Raw_Me

10/30/2017

CSCI-3656

Homework 3

Q1:

I used this Matlab code to plot the error:

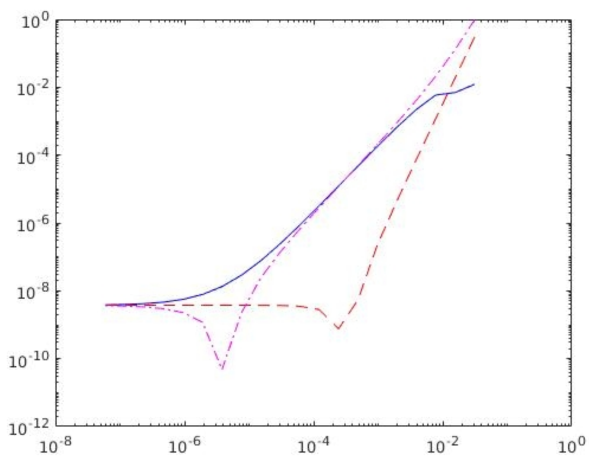
```
a = 0.2;
trueVal = 15.0796 * cos(15.0796 * a);
h = 2 .^ (-5:-1:-24);
f = @(x) sin(4.8*pi*x);

fwdDiff = (f(a+h)-f(a)) ./ h;
errFwd = abs(trueVal-fwdDiff).^2;

ctrDiff = ( f(a+h)-f(a-h) ) ./ (2*h);
errCtr = abs(trueVal-ctrDiff).^2;

bkbDiff = (f(a)-f(a-h)) ./ h;
errBkd = abs(trueVal-bkbDiff).^2;

loglog(h,errFwd,'-b', h,errCtr,'--r', h,errBkd, '-.m');
```



solid: one-sided forward difference,

dashed: one-sided backward difference,

dash-dotted: central difference.

note: As h gets smaller the loss of significance (subtracting nearly equal numbers) causes loss of accuracy.

I calculated the slope of each method at the asymptotic regime (k=12,12 Forward and Backward , k=7,6 Central) using these commands:

```
back = ( log(errBkd(8)) - log(errBkd(7) ) )/ ( log(h(8) ) - log(h(7) ) );
for = ( log(errFwd(8) ) - log(errFwd(7) ) )/(log(h(8)) - log(h(7)) );
cen = ( log(errCtr(4) ) - log(errCtr(3) ) )/(log(h(4)) - log(h(3)) );
```

which returns, back: 2.0534, for: 1.9465, cen: 4.0139

This shows that the one-sided backward difference and forward difference have the same convergence rate in the asymptotic range while the central difference is faster as we can see the slope is steeper to the double which means that the convergence rate is double the speed of the other methods.

Q2:

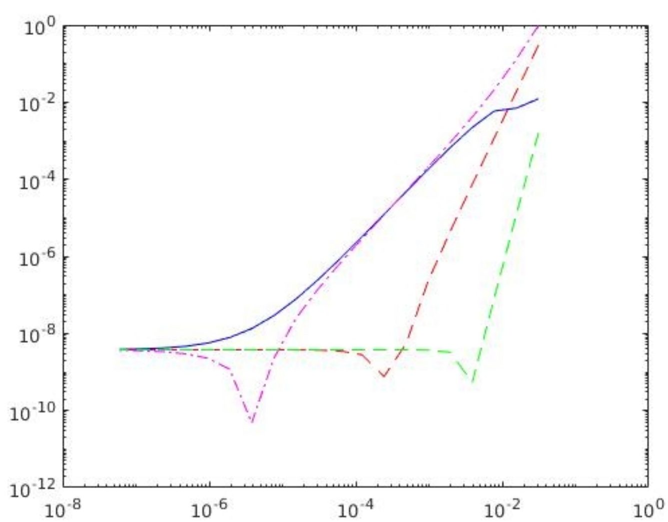
I used this Matlab code to approximate the convergence rate:

```
faf = (1./(6.*h)) .* ( 2.*f(a+h) + 3.*f(a) - 6.*f(a-h) + f(a-(2.*h)) );
errFaf = abs(trueVal-faf).^2;

slope = ( log(errFaf(3) ) - log(errFaf(2) ) )/(log(h(3)) - log(h(2)) );
```

this returns, slope: 7.1768.

The convergence rate is about 2.8 times the convergence rate of the forward difference. This is also faster than the central. As I plotted it to the same graph from Q1:



solid: one-sided forward difference,

dashed: one-sided backward difference,

dash-dotted: central difference.

dashed: Q2 approximation

Q3:

(a) The integral of f(x) = cos(3 π x) is sin(3*pi*x)/(3*pi) + constant.

And for [1,1.5] = 0.106103.

(b) I used this Matlab code and the code from the professor to plot the relative error:

```
tVal = 0.106103;
f = @(x) (cos(3.*pi.*x));
trap = 1:10;
errTrp = 1:10;
smp = 1:10;
errSmp = 1:10;
shw = 1:10;
errShw = 1:10;

% Trap
for k = 1:10
    n = 2.^(k)+1;
    a = 1;
    b = 1.5;
    h = ( (b-a)/n );
    trap(k) = 0;
    while (a<b-h)
        trap(k) = trap(k) + (h./2).*(f(a)+f(a+h) );
        shw(k) = shaw(f,1,a+h,n);
        a = a + h;
    end
    errTrp(k) = abs( (trap(k)-tVal)./tVal);
    errShw(k)= abs( (shw(k)-tVal)./tVal );
end

% simpsons & clenshaw
for k= 1:10
    smp(k) = simpsons(1,1.5,k);
    errSmp(k) = abs( (smp(k)-tVal)./tVal );
end

ln = 2.^(1:10)+1;
loglog(ln,errTrp,'-b', ln,errSmp,'--r', ln,errShw,'-.g');
```

Simpson's function:

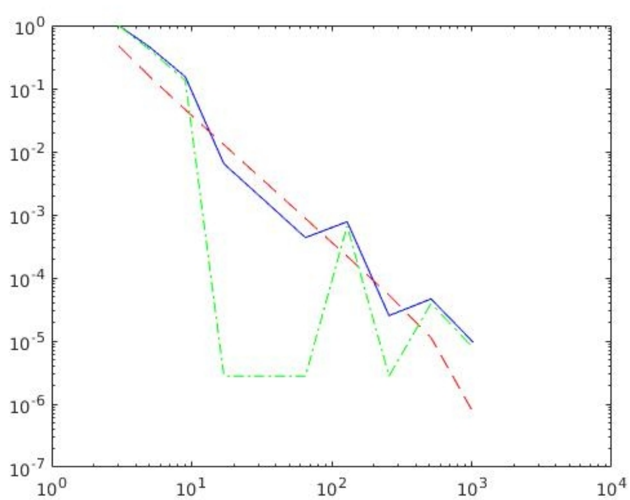
```
function int = simpsons(a,b,k)
f=@(x) (cos(3.*pi.*x));
n = 1+2.^(k);
h=(b-a)./n;
x=a:h:b;
r = f(x);
n=length(x);

st4=0;
st2=0;

for i=2:2:n
   st4 = st4 + 4*r(i);
end

for i=3:2:n-1
   st2 = st2 + 2*r(i);
end

int = h/3*(r(1)+st4+st2+r(n));
end
```



solid: Trapezoidal rule,

dashed: Simpson's rule,

dash-dotted: Clenshaw Curtis.

(c)

Neglecting the behavior of the entire n and focusing on the asymptotic regime:

Trapezoidal rule: 10^0 , 10^-1

Simpson's rule: 10^0 , 10^-1

Clenshaw Curtis: 10^-1 , 10^-1.2

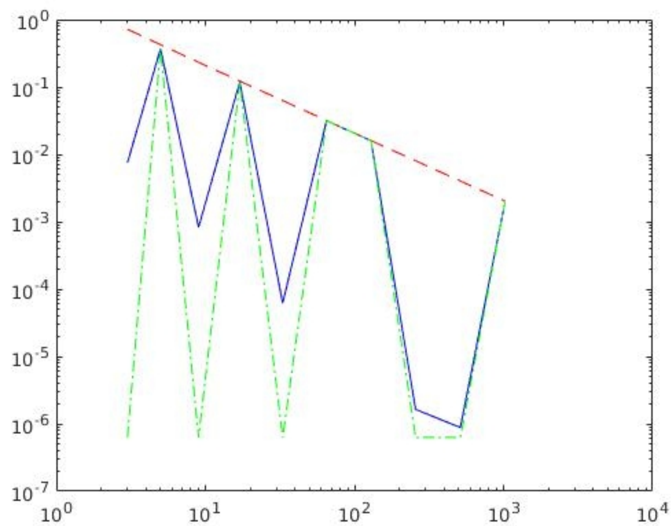
The convergence rate of the Clenshaw Curtis is faster as we can see the slope is steeper than Trapezoidal and Simpson’s. However, Simpson’s started faster and still has faster convergence rate the Trapezoidal as the slope is more steeper.

Q4:

The integral of f (x) = sign (x − 0.2) + 1 is x-cos(1/5 - x)+constant.

And for [-1,1] = 1.66565

I used the same code to plot the graph:



solid: Trapezoidal rule,

dashed: Simpson’s rule,

dash-dotted: Clenshaw Curtis.

The difference in the convergence rate due to the discontinuity in the function which causes the sharp peaks in the graph. This is because the it is approaching two different values depending on the side.