3.28 An image is filtered with three Gaussian lowpass kernels of sizes 3×3, 5×5, and 7×7, and standard deviations 1.5, 2, and 4, respectively. A composite filter, w, is formed as the convolution of these three filters.

(a)* Is the resulting filter Gaussian? Explain.

(b) What is its standard deviation?

(c) What is its size?

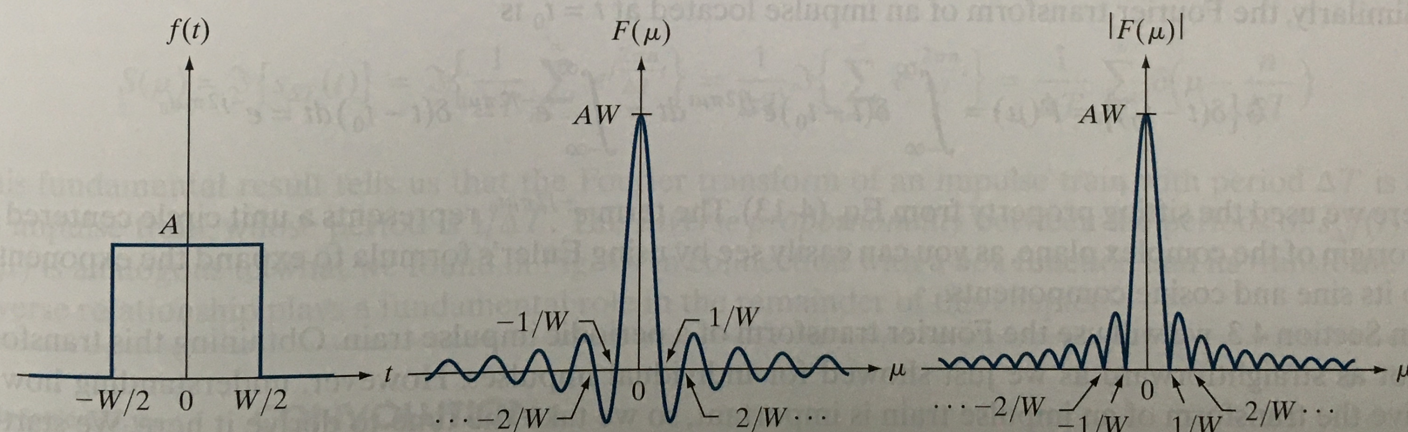
3.38 In a given application, a smoothing kernel is applied to input images to reduce noise, then a Laplacian kernel is applied to enhance fine details. Would the result be the same if the order of these operations is reversed?

4.2 Repeat Example 4.1, but using the function $f(t) = A$ for $0 \le t < T$ and $f(t) = 0$ for all other values of t. Explain the reason for any differences between your results and the results in the example.

EXAMPLE 4.1: Obtaining the Fourier transform of a simple continuous function.

The Fourier transform of the function in Fig. 4.4(a) follows from Eq. (4-20):

$$
\begin{aligned}
F(\mu) &= \int_{-\infty}^{\infty} f(t)e^{-j2\pi\mu t}\,dt = \int_{-W/2}^{W/2} Ae^{-j2\pi\mu t}\,dt \\
&= \frac{-A}{j2\pi\mu}\Big[e^{-j2\pi\mu t}\Big]_{-W/2}^{W/2} = \frac{-A}{j2\pi\mu}\Big[e^{-j\pi\mu W} - e^{j\pi\mu W}\Big] \\
&= \frac{A}{j2\pi\mu}\Big[e^{j\pi\mu W} - e^{-j\pi\mu W}\Big] \\
&= AW\frac{\sin(\pi\mu W)}{(\pi\mu W)}
\end{aligned}
$$



a b c

4.6 With reference to Fig. 4.11:

(a)* Redraw the figure, showing what the dots would look like for a sampling rate that exceeds the Nyquist rate slightly.

(b) What is the *approximate* sampling rate represented by the large dots in Fig. 4.11?

(c) *Approximately*, what would be the lowest sampling rate that you would use so that (1) the Nyquist rate is satisfied, and (2) the samples look like a sine wave?

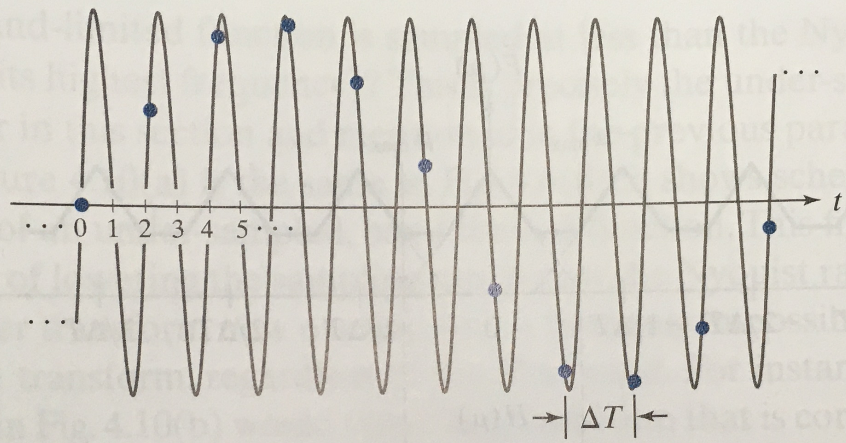


FIGURE 4.11 Illustration of aliasing. The under-sampled function (dots) looks like a sine wave having a frequency much lower than the frequency of the continuous signal. The period of the sine wave is 2 s, so the zero crossings of the horizontal axis occur every second. ΔT is the separation between samples.

4.17 Demonstrate the validity of the translation (shift) properties of the following 1-D, discrete Fourier transform pairs. (*Hint:* It is easier in part (b) to work with the IDFT.)

(a)* $f(x)e^{j2\pi u_0 x/M} \Leftrightarrow F(u-u_0)$

(b) $f(x-x_0) \Leftrightarrow F(u)e^{-j2\pi u x_0/M}$