Introduction to Machine Learning in MR Imaging

Gaël Varoquaux

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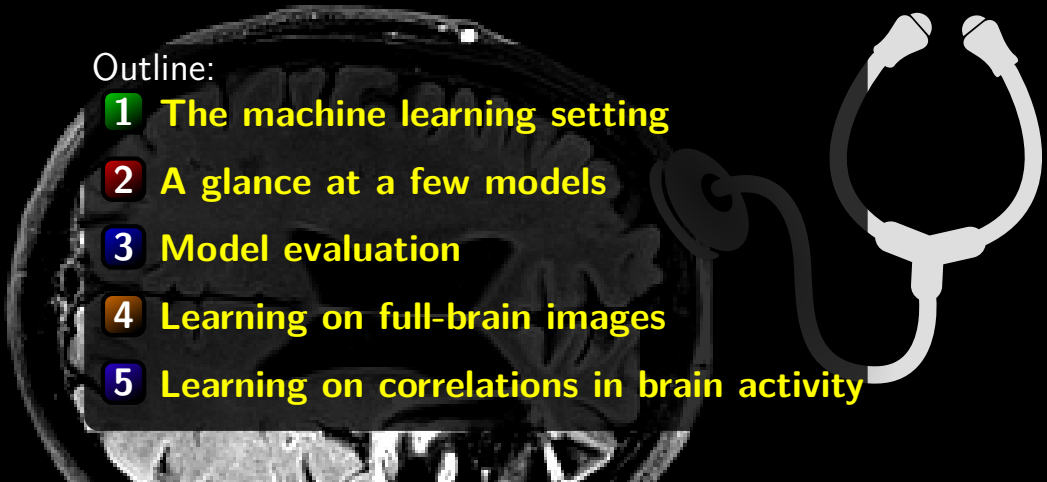

Outline:

1. The machine learning setting
2. A glance at a few models
3. Model evaluation
4. Learning on full-brain images
5. Learning on correlations in brain activity

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Declaration of Financial Interests or Relationships

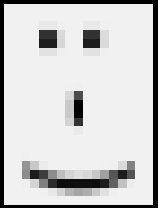
Speaker Name: Gaël Varoquaux

I have no financial interests or relationships to disclose with regard to the subject matter of this presentation.

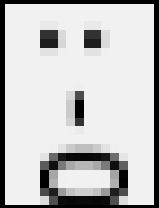
1 The machine learning setting

Adjusting models for prediction

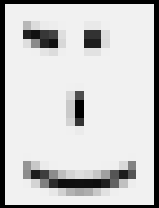
Face recognition



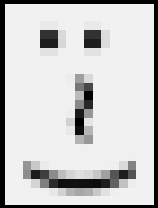
Andrew



Bill

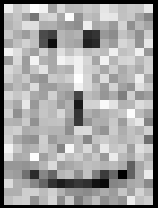


Charles



Dave

Face recognition



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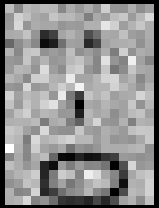

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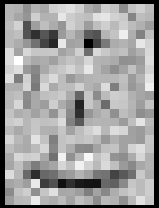

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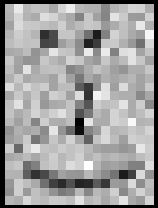
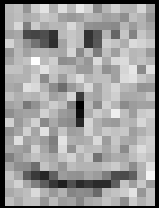


Dave



A simple method:

1. Store all the known (noisy) images and the names that go with them.
2. From a new (noisy) images, find the image that is most similar.

"Nearest neighbor" method

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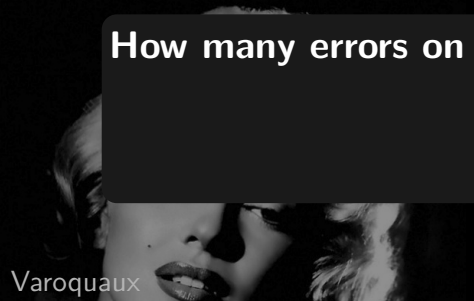
How many errors on already-known images?



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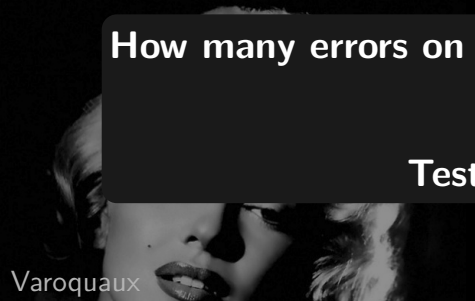
"Nearest neighbor" method

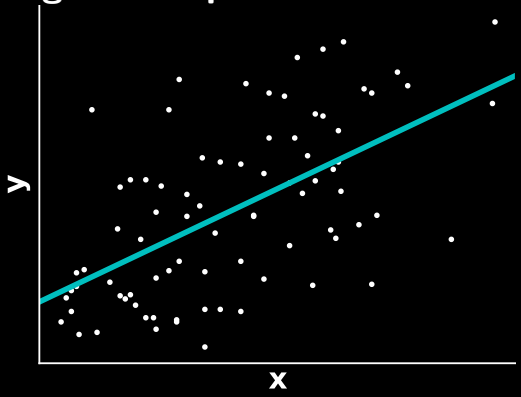
How many errors on already-known images?

... **0: no errors**

Test data $\neq$ Train data



A single descriptor: 1 dimension



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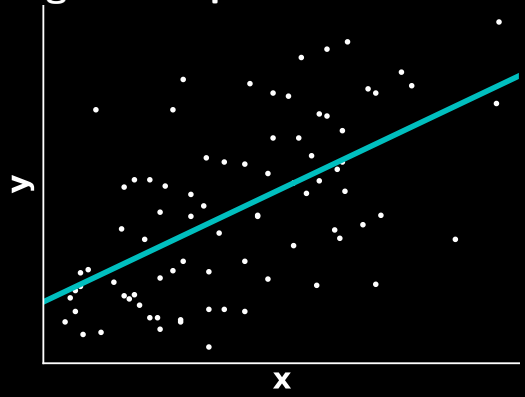
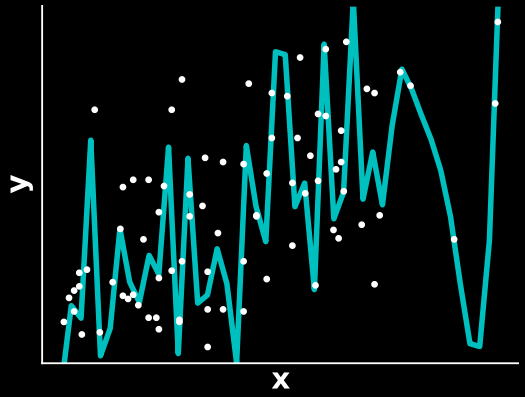
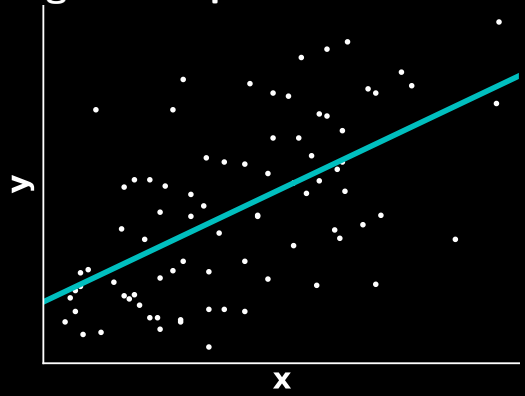
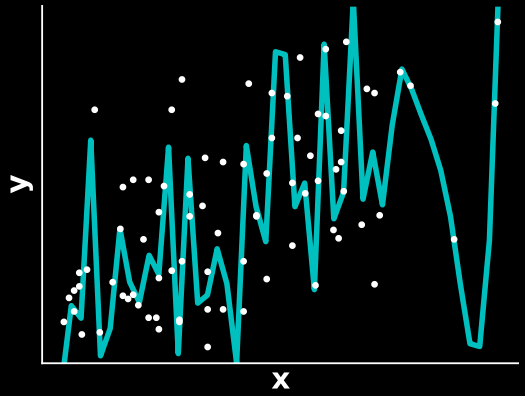




Which model to prefer?

A single descriptor: 1 dimension

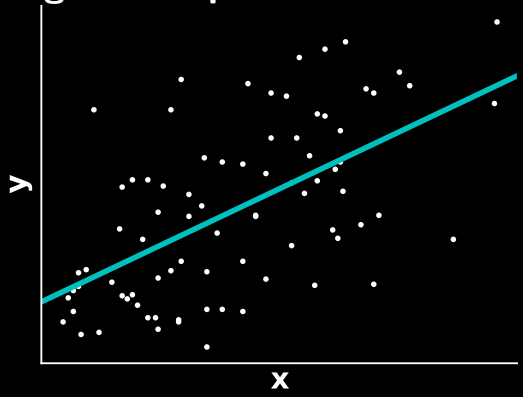


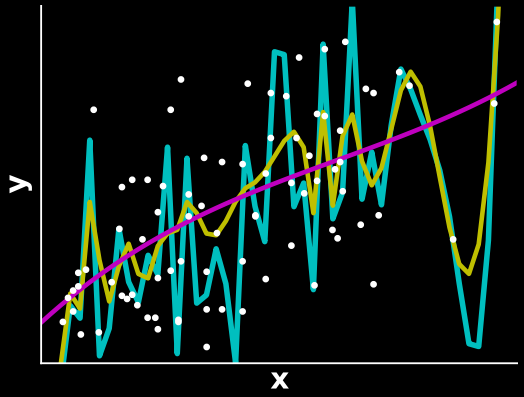


Problem of *"over-fitting"*

- Minimizing error is not always the best strategy (learning noise)
- Test data $\neq$ train data

A single descriptor: 1 dimension

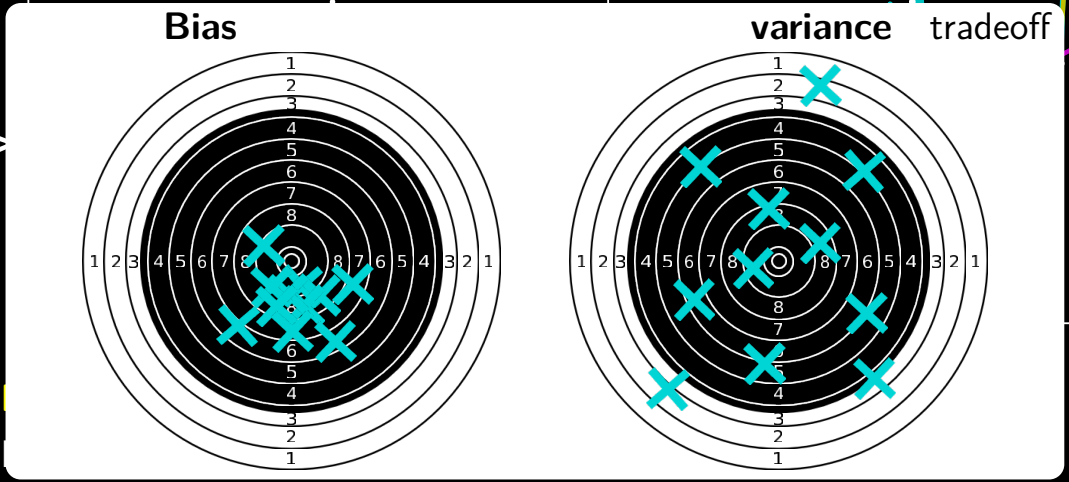




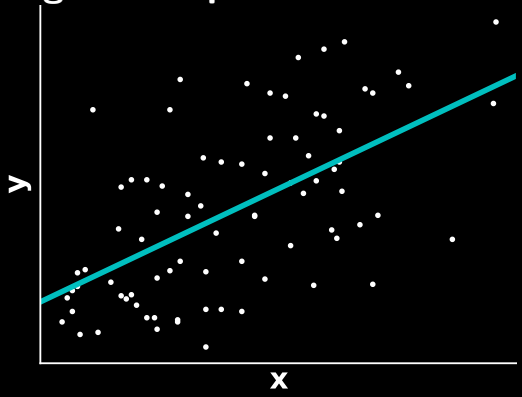
Prefer simple models = concept of *"regularization"*

Balance the number of parameters to learn with the amount of data

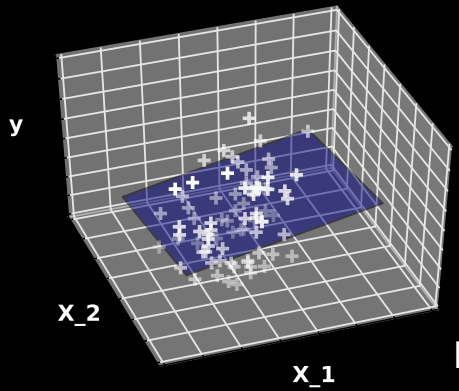
A single descriptor: 1 dimension



A single descriptor: 1 dimension

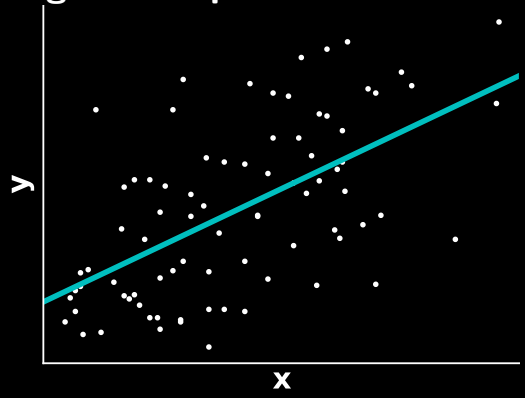


Two descriptors: 2 dimensions

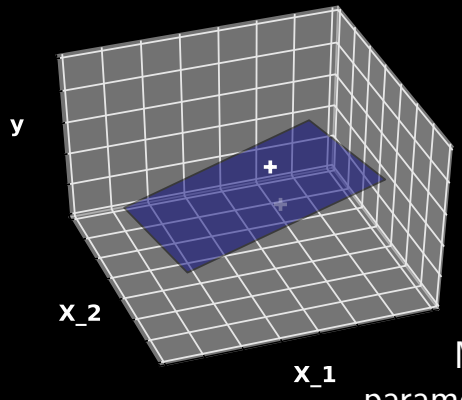


More parameters

A single descriptor: 1 dimension



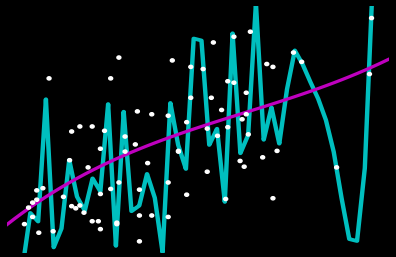
Two descriptors: 2 dimensions

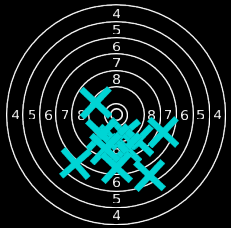


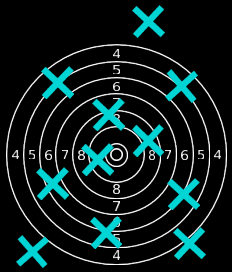
More parameters

⇒ Model with more parameters need much more data

"curse of dimensionality"

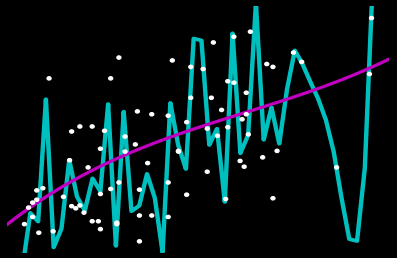


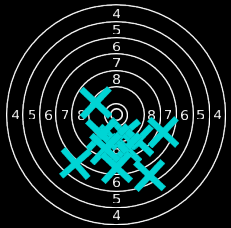


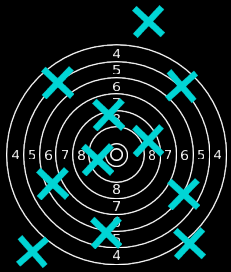


Settings: data $(\mathbf{X}, \mathbf{y})$, prediction $\mathbf{y} \sim f(\mathbf{X}, \mathbf{w})$

Our goal: $\underset{\mathbf{w}}{\text{minimize}} \quad \|\mathbf{y} - f(\mathbf{X}, \mathbf{w})\|$







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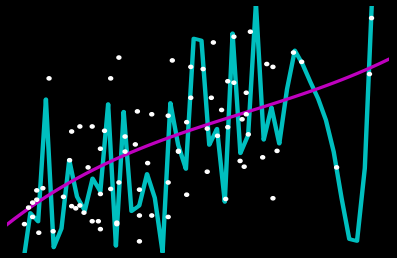
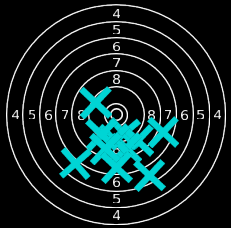
Our goal: $\underset{\mathbf{w}}{\text{minimize}}\ \mathbb{E}[\|\mathbf{y} - f(\mathbf{X}, \mathbf{w})\|\ ]$

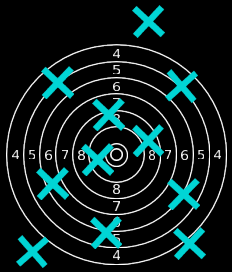
We only can measure $\|\mathbf{y} - f(\mathbf{X}, \mathbf{w})\|$

Prediction is very difficult, especially about the future.

Niels Bohr







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We only can measure $\|\mathbf{y} - f(\mathbf{X}, \mathbf{w})\|$

Solution: bias $\mathbf{w}$ to push toward a plausible solution

In a minimization framework: $\underset{\mathbf{w}}{\text{minimize}}\ \|\mathbf{y} - f(\mathbf{X}, \mathbf{w})\| + p(\mathbf{w})$

$$\mathcal{P}(\mathbf{w}|\mathbf{x},\mathbf{y}) \propto \mathcal{P}(\mathbf{y},\mathbf{x}|\mathbf{w})\ \mathcal{P}(\mathbf{w}) \qquad (\star)$$

"Posterior"
Quantity of interest

Forward model

"Prior"
Expectations on $\mathbf{w}$

- Forward model: $\mathbf{y} = f(\mathbf{x},\mathbf{w}) + \mathbf{e}$, $\mathbf{e}$: noise
  $\Rightarrow \mathcal{P}(\mathbf{x},\mathbf{y}|\mathbf{w}) \propto \exp -\mathcal{L}(\mathbf{y} - f(\mathbf{x},\mathbf{w}))$
- Prior: $\mathcal{P}(\mathbf{w}) \propto \exp -p(\mathbf{w})$

Negated log of $(\star)$: $\mathcal{L}(\mathbf{y} - f(\mathbf{X},\mathbf{w})) + p(\mathbf{w})$

Minimization framework = maximum a posteriori

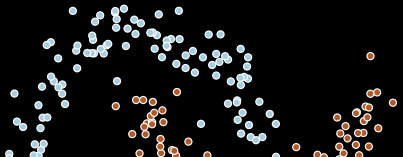
Note that the Bayesian interpretation may be more subtle [Gribonval 2011].

1 **Summary**: elements of a machine-learning method

- **A forward model**: $\mathbf{y}_{\text{pred}} = f(\mathbf{X}, \mathbf{w})$
  Numerical rules to go from $\mathbf{X}$ to $\mathbf{y}$

- **A loss**, or data fit
  A measure of error between $\mathbf{y}_{\text{true}}$ and $\mathbf{y}_{\text{pred}}$
  Can be given by a noise model

- **Regularization**:
  Any way of restricting model complexity
  - by choices in the model
  - via a penalty



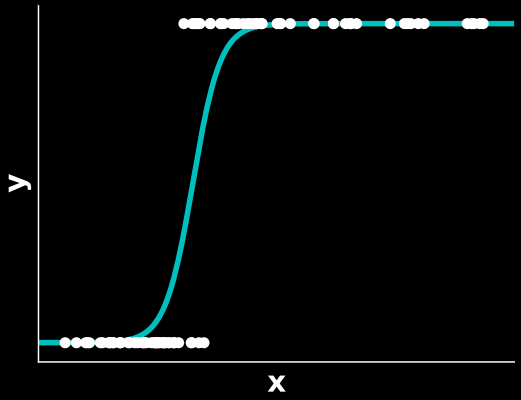
2 A glance at a few models

- Linear models
- Random forests
- Gradient boosted trees

y describes categories, say $(0, 1)$

- Fit **y** by a straight line?

y describes categories, say $(0, 1)$



- Fit **y** by a straight line?
- A sigmoid is better suited
  $\Rightarrow$ Logistic regression

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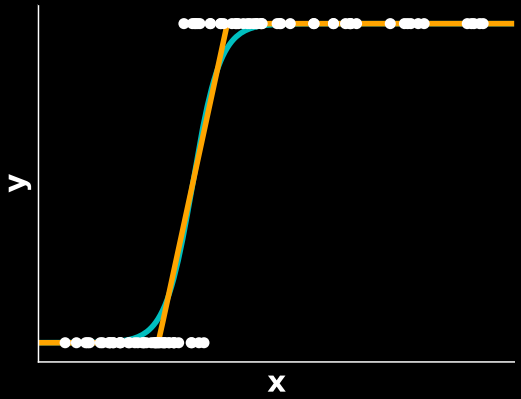
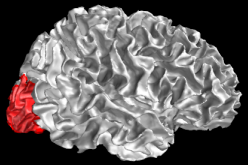
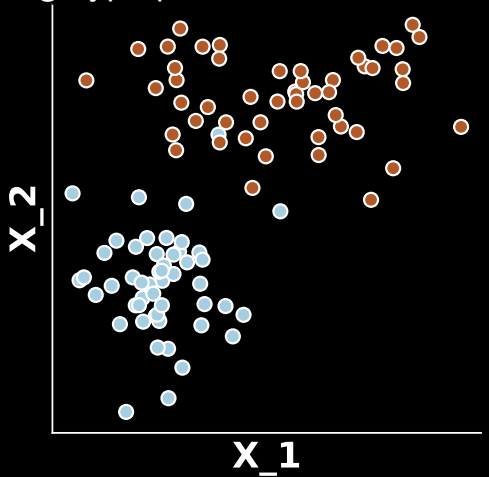


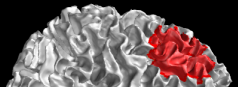
- Fit **y** by a straight line?
- A sigmoid is better suited
  $\Rightarrow$ Logistic regression
- SVM (Support Vector Machines) are similar

Note that points far from threshold don't influence the fit

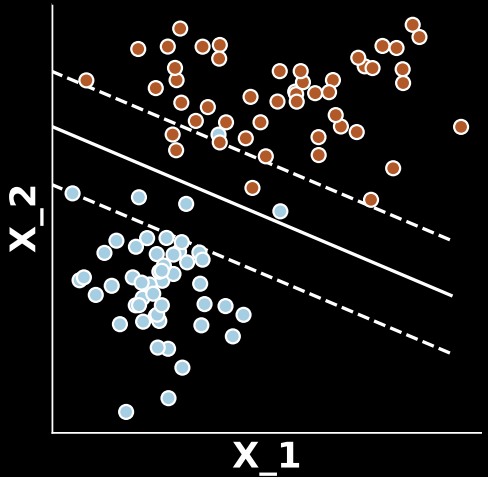
Choosing a separating hyperplane





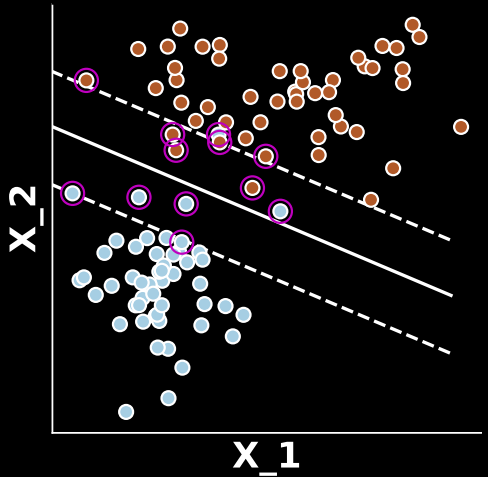


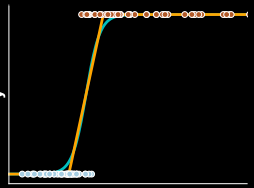
SVM: support vector machine



Discriminative method: choose hyperplane to maximize **margin**

SVM: support vector machine

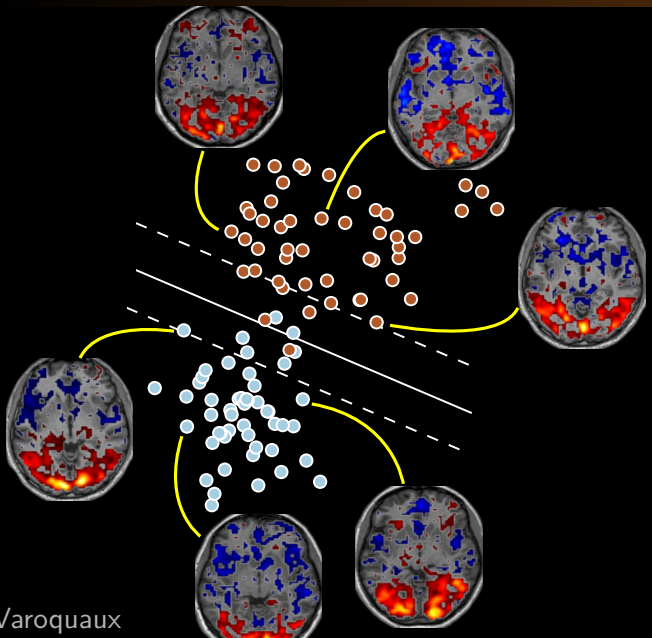




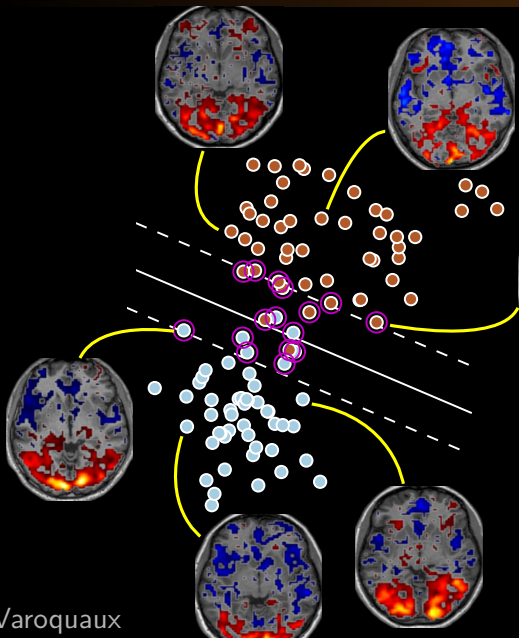
Discriminative method: choose hyperplane to maximize **margin**

use exemplars close to hyperplane (**support vectors**)

2 With brain images



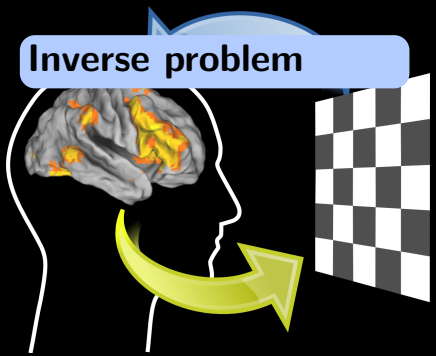
Find a separating hyperplane between images



Find a separating hyperplane between images

SVM builds it by combining exemplars

$\Rightarrow$ hyperplane looks like train images



- Minimize the error term:

$$\hat{\mathbf{w}} = \underset{\mathbf{w}}{\operatorname{argmin}}\, l(\mathbf{y} - \mathbf{X}\,\mathbf{w})$$

Ill-posed:

Many different $\mathbf{w}$ will give the same prediction error

- To choose one: inject prior with a penalty

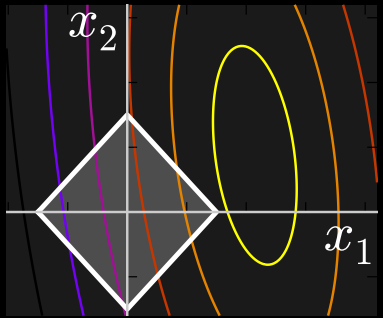
$$\hat{\mathbf{w}} = \underset{\mathbf{w}}{\operatorname{argmin}}\, l(\mathbf{y} - \mathbf{X}\,\mathbf{w}) + p(\mathbf{w})$$

- Lasso estimator

$$\min_{\mathbf{x}} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 \; + \; \lambda \, \ell_1(\mathbf{x})$$

Data fit

Penalization

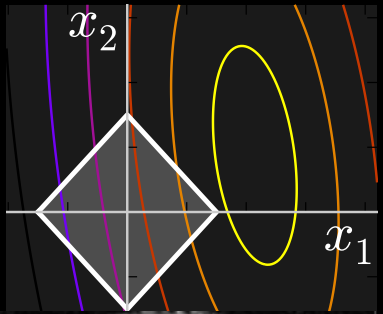


- Lasso estimator

$$\min_{\mathbf{x}} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 + \lambda \ell_1(\mathbf{x})$$

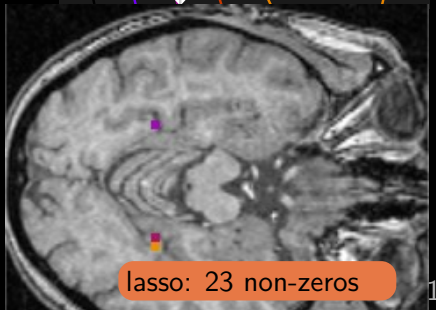
Data fit

Penalization

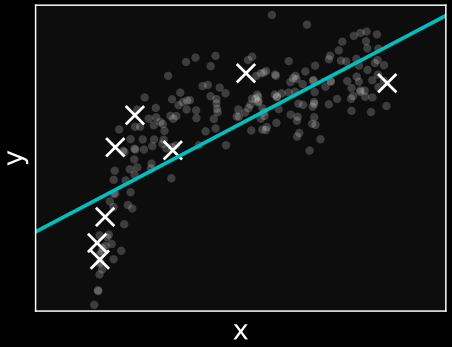


- Between correlated features, selects a random subset
  [Wainwright 2009, Varoquaux... 2012]

  Violates the restricted isometry property

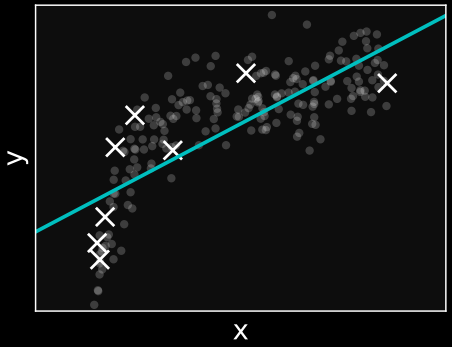


Polynome degree 1

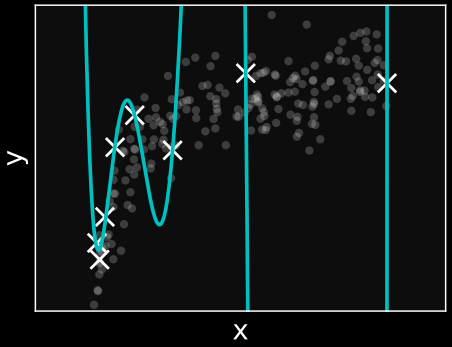


2 Underfit versus overfit

Polynome degree 1

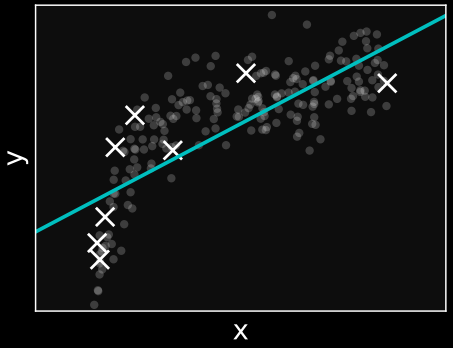


Polynome degree 9



2 Underfit versus overfit

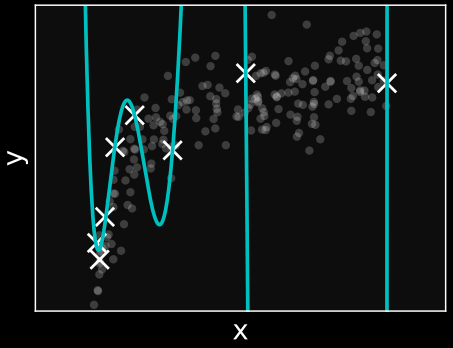
Polynome degree 1



Train explained variance: 0.56
Test explained variance: 0.56

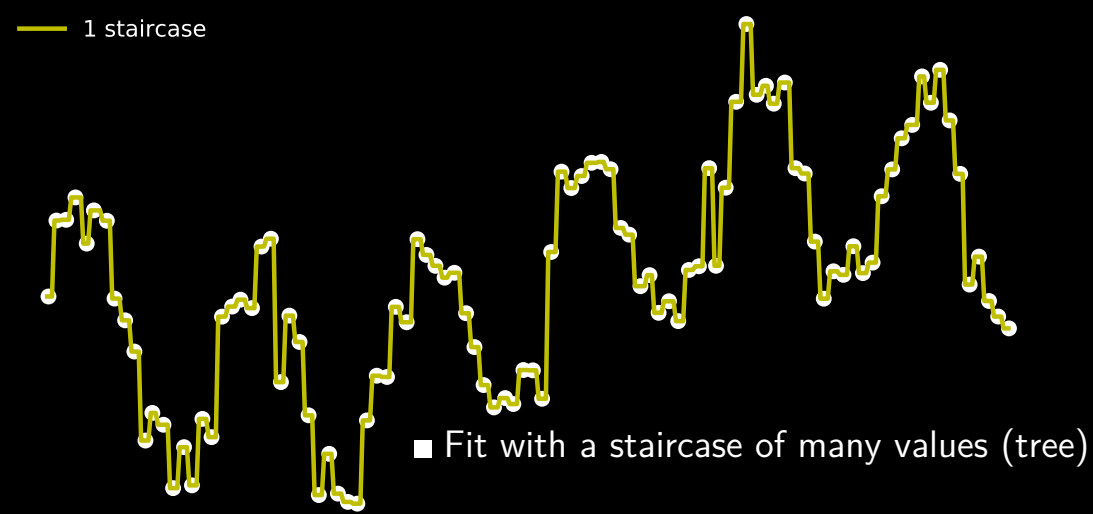
Underfit

Polynome degree 9

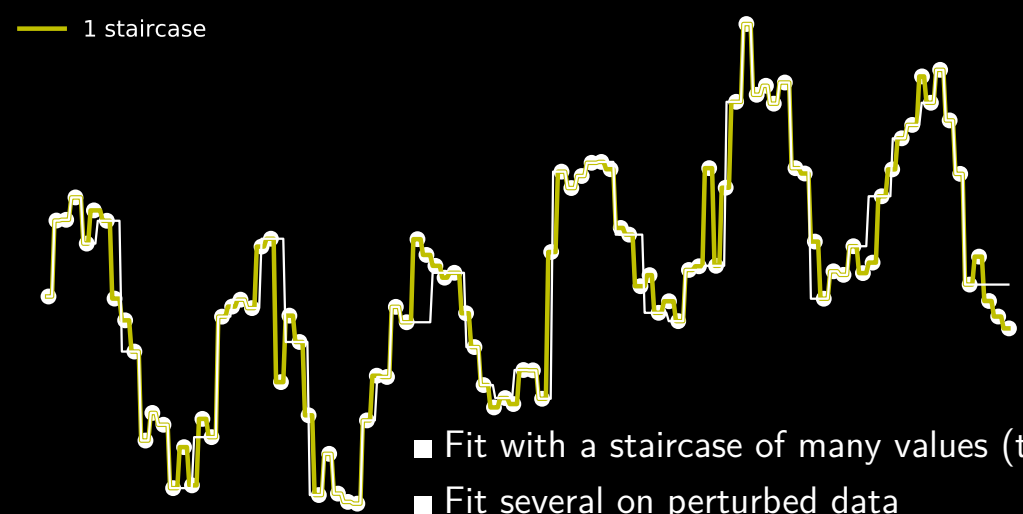


Train explained variance: 0.99
Test explained variance: -98000

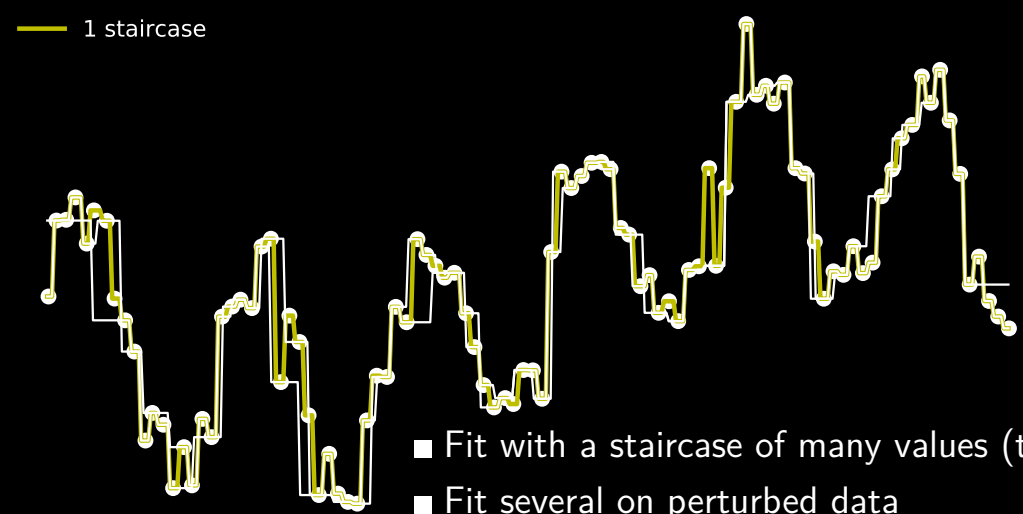
Overfit



- Fit with a staircase of many values (tree)



- Fit with a staircase of many values (tree)
- Fit several on perturbed data



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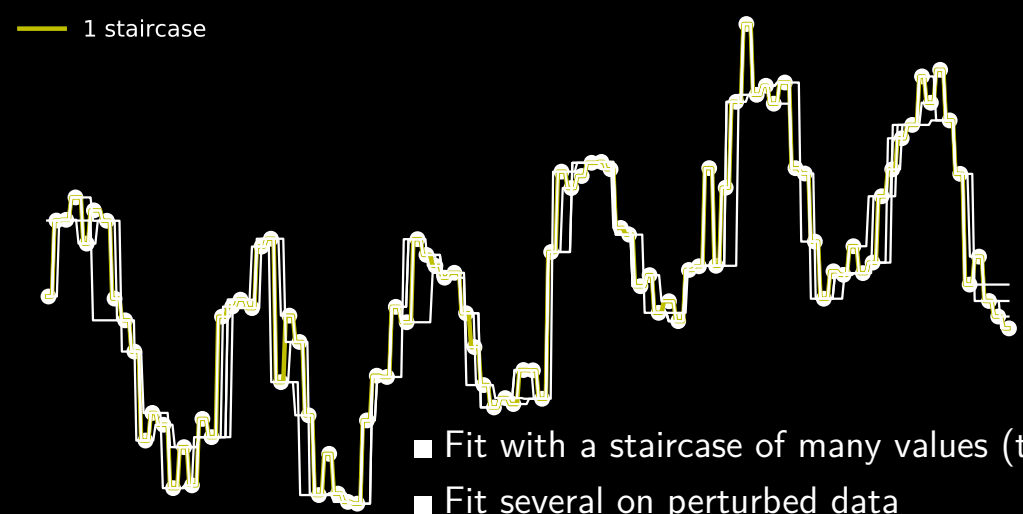
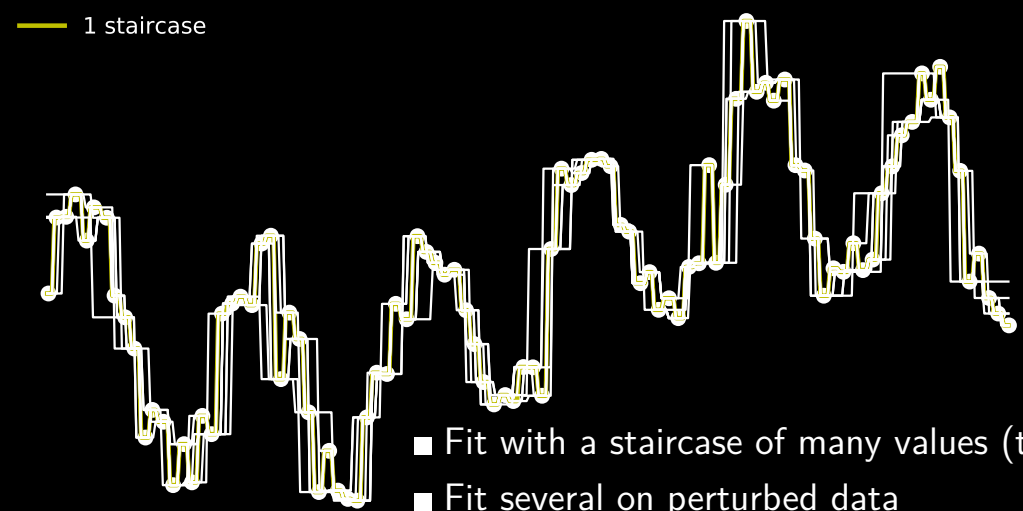
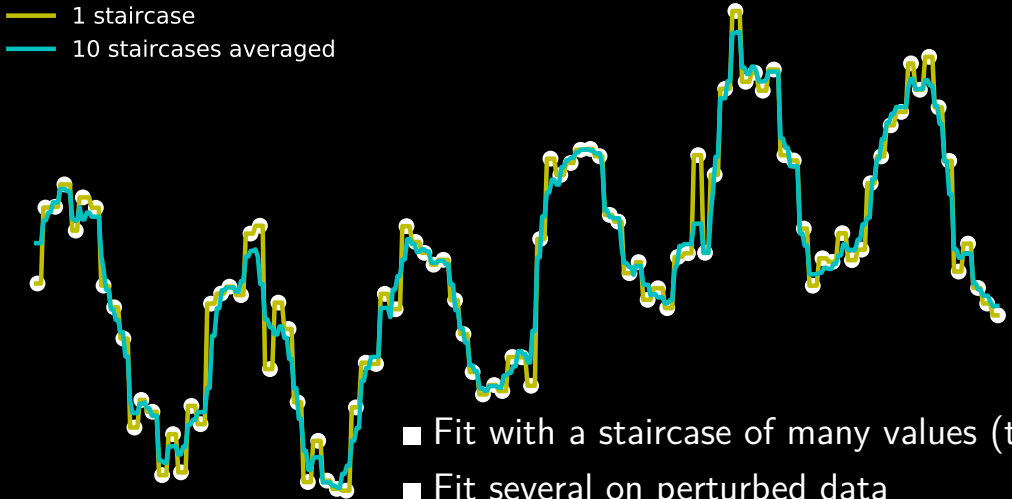


- Fit with a staircase of many values (tree)
- Fit several on perturbed data



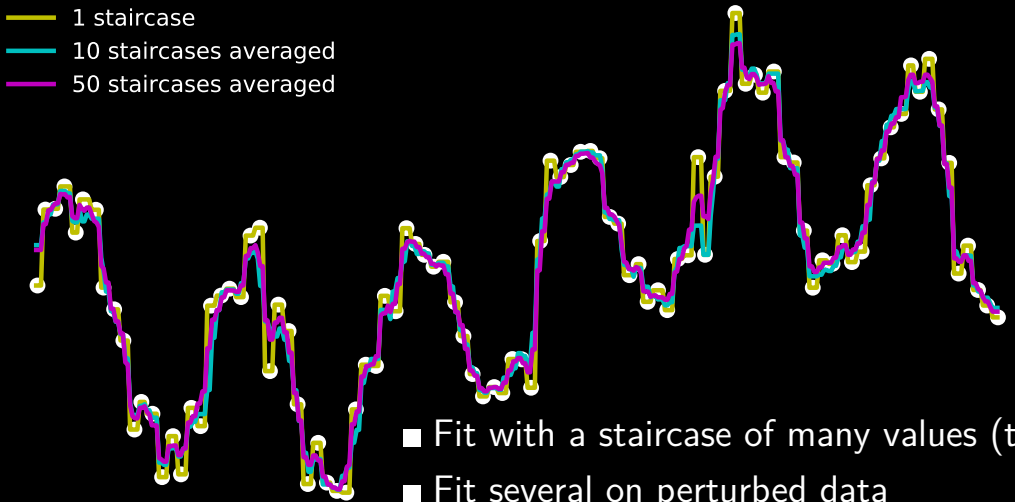
- Fit with a staircase of many values (tree)
- Fit several on perturbed data

2 Random forest



- Fit with a staircase of many values (tree)
- Fit several on perturbed data
- Average prediction

2 Random forest



- Fit with a staircase of many values (tree)
- Fit several on perturbed data
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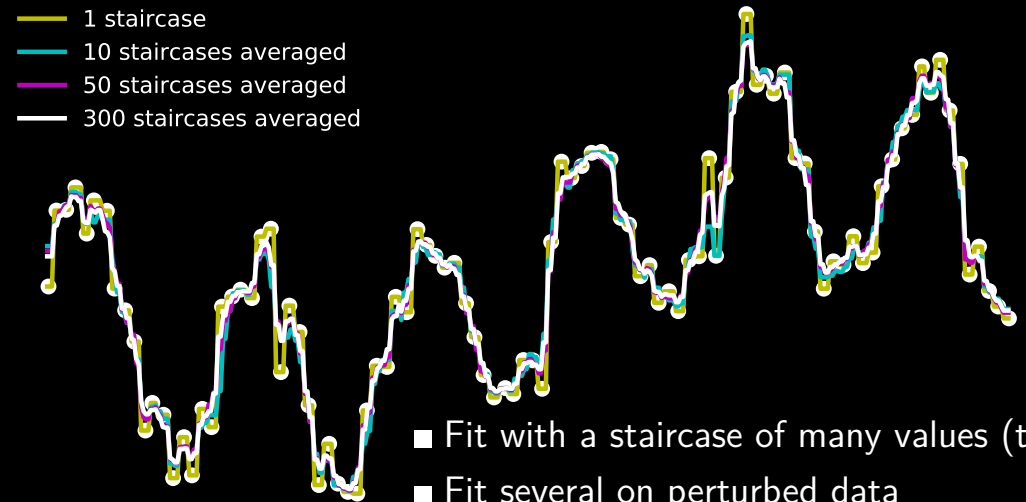
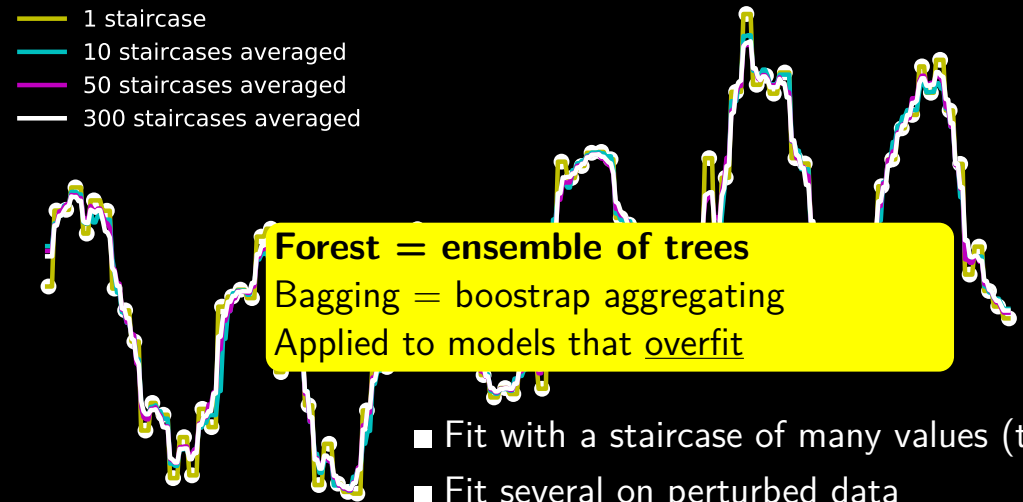
2 Random forest



- Fit with a staircase of many values (tree)
- Fit several on perturbed data
- Average prediction

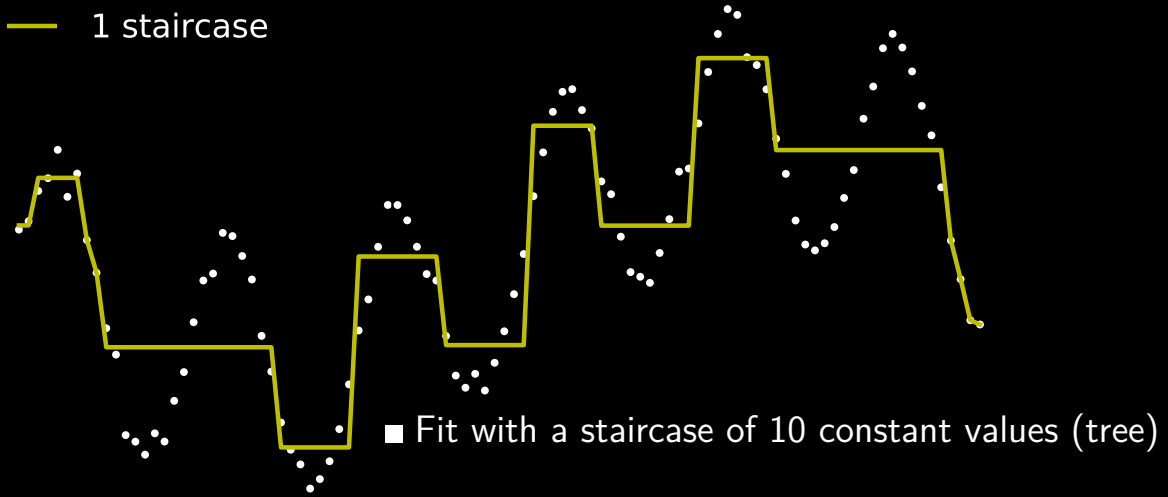


Forest = ensemble of trees

Bagging = boostrap aggregating

Applied to models that overfit

- Fit with a staircase of many values (tree)
- Fit several on perturbed data
- Average prediction

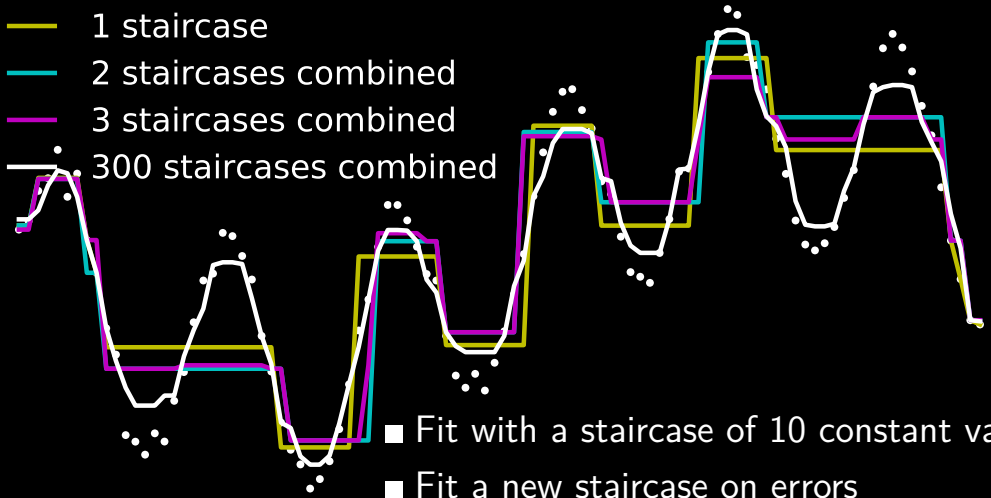

1 staircase
Fit with a staircase of 10 constant values (tree)

2 Gradient boosted trees

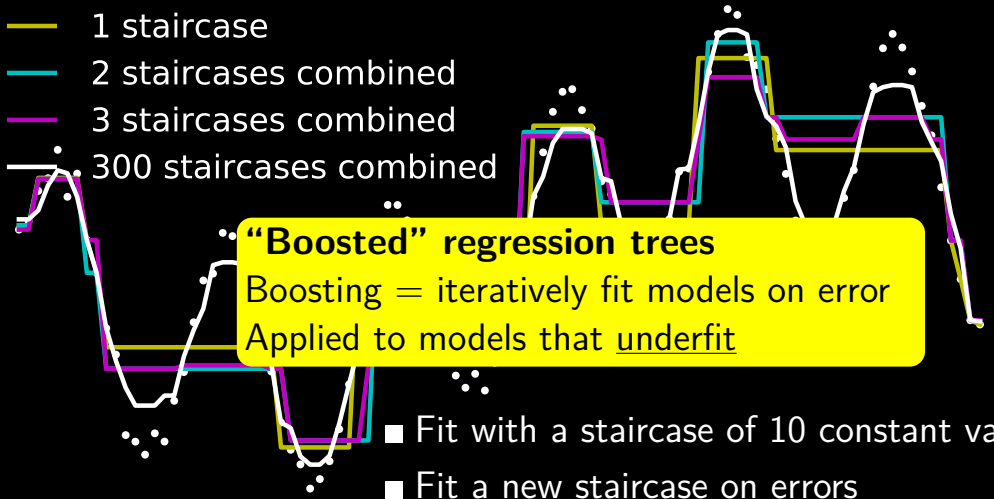
- Fit with a staircase of 10 constant values (tree)
- Fit a new staircase on errors

- Fit with a staircase of 10 constant values (tree)
- Fit a new staircase on errors
- Keep going

2 Gradient boosted trees



- Fit with a staircase of 10 constant values (tree)
- Fit a new staircase on errors
- Keep going

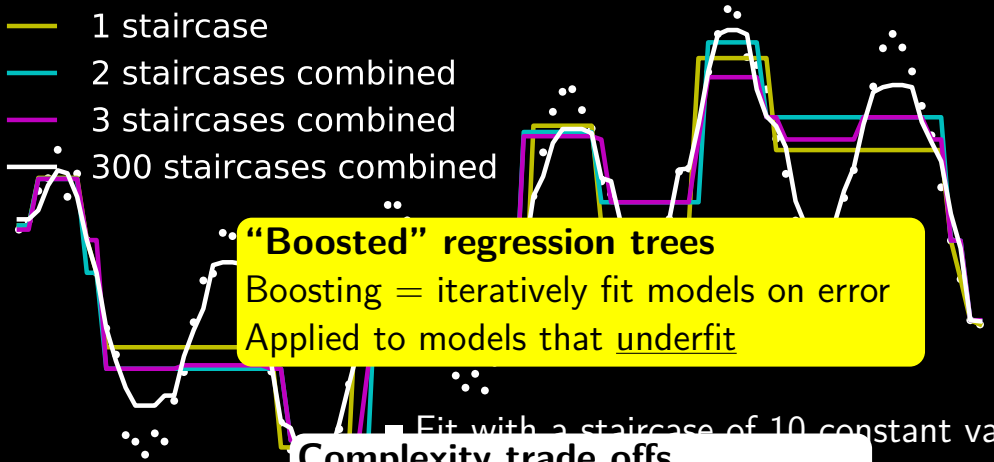


"Boosted" regression trees

Boosting = iteratively fit models on error

Applied to models that underfit

- Fit with a staircase of 10 constant values (tree)
- Fit a new staircase on errors
- Keep going



"Boosted" regression trees

Boosting = iteratively fit models on error

Applied to models that underfit

- Fit with a staircase of 10 constant values (tree)

Complexity trade offs

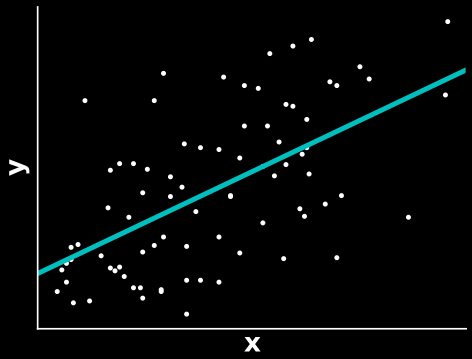
Computational + statistical

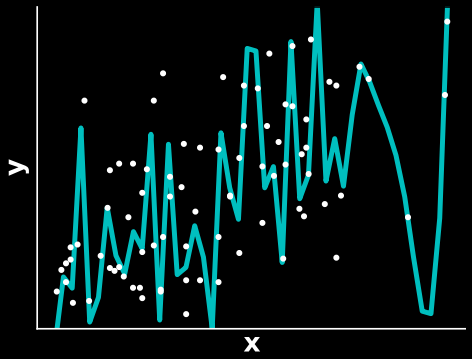
- Keep going

3 Model evaluation

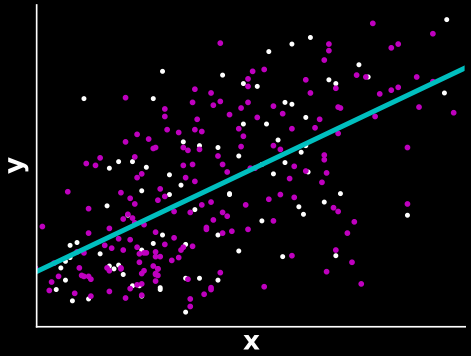
Does it predict?

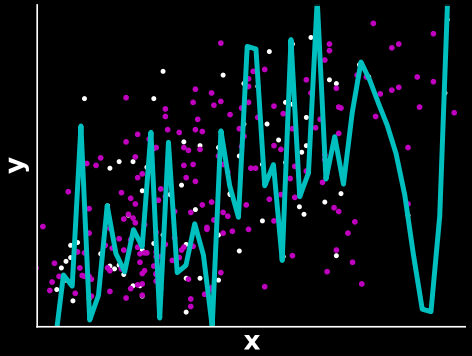
Generalization as a test: cross-validation





3 Generalization as a test: cross-validation



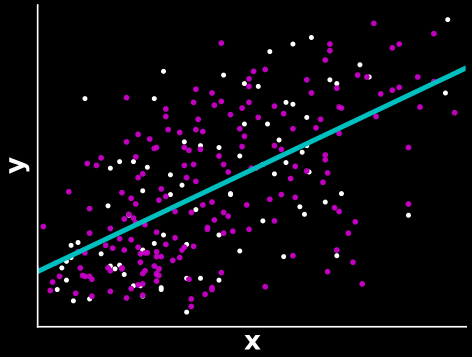


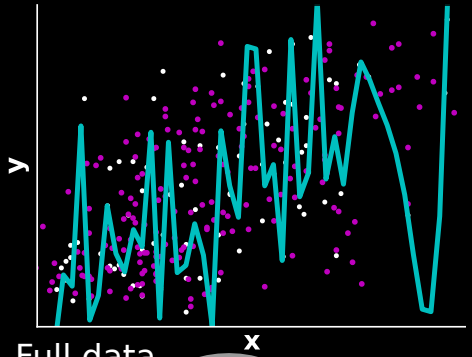
⇒ Need test on **independent** data, to control for model complexity

Measures prediction accuracy

[Varoquaux... 2017]

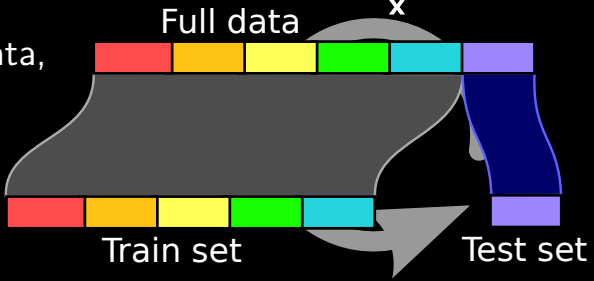
3 Generalization as a test: cross-validation



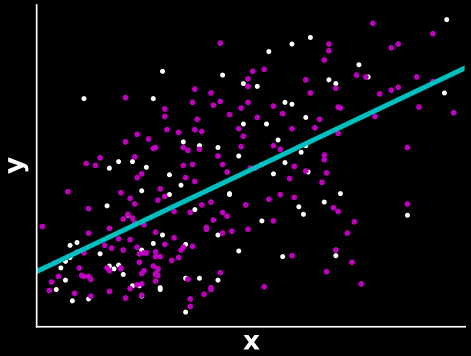


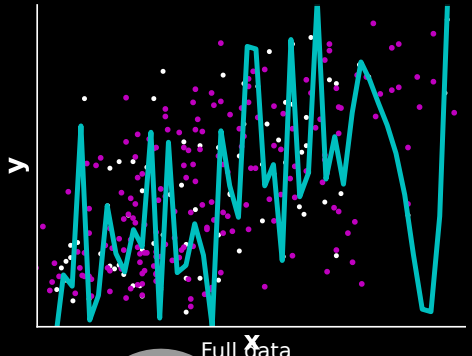
⇒ Need test on **independent** data,

Loop



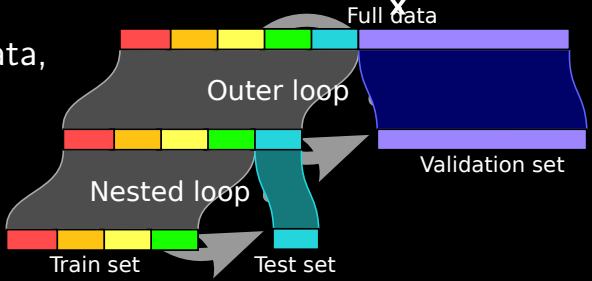
3 Generalization as a test: cross-validation



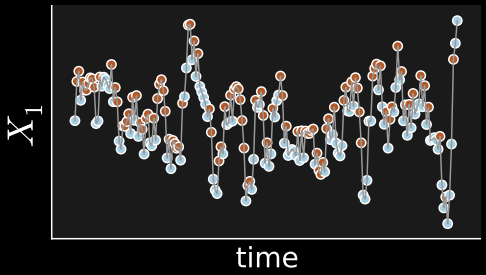


$\Rightarrow$ Need test on **independent** data,

Nested loop

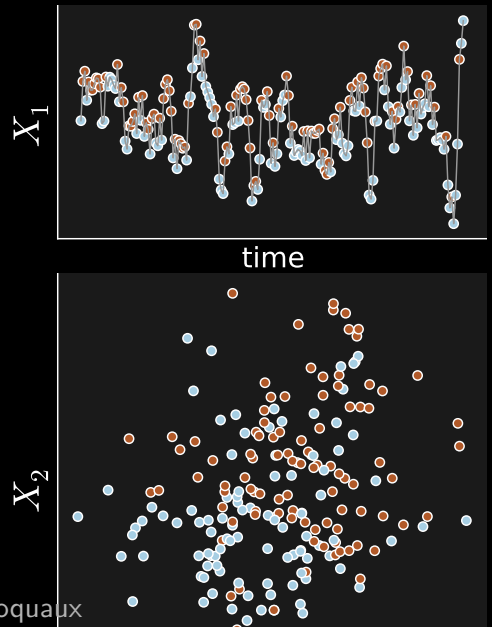


3 Confounds & dependencies



- Prediction is easy
  if you look at the last time point

3 Confounds & dependencies

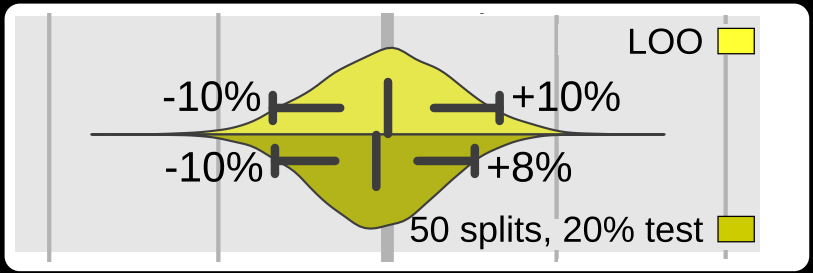


- Prediction is easy
  if you look at the last time point
- Auto-correlated noise
  Neighboring data points
  not independent
  No evidence of generalization
  No evidence of useful prediction

Validate on independent data

Not 2 images of the same subject
Unless it's a longitudinal study

[Varoquaux... 2017, Little... 2017]



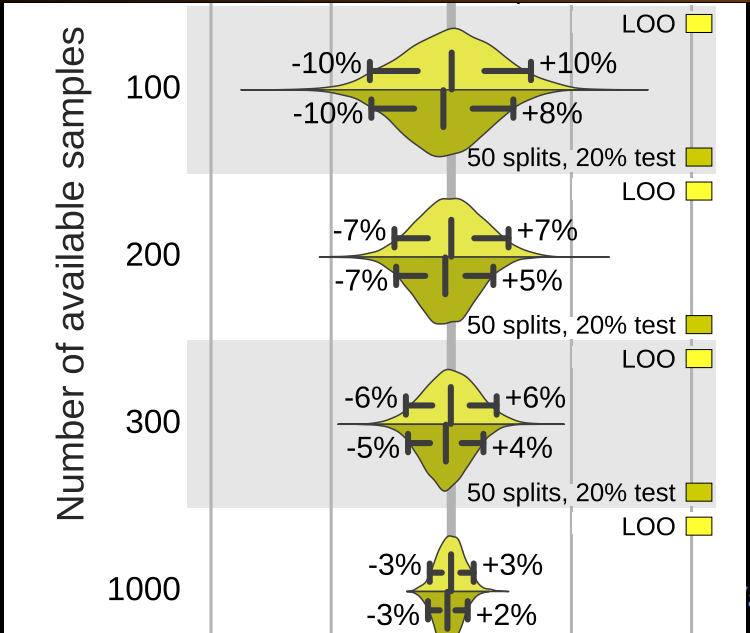
Distribution of prediction accuracies across 50 folds

- Should be seen as a posterior distribution (or bootstrap)
- $\neq$ folds are **not independent** measures

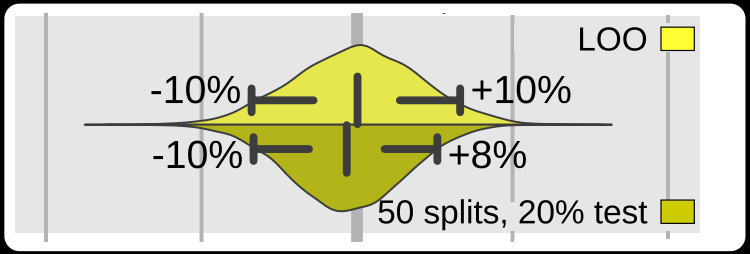
No statistical tests (eg T tests) on folds

[Varoquaux 2017]

3 Uncertainty of cross-validation



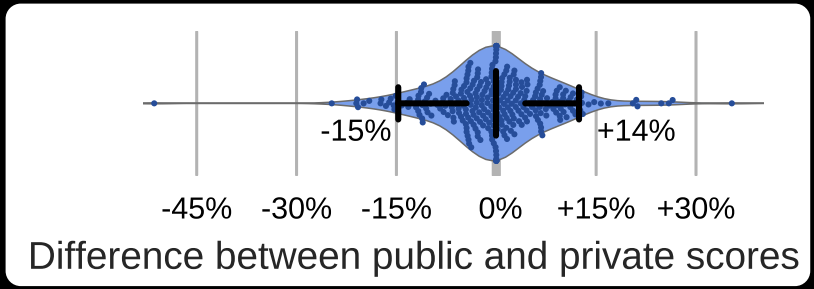
[aroquaux 2017]



Sample size	Confidence interval (binomial model)
100	18.0%
1 000	5.7%
10 000	1.8%
100 000	0.568%

Sampling noise – Decreases slowly with n [Varoquaux 2017]

3 Uncertainty of cross-validation



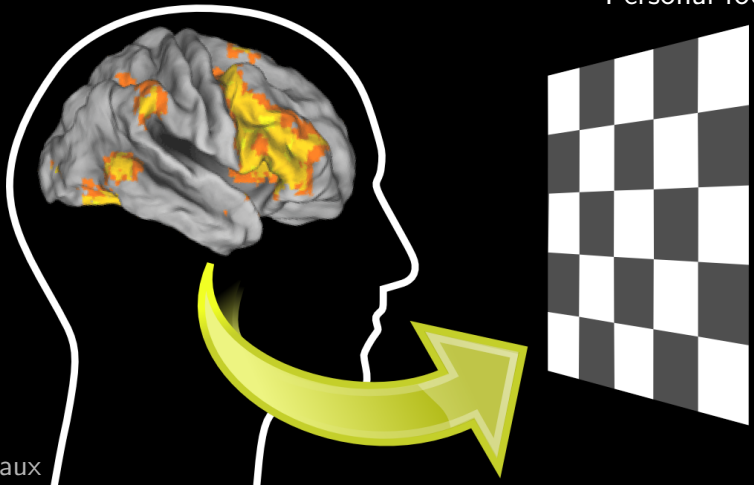
Kaggle competition on r-fMRI for Schizophrenia
2 different test sets: size 30 and 28

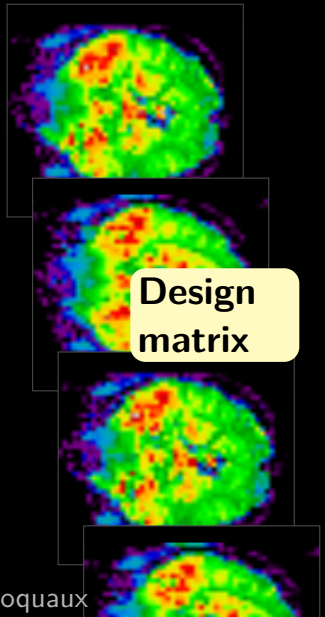
Winner of competition: first code written

[Varoquaux 2017]

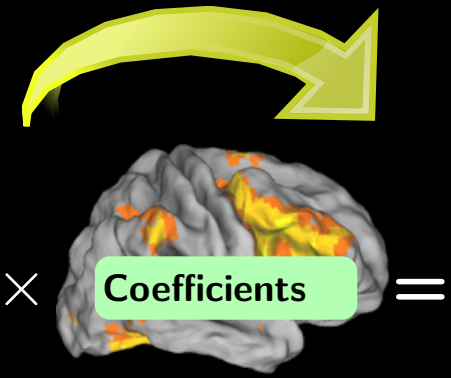
4 Learning on full-brain images

Personal focus on functional imaging





×



=

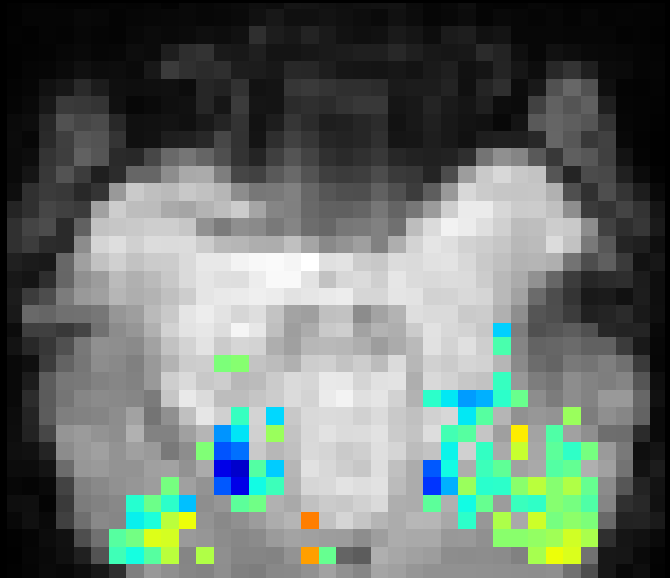
Coefficients are

brain maps

Face vs house visual recognition [Haxby... 2001]

SVM

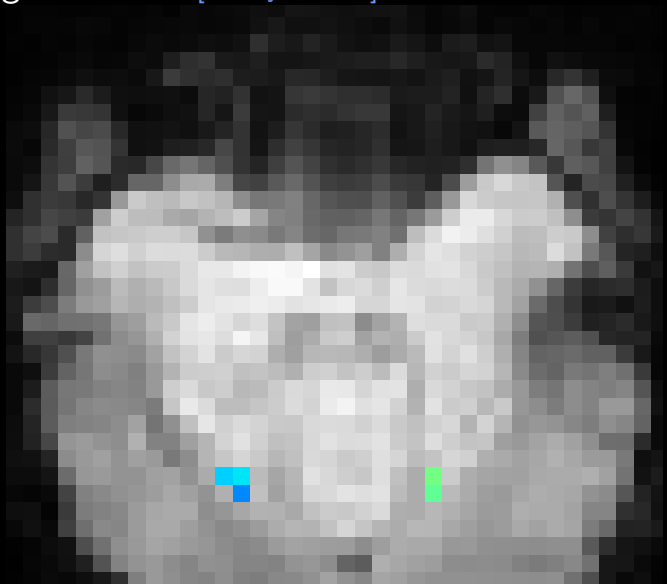
error: 26%



Face vs house visual recognition [Haxby... 2001]

Sparse model

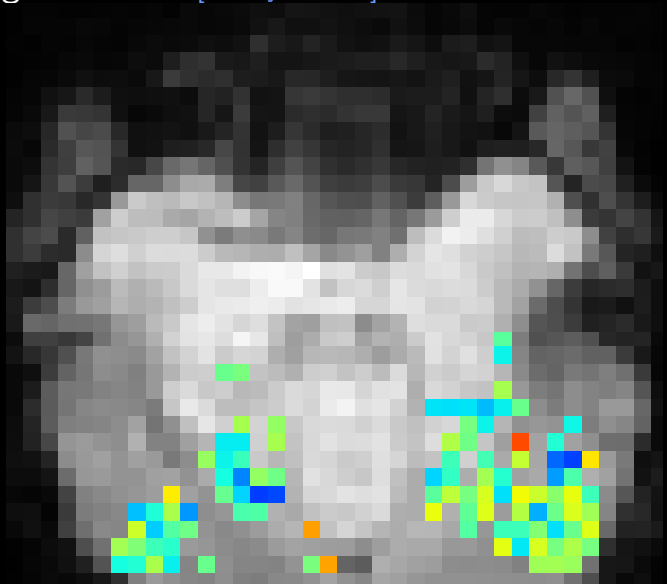
error: 19%

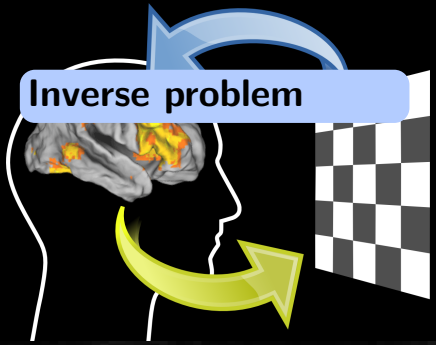


Face vs house visual recognition [Haxby... 2001]

Ridge

error: 15%





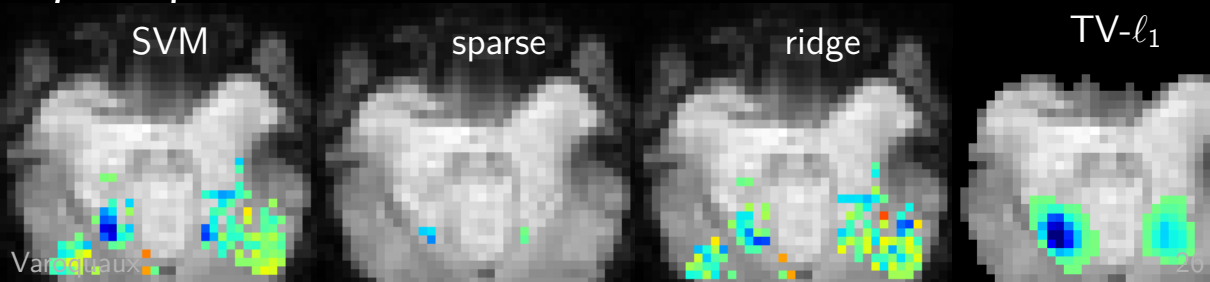
- Minimize the error term:

$$\hat{\mathbf{w}} = \underset{\mathbf{w}}{\operatorname{argmin}} \, l(\mathbf{y} - \mathbf{X}\,\mathbf{w})$$

Ill-posed:

Many different $\mathbf{w}$ will give the same prediction error

Choice driven by (implicit) priors



[Gramfort... 2013]

- Minimize the error term:

$$\hat{\mathbf{w}} = \underset{\mathbf{w}}{\operatorname{argmin}}\, l(\mathbf{y} - \mathbf{X}\,\mathbf{w})$$

Ill-posed:

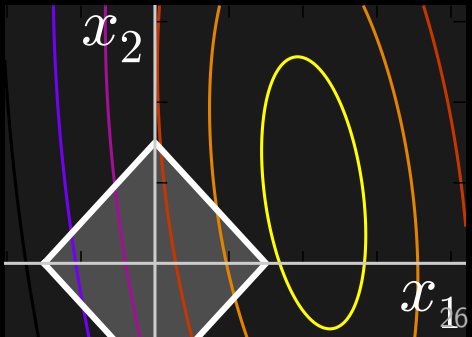
Many different $\mathbf{w}$ will give the same prediction error

Sparse estimators (Lasso)

$$\min_{\mathbf{x}} \|\mathbf{y} - \mathbf{A}\mathbf{x}\|_2^2 + \lambda\, \ell_1(\mathbf{x})$$

Data fit

Penalization



- Minimize the error term:

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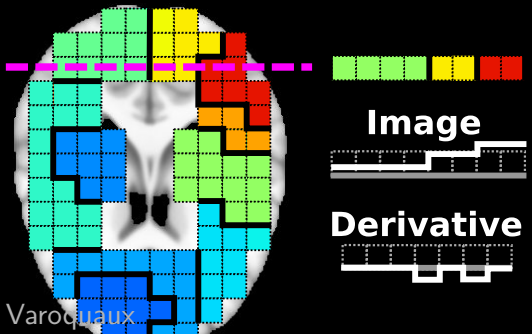
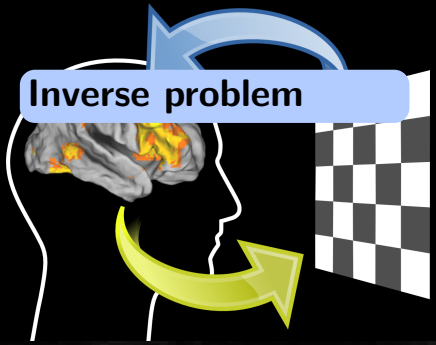


Total-variation penalization

Impose sparsity on the gradient of the image:

$$p(\mathbf{w}) = \ell_1(\nabla \mathbf{w})$$

In fMRI: [Michel... 2011]



- Minimize the error term:

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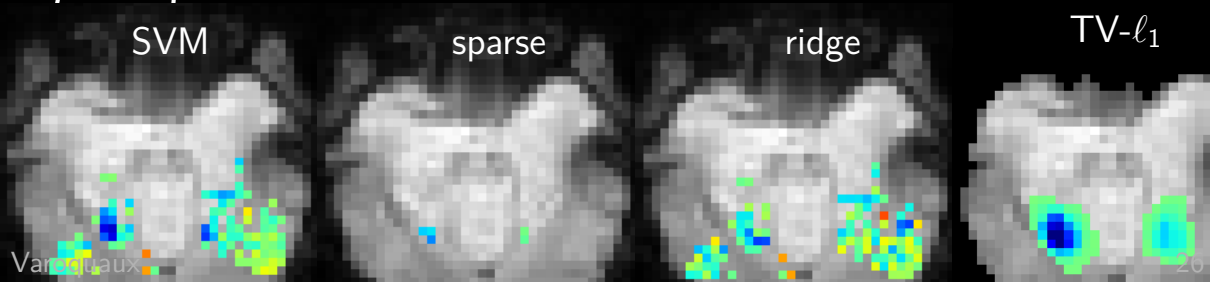
Ill-posed:

Many different $\mathbf{w}$ will give the same prediction error

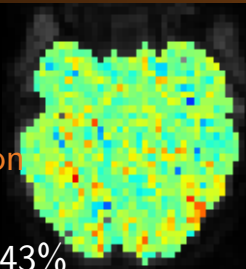
Choice driven by (implicit) priors



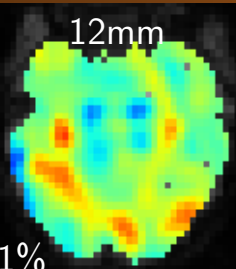
Spatial smoothing

Giving up on resolution

error rate: 43%



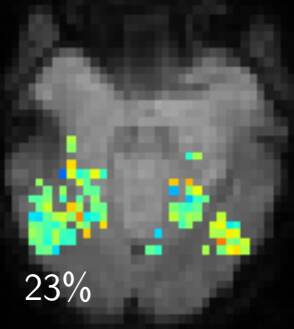
12mm



31%

Feature selection

Giving up on multivariate

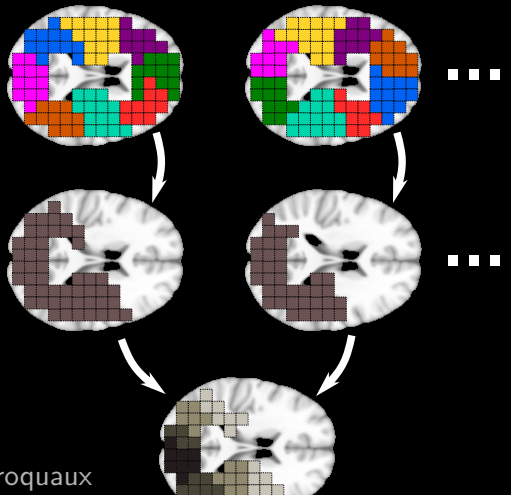


23%

4 **FREM**: Ensemble models [Hoyos-Idrobo... 2017]

- Very fast sub optimal models
- Average many of them

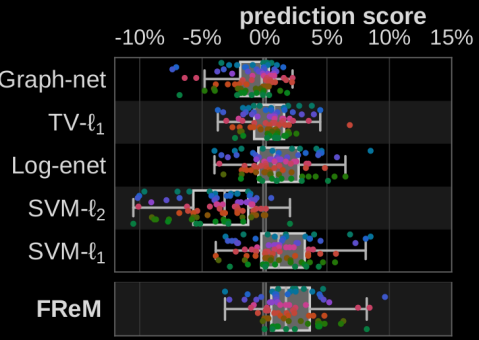
- Very fast sub optimal models
- Average many of them



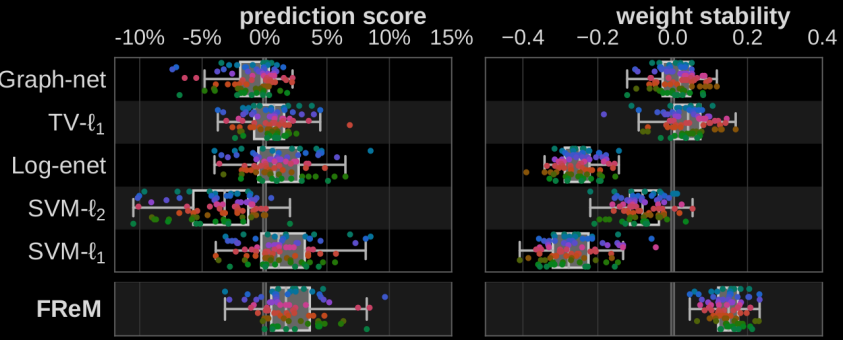
- Learn parcellation on perturbed data
- Estimate linear models

Average the results

- Very fast sub optimal models
- Average many of them



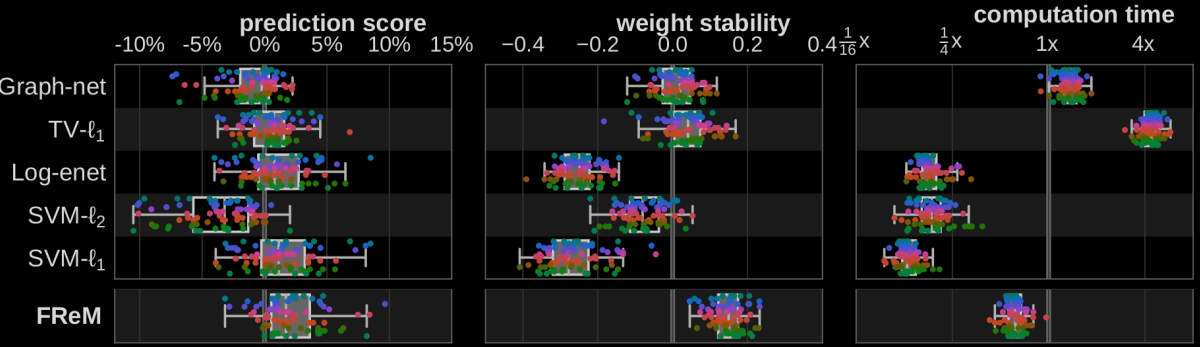
- Very fast sub optimal models
- Average many of them



4 FREM: Ensemble models

[Hoyos-Idrobo... 2017]

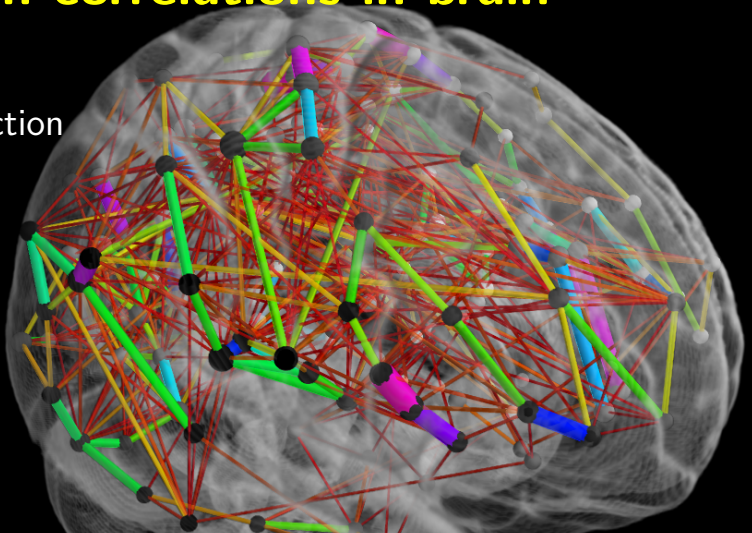
- Very fast sub optimal models
- Average many of them



5 Learning on correlations in brain activity

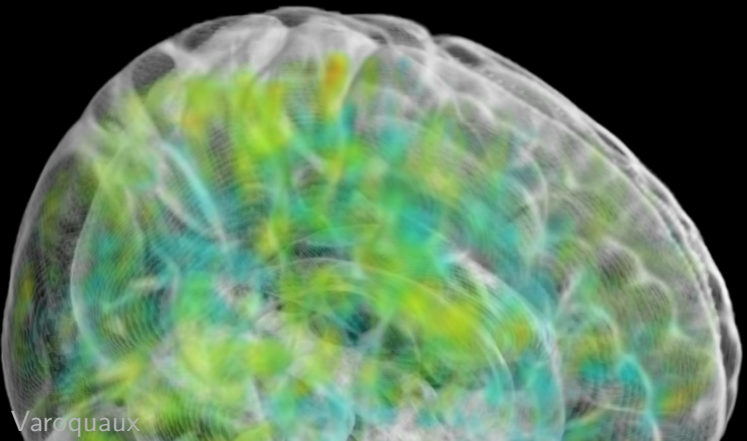
"connectome" prediction

Brain "at rest"
= easier to scan



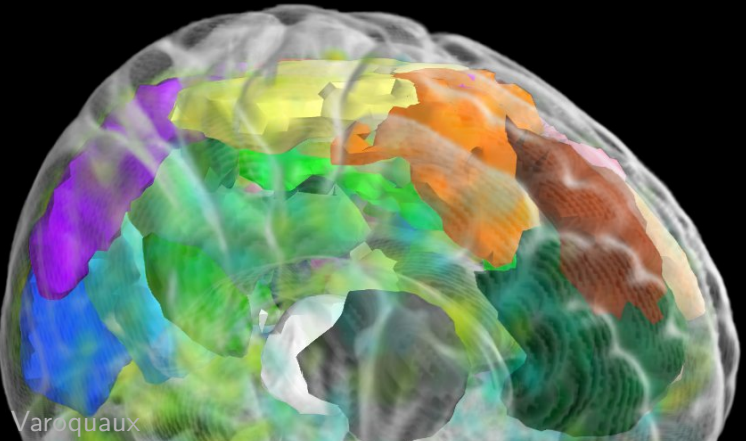
From rest-fMRI to biomarkers

No salient features in rest fMRI



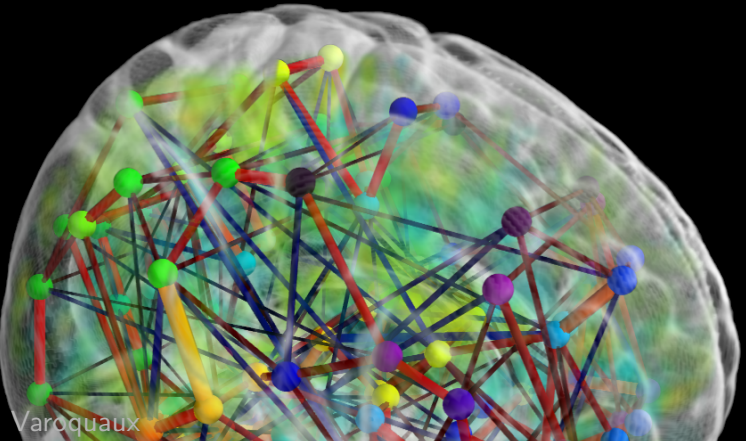
From rest-fMRI to biomarkers

- Define functional regions



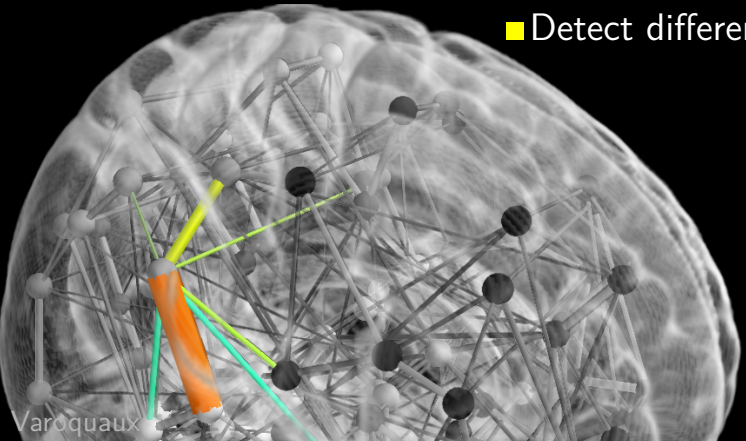
From rest-fMRI to biomarkers

- Define functional regions
- Learn interactions

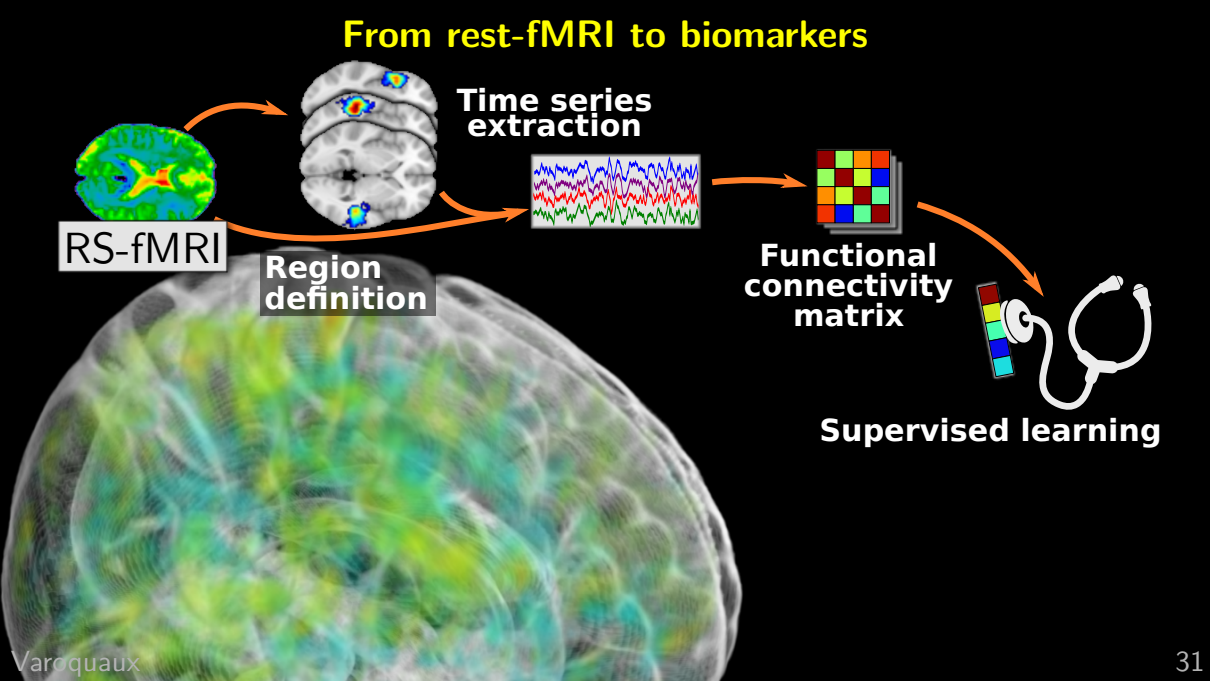


From rest-fMRI to biomarkers

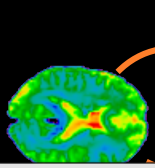
- Define functional regions
- Learn interactions
- Detect differences

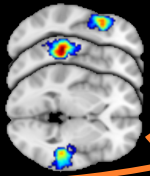


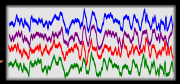
From rest-fMRI to biomarkers

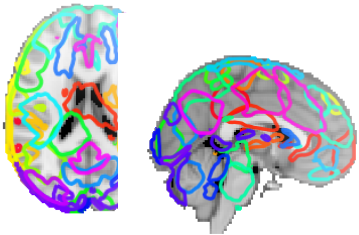


From rest-fMRI to biomarkers



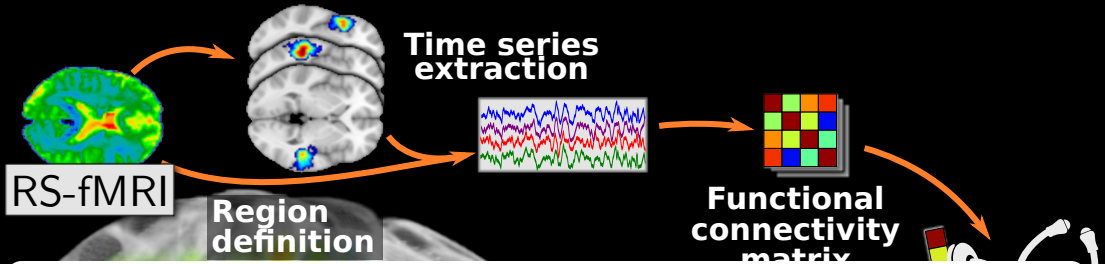


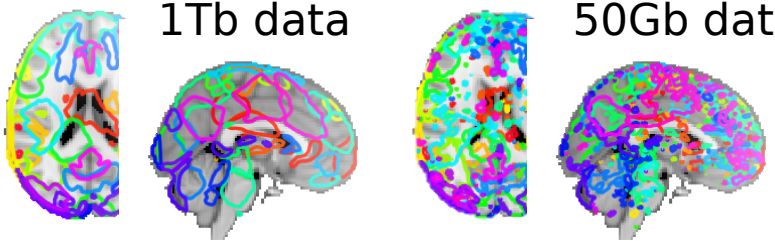




- Unsupervised learning $\Rightarrow$ adapted representations dictionary learning

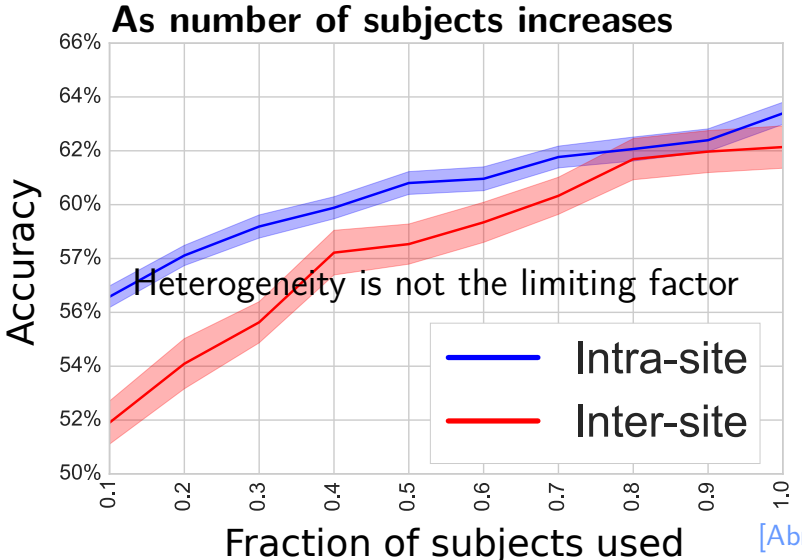
From rest-fMRI to biomarkers





- Unsupervised learning ⇒ adapted representations dictionary learning
- More data is always better computational cost [Mensch... 2016]

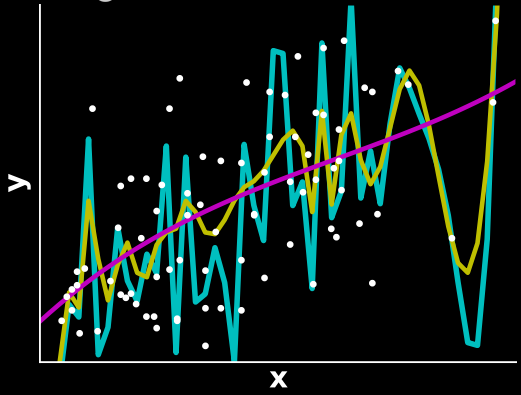
- Ill-defined pathology
- Heterogeneous dataset (ABIDE)



[Abraham... 2017]

An introduction to Machine Learning for MR imaging

- Overfit versus underfit
  - The best fit is not the best performer
  - Minimizing an observed error is the wrong solution



An introduction to Machine Learning for MR imaging

- Overfit versus underfit
- Evaluate on **independent** data

Cross-validation error bars

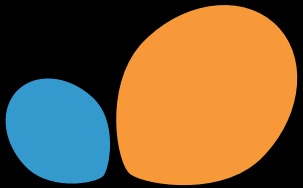
An introduction to Machine Learning for MR imaging

- Overfit versus underfit
- Evaluate on **independent** data
- More data trumps being clever

An introduction to Machine Learning for MR imaging

- Overfit versus underfit
- Evaluate on **independent** data
- More data trumps being clever
- Software: scikit-learn & nilearn

http://scikit-learn.org



In Python

http://nilearn.github.io

@GaelVaroquaux

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