


An efficient, high-order, adaptive Finite Volume solver for physical modelling in advanced manufacturing processes

Patrick Zimbrod 
Digital Manufacturing
University of Augsburg
Augsburg, Germany
patrick.zimbrod@uni-a.de

Johannes Schilp
Digital Manufacturing
University of Augsburg
Augsburg, Germany
johannes.schilp@uni-a.de

Abstract—We present a free and open source implementation of the Finite Volume Method that captures thermal and solute-driven free-surface flows, more commonly known as Marangoni flows. This type of physics commonly appears within additive manufacturing and is known to be challenging to resolve. The application is capable of performing adaptive mesh refinement coupled with high order WENO schemes in order to substantially save computational cost in multiphase flows where accurate tracking of the phase boundaries is crucial. By accurately simulating the temporal evolution of melt pools, unique insight into the physics of melt pool formation is possible that is otherwise hard to gain experimentally. We demonstrate our work by a simple two-dimensional benchmark case and outline possible applications with a mesoscopic model of Laser Powder Bed Fusion additive manufacturing.

Index Terms—finite volume method, adaptive refinement, marangoni flow, open source software

I. INTRODUCTION

This document is a model and instructions for L^AT_EX.

II. THEORY

The theory of the proposed implementation relies on the Continuum Surface Stress Method initially presented by Lafaurie et al., extending the works on modelling surface tension with the Volume of Fluid Method as described by Brackbill et al. [1], [2]. To describe the necessary linear algebra, we will use the index notation of tensors in a cartesian coordinate system alongside with the Einstein summation convention where the sum over matching indices is implied. First, the capillary stress tensor is introduced:

$$T_{ij} = -\sigma\delta_s(\delta_{ij} - n_i n_j) \quad (1)$$

Where σ is the generally non constant coefficient of surface tension, δ_s is the interface delta function, δ_{ij} is the Kronecker Delta function and n_i is the interface unit normal vector. The local coordinate system of the interface needs to be an orthogonal system in order to separate the purely geometry based capillary force and the tangential Marangoni forces. The

resulting force accounting for all surface tension effects can now be expressed as the divergence of the capillary stress tensor, yielding:

$$\frac{\partial T_{ij}}{\partial x_j} = \frac{\partial \sigma}{\partial x_j} [\delta_s(\delta_{ij} - n_i n_j)] + \frac{\partial \delta_s}{\partial x_j} [\sigma(\delta_{ij} - n_i n_j)] + \sigma \delta_s \frac{\partial}{\partial x_j} (\delta_{ij} - n_i n_j) \quad (2)$$

One can show that Eq. 2 can be re-arranged using some tensor algebra and geometric identities into a much shorter and more useful form. The derivation is given in more detail by Lafaurie et al. [1]. By performing those rearrangements, we can recover a form that separates the normal from the tangential components of the divergence vector:

$$\frac{\partial T_{ij}}{\partial x_j} = \frac{\partial \sigma}{\partial x_i} [\delta_s(\delta_{ij} - n_i n_j)] - \sigma \kappa n_i \delta_s \quad (3)$$

Here, the interface curvature κ is introduced. The second term of the right hand side corresponds to the normal capillary force directed in the normal direction of the interface, effecting a contractional movement of the interface. The first term resembles the marangoni-type forces present. By taking the derivative of the surface tension coefficient, this term does not vanish iff there are gradients present at the interface. This is normally the case when there are multiple species involved or temperature gradients present [3]. Note that we then simply evaluate the ordinary form of surface tension without any additional physics present. However, as the surface tension coefficient is otherwise not a direct local variable, one must further differentiate the term:

$$\frac{\partial \sigma(c, T)}{\partial x_j} = \frac{\partial \sigma(c, T)}{\partial T} \frac{\partial T}{\partial x_j} + \frac{\partial \sigma(c, T)}{\partial c} \frac{\partial c}{\partial x_j} \quad (4)$$

Additionally, the delta function δ_s needs to be discretized in a suitable manner in order to capture the physics within the Finite Volume framework. This can be done using the Volume of Fluid Method by taking the gradient of the phase volume fraction α [4], [5]:

$$\delta_s = \left| \frac{\partial \alpha}{\partial x_i} \right| \quad (5)$$

The interface unit normal vector n_i can be computed in a similar manner by using the previously computed interface function:

$$n_i = \frac{1}{\delta_s} \frac{\partial \alpha}{\partial x_i} \quad (6)$$

To perform the finite volume simulations, the Open Source C++ Library OpenFOAM is used in Version 2106. In order to accommodate the extended surface stress formulation, some of the existing solver capabilities need to be modified. The changes necessary are elaborated on in more detail in the following chapter.

III. RESULTS

A. Implementation

This foundation is modified and implemented based on the open source library OpenFOAM based on version 2106, which is a popular software library that implements the Finite Volume Method [6]. The modifications rely on the works of Gueyffier et al. in order to account for the additional Marangoni stresses [4].

In our proposed implementation, we compute the local gradient of the surface tension explicitly, after it is evaluated for every cell. This means that the gradients with respect to temperature and species concentration are interpolated from the given material properties and the field derivatives are computed separately, unlike Eq. 4 suggests. As a result, the capillary stress is evaluated regardless of the type of Marangoni flow. That is, temperature as well as species gradients or in other effects causing spacial gradients are regarded in this model and contribute to a resulting tangential stress on the interface. Using this approach, the solver configuration remains flexible regarding the future implementation of other or different type physics. However, an appropriate modelling of the surface tension coefficient is required.

The proposed code computes the divergence of the capillary stress tensor for finite volume cell centers. However, to handle source terms within the PIMPLE loop, it is necessary to also supply the surface tension force as a scalar interpolated to the cell face. Therefore, an additional method is implemented that computes the L_2 norm of the surface tension force vector and interpolates the scalar value via finite differencing.

For powder bed fusion applications where large spacial temperature gradients in the order of $1 \times 10^7 \frac{\text{K}}{\text{m}}$ can be present [7], it is advisable to use a second order accurate and stable discretisation scheme for the temperature flux, such as the already within OpenFOAM implemented TVD schemes minmod, superBee or vanLeer [8], [9].

A very useful property of this approach is that not only thermally driven surface tension effects can be incorporated, but also gradients arising from any kind of inhomogeneity. This means that among others solute-driven effects can also be modelled. This scenario is common in manufacturing

processes involving mixing of multiple solvents. Within the context of additive manufacturing, in-situ alloying is a scenario where such effects are not negligible [10].

To account for thermal and solute-driven effects simultaneously, we need to incorporate the temperature dependence of the surface tension as well as interfacial tension between the alloying elements in the liquid state. This allows us to capture not only flow patterns on the liquid-gas interface, but also between liquid interfaces of dissimilar species. We may find accurate values for temperature dependent surface tension in the literature. However, determining interfacial tension poses a non-trivial problem that has been discussed in the literature [11].

Naturally, the occurrence of interfaces between dissimilar metals is nothing unique to the setting of in-situ alloying. This effect should be observable in pre-alloyed powders as well and hence also play a role in Powder Bed Fusion processes in general. However, the mixing paths in this case will be in the range of less than the typical powder diameter and thus in the order of $1 \mu\text{m}$ to $10 \mu\text{m}$. In-situ alloying however leads to a distribution of alloying species much further apart from each other, i.e. about 10x in comparison. This consequently increases the time the flow takes to homogenize the melt notably and thus should not be considered instantaneous and hence neglected anymore.

In a pursuit to keep model evaluation simple and not rely on Molecular Dynamics calculations, we use the following, relatively simple approximation for interfacial tension given by Girifalco and Good [12]:

$$\sigma_{1,2} = \sigma_1 + \sigma_2 + 2C(\sigma_1\sigma_2)^{1/2} \quad (7)$$

Here, $\sigma_{1,2}$ denotes the interfacial tension, σ_1 and σ_2 are the surface tensions of the involved species and C is a constant which we assume to be unity as a reasonable approximation [11].

In order to approximate the differential terms in the governing equations efficiently, we use high order weighted essentially non-oscillatory (WENO) schemes that have been developed separately for OpenFOAM by Gärtner et al. [13], [14].

B. Oscillating Droplet in a Temperature Gradient

We now investigate a simple, two-dimensional benchmark case that illustrates the ability to capture Marangoni effects for the present work.

C. Powderbed-scale physics during selective laser melting

We now turn to a more complex and relevant simulation setup that appears regularly in additive manufacturing. We aim to investigate the melting, solidification and evaporation behavior of molten metal during the Laser Powder Bed Fusion Process (PBF-LB/M).

IV. SUMMARY

We expect that the results of this work will help to improve understanding of the complex physics in metal-additive manufacturing substantially. Furthermore, the high fidelity data that can be generated using the proposed solver can be used in order to train physics informed machine learning models. It has previously been shown that such models can speed up simulations by order of magnitudes by execution of a simple forward pass. However, accurate and rich training data are needed in order to train such models. We anticipate that this work can help supply this data and further enhance the field of modelling for complex manufacturing processes.

V. DATA AVAILABILITY

The presented solver is available via Github at [pzimbrod/thermocapillaryInterFoam](https://github.com/pzimbrod/thermocapillaryInterFoam). All data used in this article can be accessed at [pzimbrod/ieee-iceccme-2022](https://github.com/pzimbrod/ieee-iceccme-2022).

REFERENCES

- [1] B. Lafaurie, C. Nardone, R. Scardovelli, S. Zaleski, and G. Zanetti, "Modelling Merging and Fragmentation in Multiphase Flows with SURFER," *Journal of Computational Physics*, vol. 113, pp. 134–147, July 1994.
- [2] J. Brackbill, D. Kothe, and C. Zemach, "A continuum method for modeling surface tension," *Journal of Computational Physics*, vol. 100, pp. 335–354, June 1992.
- [3] J. Straub, A. Weinzierl, and M. Zell, "Thermokapillare Grenzflächenkonvektion an Gasblasen in einem Temperaturgradientenfeld," *Wärme- und Stoffübertragung*, vol. 25, p. 281288, Jan. 1990.
- [4] D. Gueyffier, J. Li, A. Nadim, R. Scardovelli, and S. Zaleski, "Volume-of-Fluid Interface Tracking with Smoothed Surface Stress Methods for Three-Dimensional Flows," *Journal of Computational Physics*, vol. 152, pp. 423–456, July 1999.
- [5] C. W. Hirt and B. D. Nichols, "Volume of fluid (VOF) method for the dynamics of free boundaries," *Journal of Computational Physics*, vol. 39, pp. 201–225, Jan. 1981.
- [6] H. Jasak, A. Jemcov, and Z. Tukovic, "OpenFOAM: A C++ Library for Complex Physics Simulations," *International Workshop on Coupled Methods in Numerical Dynamics*, p. 20, Sept. 2007.
- [7] P. A. Hooper, "Melt pool temperature and cooling rates in laser powder bed fusion," *Additive Manufacturing*, vol. 22, pp. 548–559, Jan. 2018.
- [8] P. L. Roe, "Characteristic-Based Schemes for the Euler Equations," *Annual Review of Fluid Mechanics*, vol. 18, pp. 337–365, Jan. 1986.
- [9] B. van Leer, "Towards the ultimate conservative difference scheme. II. Monotonicity and conservation combined in a second-order scheme," *Journal of Computational Physics*, vol. 14, pp. 361–370, Mar. 1974.
- [10] A. Katz-Demyanetz, A. Koptiyug, and V. V. Popov, "In-situ Alloying as a Novel Methodology in Additive Manufacturing," in *2020 IEEE 10th International Conference Nanomaterials: Applications Properties (NAP)*, pp. 02SAMA05–1–02SAMA05–4, Nov. 2020.
- [11] A. Marmur and D. Valal, "Correlating Interfacial Tensions with Surface Tensions: A Gibbsian Approach," *Langmuir*, vol. 26, pp. 5568–5575, Apr. 2010.
- [12] L. A. Girifalco and R. J. Good, "A Theory for the Estimation of Surface and Interfacial Energies. I. Derivation and Application to Interfacial Tension," *The Journal of Physical Chemistry*, vol. 61, pp. 904–909, July 1957.
- [13] J. W. Gärtner, A. Kronenburg, and T. Martin, "Efficient WENO library for OpenFOAM," *SoftwareX*, vol. 12, p. 100611, July 2020.
- [14] T. Martin and I. Shevchuk, "Implementation and Validation of Semi-Implicit WENO Schemes Using OpenFOAM®," *Computation*, vol. 6, p. 6, Jan. 2018.