Applied Mathematics Letters 41 (2015) 35–40

Contents lists available at ScienceDirect

Applied Mathematics Letters

journal homepage: www.elsevier.com/locate/aml

A mean value algorithm for Toeplitz matrix completion[☆]

Chuan-Long Wang[a,*], Chao Li[b]

[a] Higher Education Key Laboratory of Engineering Science Computing in Shanxi Province, Taiyuan Normal University, Taiyuan 030012, PR China

[b] Institution of Mathematics, Taiyuan University of Technology, Taiyuan 030024, PR China

ARTICLE INFO

Article history:
Received 8 September 2014
Received in revised form 21 October 2014
Accepted 22 October 2014
Available online 31 October 2014

Keywords:
Matrix completion
Toeplitz matrix
Mean value

ABSTRACT

In this paper, we propose a new mean value algorithm for the Toeplitz matrix completion based on the singular value thresholding (SVT) algorithm. The completion matrices generated by the new algorithm keep a feasible Toeplitz structure. Meanwhile, we prove the convergence of the new algorithm under some reasonal conditions. Finally, we show the new algorithm is much more effective than the ALM (augmented Lagrange multiplier) algorithm through numerical experiments and image inpainting.

1. Introduction

Recovering an unknown low-rank or approximately low-rank matrix from a sampling of its entries has aroused general interest. This problem is well known as the matrix completion (MC) problem. It occurs in many areas, such as machine learning [1,2], control [3], image inpainting [4], computer vision [5] and so on.

MC problem was first proposed by Candès and Recht [6]. In [6] they showed that most low-rank matrices can be recovered from a sampling of its entries by solving the optimization problem,

$$\begin{aligned} \min \quad & \|X\|_* \\ \text{s.t.} \quad & X_{ij} = M_{ij}, \quad (i,j) \in \Omega \end{aligned} \tag{1.1}$$

where $\|X\|_* = \sum_{k=1}^{r} \sigma_k(X)$, $\sigma_k(X)$ denotes the kth largest singular value of $X \in \mathbb{R}^{n_1 \times n_2}$ of rank r. $M \in \mathbb{R}^{n_1 \times n_2}$, and $\{M_{ij} : (i,j) \in \Omega\}$ is known. Ω is a random subset of cardinality m which is the number of sampled entries. They also proved that if m obeys $m \geq Cn^{1.2} r \log n$ for some positive numerical constant C, most matrices of rank r can be recovered with high probability. Then Candès and Tao [7], Keshavan, Montanari, and Oh [8], and Recht [9] improved the bound. Many algorithms have been proposed to solve the optimization problem (1.1), such as the accelerated proximal gradient (APG) algorithm [10], the singular value thresholding (SVT) algorithm [11], and the augmented Lagrange multiplier (ALM) algorithm [12]. For many other algorithms we can refer to [13–18].

☆ This work was supported by National Natural Science Foundation of China (Grant No. 11371275).

* Corresponding author.
E-mail address: clwang1964@163.com (C.-L. Wang).

http://dx.doi.org/10.1016/j.aml.2014.10.013

On the other hand, Toeplitz matrices play an important role in signal and image processing [19–22]. Shaw et al. discussed the approximation of Toeplitz and Hankel matrices [23]. A $n \times n$ Toeplitz matrix $T \in \mathbb{R}^{n\times n}$ is of the form

$$T = \begin{pmatrix} t_0 & t_1 & \cdots & t_{n-2} & t_{n-1} \\ t_{-1} & t_0 & \cdots & t_{n-3} & t_{n-2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ t_{-n+2} & t_{-n+3} & \cdots & t_0 & t_1 \\ t_{-n+1} & t_{-n+2} & \cdots & t_{-1} & t_0 \end{pmatrix},$$

which is determined by $2n - 1$ entries, that is the first row and first column. In [24,25], Qiao et al. proposed an $O(n^2 \log n)$ algorithm for the SVD of Hankel matrices by using the Lanczos method [26] and FFT technique [27], in contrast with existing $O(n^3)$ SVD algorithms. Since a Toeplitz matrix can be transformed into a Hankel matrix by reversing the columns or rows [28], we can straightforwardly obtain an $O(n^2 \log n)$ algorithm for computing the SVD of Toeplitz matrices. Thus, it has great significance to the study of Toeplitz matrix completion.

In this paper, we focus on the completion of Toeplitz matrices and present a mean value algorithm for Toeplitz matrix completion based on the SVT algorithm [11]. First, we give some definitions.

Definition 1 (*Singular Value Decomposition (SVD) [26]*)**.** The singular value decomposition of a matrix $X \in \mathbb{R}^{n_1\times n_2}$ of rank r is:

$$X = U\Sigma_r V^*, \quad \Sigma_r = diag(\sigma_1, \dots, \sigma_r),$$

where $U \in \mathbb{R}^{n_1\times r}$ and $V \in \mathbb{R}^{n_2\times r}$ are orthogonal, $\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r > 0$.

Definition 2 (*Singular Value Thresholding Operator [11]*)**.** For each $\tau \geq 0$, the singular value thresholding operator $\mathcal{D}_\tau$ is defined as follows:

$$\mathcal{D}_\tau(X) := U\mathcal{D}_\tau(\Sigma)V^*, \quad \mathcal{D}_\tau(\Sigma) = diag(\{\sigma_i - \tau\}_+)$$

where $X = U\Sigma_r V^*$ is the SVD of a matrix X of rank r, $\{\sigma_i - \tau\}_+ = \begin{cases} \sigma_i - \tau, & \text{if } \sigma_i > \tau \\ 0, & \text{if } \sigma_i \leq \tau. \end{cases}$

The rest of this paper is organized as follows. In Section 2, we give the proposed algorithm for Toeplitz matrices in detail. Convergence analysis is given in Section 3. In Section 4, we compare our algorithm with the ALM algorithm through numerical experiments. Finally, we conclude the paper.

Notation. For convenience, **R** denotes the set of real numbers. $\mathbb{R}^{n_1\times n_2}$ denotes $n_1 \times n_2$ real matrices set. $r(X)$ denotes the rank of a matrix X. X_{ij} denotes the (i, j)th entry of a matrix X. $X = (x_1, \dots, x_n)$ is a column partitioning of a matrix X. The nuclear norm of a matrix is denoted by $\|X\|_*$, and the Frobenius norm by $\|X\|_F$. X^* is the conjugate transpose of a matrix X. The standard inner product between two matrices is denoted by $\langle X, Y\rangle = \mathrm{trace}(X^*Y)$. The Cauchy–Schwarz inequality gives $\langle X, Y\rangle \leq \|X\|_F\|Y\|_F$. $\Omega \subset \{-n_1+1, \dots, n_2-1\}$ is the indices of observed diagonals of a matrix X, $\bar{\Omega}$ is the complementary set of Ω. Vector $diag(X, l)$ denotes the l th diagonal of a Toeplitz matrix X, $l \in \{-n_1 + 1, \dots, n_2 - 1\}$. P_Ω is the orthogonal projector on Ω, which is

$$diag(P_\Omega(X), l) = \begin{cases} diag(X, l), & l \in \Omega \\ \mathbf{0}, & l \notin \Omega. \end{cases} \quad (\mathbf{0} \text{ is a vector})$$

2. Algorithm

In this section, we focus on the completion of a Toeplitz matrix, then our problem is expressed as the following convex programming,

$$\begin{array}{ll} \min & \|X\|_* \\ \text{s.t.} & P_\Omega(X) = P_\Omega(M) \end{array} \tag{2.1}$$

where X, M are Toeplitz matrices, $\Omega \subset \{-n_1 + 1, \cdots, n_2 - 1\}$.

For convenience, $[U_k, \Sigma_k, V_k]_{\tau_k} = lansvd(Y_k)$, denotes the SVD of the matrix Y_k using the Lanczos method. And let $R_l = (r_{ij})_{n\times n} = \begin{cases} 1, & j-i=l \\ 0, & j-i\neq l \end{cases}$, $l = -n + 1, \dots, n - 1$.

Algorithm 2.1 (*The Mean Value (MV) Algorithm for Toeplitz Matrix Completion*)**.** Step 0. Set sampled set Ω and sampled entries $P_\Omega(M)$, parameters τ_0, $0 < c < 1$, tolerance ϵ, set initial matrix $Y_0 = P_\Omega(M)$, $k := 0$;

Step 1. Compute the SVD of Y_k

$$[U_k, \Sigma_k, V_k]_{\tau_k} = lansvd(Y_k),$$

set $X_{k+1} = U_k\mathcal{D}_{\tau_k}(\Sigma_k)V_k^*$,

Step 2. Compute $a_l = average(diag(P_{\bar{\Omega}}(X_{k+1}), l)), l \in \bar{\Omega}$,
set

$$\bar{X} = \sum_{l \in \bar{\Omega}} a_l R_l,$$

set

$$Y_{k+1} = \bar{X} + P_{\Omega}(M),$$

Step 3. If $\|Y_{k+1} - Y_k\|_F / \|Y_k\|_F < \epsilon$, stop; otherwise set $\tau_{k+1} = c\tau_k$, $k := k+1$; go to Step 1.

3. Convergence analysis

In this section, we prove the convergence of Algorithm 2.1 under some reasonal conditions.

Lemma 3.1. *For any matrix $X = U\Sigma V^*$,*

$$\langle UV^*, X\rangle = \langle UV^*, UU^*XVV^*\rangle \tag{3.1}$$

Proof. Firstly, we prove that the matrix $(I - UU^*)(I - VV^*)$ is orthogonal to U in column and V in row. Obviously,

$$U^*(I - UU^*)(I - VV^*) = 0, \qquad (I - UU^*)(I - VV^*)V = 0.$$

Secondly,

$$\begin{aligned}\langle UV^*, X\rangle &= tr(U^*XV) \\ &= tr(U^*UU^*XVV^*V) \\ &= \langle UV^*, UU^*XVV^*\rangle. \quad \square\end{aligned}$$

Theorem 3.2. *Let $\hat{X} \in \mathbb{R}^{n\times n}$ be a Toeplitz matrix with $\hat{X} = U\Sigma V^*$. Then $\hat{X}$ is the solution of (2.1) if and only if*

$$\sum_{i\in\bar{\Omega}} y_i tr(U^*R_iV) + \left|\sum_{i\in\bar{\Omega}} y_i tr((I - UU^*)R_i(I - VV^*))\right| \geq 0, \quad \forall y_i \in \mathbf{R}. \tag{3.2}$$

Proof. For any Toeplitz matrix X satisfies $P_\Omega(X) = P_\Omega(M)$, then

$$\|X\|_* \geq \|\hat{X}\|_* + \langle \partial\|\hat{X}\|_*, X - \hat{X}\rangle,$$

$\hat{X}$ is the optimal completion matrix, if and only if there exists a subgradient $UV^* + W_1 \in \partial\|\hat{X}\|_*$, such that $\langle UV^* + W_1, X - \hat{X}\rangle \geq 0$, where the row of W_1 is orthogonal to the row of V, the column of W_1 is orthogonal to the column of U, and $\|W_1\|_2 \leq 1$.

Let $X - \hat{X} = \sum_{i\in\bar{\Omega}} y_i R_i$. For the arbitrariness of W_1,

$$\begin{aligned}\langle UV^* + W_1, X - \hat{X}\rangle &= \langle UV^*, X - \hat{X}\rangle + \langle W_1, X - \hat{X}\rangle \\ &= \langle UV^*, UU^*(X - \hat{X})VV^*\rangle + \langle W_1, (I - UU^*)(X - \hat{X})(I - VV^*)\rangle \\ &= \sum_{i\in\bar{\Omega}} y_i tr(U^*R_iV) + \left|\sum_{i\in\bar{\Omega}} y_i tr((I - UU^*)R_i(I - VV^*))\right|.\end{aligned}$$

Hence, (3.2) holds. $\square$

Corollary 3.1. *If $\forall i \in \bar{\Omega}$,*

$$tr(U^*R_iV) = 0, \tag{3.3}$$

then $\hat{X}$ is an optimal completion matrix of (2.1).

Example 3.1. Let $X = \begin{pmatrix} 1 & x \\ 1 & 1 \end{pmatrix}$, where x is unknown.

Obviously, the optimal completion matrix is $\hat{X} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, and the SVD of $\hat{X} = U\Sigma V^*$ is $\hat{X} = 2\begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$. By calculation, we have:

- $tr(U^*R_iV) = \frac{1}{2}$. So (3.3) does not hold;
- $\forall y_i \in \mathbf{R}$, $y_i tr(U^*R_iV) + |y_i tr(I - UU^*)R_i(I - VV^*)| = \frac{1}{2}y_i + |\frac{1}{2}y_i| \geq 0$. Hence, (3.2) holds.

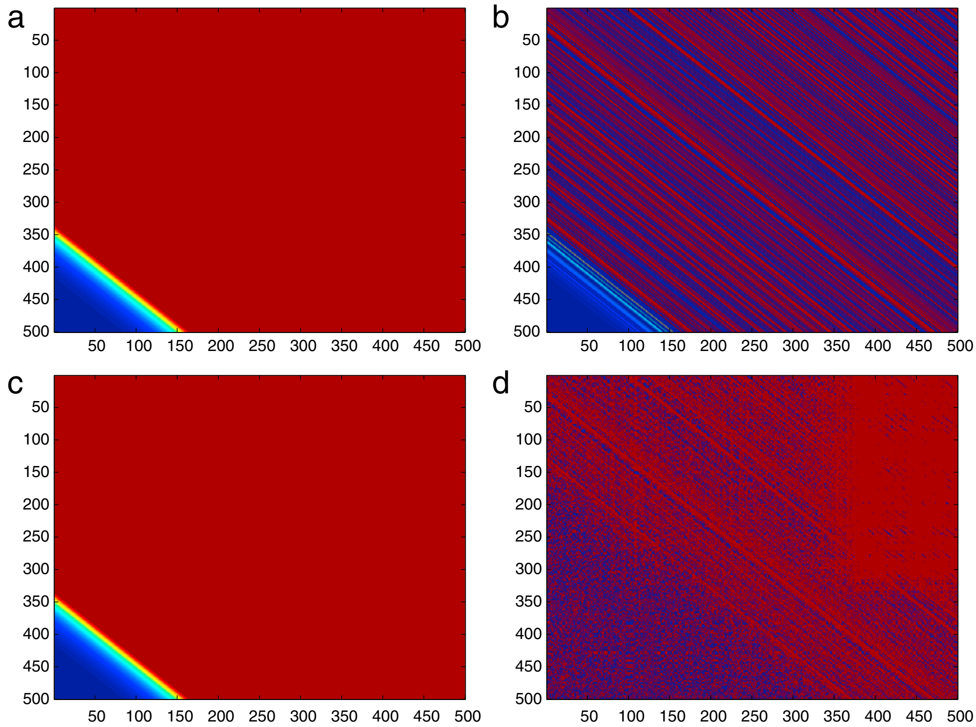

Fig. 1. (a) Original image, (b) damaged image, (c) inpainted image by our algorithm, (d) inpainted image by the ALM algorithm.

Corollary 3.2. *Assume $\hat{X}$ is an optimal completion matrix of (2.1). Then $\hat{X}$ is a sole feasible completion matrix in the subspace $\{UYV^*, Y \in \mathbb{R}^{n\times n}\}$ or $tr(U^*R_iV) = 0,\ i \in \bar{\Omega}$.*

Lemma 3.3. *Let Y_k be the kth completion matrix, $X_k = \mathcal{D}_{\tau_k}(Y_k)$. Then*

$$\|Y_k - X_k\|_F^2 = \|Y_k - Y_{k+1}\|_F^2 + \|Y_{k+1} - X_k\|_F^2.$$

Proof.

$$\begin{aligned} Y_k - X_k &= P_\Omega(Y_k - X_k) + P_{\bar{\Omega}}(Y_k - X_k) \\ &= P_\Omega(Y_{k+1} - X_k) + P_{\bar{\Omega}}(Y_k - Y_{k+1} + Y_{k+1} - X_k) \end{aligned}$$

$$\|Y_k - X_k\|_F^2 = \|P_\Omega(Y_{k+1} - X_k)\|_F^2 + \|P_{\bar{\Omega}}(Y_k - Y_{k+1} + Y_{k+1} - X_k)\|_F^2$$

$$\begin{aligned} \|P_{\bar{\Omega}}(Y_k - Y_{k+1} + Y_{k+1} - X_k)\|_F^2 &= \langle P_{\bar{\Omega}}(Y_k - Y_{k+1}) + P_{\bar{\Omega}}(Y_{k+1} - X_k), P_{\bar{\Omega}}(Y_k - Y_{k+1}) + P_{\bar{\Omega}}(Y_{k+1} - X_k)\rangle \\ &= \langle P_{\bar{\Omega}}(Y_k - Y_{k+1}), P_{\bar{\Omega}}(Y_k - Y_{k+1})\rangle + 2\langle P_{\bar{\Omega}}(Y_{k+1} - X_k), P_{\bar{\Omega}}(Y_k - Y_{k+1})\rangle \\ &\quad + \langle P_{\bar{\Omega}}(Y_{k+1} - X_k), P_{\bar{\Omega}}(Y_{k+1} - X_k)\rangle. \end{aligned}$$

Because, $P_{\bar{\Omega}}(Y_k - Y_{k+1})$ is a Toeplitz matrix and $\sum_{i,j}(P_{\bar{\Omega}}(Y_{k+1} - X_k))_{ij} = 0, i, j = 1, \ldots, n$.So

$$\langle P_{\bar{\Omega}}(Y_{k+1} - X_k), P_{\bar{\Omega}}(Y_k - Y_{k+1})\rangle = 0$$

which implies,

$$\begin{aligned} \|Y_k - X_k\|_F^2 &= \|P_\Omega(Y_{k+1} - X_k)\|_F^2 + \|P_{\bar{\Omega}}(Y_k - Y_{k+1})\|_F^2 + \|P_{\bar{\Omega}}(Y_{k+1} - X_k)\|_F^2 \\ &= \|Y_k - Y_{k+1}\|_F^2 + \|Y_{k+1} - X_k\|_F^2. \quad \square \end{aligned}$$

Theorem 3.4. *Let $\{Y_k\}$ be the completion matrices generated by Algorithm 2.1. Assume that $\lim_{k\to\infty}\langle U_kV_k^*, R_i\rangle = 0, i \in \bar{\Omega}$, then Y_k converges to the solution of (2.1), if $r(Y_{k+1}) \le \frac{r(\mathcal{D}_{\tau_k}(Y_k))}{1-\sin^2\theta}$, where $\sin\theta = \|Y_{k+1} - Y_k\|_F / \|Y_k - \mathcal{D}_{\tau_k}(Y_k)\|_F$.*

Proof. For Y_{k+1}, we have

$$\|\mathcal{D}_{\tau_k}(Y_k)\|_* \ge \|Y_{k+1}\|_* + \langle \partial\|Y_{k+1}\|_*, \mathcal{D}_{\tau_k}(Y_k) - Y_{k+1}\rangle$$

Table 1

Size ($n \times n$)	Rank (r)	$m/(2n-1)$	Algorithm	#iter	Time (s)	t(SVD)	$\frac{\Vert\hat{Y}-M\Vert_F}{\Vert M\Vert_F}$
500	10	0.4	MV	55	10.0139	2.227	4.0376e−5
			ALM	40	64.9795	62.060	0.0190
500	10	0.5	MV	54	9.1861	2.265	6.5712e−6
			ALM	40	48.9684	46.348	0.0051
800	10	0.4	MV	57	22.1955	3.685	1.2519e−5
			ALM	47	345.8164	337.090	0.0094
800	10	0.5	MV	56	20.5786	3.039	8.7391e−5
			ALM	47	354.8654	347.305	0.0124
1000	10	0.4	MV	53	30.7834	5.645	4.8855e−6
			ALM	51	380.1966	366.026	0.0015
1000	10	0.5	MV	53	31.5101	5.459	6.4956e−5
			ALM	52	117.1756	104.489	4.5571e−4
1500	10	0.4	MV	53	65.1266	8.269	4.3124e−6
			ALM	62	1.2992e+3	1262.200	0.0038
1500	10	0.5	MV	53	63.9595	8.940	4.0222e−5
			ALM	61	387.3567	355.042	4.5571e−4
2000	10	0.4	MV	52	101.8786	7.639	7.0815e−6
			ALM	67	265.0791	194.407	1.0680e−4
2000	10	0.5	MV	52	102.0252	7.798	4.0742e−6
			ALM	67	226.2102	163.678	1.1530e−4
2500	10	0.4	MV	55	175.3766	14.275	4.1223e−6
			ALM	74	1.5442e+3	1421.690	0.0013
2500	10	0.5	MV	53	167.8779	15.960	4.0833e−6
			ALM	72	242.8386	141.612	5.4778e−4
3000	10	0.4	MV	51	422.3507	76.208	3.6895e−6
			ALM	75	828.7169	612.028	0.0038
3000	10	0.5	MV	52	315.4619	40.250	3.7480e−6
			ALM	75	516.7757	323.769	0.0011

that is,

$$\begin{aligned}
\Vert Y_{k+1}\Vert_* &\le \Vert \mathcal{D}_{\tau_k}(Y_k)\Vert_* - \langle \partial\Vert Y_{k+1}\Vert_*, \mathcal{D}_{\tau_k}(Y_k) - Y_{k+1}\rangle \\
&\le \Vert \mathcal{D}_{\tau_k}(Y_k)\Vert_* + \Vert \partial\Vert Y_{k+1}\Vert_*\Vert_F \cdot \Vert \mathcal{D}_{\tau_k}(Y_k) - Y_{k+1}\Vert_F \\
&\le \Vert \mathcal{D}_{\tau_k}(Y_k)\Vert_* + \sqrt{r(Y_{k+1})}\cdot\sqrt{\Vert \mathcal{D}_{\tau_k}(Y_k) - Y_{k+1}\Vert_F^2} \\
&= \sum_{i=1}^{r(\mathcal{D}_{\tau_k}(Y_k))}(\sigma_i - \tau_k) + \sqrt{r(Y_{k+1})}\cdot\sqrt{\Vert Y_k - \mathcal{D}_{\tau_k}(Y_k)\Vert_F^2 - \Vert Y_k - Y_{k+1}\Vert_F^2} \\
&= \sum_{i=1}^{r(\mathcal{D}_{\tau_k}(Y_k))}(\sigma_i - \tau_k) + \sqrt{r(\mathcal{D}_{\tau_k}(Y_k))\cdot\Vert Y_k - \mathcal{D}_{\tau_k}(Y_k)\Vert_F^2} \\
&= \sum_{i=1}^{r(\mathcal{D}_{\tau_k}(Y_k))}(\sigma_i - \tau_k) + \sqrt{r^2(\mathcal{D}_{\tau_k}(Y_k))\cdot\tau_k^2 + r(\mathcal{D}_{\tau_k}(Y_k))\cdot\sum_{i=r(\mathcal{D}_{\tau_k}(Y_k))+1}^{r(Y_k)}\sigma_i^2}.
\end{aligned}$$

Since $0 < \sigma_i \le \tau_k$, $r(D_{\tau_k}(Y_k)) < i \le r(Y_k)$, which implies

$$\sqrt{r^2(\mathcal{D}_{\tau_k}(Y_k))\cdot\tau_k^2 + r(\mathcal{D}_{\tau_k}(Y_k))\cdot\sum_{i=r(\mathcal{D}_{\tau_k}(Y_k))+1}^{r(Y_k)}\sigma_i^2} < r(\mathcal{D}_{\tau_k}(Y_k))\cdot\tau_k + \sum_{i=r(\mathcal{D}_{\tau_k}(Y_k))+1}^{r(Y_k)}\sigma_i.$$

So

$$\|Y_{k+1}\|_* < \sum_{i=1}^{r(\mathcal{D}_{\tau_k}(Y_k))} (\sigma_i - \tau_k) + r(\mathcal{D}_{\tau_k}(Y_k)) \cdot \tau_k + \sum_{i=r(\mathcal{D}_{\tau_k}(Y_k))+1}^{r(Y_k)} \sigma_i = \|Y_k\|_*.$$

Thus, $\lim_{k\to\infty} \|Y_k\|_*$ exists.

Let $\lim_{k\to\infty} \|Y_k\|_* = \|\hat{Y}\|_*$, and $\hat{Y} = \hat{U}\Sigma\hat{V}^*$. From the assumption $\lim_{k\to\infty}\langle U_k V_k^*, R_i\rangle = 0,\ i \in \bar{\Omega}$, hence $\langle \hat{U}\hat{V}^*, R_i\rangle = 0$, $i \in \bar{\Omega}$. From Corollary 3.3, $\hat{Y}$ is the optimal completion matrix of (2.1). □

4. Numerical experiments

In this section, we evaluate the performance of our mean value (MV) algorithm with the ALM algorithm in [12] through numerical experiments, and all the experiments are conducted on the same workstation.

In the experiments, we suggest $p = m/(2n-1)$ as the sampling density, where m is the number of observed entries. For the special structure of Toeplitz matrix, we have $0 \le m \le 2n-1$. We denote the output by $\hat{Y}$, the true Toeplitz matrix by M, and the time of SVD by $t(\text{SVD})$. And we empirically set the parameters $\tau_0 = 0.2\|P_\Omega(M)\|_2$, $c = 0.8$, $\epsilon = 10^{-8}$ in our experiments. The choosing of parameters for the ALM algorithm is suggested in [12], except the parameters of the stopping criteria are $\varepsilon_1 = 10^{-9}$ and $\varepsilon_2 = 10^{-7}$.

A brief comparison of the two algorithms is presented in Table 1. Besides, we apply our algorithm to simply image processing using one of the experimental data. We demonstrate this simulation in Fig. 1.

5. Conclusion

In this paper, we develop a mean value algorithm for Toeplitz matrix completion based on the SVT algorithm. Obviously, our algorithm performs better than ALM algorithm of much higher accuracy. Throughout the iterative process, the completion matrices keep the Toeplitz structure that ensure the fast SVD of Toeplitz matrices. The time of SVD in our algorithm is much less than in the ALM algorithm. In spite of smaller stopping tolerance ($\varepsilon = 10^{-9}$), the ALM algorithm does not converge to the exact solution of the problem well. Besides, the completion matrices of the ALM algorithm do not keep the Toeplitz structure.

References

[1] Y. Amit, M. Fink, N. Srebro, S. Ullman, Uncovering shared structures in multiclass classification, in: Processings of the 24th International Conference on Machine Learing, ACM, 2007, pp. 17–24.
[2] A. Argyriou, T. Evgeniou, M. Pontil, Multi-task feature learing, Adv. Neural Inf. Process. Syst. 19 (2007) 41–48.
[3] M. Mesbahi, G.P. Papavassilopoulos, On the rank minimization problem over a positive semidefinite linear matrix inequality, IEEE Trans. Automat. Control 42 (1997) 239–243.
[4] M. Bertalmio, G. Sapiro, V. Caselles, C. Ballester, Image inpainting, Comput. Graph. 34 (2000) 417–424. SIGGRAPH 2000.
[5] C. Tomasi, T. Kanade, Shape and motion from image streams under orthography: a factorization method, Int. J. Comput. Vis. 9 (1992) 137–154.
[6] E.J. Candès, B. Recht, Exact matrix completion via convex optimization, Found. Comput. Math. 9 (2009) 717–772.
[7] E.J. Candès, T. Tao, The power of convex relaxation: Near-optimal matrix completion, IEEE Trans. Inform. Theory 56 (2010) 2053–2080.
[8] R.H. Keshavan, A. Montanari, S. Oh, Matrix completion from a few entries, IEEE Trans. Inform. Theory 56 (2010) 2980–2998.
[9] B. Recht, A simpler approach to matrix completion, J. Mach. Learn. Res. 12 (2011) 3413–3430.
[10] K.C. Toh, S. Yun, An accelerated proximal gradient algorithm for nuclear norm regularized least squares problems, Pac. J. Optim. 6 (2010) 615–640.
[11] J.-F. Cai, E.J. Candès, Z. Shen, A singular value thresholding algorithm for matrix completion, SIAM J. Optim. 20 (2010) 1956–1982.
[12] Z. Lin, M. Chen, L. Wu, Y. Ma, The augmented lagrange multiplier method for exact recovery of corrupted low-rank matrices, UIUC Technicial Report UIUL-ENG-09-2214, 2010.
[13] M. Fazel, Matrix rank minimization with applications (Ph.D. thesis), Stanford University.
[14] M. Fazel, H. Hindi, S.P. Boyd, Log-det heuristic for matrix rank minimization with applications to Hankel and Euclidean distance matrices, in: Proceedings of the American Control Conference, vol. 3, 2003, pp. 2156–2162.
[15] S. Ma, D. Goldfarb, L. Chen, Fixed point and bregman iterative methods for matrix rank minimization, Math. Program. 128 (2011) 321–353.
[16] E.J. Candès, Y. Plan, Matrix completion with noise, Proc. IEEE (2009).
[17] J.-F. Cai, S. Osher, Fast singular value thresholding without singular value decomposition, Methods Appl. Anal. 20 (2013) 335–352.
[18] Y. Hu, D.-B. Zhang, J. Liu, J.-P. Ye, X.-F. He, Accelerated singular value thresholding for matrix completion, in: KDD'12: Proceedings of the 18th ACM SIGKDD international conference on Knowledge discovery and data mining, 2012, pp. 298–306.
[19] W.U. Bajwa, J.D. Haupt, G.M. Raz, S.J. Wright, R.D. Nowak, Toeplitz-structures compressed sensing matrices, in: Statistical Signal Processing, 2007, SSP'07. IEEE/SP 14th Workshop on, 2007, pp. 294–298.
[20] J. Huang, T.-Z. Huang, X.-L. Zhao, Z.-B. Xu, Image restoration with shifting reflective boundary conditions, Sci. China 42 (2012) 504–519.
[21] J. Gutiérrez-Gutiérrez, P.M. Crespo, A. Böttcher, Functions of banded Hermitian block Toeplitz matrices in signal processing, Linear Algebra Appl. 422 (2007) 788–807.
[22] F. Sebert, Y.-M. Zhou, L. Ying, Toeplitz block matrices in compressed sensing and their applications in imaging, in: Information Technology and Applications in Biomedicine (ITAB), International Conference on, IEEE, 2008, pp. 47–50.
[23] A.K. Shaw, S. Pokala, R. Kumaresan, Toeplitz and Hankel approximation using structured approach, in: acoustics, speech and signal processing, in: Processing of the IEEE International Conference, vol. 2349, 1998, pp. 12–15.
[24] F.T. Luk, S. Qiao, A fast singular value algorithm for Hankel matrices, in: V. Olshevsky (Ed.), Fast Algorithms for Structured Matrices: Theory and Applications, Contemporary Mathematics, vol. 323, American Mathematical Society, 2003.
[25] W. Xu, S. Qiao, A fast SVD algorithm for square Hankel matrices, Linear Algebra Appl. 428 (2008) 550–563.
[26] G.H. Golub, C.F. VanLoan, Matrix Computations, third ed. The Johns Hopkins University Press, Baltimore, MD, 1996.
[27] C.F. VanLoan, Computational Frameworks for the Fast Fourier Tansform, Society for Industrial and Applied Mathematics (SIAM), 1992.
[28] T. Kailath, A.H. Sayed (Eds.), Fast Reliable Algorithms for Matrices with Structure, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1999.