Raw_Me

9/13/2017

CSCI-3656

Homework 1

Q1:

When I execute the commands I get:

$z = 3.330669073875470e\text{-}16$

This is because the first command is:

>> format long

This command makes the computer use the 64 bit representation. In this case 9.4 and 0.4 are represented in(respectively):

1. 0010110011001100110011001100110011001100110011001101 × 2^3

1. 1001100110011001100110011001100110011001100110011010 × 2^−2

Since long uses the rounding to the nearest rule, we can see that both numbers have rounding errors. The 9.4 representation makes a rounding error equals to 0.2 × 2^−49. The 0.4 representation makes a rounding error equals to 0.1 × 2^−52. Subtracting the two rounding errors equals z.

Q2:

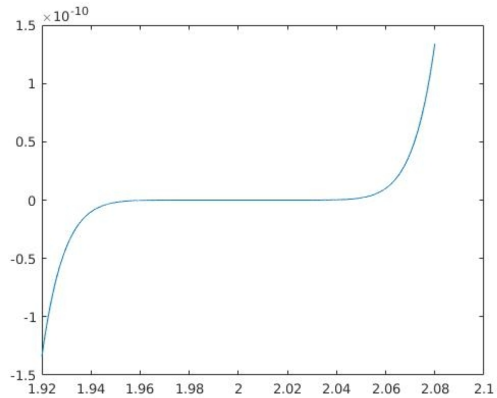
The implementation in Matlab:

```
function x = horner(a,z)
m = length(z);
n = length(a);
p = a(1);
for j = 2:n
   p = p.*z + a(j);
end
x = p;
```

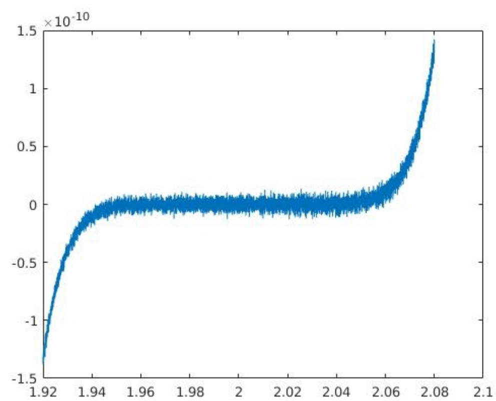
Then executing the following commands:

```
>>z = linspace(1.92, 2.08, 8000);
>>p1 = (z-2).^9;
>>plot(z,p1)
>>plot(z, horner([1 -18 144 -672 2016 -4032 5376 -4608 2304 -512],z))
```

p1(x) graph:



p2(x) graph:



The difference is clear. It is only happening as the polynomial is reaching 0 using Horner's Algorithm.

I think that it is because when reaching 0 the cancellation of big numbers, compared to the result (≈ 0), causes errors due to the rounding rule.

Q3:

proving they are equal:

$$f_1(x)=\frac{1-\cos(x)}{\sin^2(x)}=\frac{1-\cos(x)}{1-\cos^2(x)}=\frac{1-\cos(x)}{1^2-\cos^2(x)}=\frac{1-\cos(x)}{(1+\cos(x))(1-\cos(x))}=\frac{1}{1+\cos(x)}=f_2(x)$$

The Matlab implementation:

```
k = linspace(0,12,13);
x = 10.^(-k);
f1 = (1 - cos(x))./(sin(x).^2)
f2 = 1./(1 + cos(x))
```

The results:

f1 = [0.649223205204762   0.501252086288577   0.500012500208481   0.500000124992189   0.499999998627931   0.500000041386852   0.500044450291337   0.499600361081322   0   0   0   0   0]

f2 = [0.649223205204762   0.501252086288571   0.500012500208336   0.500000125000021   0.500000001250000   0.500000000012500   0.500000000000125   0.500000000000001   0.500000000000000   0.500000000000000   0.500000000000000   0.500000000000000   0.500000000000000]

The reason behind this difference is as x goes to 0 cos(x) gets closer to 1. This affects only f1(x) because of 1 - cos(x). In this case, the subtraction of nearly equal numbers causes loss of significance.