

# A Second Order Quadrupole Doublet Achromat

## Recommended readings

- Sections 3.3 and 3.4 in “Generalization of the Zgoubi method for ray-tracing to include electric fields”, NIM A 340 (1994) 594-604.
- Zgoubi Users’ Guide, regarding the keywords mentioned below (EBMULT, OBJET/KOPT=6,6[.1], etc.). Hint: use the index (last 3 pages of the Guide) to locate the related sections in Part A and Part B.
- During the exercise, it is recommended to keep 2 copies of the guide at hand, with one copy opened at the Index (last 3 pages of the document).

## Keywords we play with in this exercise

EBMULT is the optical element of concern (pp. 129 and 224 in the Users’ Guide).  $\vec{B}$  and  $\vec{E}$  are set so to ensure focal distance as desired, and cancellation of the second order chromatic aberrations inherent to magnetic quadrupoles.

OBJET/KOBJ=5, 6[.1] and MATRIX will allow computing the first, second and third order transport matrices, and series of higher order transport coefficients.

REBELOTE is used to track 100,000s of particles, for statistics purposes, and FAISTORE to log them to zgoubi.fai.

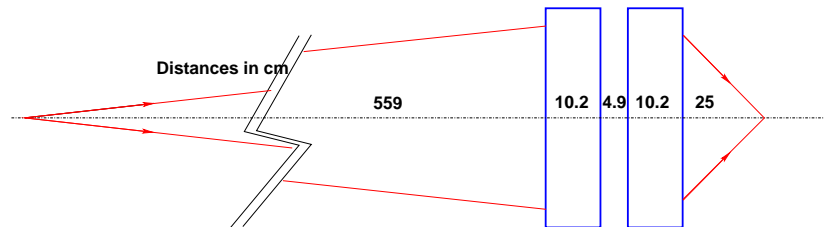
Imaging: IMAGE is used to locate the beam waist, HISTO is used to get transverse beam densities (zpop allows the latter as well, it can be used instead - or concurrently).

## Simulation files

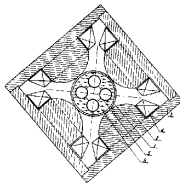
They may all be found in zgoubiFiles folder. Depending on the questions, we’ll use one or the other of bqpole\_image.res, bqpole\_matrix.res, microProbe\_matrix\_2rdOrder.res, microProbe\_matrix\_3rdOrder.res.

## Working hypotheses

- The arrangement of a pair of “E×B” quadrupoles, to form the front end of a microprobe, is sketched in the figure below. The probe is assumed to be a 20 keV proton beam.



- A cross-section of the E×B quadrupole:



[Ref.: S.Ya. Yavor, NIM A 26 (1964)]

The electrostatic component (the cylindrical rods, at  $\pi/4$  from the magnetic poles) satisfies:

$$\begin{cases} E_Y = -KY = -\partial V_{el}/\partial Y \\ E_Z = +KZ = -\partial V_{el}/\partial Z \end{cases}$$

so deriving, in the upright (Y, Z) frame, from the scalar potential

$$V_{el} = \frac{K}{2}(Y^2 - Z^2) \quad (+Const)$$

In the  $45^\circ$ -rotated (U, V) frame, defined by

$$\begin{bmatrix} U \\ V \end{bmatrix} = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix} \begin{bmatrix} Z \\ Y \end{bmatrix},$$

the equipotentials satisfy  $V_{el} = KUV$ .

It means that the field from an electrostatic quadrupole can be computed using the same equations as for an upright magnetic multipole (found in multip.f in zgoubi), after what the vector field so obtained must be  $45^\circ$ -rotated (a call to XROT at the end of multip.f).

## Numerical experience

NOTE: Numerical parameters in the zgoubi sequence are as per sections 3.3 and 3.4 in NIM A 340 (1994) 594-604.

1/ First, let's walk through the Fortran, see what happens when the execution pointer meets 'EBMULT' in zgoubi.dat sequence...

- Open zgoubi\_main.f : it calls zgoubi. The rest of the it essentially manages the 'FIT' procedure.
- Open zgoubi.f
  - find include/LSTKEY.H: take a look into it, spot EBMULT - and all of zgoubi keywords! If you need to add a keyword, that's here
  - find 'CALL REBMUL', look into rebmul.f. What does it do?
  - it is followed by 'CALL QUASEX'. Optical elements call either QUASEX, or AIMANT (check that statement, scrolling through zgoubi.f), what's the difference between the two?
  - Open quaxex.f. In there, find 'CALL CHXC'. What does it do?
    - \* open chxc.f
      - No "EBMULT" therein! (by contrast for instance with 'MULTIPOL', 'SOLENOID'. Figure out why
    - \* find 'CALL TRANSF' in quaxex, open transf.f: it pushes the particles, one by one, through 'INTEGR'
      - find 'CALL INTEGR', open integr.f: it pushes a particle, step by step, through EBMULT (or any other element)
      - In integer.f look for (i) 'CALL CHAMC', (ii) 'CALL CHAMK', (iii) 'CALL DEVTRA': Figure out what each does.

2/ Install in Zgoubi the optical sequence sketched above, leaving first the electrostatic component zero, with B value set to provide the image distance indicated in the figure.

Produce the transport matrices of this system, to 2nd order: we want to see the chromatic coefficient, they are not all zero.

3/ Now switch on the E component, setting E and B to (i) preserve the image distance and (ii) ensure cancellation of second order chromatic aberrations.

Produce the transport matrices to 2nd order, compare with the previous case.

4/ Array sizes limited the number of particles tracked in one go ()would be increased (in the include files),

Track 30,000 particles through the achromat in both cases above, E alone or  $E \times B$ . Proceed in the following way:

- using MCOBJET, create a point object with 0.2 mrad *rms* divergence, Gaussian, in both planes (thus, in the absence of any optical aberrations, the image would be a point). Add  $10^{-3}$  *rms* momentum spread, Gaussian,
- use HISTO to display D (relative rigidity) density, as well as the initial  $Y_0$  and  $Z_0$  transverse beam densities,
- use REBELOTE to iterate to  $3 \times 10^4$  particles, as the maximum number of particles acceptable in MCOBJET is less (a matter of array sizing in the source code, this can be changed in [pathTo]/zgoubi-code/include/MAXTRA.H) - by the way, how much is it?)
- use IMAGE to localize the image waist downstream of the doublet; set the final straight length to the waist distance,
- use HISTO to display the Y and Z transverse beam densities at the image.

5/ Repeat 4/ with B alone (as in 2/,  $E=0$ ). Compare the image widths from HISTO for both cases: pure B and combined E, B.