4l1, f2]]],
  Row[FullSimplify[Coefficient[D1ord4l1, f3]]],
  Row[FullSimplify[Coefficient[D1ord4l1, f4]]]}
```

$$\text{Out[]} = \left\{ \frac{h1 h2 h3}{(h1 - hm1) hm1 (-h2 + hm1) (-h3 + hm1)}, \right. \\ \left. -\frac{1}{h1} - \frac{1}{h2} - \frac{h3 + hm1}{h3 hm1}, \frac{h2 h3 hm1}{h1 (-h1 + h2) (-h1 + h3) (-h1 + hm1)}, \right. \\ \left. \frac{h1 h3 hm1}{(h1 - h2) h2 (h2 - h3) (h2 - hm1)}, \frac{h1 h2 hm1}{(h1 - h3) h3 (-h2 + h3) (h3 - hm1)} \right\}$$

implement these

$$\text{Out[]} = \left\{ \frac{h1 h2 h3}{(h1 - hm1) hm1 (-h2 + hm1) (-h3 + hm1)}, \right. \\ \left. -\frac{1}{h1} - \frac{1}{h2} - \frac{h3 + hm1}{h3 hm1}, \frac{h2 h3 hm1}{h1 (-h1 + h2) (-h1 + h3) (-h1 + hm1)}, \right. \\ \left. \frac{h1 h3 hm1}{(h1 - h2) h2 (h2 - h3) (h2 - hm1)}, \frac{h1 h2 hm1}{(h1 - h3) h3 (-h2 + h3) (h3 - hm1)} \right\}$$

```
In[ ]:= FullSimplify[c1ord4l1 /. {hm1 -> -h, h1 -> h, h2 -> 2 h, h3 -> 3 h}]
```

$$\text{Out[]} = \left\{ -\frac{1}{4 h}, -\frac{5}{6 h}, \frac{3}{2 h}, -\frac{1}{2 h}, \frac{1}{12 h} \right\}$$

second derivative at first point left

```
In[ ]:= c2ord4l1 = {Row[FullSimplify[Coefficient[D2ord4l1, fm2]]],
  Row[FullSimplify[Coefficient[D2ord4l1, fm1]]],
  Row[FullSimplify[Coefficient[D2ord4l1, f0]]],
  Row[FullSimplify[Coefficient[D2ord4l1, fp1]]],
  Row[FullSimplify[Coefficient[D2ord4l1, fp2]]]}
```

$$\text{Out[]} = \left\{ -\frac{2(3hm1^2 + hp1hp2 + 2hm1(hp1 + hp2))}{(hm1 - hm2)hm2(hm2 + hp1)(hm2 + hp2)}, \right. \\ \frac{(2(6hm1^2 + hp1hp2 - hm2(hp1 + hp2) + 3hm1(-hm2 + hp1 + hp2)))}{(hm1(hm1 - hm2)(hm1 + hp1)(hm1 + hp2))}, \\ \frac{(6hm1^2 + 2hp1hp2 - 2hm2(hp1 + hp2) + 4hm1(-hm2 + hp1 + hp2))}{(hm1hm2hp1hp2)}, \\ \left. \frac{6hm1^2 - 2hm2hp2 + 4hm1(-hm2 + hp2)}{hp1(hm1 + hp1)(hm2 + hp1)(hp1 - hp2)}, \frac{-6hm1^2 + 4hm1(hm2 - hp1) + 2hm2hp1}{(hp1 - hp2)hp2(hm1 + hp2)(hm2 + hp2)} \right\}$$

```
In[ ]:= FullSimplify[c2ord4l1 /. {hm2 -> 2 h, hm1 -> h, hp1 -> h, hp2 -> 2 h}]
```

$$\text{Out[]} = \left\{ \frac{11}{12h^2}, -\frac{5}{3h^2}, \frac{1}{2h^2}, \frac{1}{3h^2}, -\frac{1}{12h^2} \right\}$$

```
In[ ]:= D2ord4l1 = FullSimplify[ford4''[0] /. sol4l1];
c2ord4l1 = {Row[FullSimplify[Coefficient[D2ord4l1, f1]]],
  Row[FullSimplify[Coefficient[D2ord4l1, f0]]],
  Row[FullSimplify[Coefficient[D2ord4l1, f2]]],
  Row[FullSimplify[Coefficient[D2ord4l1, f3]]],
  Row[FullSimplify[Coefficient[D2ord4l1, f4]]]}
```

$$\text{Out[]} = \left\{ -\frac{2(h2h3 + h1(h2 + h3))}{(h1 - hm1)hm1(-h2 + hm1)(-h3 + hm1)}, \right. \\ \frac{2(h3hm1 + h2(h3 + hm1) + h1(h2 + h3 + hm1))}{h1h2h3hm1}, \frac{2(h3hm1 + h2(h3 + hm1))}{h1(h1 - h2)(h1 - h3)(h1 - hm1)}, \\ \left. -\frac{2(h3hm1 + h1(h3 + hm1))}{(h1 - h2)h2(h2 - h3)(h2 - hm1)}, -\frac{2(h2hm1 + h1(h2 + hm1))}{(h1 - h3)h3(-h2 + h3)(h3 - hm1)} \right\}$$

implement these

$$\text{Out[]} = \left\{ -\frac{2(h2h3 + h1(h2 + h3))}{(h1 - hm1)hm1(-h2 + hm1)(-h3 + hm1)}, \right. \\ \frac{2(h3hm1 + h2(h3 + hm1) + h1(h2 + h3 + hm1))}{h1h2h3hm1}, \frac{2(h3hm1 + h2(h3 + hm1))}{h1(h1 - h2)(h1 - h3)(h1 - hm1)}, \\ \left. -\frac{2(h3hm1 + h1(h3 + hm1))}{(h1 - h2)h2(h2 - h3)(h2 - hm1)}, -\frac{2(h2hm1 + h1(h2 + hm1))}{(h1 - h3)h3(-h2 + h3)(h3 - hm1)} \right\}$$

```
In[ ]:= FullSimplify[c2ord4l1 /. {hm1 -> -h, h1 -> h, h2 -> 2 h, h3 -> 3 h}]
```

$$\text{Out[]} = \left\{ \frac{11}{12h^2}, -\frac{5}{3h^2}, \frac{1}{2h^2}, \frac{1}{3h^2}, -\frac{1}{12h^2} \right\}$$

In[]:=

d) Coefficients for first derivative at second point left

In[16]:= D1ord4l2 = FullSimplify[ford4'[-hm2] /. sol4];

```
In[17]:= c1ord4l2 = {Row[FullSimplify[Coefficient[D1ord4l2, fm2]]],
  Row[FullSimplify[Coefficient[D1ord4l2, fm1]]],
  Row[FullSimplify[Coefficient[D1ord4l2, f0]]],
  Row[FullSimplify[Coefficient[D1ord4l2, fp1]]],
  Row[FullSimplify[Coefficient[D1ord4l2, fp2]]]}
```

$$\text{Out[17]} = \left\{ \frac{hm1 - 2 hm2}{hm2 (-hm1 + hm2)} - \frac{1}{hm2 + hp1} - \frac{1}{hm2 + hp2}, \right. \\ \left. - \frac{hm2 (hm2 + hp1) (hm2 + hp2)}{hm1 (hm1 - hm2) (hm1 + hp1) (hm1 + hp2)}, \frac{(hm1 - hm2) (hm2 + hp1) (hm2 + hp2)}{hm1 hm2 hp1 hp2}, \right. \\ \left. \frac{(hm1 - hm2) hm2 (hm2 + hp2)}{hp1 (hm1 + hp1) (hm2 + hp1) (hp1 - hp2)}, - \frac{(hm1 - hm2) hm2 (hm2 + hp1)}{(hp1 - hp2) hp2 (hm1 + hp2) (hm2 + hp2)} \right\}$$

In[]:= FullSimplify[c1ord4l2 /. {hm2 → 2 h, hm1 → h, hp1 → h, hp2 → 2 h}]

$$\text{Out[]} = \left\{ -\frac{25}{12 h}, \frac{4}{h}, -\frac{3}{h}, \frac{4}{3 h}, -\frac{1}{4 h} \right\}$$

```
In[18]:= sol4l2 = FullSimplify[Solve[
  {ford4[h1] == f1, ford4[h2] == f2, ford4[h3] == f3, ford4[h4] == f4}, {a, b, c, d}]];
```

In[]:= D1ord4l2 = FullSimplify[ford4'[0] /. sol4l2]

$$\text{Out[]} = \left\{ f0 \left(-\frac{1}{h1} - \frac{1}{h2} - \frac{1}{h3} - \frac{1}{h4} \right) + \frac{f2 h1 h3 h4}{(h1 - h2) h2 (h2 - h3) (h2 - h4)} + \frac{f3 h1 h2 h4}{(h1 - h3) h3 (-h2 + h3) (h3 - h4)} + \right. \\ \left. \frac{f1 h2 h3 h4}{h1 (-h1 + h2) (-h1 + h3) (-h1 + h4)} + \frac{f4 h1 h2 h3}{(h1 - h4) h4 (-h2 + h4) (-h3 + h4)} \right\}$$

```
In[ ]:= c1ord4l2 = {Row[FullSimplify[Coefficient[D1ord4l2, f0]]],
  Row[FullSimplify[Coefficient[D1ord4l2, f1]]],
  Row[FullSimplify[Coefficient[D1ord4l2, f2]]],
  Row[FullSimplify[Coefficient[D1ord4l2, f3]]],
  Row[FullSimplify[Coefficient[D1ord4l2, f4]]]}
```

$$\text{Out[]} = \left\{ -\frac{1}{h1} - \frac{1}{h2} - \frac{h3 + h4}{h3 h4}, \frac{h2 h3 h4}{h1 (-h1 + h2) (-h1 + h3) (-h1 + h4)}, \frac{h1 h3 h4}{(h1 - h2) h2 (h2 - h3) (h2 - h4)}, \right. \\ \left. \frac{h1 h2 h4}{(h1 - h3) h3 (-h2 + h3) (h3 - h4)}, \frac{h1 h2 h3}{(h1 - h4) h4 (-h2 + h4) (-h3 + h4)} \right\}$$

implement these

$$\text{Out}[*]= \left\{ -\frac{1}{h_1} - \frac{1}{h_2} - \frac{h_3 + h_4}{h_3 h_4}, \frac{h_2 h_3 h_4}{h_1 (-h_1 + h_2) (-h_1 + h_3) (-h_1 + h_4)}, \frac{h_1 h_3 h_4}{(h_1 - h_2) h_2 (h_2 - h_3) (h_2 - h_4)}, \right. \\ \left. \frac{h_1 h_2 h_4}{(h_1 - h_3) h_3 (-h_2 + h_3) (h_3 - h_4)}, \frac{h_1 h_2 h_3}{(h_1 - h_4) h_4 (-h_2 + h_4) (-h_3 + h_4)} \right\}$$

`In[*]:= c1ord4l2 /. {h1 -> h, h2 -> 2 h, h3 -> 3 h, h4 -> 4 h}`

$$\text{Out}[*]= \left\{ -\frac{25}{12 h}, \frac{4}{h}, -\frac{3}{h}, \frac{4}{3 h}, -\frac{1}{4 h} \right\}$$

e) Coefficients for first and second derivative at first point right

`In[*]:= D1ord4r1 = FullSimplify[ford4'[hp1] /. sol4];`

`In[*]:= D2ord4r1 = FullSimplify[ford4''[hp1] /. sol4];`

`In[*]:= c1ord4r1 = {Row[FullSimplify[Coefficient[D1ord4r1, fm2]]],
Row[FullSimplify[Coefficient[D1ord4r1, fm1]]],
Row[FullSimplify[Coefficient[D1ord4r1, f0]]],
Row[FullSimplify[Coefficient[D1ord4r1, fp1]]],
Row[FullSimplify[Coefficient[D1ord4r1, fp2]]]}`

$$\text{Out}[*]= \left\{ -\frac{h p_1 (h m_1 + h p_1) (h p_1 - h p_2)}{(h m_1 - h m_2) h m_2 (h m_2 + h p_1) (h m_2 + h p_2)}, \right. \\ \frac{h p_1 (h m_2 + h p_1) (h p_1 - h p_2)}{h m_1 (h m_1 - h m_2) (h m_1 + h p_1) (h m_1 + h p_2)}, \frac{(h m_1 + h p_1) (h m_2 + h p_1) (h p_1 - h p_2)}{h m_1 h m_2 h p_1 h p_2}, \\ \left. \frac{1}{h p_1} + \frac{1}{h m_1 + h p_1} + \frac{1}{h m_2 + h p_1} + \frac{1}{h p_1 - h p_2}, -\frac{h p_1 (h m_1 + h p_1) (h m_2 + h p_1)}{(h p_1 - h p_2) h p_2 (h m_1 + h p_2) (h m_2 + h p_2)} \right\}$$

`In[*]:= FullSimplify[c1ord4r1 /. {hm2 -> 2 h, hm1 -> h, hp1 -> h, hp2 -> 2 h}]`

$$\text{Out}[*]= \left\{ -\frac{1}{12 h}, \frac{1}{2 h}, -\frac{3}{2 h}, \frac{5}{6 h}, \frac{1}{4 h} \right\}$$

`In[*]:= sol4r1 = FullSimplify[Solve[
{ford4[h3] == f1, ford4[h2] == f2, ford4[h1] == f3, ford4[hp1] == f4}, {a, b, c, d}]];
D1ord4r1 = FullSimplify[ford4'[0] /. sol4r1];`


```
In[ ]:= c1ord4r1 = {Row[FullSimplify[Coefficient[D1ord4r1, f1]]],
  Row[FullSimplify[Coefficient[D1ord4r1, f2]]],
  Row[FullSimplify[Coefficient[D1ord4r1, f3]]],
  Row[FullSimplify[Coefficient[D1ord4r1, f0]]],
  Row[FullSimplify[Coefficient[D1ord4r1, f4]]]}
```

$$\text{Out[]} = \left\{ \frac{h_1 h_2 h_{p1}}{(h_1 - h_3) h_3 (-h_2 + h_3) (h_3 - h_{p1})}, \right. \\ \frac{h_1 h_3 h_{p1}}{(h_1 - h_2) h_2 (h_2 - h_3) (h_2 - h_{p1})}, \frac{h_2 h_3 h_{p1}}{h_1 (-h_1 + h_2) (-h_1 + h_3) (-h_1 + h_{p1})}, \\ \left. -\frac{1}{h_1} - \frac{1}{h_2} - \frac{h_3 + h_{p1}}{h_3 h_{p1}}, \frac{h_1 h_2 h_3}{(h_1 - h_{p1}) h_{p1} (-h_2 + h_{p1}) (-h_3 + h_{p1})} \right\}$$

implement these

$$\text{Out[]} = \left\{ \frac{h_1 h_2 h_{p1}}{(h_1 - h_3) h_3 (-h_2 + h_3) (h_3 - h_{p1})}, \right. \\ \frac{h_1 h_3 h_{p1}}{(h_1 - h_2) h_2 (h_2 - h_3) (h_2 - h_{p1})}, \frac{h_2 h_3 h_{p1}}{h_1 (-h_1 + h_2) (-h_1 + h_3) (-h_1 + h_{p1})}, \\ \left. -\frac{1}{h_1} - \frac{1}{h_2} - \frac{h_3 + h_{p1}}{h_3 h_{p1}}, \frac{h_1 h_2 h_3}{(h_1 - h_{p1}) h_{p1} (-h_2 + h_{p1}) (-h_3 + h_{p1})} \right\}$$

```
In[ ]:= FullSimplify[c1ord4r1 /. {h3 -> -3 h, h2 -> -2 h, h1 -> -h, hp1 -> h}]
```

$$\text{Out[]} = \left\{ -\frac{1}{12 h}, \frac{1}{2 h}, -\frac{3}{2 h}, \frac{5}{6 h}, \frac{1}{4 h} \right\}$$

$$\text{In[]} = \left\{ -\frac{1}{12 h}, \frac{1}{2 h}, -\frac{3}{2 h}, \frac{5}{6 h}, \frac{1}{4 h} \right\}$$

$$\text{Out[]} = \left\{ -\frac{1}{12 h}, \frac{1}{2 h}, -\frac{3}{2 h}, \frac{5}{6 h}, \frac{1}{4 h} \right\}$$

second derivative

```
In[ ]:= c2ord4r1 = {Row[FullSimplify[Coefficient[D2ord4r1, fm2]]],
  Row[FullSimplify[Coefficient[D2ord4r1, fm1]]],
  Row[FullSimplify[Coefficient[D2ord4r1, f0]]],
  Row[FullSimplify[Coefficient[D2ord4r1, fp1]]],
  Row[FullSimplify[Coefficient[D2ord4r1, fp2]]]}
```

$$\text{Out[]} = \left\{ \frac{-6 h_{p1}^2 + 4 h_{p1} h_{p2} + 2 h_{m1} (-2 h_{p1} + h_{p2})}{(h_{m1} - h_{m2}) h_{m2} (h_{m2} + h_{p1}) (h_{m2} + h_{p2})}, \frac{4 h_{m2} h_{p1} + 6 h_{p1}^2 - 2 h_{m2} h_{p2} - 4 h_{p1} h_{p2}}{h_{m1} (h_{m1} - h_{m2}) (h_{m1} + h_{p1}) (h_{m1} + h_{p2})}, \right. \\ \frac{(2 h_{m1} h_{m2} + 4 h_{m1} h_{p1} + 4 h_{m2} h_{p1} + 6 h_{p1}^2 - 2 (h_{m1} + h_{m2} + 2 h_{p1}) h_{p2})}{(2 (h_{m1} h_{m2} + 3 h_{m1} h_{p1} + 3 h_{m2} h_{p1} + 6 h_{p1}^2 - (h_{m1} + h_{m2} + 3 h_{p1}) h_{p2}))} / (h_{m1} h_{m2} h_{p1} h_{p2}), \\ \left. \frac{(h_{p1} (h_{m1} + h_{p1}) (h_{m2} + h_{p1}) (h_{p1} - h_{p2}))}{(h_{p1} - h_{p2}) h_{p2} (h_{m1} + h_{p2}) (h_{m2} + h_{p2})} - \frac{2 (h_{m1} h_{m2} + 2 (h_{m1} + h_{m2}) h_{p1} + 3 h_{p1}^2)}{(h_{p1} - h_{p2}) h_{p2} (h_{m1} + h_{p2}) (h_{m2} + h_{p2})} \right\}$$

```
In[8]:= FullSimplify[c2ord4r1 /. {hm2 → 2 h, hm1 → h, hp1 → h, hp2 → 2 h}]
```

$$\text{Out[8]} = \left\{ -\frac{1}{12 h^2}, \frac{1}{3 h^2}, \frac{1}{2 h^2}, -\frac{5}{3 h^2}, \frac{11}{12 h^2} \right\}$$

```
D2ord4r1 = FullSimplify[ford4'[0] /. sol4r1];
c2ord4r1 = {Row[FullSimplify[Coefficient[D2ord4r1, f1]]],
  Row[FullSimplify[Coefficient[D2ord4r1, f2]]],
  Row[FullSimplify[Coefficient[D2ord4r1, f3]]],
  Row[FullSimplify[Coefficient[D2ord4r1, f0]]],
  Row[FullSimplify[Coefficient[D2ord4r1, f4]]]}
```

implement these

$$\begin{aligned} \text{Out[8]} = & \left\{ -\frac{2 (h_2 h_{p1} + h_1 (h_2 + h_{p1}))}{(h_1 - h_3) h_3 (-h_2 + h_3) (h_3 - h_{p1})}, \right. \\ & -\frac{2 (h_3 h_{p1} + h_1 (h_3 + h_{p1}))}{(h_1 - h_2) h_2 (h_2 - h_3) (h_2 - h_{p1})}, \frac{2 (h_3 h_{p1} + h_2 (h_3 + h_{p1}))}{h_1 (h_1 - h_2) (h_1 - h_3) (h_1 - h_{p1})}, \\ & \left. \frac{2 (h_3 h_{p1} + h_2 (h_3 + h_{p1}) + h_1 (h_2 + h_3 + h_{p1}))}{h_1 h_2 h_3 h_{p1}}, -\frac{2 (h_2 h_3 + h_1 (h_2 + h_3))}{(h_1 - h_{p1}) h_{p1} (-h_2 + h_{p1}) (-h_3 + h_{p1})} \right\} \end{aligned}$$

```
In[9]:= FullSimplify[c2ord4r1 /. {h3 → -3 h, h2 → -2 h, h1 → -h, hp1 → h}]
```

$$\text{Out[9]} = \left\{ -\frac{1}{12 h^2}, \frac{1}{3 h^2}, \frac{1}{2 h^2}, -\frac{5}{3 h^2}, \frac{11}{12 h^2} \right\}$$

f) Coefficients for first derivative at second point right

```
In[20]:= D1ord4r2 = FullSimplify[ford4'[hp2] /. sol4];
```

```
In[21]:= c1ord4r2 = {Row[FullSimplify[Coefficient[D1ord4r2, fm2]]],
  Row[FullSimplify[Coefficient[D1ord4r2, fm1]]],
  Row[FullSimplify[Coefficient[D1ord4r2, f0]]],
  Row[FullSimplify[Coefficient[D1ord4r2, fp1]]],
  Row[FullSimplify[Coefficient[D1ord4r2, fp2]]]}
```

$$\begin{aligned} \text{Out[21]} = & \left\{ \frac{(h_{p1} - h_{p2}) h_{p2} (h_{m1} + h_{p2})}{(h_{m1} - h_{m2}) h_{m2} (h_{m2} + h_{p1}) (h_{m2} + h_{p2})}, \right. \\ & -\frac{(h_{p1} - h_{p2}) h_{p2} (h_{m2} + h_{p2})}{h_{m1} (h_{m1} - h_{m2}) (h_{m1} + h_{p1}) (h_{m1} + h_{p2})}, -\frac{(h_{p1} - h_{p2}) (h_{m1} + h_{p2}) (h_{m2} + h_{p2})}{h_{m1} h_{m2} h_{p1} h_{p2}}, \\ & \left. \frac{h_{p2} (h_{m1} + h_{p2}) (h_{m2} + h_{p2})}{h_{p1} (h_{m1} + h_{p1}) (h_{m2} + h_{p1}) (h_{p1} - h_{p2})}, \frac{1}{h_{p2}} + \frac{1}{h_{m1} + h_{p2}} + \frac{1}{h_{m2} + h_{p2}} + \frac{1}{-h_{p1} + h_{p2}} \right\} \end{aligned}$$

```
In[10]:= FullSimplify[c1ord4r2 /. {hm2 → 2 h, hm1 → h, hp1 → h, hp2 → 2 h}]
```

$$\text{Out[10]} = \left\{ \frac{1}{4 h}, -\frac{4}{3 h}, \frac{3}{h}, -\frac{4}{h}, \frac{25}{12 h} \right\}$$

In[26]:= **FortranForm**[c1ord4l2]

Out[26]//FortranForm=

```
List(Row(List((hm1 - 2*hm2)/(hm2*(-hm1 + hm2)) - 1/(hm2 + hp1) -
- 1/(hm2 + hp2))),Row(List(-((hm2*(hm2 + hp1)*(hm2 + hp2))/
- (hm1*(hm1 - hm2)*(hm1 + hp1)*(hm1 + hp2))))) ,
- Row(List((hm1 - hm2)*(hm2 + hp1)*(hm2 + hp2)/
- (hm1*hm2*hp1*hp2))),
- Row(List((hm1 - hm2)*hm2*(hm2 + hp2)/
- (hp1*(hm1 + hp1)*(hm2 + hp1)*(hp1 - hp2))))) ,
- Row(List(-((hm1 - hm2)*hm2*(hm2 + hp1)/
- ((hp1 - hp2)*hp2*(hm1 + hp2)*(hm2 + hp2)))))
```

In[27]:= **FortranForm**[c1ord4r2]

Out[27]//FortranForm=

```
List(Row(List((hp1 - hp2)*hp2*(hm1 + hp2))/
- ((hm1 - hm2)*hm2*(hm2 + hp1)*(hm2 + hp2))))) ,
- Row(List(-((hp1 - hp2)*hp2*(hm2 + hp2))/
- (hm1*(hm1 - hm2)*(hm1 + hp1)*(hm1 + hp2))))) ,
- Row(List(-((hp1 - hp2)*(hm1 + hp2)*(hm2 + hp2))/
- (hm1*hm2*hp1*hp2))))) ,
- Row(List((hp2*(hm1 + hp2)*(hm2 + hp2))/
- (hp1*(hm1 + hp1)*(hm2 + hp1)*(hp1 - hp2))))) ,
- Row(List(1/hp2 + 1/(hm1 + hp2) + 1/(hm2 + hp2) + 1/(-hp1 + hp2)))
```

```
In[28]:= sol4r2 = FullSimplify[Solve[
  {ford4[h4] == f1, ford4[h3] == f2, ford4[h2] == f3, ford4[h1] == f4}, {a, b, c, d}]];
D1ord4r2 = FullSimplify[ford4'[0] /. sol4r2];
```

```
In[29]:= c1ord4r2 = {Row[FullSimplify[Coefficient[D1ord4r2, f1]]],
  Row[FullSimplify[Coefficient[D1ord4r2, f2]]],
  Row[FullSimplify[Coefficient[D1ord4r2, f3]]],
  Row[FullSimplify[Coefficient[D1ord4r2, f4]]],
  Row[FullSimplify[Coefficient[D1ord4r2, f0]]]}
```

Out[29]= $\left\{ \frac{h_1 h_2 h_3}{(h_1 - h_4) h_4 (-h_2 + h_4) (-h_3 + h_4)}, \frac{h_1 h_2 h_4}{(h_1 - h_3) h_3 (-h_2 + h_3) (h_3 - h_4)}, \right.$
 $\left. \frac{h_1 h_3 h_4}{(h_1 - h_2) h_2 (h_2 - h_3) (h_2 - h_4)}, \frac{h_2 h_3 h_4}{h_1 (-h_1 + h_2) (-h_1 + h_3) (-h_1 + h_4)}, -\frac{1}{h_1} - \frac{1}{h_2} - \frac{h_3 + h_4}{h_3 h_4} \right\}$

implement these

Out[29]= $\left\{ \frac{h_1 h_2 h_3}{(h_1 - h_4) h_4 (-h_2 + h_4) (-h_3 + h_4)}, \frac{h_1 h_2 h_4}{(h_1 - h_3) h_3 (-h_2 + h_3) (h_3 - h_4)}, \right.$
 $\left. \frac{h_1 h_3 h_4}{(h_1 - h_2) h_2 (h_2 - h_3) (h_2 - h_4)}, \frac{h_2 h_3 h_4}{h_1 (-h_1 + h_2) (-h_1 + h_3) (-h_1 + h_4)}, -\frac{1}{h_1} - \frac{1}{h_2} - \frac{h_3 + h_4}{h_3 h_4} \right\}$

```
In[30]:= FullSimplify[c1ord4r2 /. {h4 -> -4 h, h3 -> -3 h, h2 -> -2 h, h1 -> -h}]
```

Out[30]= $\left\{ \frac{1}{4 h}, -\frac{4}{3 h}, \frac{3}{h}, -\frac{4}{h}, \frac{25}{12 h} \right\}$