Measuring Local

Free Air Gradient

GOPH 547

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Abstract

The investigation made use of a Worden relative gravimeter to measure vertical gradient on each of the nine floors of a certain building. The ground floor was used as a 'base station' where repeated measurements were averaged to produce drift curve. Using the base station measurements and drift curve, instrument drift within the relative gravimeter itself (such as changes in the spring stiffness) can be minimized. After doing corrections, a measured local free air gradient is calculated which is then compared with the theoretical value and sources of error/deviation are discussed.

Background and Theory

Examining change of gravity with elevation involves two corrections; free air and Bougeur plate correction. This laboratory will focus solely on the free air correction. The free air correction is needed to examine the variations of gravity due to elevation increases or decreases from the Earth's barycentre[1]. As such, the following background equation will be necessary;

$$g(r) = \frac{GM_e}{r^2} \tag{1}$$

where r, in this specific case, represents the radius of the Earth.

However, the radius of the Earth changes with latitude. As such, the International Gravity Formula (IGF) is used along with its vertical derivative;

$$g(\theta) = 9.78327\left(1 + 0.0053024\sin^2\theta - 0.0000058\sin^2 2\theta\right)\ \mathrm{m/s^2} \tag{2}$$

$$\frac{dg}{dr} = -0.308558\left(1 + 0.000704\cos 2\theta\right)\ \mathrm{mGal/m} \tag{3}$$

which then provides us with a more local result. In Calgary, a latitude of 51.1 degrees is used and as such, when plugging it into equation (3), a value of -0.3085 mGal/m is found. This is known as the theoretical local free air gradient and will be used to compare computed/ measured values outputted as results during the investigation.

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[1] Karchewski, B. (2016). GOPH 547 - Gravity and magnetics. Retrieved from https://d2l.ucalgary.ca/d2l/le/content/169330/viewContent/2408230/View

The actual free air gradient will however differ from this theoretical value due to the presence of mass anomalies within the subsurface locally. A key assumption, for the purposes of this laboratory, is to neglect the minuscule variations since they may be smaller than the measurement errors themselves. Therefore, the theoretical value of -0.3085 mGal/m will be sufficient in calculating our percentage errors.

Method and Algorithm

The laboratory began by setting up the relative gravimeter on the ground floor of the Earth Sciences building located at the University of Calgary and letting a timer run through the entire duration of the 'survey'. A full procedure regarding the operation of the gravimeter itself is located in the references. Three readings of extremely close proximity were first taken at the basement floor, which constituted as our base station. The relative gravimeter was then moved up to take readings of floors 1,2,3 and back down to the ground floor 0. This was due to correcting for instrument drift by taking the survey in a loop. As such, floors 4,5,6 were taken along with bringing it to the ground again, and then 7,8,9 and back to ground floor 0.

Once gravimeter readings are recorded along with the stopwatch time laps, floor 'thickness' or 'elevation changes' are measured using a 30m tape ruler. The recorded data is composed in columns and processed in Microsoft Excel spreadsheet software. Since the data is in instrument units, a multiplication by 0.10254 (which was the dial constant) was necessary to convert into meaningful mGal units. Being looping always back to ground floor 0, a drift curve was plotted who's slope was calculated in Excel. This drift curve slope essentially showed instrument drift which was corrected by multiplying the slope to get corrected time readings. The corrected time readings were subtracted from the corrected readings to produce final drift corrected measurements.

Once final drift corrected measurements were put in a column, they could be plotted against elevation changes between floors to show a correlation of variance in vertical gravity gradient. Excel used a linear least squares regression function built on these plots to produce best fitting slopes whose value can be regarded as the calculated local free air gradient. The local free air gradient that was calculated with the raw data (after corrections) was finally compared with the theoretical value of the free air gradient at a latitude of 51.7 degrees (for Calgary) to provide with some context and possible sources of error.

Results and Discussion

To begin, the first figure shows a graph of the base station readings for each of the times the relative gravimeter was looped back. The three grouped readings correspond to the three readings taken each time at the floor which were in close proximity to one another. This is plotted against the time which is the x axis. The y axis represents the change in relative gravimeter readings subtracted from the average of the base station reading. From these aggregated values plotted in groups, a line of best fit is plotted who's slope is then calculated.

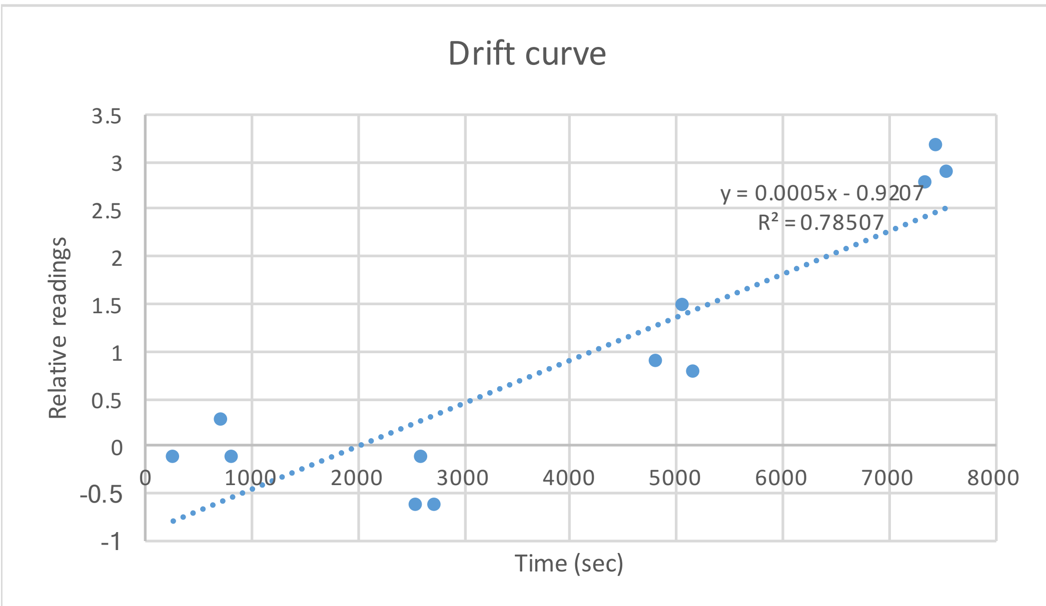

Figure 1: relative gravimeter reading vs time, a line of best fit along with its slope provides drift curve

This figure also illustrates how the instrument varies with time. The slope of this curve, 0.78507, is then used to perform drift correction.

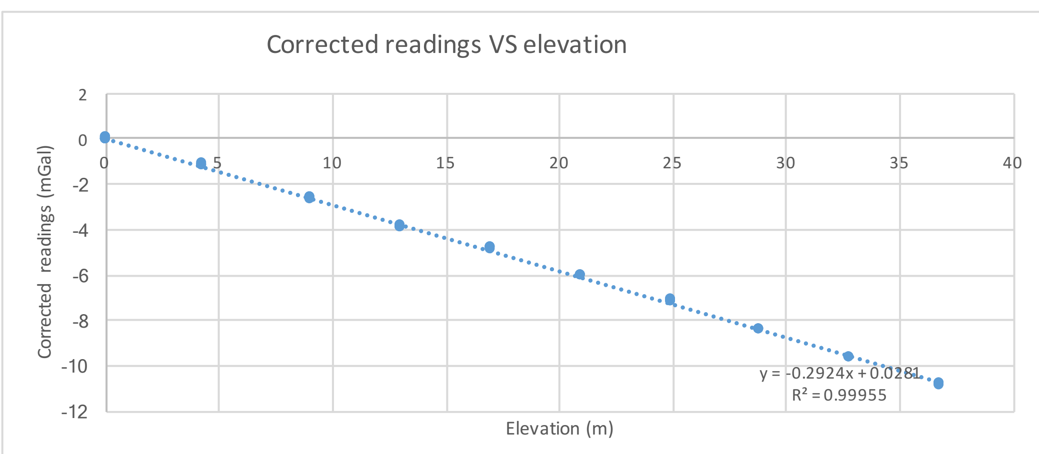

Figure 2: corrected readings vs elevation

Figure two shows the plot for corrected readings against elevation. The corrected readings are found by subtracting each elevation reading by the average and then multiplying that new column with the dial constant to get final corrected readings, and then that is plotted against the elevation. From here, a free air gradient that is not completely corrected can be calculated by the slope which is -0.2924.

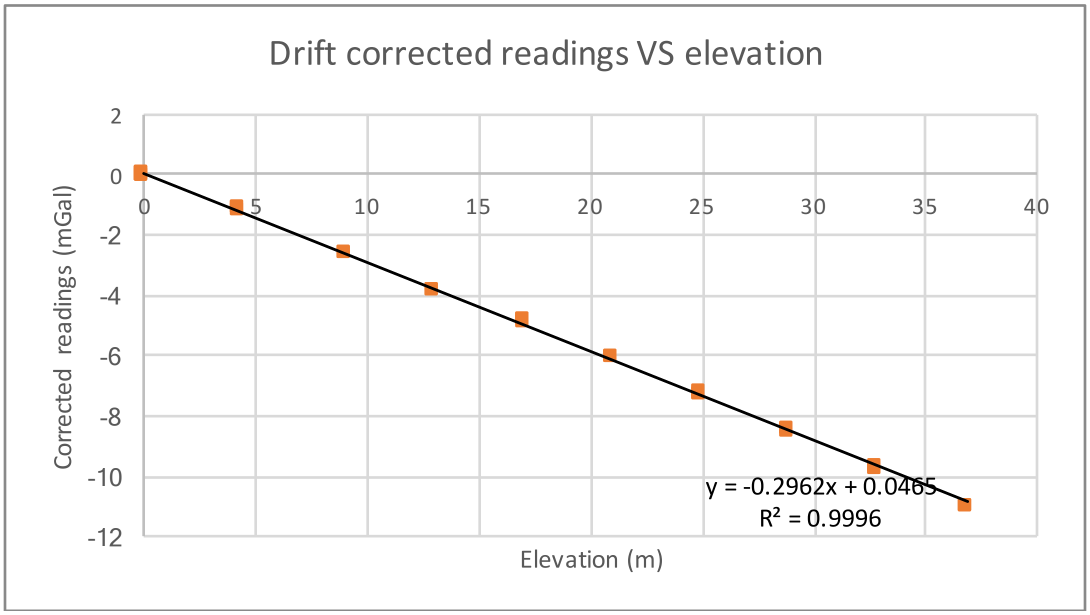

Figure 3: final drift corrected vertical gradient measurements against elevation

The final drift corrected readings vs elevation are plotted in figure 3. This provides us with our local free air gradient which is quite notably similar to the theoretical. The y axis is determined by taking our corrected readings found earlier above and subtracting those values by the measured free air gradient (not time corrected) which itself is subtracted by the average time of the specific loop. That value is multiplied by the subtraction of the averaged gravimeter readings for the two individual loops. The entire numerator is divided by averaged time interval values for the individual loops. A new column is made by those values and it is finally multiplied by 0.1025 which is the instrumental dial constant.

Once the slope of the final plot is found, this is regarded as the local free air gradient calculated by processing the raw data. In this case, -0.2962 mGal/m when compared to the theoretical -0.3085 mGal/m produces an error of 3.987%. Several key assumptions are made that can affect the measurements and be a source of error. Like mentioned previously, local mass anomalies within the subsurface were ignored, mainly assumed to have a minimal effect. It was also assumed that change in vertical elevation was the only variable affecting gravity readings though in reality that is not entirely true. Tidal and celestial bodies of large mass also effect readings. Mass of the building itself, along with mass distribution of students as they walk and shake the ground, especially during the latter stages of the survey once more came into the building, could contribute to error as well. Measuring the length of the floors may also have been slightly inconsistent. There was a small angle between floors so it was not directly straight down vertical in length. Technically, the hypotenuse should be taken which constituted a very small error as well possibly. The final error may be constituted by each of the three students handling the gravimeter. They may have their own style for calibrating the instrument and as such, a deviation may occur whereas if one student was consistently using the gravimeter throughout the entire survey, it may have been slightly more consistent.

Although it is important to recognize these errors, it is also important to realize that the error produced was essentially quite small. As such, this survey method and technique would probably invaluable on the field to extract vertical gradient data and could be used in geophysical interpretation of the subsurface quite well.

Conclusion

This investigation into measuring the vertical gradient of gravity via relative gravimeter highlighted importance of performing corrections on measured gravity data. The process of refining data was clearly seen in the figures, as the final drift corrected data slope was closer to the theoretical value. From the raw gravity data, after applying instrument drift corrections, the produced graphs slope was -0.2962 mGal/m. This deviated from the theoretical value of -0.3085 mGal/m by an error of 3.987%. As discussed previously, the deviation may be due the errors such as mass of building and mass distribution within the building during the later stages of the survey. Calculations of elevation change between floors may also be of influence as they were not measured vertically straight down due to slight angle and for further refinement, the hypotenuse may have been better used. Although these errors are indeed notable, a sound case can be made that they don't seem to influence our readings enough and the error produced is small. As such, this data can be very useful on a real world basis.

References

Karchewski, B. (2017). GOPH 547 - Gravity and magnetics. Retrieved from https://d2l.ucalgary.ca/d2l/le/content/169330/viewContent/2406931/View?ou=169330

W.M. Telford, L.P. Geldart and R.E. Sheriff. (1990). *Applied Geophysics, Second Edition*. Cambridge University Press.

"Theoretical gravity" from *Wikipedia: The Free Encyclopedia*. Link: https://en.wikipedia.org/wiki/Theoretical_gravity.