Relative Performance and the Market

Perspectives on Economic Studies TA session

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Providing Incentives vs. Risk-Sharing

- Alternative ways of organizing agricultural production: pay wages to farmers or rent out land to tenants?
- Paying fixed wages eliminates risks for farmers, but provides no incentive for effort when effort is not observable. Renting out land gives tenants full incentives, but makes them bear all risks.
- Share-cropping, a widely-adopted practice, serves as a reconciliation of the tradeoff. Share-holding managers in corporations are an analogue.

Today

- Nalebuff and Stiglitz, 1983, Information, Competition, and Markets
- Effort (input) is unobservable. Therefore compensation can only be a function of observables (output).
- There is a random variable that affects the mapping from input to output. Otherwise it would be possible to infer input from output.
- When there are multiple agents whose random disturbances are correlated, there may be another way to tackle the incentive-risk sharing tradeoff: relative performance based compensation.
- Intuition: other agents' performance provides information about the common random shock.
- They then argue that a competitive market has virtues of a relative performance based compensation scheme.

A Simple Example

- Suppose the government wants to develop a bomber, the cost of which is subject to a random variable. Contractors can reduce cost by expending more effort.
- If the government signs contract with just one contractor:
  - Fixed-fee contract: the contractor bears all the cost. Maximum incentives but high risk on the part of the contractor. The contractor will demand a high risk premium from the government.
  - Sharing a fraction of the cost: now the contractor has less incentive to reduce cost.
- But we can sign contract simultaneously with two contractors. Pay each firm a fixed fee plus the cost of the other firm.
  - Assuming the two firms face the same shock, they are fully insured under this scheme.
  - They also have incentive to reduce cost.

A More General Treatment

- An individual's output Q_i is determined by effort, μ_i, a shock common to all, θ, and an individual shock, ϵ_i, in the following way:

$$Q_i = \mu_i \theta + \epsilon_i$$

- Individuals choose effort after observing θ but before observing ϵ_i. Their payoff is given by

$$U(Y_i) - V(\mu_i)$$

The First Best

- Suppose the Principal can observe both effort and the environmental shock. She can therefore stipulate effort for each realization of θ.
- Then there is no need to provide incentives. Assuming the Principal is risk neutral and Agents risk averse, it is optimal for Agents to receive a constant income $\bar{Y}$ regardless of their output.
- Next determine optimal effort level in each state (realization of θ).
- Marginal benefit of effort equal to marginal cost:

$$\theta U'(\bar{Y}) = V'(\mu)$$

- This gives optimal effort as a function of θ: $\mu^*(\theta)$.

A Mechanism that Induces Revelation of θ

- Now assume the Principal can observe neither θ or μ_i. Let's design a mechanism that induces first-best effort and fully insures the Agents.
- After an Agent observes θ, she is to announce what θ is (Agent i's announcement: $\hat{\theta}_i$).
- Agent is paid only if her output is close to a target, which depends on $\hat{\theta}_i$. Otherwise the Agent pays a penalty of infinity (sufficiently large).

A Mechanism that Induces Revelation of θ

- Suppose the idiosyncratic shock ϵ_i has support $[-1, 1]$.
- The compensation scheme is

$$Y_i = \frac{\phi(\hat{\theta}_i)}{U'(\bar{Y})} - \frac{\phi(\bar{\theta}_{-i})}{U'(\bar{Y})} + \bar{Y} \text{ if } Q_i \geq \mu^*(\hat{\theta}_i)\hat{\theta}_i - 1$$

$$Y_i = -\infty \text{ otherwise}$$

  where $\bar{\theta}_{-i}$ is average announcement of other individuals.
- Now let's see that we can find a function ϕ such that it is a best strategy for each Agent to announce the true θ and expend first-best effort.
- The red term provides full incentives. The blue term kills risks. Similar to the bomber example.

A Mechanism that Induces Revelation of θ

- First let's see that if I announce true θ, I will want to expend first-best effort $\mu^*(\theta)$. Lower effort $\implies$ positive probability of negative infinity income. Higher effort $\implies$ unnecessary because $\mu^*(\theta)$ already guarantees the compensation.
- If instead of true θ, I announce a higher $\hat{\theta}$, I will receive higher Y, but the target also becomes higher.
- Marginal benefit of announcing a higher $\hat{\theta}$, evaluated at true θ:

$$U'(\bar{Y})\frac{\phi'(\theta)}{U'(\bar{Y})} = \phi'(\theta)$$

- Have to adjust effort to ensure $\mu\theta = \mu^*(\hat{\theta}_i)\hat{\theta}_i \implies \mu(\hat{\theta}_i) = \mu^*(\hat{\theta}_i)\hat{\theta}_i/\theta$
- Marginal cost of higher $\hat{\theta}$, evaluated at true θ:

$$V'\frac{d\mu}{d\hat{\theta}_i}\bigg|_{\hat{\theta}_i=\theta} = V'\frac{1}{\theta}(\mu^{*'}(\theta)\theta + \mu^*(\theta))$$

A Mechanism that Induces Revelation of θ

- When marginal benefit equals marginal cost, it is optimal to announce true θ:

$$\phi'(\theta) = V' \frac{1}{\theta}(\mu^{*'}(\theta)\theta + \mu^{*}(\theta))$$

- Can then integrate for ϕ.
- Notice that when everyone announces true θ, income is always $\bar{Y}$. Introducing $\bar{\theta}_{-i}$ makes Agents fully insured.

Connection between Competitive Market and Relative Performance

Interesting idea:

- In a market, profit of a firm depends not only on its own cost, but also on costs of other firms (which for example determines the equilibrium price). This is analogous to a relative performance based compensation scheme.
- Suppose cost
$$c_i = k - \theta\mu_i$$
- Then the profit of firm net of effort cost is
$$\hat{\pi}_i = (P - k + \theta\mu_i - V(\mu_i))Q_i$$
- Competition ensures that every firm makes zero profit and expends optimal effort:
$$P = \min\ k - \mu_i\theta + V(\mu_i)$$
- Profit always zero $\implies$ no risk.

Competitive Market vs. Monopoly

- Now consider the structure of a firm: owner and manager.
- No risk in the competitive market $\Longrightarrow$ can let the manager own the firm to provide maximum incentives.
- A monopoly market akin to an individualistic compensation scheme with no peers to kill the uncertainty. Therefore profit is variable.
- In this situation cannot let the manager own the firm because the manager will bear all the risk. Have to resort to "share-cropping". Suppose pay of manager is $\alpha\pi_i + \beta, 0 < \alpha < 1$.
- Manager will set $\alpha\theta = V'(\mu)$, while first-best effort is $\theta = V'(\mu)$.
- Managerial effort does not adjust in response to the environment as fully as in a competitive market. They call this "managerial slack".