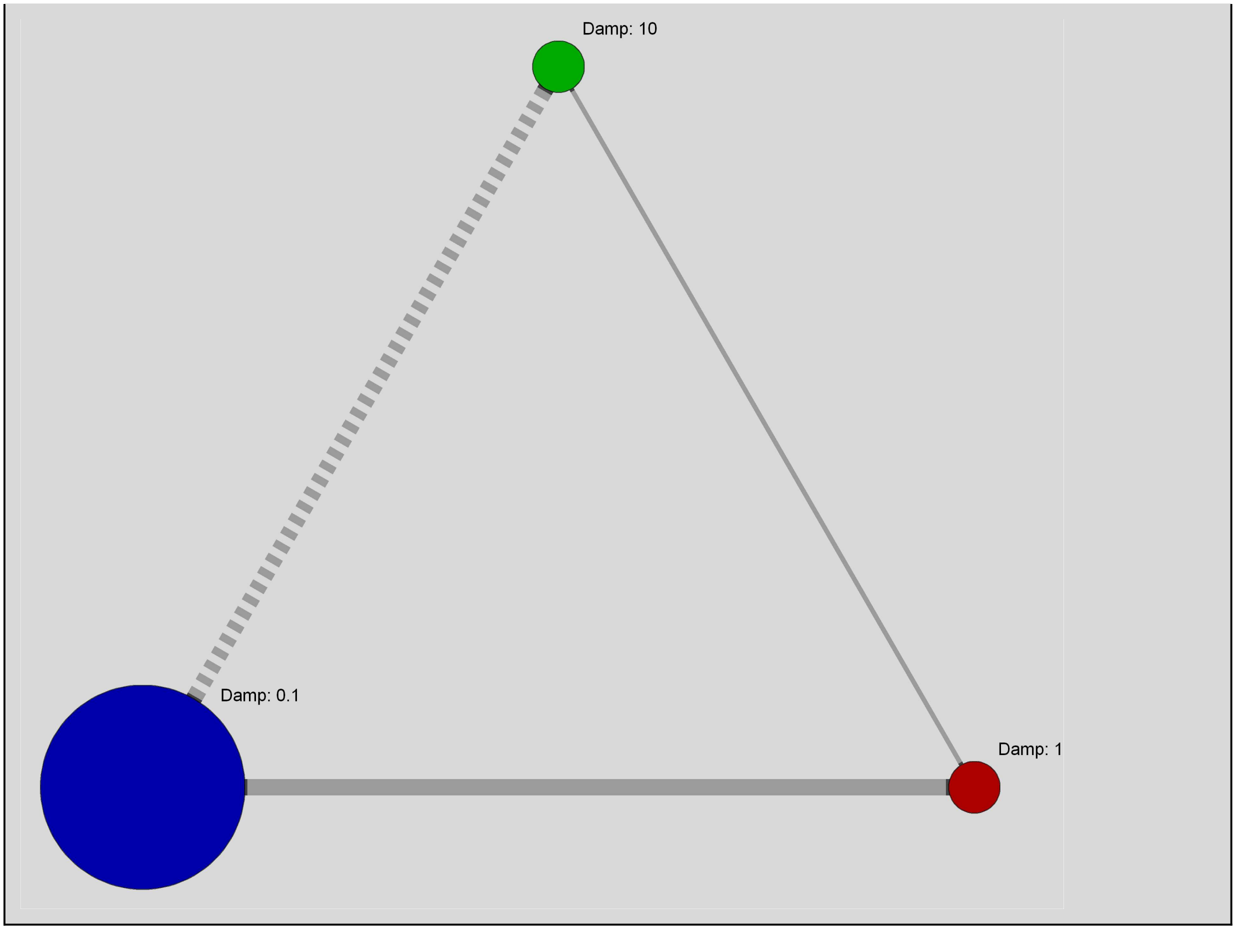
Synthetic Test Case Overview

In[1]:=

```
versca1 = 0.01;
versca2 = 0.05;
edgsca1 = 0.007;
edgsca2 = 0.02;
Graph[{Style[1, Darker@Blue], Style[2, Darker@Red], Style[3, Darker@Green]},
       style     plus fo…  bleu   style    plus fo…  rouge  style    plus fo…  vert
  {Style[1 ⟷ 2, Thickness[40/17 edgsca1], Gray], Style[2 ⟷ 3, Thickness[4/17 edgsca2], Gray],
   style        épaisseur                 gris    style        épaisseur                gris
   Style[3 ⟷ 1, Thickness[40/17 edgsca1], Dashed, Gray]}, VertexShapeFunction → "Circle",
   style        épaisseur                 rayé    gris     fonction de forme de noeud  cercle
  VertexSize → {1 -> 417/17 versca1, 2 -> 21/17 versca2, 3 -> 21/17 versca2},
  taille de noeud
  VertexLabels → {1 → "Damp: 0.1", 2 → "Damp: 1", 3 → "Damp: 10"}]
  étiquettes des sommets
```

Out[1]=



In[2]:=

$$H = \begin{pmatrix} 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ \frac{417}{17} & -\frac{40}{17} & \frac{40}{17} & \frac{1}{10} & 0 & 0 \\ -\frac{40}{17} & \frac{21}{17} & -\frac{4}{17} & 0 & 1 & 0 \\ \frac{40}{17} & -\frac{4}{17} & \frac{21}{17} & 0 & 0 & 10 \end{pmatrix};$$

In[3]:=

$$\text{eqnham} = \text{Flatten}\left[\text{FullSimplify}\left[\begin{pmatrix} x_1'[t] \\ x_2'[t] \\ x_3'[t] \\ p_1'[t] \\ p_2'[t] \\ p_3'[t] \end{pmatrix} + H.\begin{pmatrix} x_1[t] \\ x_2[t] \\ x_3[t] \\ p_1[t] \\ p_2[t] \\ p_3[t] \end{pmatrix}\right]\right];$$

aplatis   simplifie complètement

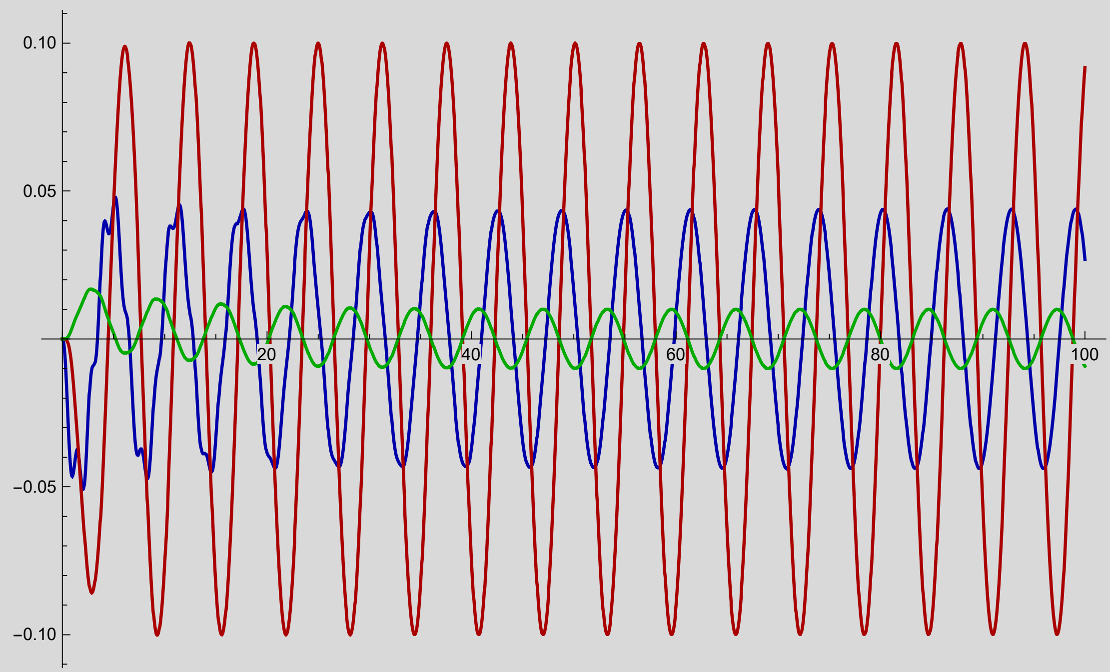
In[5]:=

```
solham = NDSolve[
            résolveur numérique d'équations différentielles
  {eqnham⟦1⟧ == 0, eqnham⟦2⟧ == 0, eqnham⟦3⟧ == 0, eqnham⟦4⟧ == Cos[t + 1.7], eqnham⟦5⟧ == 0,
                                                                  cosinus
   eqnham⟦6⟧ == 0, x1[0] == 0, x2[0] == 0, x3[0] == 0, p1[0] == 0, p2[0] == 0, p3[0] == 0},
  {x1, x2, x3, p1, p2, p3}, {t, 0, 100}, Method → "ImplicitRungeKutta"];
                                          méthode
```

In[6]:=

```
Plot[{x1[t] /. solham, x2[t] /. solham, x3[t] /. solham}, {t, 0, 100}, PlotRange → All,
tracé de courbes                                                        zone de tracé  tout
 PlotStyle → {Darker@Blue, Darker@Red, Darker@Green}, ImageSize → 570]
 style de tracé plus fo… bleu plus fo… ro… plus fo… vert  taille d'image
```

Out[6]=



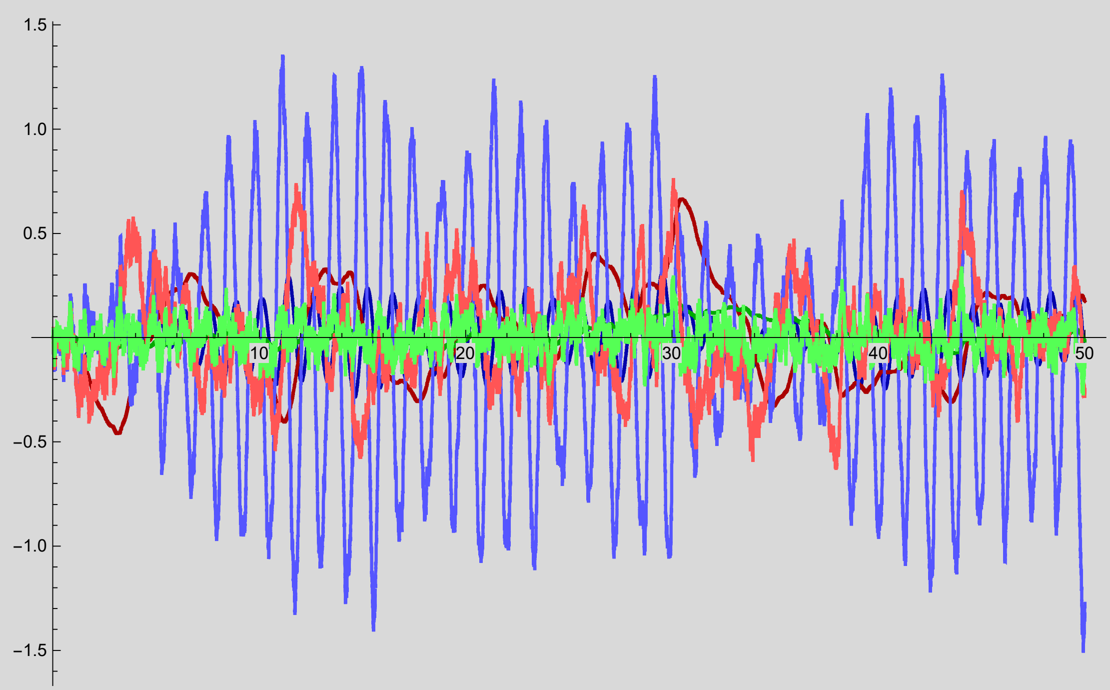
Stochastic ODE & Data Generation

In[65]:=

```
proc = ItoProcess[{{a[t], b[t], c[t], -a[t]/10 - 417 x[t]/17 + 40 y[t]/17 - 40 z[t]/17 + Cos[t + 1.7],
    -b[t] + 40 x[t]/17 - 21 y[t]/17 + 4 z[t]/17, -10 c[t] - 40 x[t]/17 + 4 y[t]/17 - 21 z[t]/17},
   {{0}, {0}, {0}, {1/3}, {1/3}, {1/3}}, {x[t], y[t], z[t], a[t], b[t], c[t]}},
  {{x, y, z, a, b, c}, {0, 0, 0, 0, 0, 0}}, {t, 0}];
time = 50;
resolution = 10^-3;
path = RandomFunction[proc, {0., time, resolution}, Method → "StochasticRungeKutta"];
ListLinePlot[path, PlotStyle → {Darker@Blue, Darker@Red, Darker@Green,
   Lighter@Blue, Lighter@Red, Lighter@Green}, PlotRange → All, ImageSize → 570]
```

processus Ito, cosinus, fonction aléatoire, méthode, tracé de liste de ligne, style de tracé, plus fo···, bleu, plus fo···, ro···, plus fo···, vert, plus clair, bleu, plus clair, ro···, plus clair, vert, zone de tracé, tout, taille d'image

Out[65]=



Discrete Frequencies

In[8]:=

```
N@ time/(2 π) {1, 5}
```

valeur numérique

Out[8]=

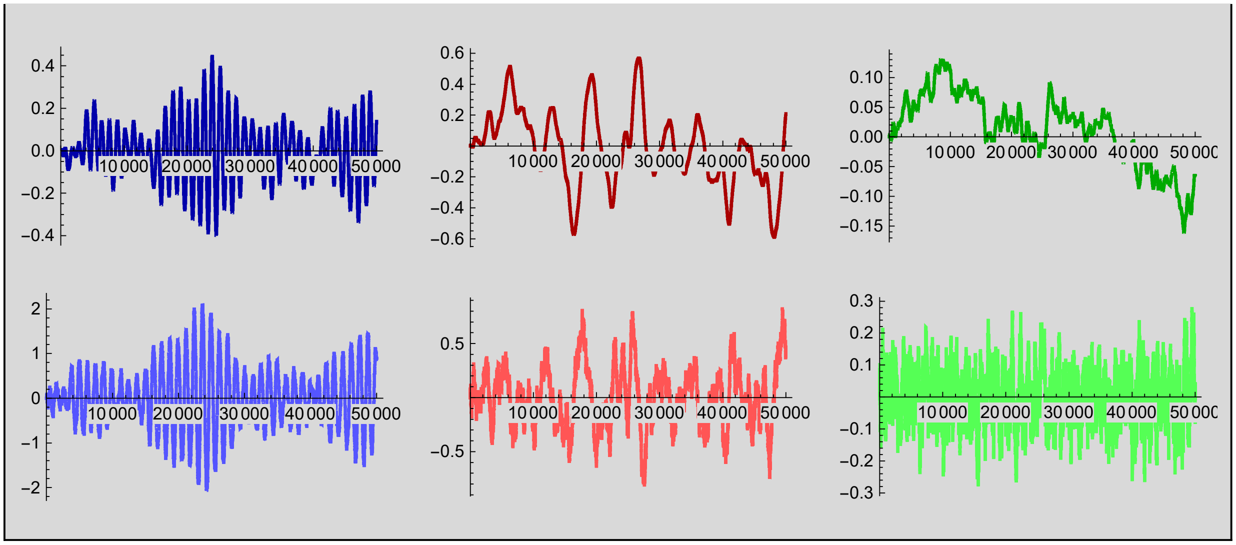
```
{7.95775, 39.7887}
```

In[81]:=

```
tsfulldata = Flatten[Normal@
             aplatis  forme normale
   RandomFunction[proc, {0., time, resolution}, Method → "StochasticRungeKutta"], 1];
   fonction aléatoire                              méthode
samples = Length[tsfulldata] - 1;
          longueur
tssignal = Table[tsfulldata⟦k, 2, i⟧, {i, 1, 6}, {k, 1, samples}];
           table
tsderivative =
  Table[(tsfulldata⟦k + 1, 2, i⟧ - tsfulldata⟦k, 2, i⟧) / (tsfulldata⟦k + 1, 1⟧ - tsfulldata⟦k, 1⟧), {i, 1, 6}, {k, 1, samples}];
  table

matrixsignal = Sum[{tssignal⟦ ;; , k⟧}ᵀ.{tssignal⟦ ;; , k⟧}, {k, 1, samples}] / samples;
               somme
matrixderivativesignal =
  Sum[{tssignal⟦ ;; , k⟧}ᵀ.{tsderivative⟦4 ;; 6, k⟧}, {k, 1, samples}] / samples;
  somme
colorlist = {Darker@Blue, Darker@Red, Darker@Green,
             plus fo⋯ bleu plus fo⋯ ro⋯ plus fo⋯ vert
   Lighter@Blue, Lighter@Red, Lighter@Green};
   plus clair bleu plus clair ro⋯ plus clair vert
fouriersignal = Table[Fourier[tssignal⟦i⟧], {i, 1, 6}];
                table transformée de Fourier discrète
fourierderivative = Table[Fourier[tsderivative⟦i⟧], {i, 1, 6}];
                    table transformée de Fourier discrète
fouriermatrixsignal = Table[
                      table
   Re[Conjugate[{fouriersignal⟦ ;; , k⟧}ᵀ].{fouriersignal⟦ ;; , k⟧}], {k, 1, samples}];
      conjugué
fouriervectorderivativesignal = Table[Re[Conjugate[fourierderivative⟦i, k⟧]
                                table p⋯ conjugué
     fouriersignal⟦ ;; , k⟧], {i, 1, 6}, {k, 1, samples}];
fourierdynamic = Table[Abs[fourierderivative⟦i + 3, k⟧ +
                 table valeur absolue
     H⟦4 ;; 6, ;;⟧⟦i, ;;⟧.fouriersignal⟦ ;; , k⟧], {i, 1, 3}, {k, 1, samples}];
GraphicsGrid[Partition[Table[ListPlot[tssignal⟦i⟧, PlotRange → {{0, All}, All},
             partitionne table tracé de liste         zone de tracé   tout  tout
     Joined → True, PlotStyle → colorlist⟦i⟧, ImageSize → 190], {i, 1, 6}], 3]]
              vrai   style de tracé                    taille d'image
```

Out[81]=

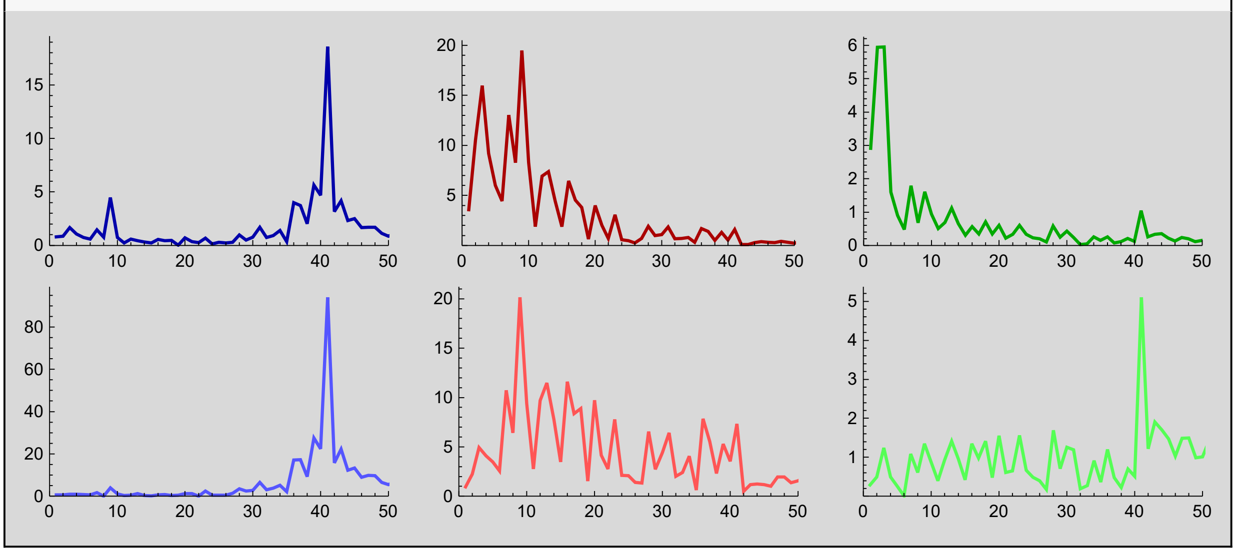


In[82]:=

```
GraphicsGrid[
 Partition[Table[ListPlot[{Abs[fouriersignal[[i]]]}, PlotRange → {{0, 50}, {0, All}},
    Joined → True, PlotStyle → colorlist[[i]], ImageSize → 190], {i, 1, 6}], 3]]
```

grille de graphiques · partitionne · table · tracé de liste · valeur absolue · zone de tracé · tout · vrai · style de tracé · taille d'image

Out[82]=

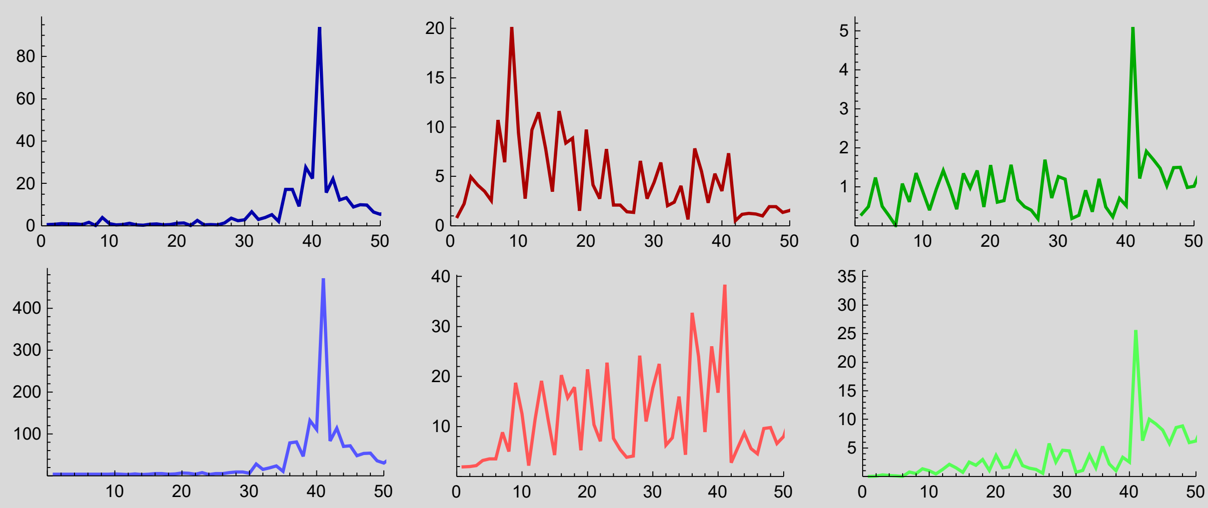


In[83]:=

```
GraphicsGrid[Partition[
  Table[ListPlot[{Abs[fourierderivative[[i]]]}, PlotRange → {{0, 50}, {0, All}},
    Joined → True, PlotStyle → colorlist[[i]], ImageSize → 190], {i, 1, 6}], 3]]
```

grille de graphi··· partitionne tracé de liste valeur absolue zone de tracé tout vrai style de tracé taille d'image

Out[83]=

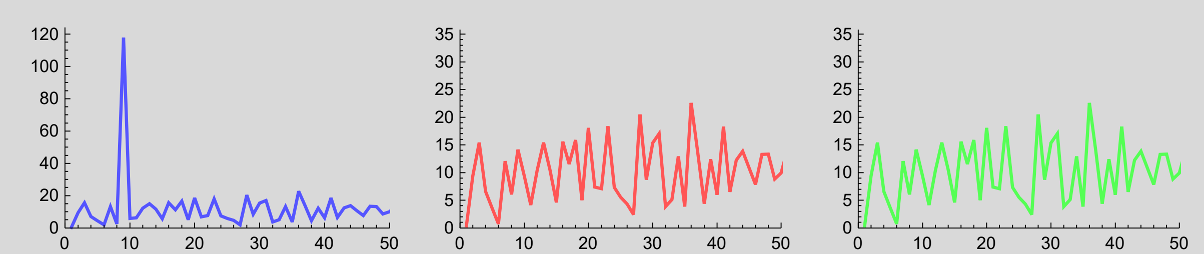


In[84]:=

```
GraphicsGrid[
 {Table[ListPlot[{Abs[fourierdynamic[[i]]]}, PlotRange → {{0, 50}, {0, All}},
    Joined → True, PlotStyle → colorlist[[i + 3]], ImageSize → 190], {i, 1, 3}]}]
```

grille de graphiques table tracé de liste valeur absolue zone de tracé tout joint vrai style de tracé taille d'image

Out[84]=



Log-Likelihood Regression

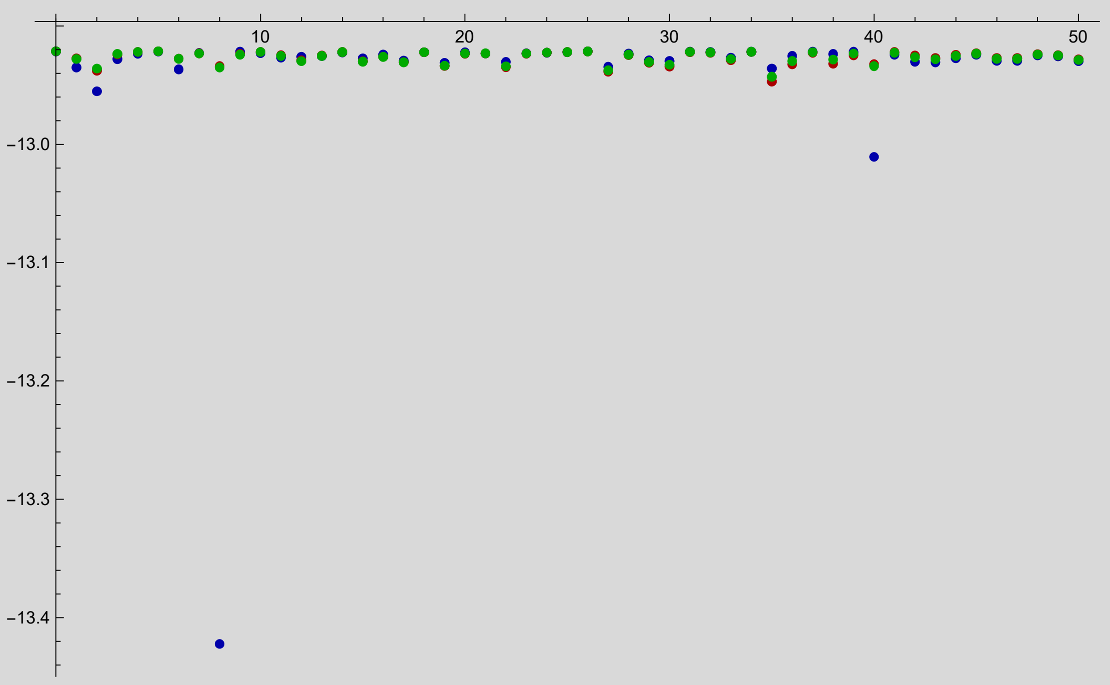
In[88]:=

```
localizationreg = Table[varmat = {{a1, b1, b2, c1, 0, 0}, {b1, a2, b3, 0, c2, 0}, {b2, b3, a3, 0, 0, c3}};
  NMinimize[{Tr[varmat^T.varmat.matrixsignal] + 2 Tr[varmat.matrixderivativesignal] +
      1/2 γ^2 - (2 γ)/Sqrt[samples] (Tr[{varmat[[l, ;;]]}^T.{varmat[[l, ;;]]}.fouriermatrixsignal[[
            k + 1]]] + 2 fouriervectorderivativesignal[[3 + l, k + 1]].varmat[[l, ;;]] +
          Abs[fourierderivative[[3 + l, k + 1]]]^2)^(1/2), γ ≥ 0 && c1 ≥ 0 && c2 ≥ 0 && c3 ≥ 0 && a1 ≥
      0 && a2 ≥ 0 && a3 ≥ 0}, {a1, a2, a3, b1, b2, b3, c1, c2, c3, γ}], {l, 1, 3}, {k, 0, 50}];
freqreg = Table[{k, localizationreg[[l, k + 1, 1]]}, {l, 1, 3}, {k, 0, 50}];
```

In[89]:=

```
ListPlot[freqreg, PlotRange → All, PlotStyle → colorlist[[1 ;; 3]], ImageSize → 570]
```

Out[89]=



Estimated Matrix & Amplitude

In[90]:=
```
siteest = 1;
frequest = 8;
Grid[{{MatrixForm[varmat /. localizationreg[[siteest, frequest + 1, 2]]],
    "γ = " ~~ ToString[γ /. localizationreg[[siteest, frequest + 1, 2]]]}}]
```
grille · apparence matricielle · convertis en chaîne de caractères

Out[90]=

$$\begin{pmatrix} 24.7743 & -2.07591 & 2.3514 & 0.0698741 & 0 & 0 \\ -2.07591 & 1.35923 & -0.101915 & 0 & 0.925181 & 0 \\ 2.3514 & -0.101915 & 1.25028 & 0 & 0 & 9.59457 \end{pmatrix} \quad \gamma = 1.04081$$

Ground Truth

In[91]:=
```
Grid[{{MatrixForm[N@H[[4 ;; 6, ;;]]], "γ = 1"}}]
```
grille · apparence m·· · valeur numérique

Out[91]=

$$\begin{pmatrix} 24.5294 & -2.35294 & 2.35294 & 0.1 & 0. & 0. \\ -2.35294 & 1.23529 & -0.235294 & 0. & 1. & 0. \\ 2.35294 & -0.235294 & 1.23529 & 0. & 0. & 10. \end{pmatrix} \quad \gamma = 1$$

Log-Likelihood Spatial Relaxation

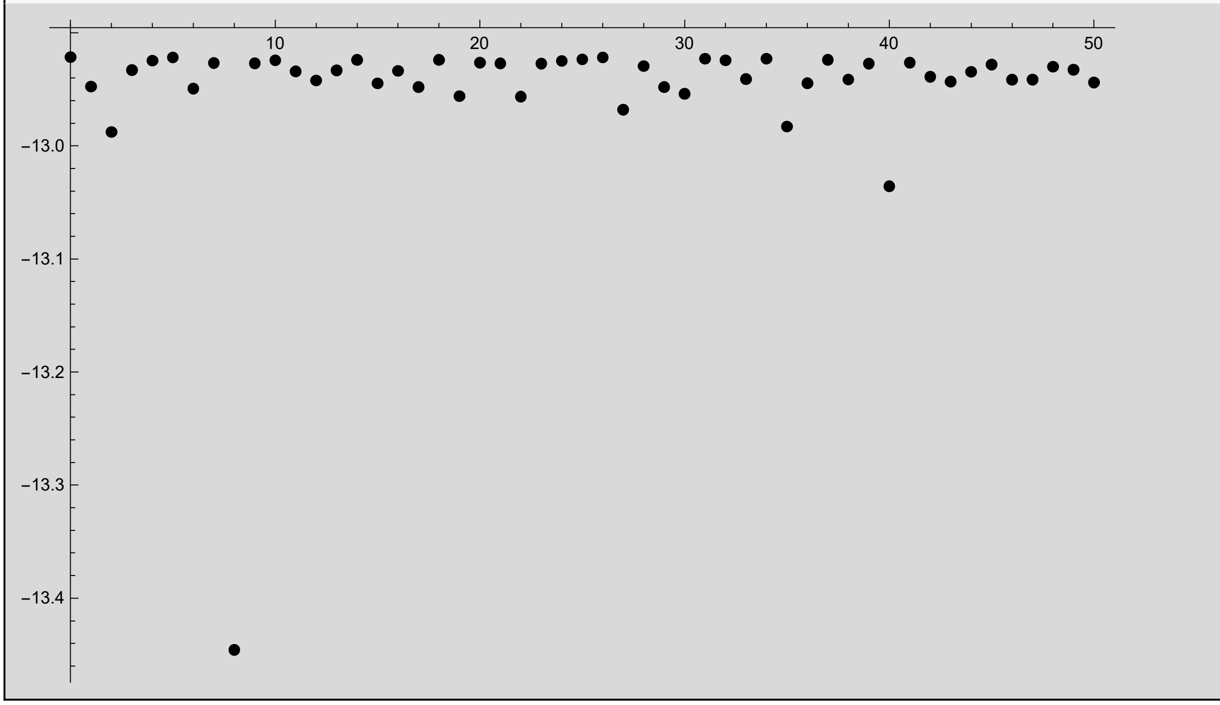
In[92]:=

```
relocalizationreg = Table[varmat = {{a1, b1, b2, c1, 0, 0}, {b1, a2, b3, 0, c2, 0}, {b2, b3, a3, 0, 0, c3}};

  NMinimize[{Tr[varmat^T.varmat.matrixsignal] + 2 Tr[varmat.matrixderivativesignal] +
      1/2 (γ1^2 + γ2^2 + γ3^2) - (2 γ1/Sqrt[samples] (Tr[{varmat[[1, ;;]]}^T.{varmat[[1, ;;]]}.
             fouriermatrixsignal[[k + 1]]] + 2 fouriervectorderivativesignal[[3 +
               1, k + 1]].varmat[[1, ;;]] + Abs[fourierderivative[[3 + 1, k + 1]]]^2)^(1/2) +
        2 γ2/Sqrt[samples] (Tr[{varmat[[2, ;;]]}^T.{varmat[[2, ;;]]}.fouriermatrixsignal[[k + 1]]] +
            2 fouriervectorderivativesignal[[3 + 2, k + 1]].varmat[[2, ;;]] +
            Abs[fourierderivative[[3 + 2, k + 1]]]^2)^(1/2) + 2 γ3/Sqrt[samples]
         (Tr[{varmat[[3, ;;]]}^T.{varmat[[3, ;;]]}.fouriermatrixsignal[[k + 1]]] +
            2 fouriervectorderivativesignal[[3 + 3, k + 1]].varmat[[3, ;;]] +
            Abs[fourierderivative[[3 + 3, k + 1]]]^2)^(1/2)),
    γ1 ≥ 0 && γ2 ≥ 0 && γ3 ≥ 0 && c1 ≥ 0 && c2 ≥ 0 && c3 ≥ 0 && a1 ≥ 0 && a2 ≥ 0 && a3 ≥ 0},
   {a1, a2, a3, b1, b2, b3, c1, c2, c3, γ1, γ2, γ3}], {k, 0, 50}];
refreqreg = Table[{k, relocalizationreg[[k + 1, 1]]}, {k, 0, 50}];
```

In[93]:=
```
ListPlot[refreqreg, PlotRange → All, PlotStyle → Black, ImageSize → 570]
```
tracé de liste, zone de tracé, tout, style de tracé, noir, taille d'image

Out[93]=



Estimated Matrix & Amplitude

In[94]:=
```
frequest = 8;
Grid[{{MatrixForm[varmat /. relocalizationreg[[frequest + 1, 2]]],
    "(γ1,γ2,γ3) = " ~~ ToString[{γ1, γ2, γ3} /. relocalizationreg[[frequest + 1, 2]]]}}]
```
grille, apparence matricielle, convertis en chaîne de caractères

Out[94]=

$$\begin{pmatrix} 24.7661 & -2.08956 & 2.49514 & 0.0687766 & 0 & 0 \\ -2.08956 & 1.48441 & 0.169378 & 0 & 1.07486 & 0 \\ 2.49514 & 0.169378 & 0.413976 & 0 & 0 & 9.81136 \end{pmatrix} \quad (\gamma_1,\gamma_2,\gamma_3) = \{1.03953, 0.167$$

Ground Truth

In[95]:=
```
Grid[{{MatrixForm[N@H[[4 ;; 6, ;;]]], "γ = 1"}}]
```
grille, apparence m··, valeur numérique

Out[95]=

$$\begin{pmatrix} 24.5294 & -2.35294 & 2.35294 & 0.1 & 0. & 0. \\ -2.35294 & 1.23529 & -0.235294 & 0. & 1. & 0. \\ 2.35294 & -0.235294 & 1.23529 & 0. & 0. & 10. \end{pmatrix} \quad \gamma = 1$$