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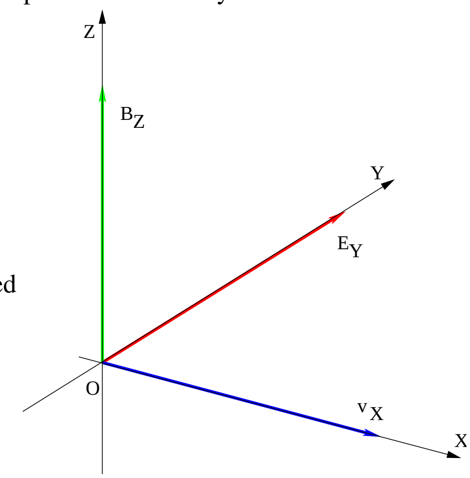
An Introduction to the Wien Filter
A Parallel Plate Electrostatic Deflector

`WIENFILTER` is used in this exercise (pp. 168 and 306 in the Users' Guide), with B=0 so that the Wien filter operates as a simple parallel plate electrostatic deflector.

Note: `ELMIR, ELMIRC` and even `ELMULT` or `EBMULT` could be used as well, they would be able to provide this dipole $\vec{E}$ field simulations (see "Optical elements versus keywords" in the Users' Guide).

Working hypotheses

• The Wien filter length is L. Take $\vec{E} = \vec{E}_Y \parallel \vec{Y}$. Take hard-edge E-field model so to allow tight comparison with theory [1].



• In order to check numerical outcomes: the expected trajectory is a catenary:

$$Y_{th}(X) = \frac{E_i}{qE_Y}\left(\cosh\frac{qE_Y X}{\beta_i E_i} - 1\right)$$

with E_i initial total energy, $c\beta_i$ initial velocity, q particle charge

• Spin rotation: assume we manage to maintain $\vec{E} \perp \vec{v}$, namely, $\vec{v} = v_X$, always. Thus expected spin rotation over L is (Eq. 2.1.4 in Zgoubi Users' Guide)

$$\theta_s = (a\gamma + \frac{\gamma}{1+a\gamma})\beta_X^2 \frac{E_Y cL}{B\rho}$$

Numerical experiments

1/ Set up a Wien filter in zgoubi with $E_Y = 0.98$ MV/m, $L = 0.5$ m. Consider an electron with 350 keV energy.
Run zgoubi and check (in zgoubi.res) final $Y(X \equiv L)$, particle deviation, compare with result from catenary equation above.

2/ Check the effect of step size:
Using REBELOTE, get a scan of Y values for $\Delta s = .001 : 10$ cm. Plot $(Y - Y_{th})/Y_{th}$ versus step size (can use gnuplot to plot data read from zgoubi.fai).

3/ Force Y=0 across the Wien filter, by means of OPTION/CONSTY.
Check the rotation of an initial $\vec{S} \equiv \vec{S}_X$, at the downstream end of the condenser (X=L), compare to expected θ_s.
Check convergence of result versus integration step size.

4/ Add fringe-fields.
3.a Plot the particle trajectory and the E_Y and B_Z fields along the trajectory.
3.b Plot particle energy versus distance, conclude on the importance of fringe fields as to 6-D symplecticity.

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[1] G. Leleux, INSTN Saclay, 1978 (unpublished)

Answers

1/

• A xcell sheet can be used...

$$\text{Expected } Y(X=L) = \frac{511+350}{-980}\left(\cosh\frac{-980\times 0.5}{\sqrt{((511+350)^2-511^2}} - 1\right) = -22.8948628 \quad (E_Y > 0 \text{ and } q<0 \text{ so } Y_{final}<0.$$

• Expected spin rottion is $\theta_s \approx (1.68\times 1.16\,10^{-3} + \frac{1.68}{2.68})\times 0.8^2\frac{0.8\times 3\,10^8\times 0.5}{2.31\,10^{-3}} * ** \approx 20.526$

• Input data file (it can be copy-pasted, as is, and run):

```
E field only
 'OBJET'                                                                                        1
2.3114795386518345           ! Rigidity of a 350 keV electron.
2
3  1                         ! 3 electrons, reason: see SPNTRK below.
0.  0. 0. 0. 0. 1. 'o'
0.  0. 0. 0. 0. 1. 'o'
0.  0. 0. 0. 0. 1. 'o'
1 1 1

 'PARTICUL'                                                                                     2
ELECTRON
 'SPNTRK'                      ! Allows chceking rotation of all 3 spin components.             3
4                             ! (they are computed independently by zgoubi)
1. 0. 0.
0. 1. 0.
0. 0. 1.

 'WIENFILT'                                                                                     4
2                             ! Log to zgoubi.plt, every other 10 step.
0.5  0.98e6 0.    1
0. 0. 0.     ! 20. 5. 5.      ! Hard-edge entrance face.
0.2401  1.8639  -0.5572  0.3904 0. 0.
0.2401  1.8639  -0.5572  0.3904 0. 0.
0. 0. 0.     ! 20. 5. 5.      ! Hard-edge exit face.
0.2401  1.8639  -0.5572  0.3904 0. 0.
0.2401  1.8639  -0.5572  0.3904 0. 0.
.1
1. 0. 0. 0.
 'FAISCEAU'     ! Get some trajectory and some                                                  5

 'SPNPRT'  MATRIX                                                                               6

 'SYSTEM'                                                                                       7
1
gnuplot <./gnuplot_trajectory.gnu &
 'END'                                                                                          8
```

• Excerpts, from zgoubi.res:

- Particle and kinematics data:

```
*******************************************************************************************************************************
      2  Keyword, label(s) :  PARTICUL                                                                              IPASS= 1

     Particle  properties :
     ELECTRON
                     Mass          =   0.510999        MeV/c2
                     Charge        =  -1.602176E-19    C
                     G  factor     =   1.159652E-03
                     COM life-time =   1.000000E+99    s

              Reference  data :
                    mag. rigidity (kG.cm)   :   2.3114795      =p/q, such that dev.=B*L/rigidity
                    mass (MeV/c2)           :  0.51099895
                    momentum (MeV/c)        : -0.69296413
                    energy, total (MeV)     :  0.86099896
                    energy, kinetic (MeV)   :  0.35000002
                    beta = v/c              : -0.8048373615
                    gamma                   :   1.684932950
                    beta*gamma              :  -1.356096989
                    G*gamma                 :  1.9539361699E-03
                    electric rigidity (MeV) : -0.5577234240    =T[eV]*(gamma+1)/gamma, such that dev.=E*L/rigidity
*******************************************************************************************************************************
```

- final coordinates:

```
*******************************************************************************************************************************
      5  Keyword, label(s) :  FAISCEAU                                                                              IPASS= 1

0                                             TRACE DU FAISCEAU
                                           (follows element #      4)
                                                  3 TRAJECTOIRES

                                   OBJET                                                  FAISCEAU

          D       Y(cm)     T(mr)     Z(cm)     P(mr)       S(cm)       D-1     Y(cm)    T(mr)     Z(cm)     P(mr)       S(cm)

o  1   1.0000     0.000     0.000     0.000     0.000      0.0000    0.3817  -22.893 -761.600    0.000    0.000    5.637547E+01     1
               Time of flight (mus) :  2.24897868E-03 mass (MeV/c2) :  0.510999
o  1   1.0000     0.000     0.000     0.000     0.000      0.0000    0.3817  -22.893 -761.600    0.000    0.000    5.637547E+01     2
               Time of flight (mus) :  2.24897868E-03 mass (MeV/c2) :  0.510999
o  1   1.0000     0.000     0.000     0.000     0.000      0.0000    0.3817  -22.893 -761.600    0.000    0.000    5.637547E+01     3
               Time of flight (mus) :  2.24897868E-03 mass (MeV/c2) :  0.510999
*******************************************************************************************************************************
```

2/

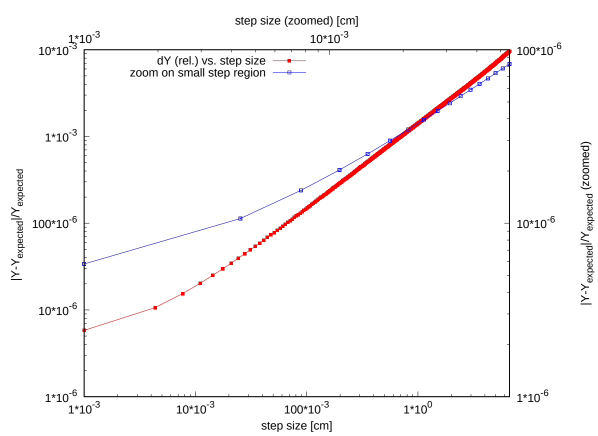
• Input data file (it can be copy-pasted, as is, and run):

REBELOTE does the scan. Its option “1” ensures the change, parameter “80” in WIENFILT, takes NPASS = 60 different values in [0.01,10.].

```
E field only
 'OBJET'                                                                                       1
2.3114795386518345           ! Rigidity of a 350 keV electron.
2
3  1                         ! 3 electrons, reason: see SPNTRK below.
0.  0. 0. 0. 0. 1. 'o'
0.  0. 0. 0. 0. 1. 'o'
0.  0. 0. 0. 0. 1. 'o'
1 1 1

 'PARTICUL'                                                                                    2
ELECTRON

 'WIENFILT'                                                                                    3
0                             ! Log to zgoubi.plt, every other 10 step.
0.5  0.98e6 0.    1
0. 0. 0.      ! 20. 5. 5.      ! Hard-edge entrance face.
0.2401  1.8639  -0.5572  0.3904 0. 0.
0.2401  1.8639  -0.5572  0.3904 0. 0.
0. 0. 0.      ! 20. 5. 5.      ! Hard-edge exit face.
0.2401  1.8639  -0.5572  0.3904 0. 0.
0.2401  1.8639  -0.5572  0.3904 0. 0.
.1
1. 0. 0. 0.

 'FAISCEAU'    ! Get some trajectory data
 'DRIFT'       ! This gives more digits on coordinates (printing to zgoubi.fai would, as well)      5
0.
 'FAISTORE'    ! For use by gnuplot.                                                           6
zgoubi.fai
1

 'REBELOTE'                                                                                    7
2000  1.1  0 1
1
WIENFILT 80 0.001:10.      ! Step size is parameter #80 in WIENFILT

 'SYSTEM'                                                                                      8
1
gnuplot <./gnuplot_scanStepSize.gnu &

 'END'                                                                                         9
```

A convenient gnuplot file to plot the step size scan:

```
set xlabel "step size [cm]"
set x2label "step size (zoomed) [cm]"
set ylabel "|Y-Y_{expected}|/Y_{expected} "
set y2label "|Y-Y_{expected}|/Y_{expected} (zoomed)"

set k t l

set logscale x  ; set format x '%.0s*10^{%S}'
set logscale x2  ; set format x2 '%.0s*10^{%S}'

set logscale y  ; set format y '%.0s*10^{%S}'
set logscale y2 ; set format y2 '%.0s*10^{%S}'

set xtics nomirror
set x2tics
set ytics nomirror
set y2tics

Yexpect = -22.8948628112063  # cm for E=0.98MV/m, from catenary equation.
stp_i = 0.001 ; stp_f = 10. ; NPASS = 3000         # from REBELOTE
dStep = (stp_f-stp_i)/(NPASS-1.)                 # this is what REBELOTE computes

plot  \
"zgoubi.fai" u ($38 >= 3 ? stp_i + ($38-3)*dStep : 1/0):(abs(($10-Yexpect)/Yexpect)) w lp pt 5 ps .6 lc rgb "red" tit "dY (rel.) vs. step size"  ,\
"zgoubi.fai" u ($38 >= 3 && $38 < 20 ? stp_i + ($38-3)*dStep : 1/0):(abs(($10-Yexpect)/Yexpect)) axes x2y2 w lp pt 4 ps .6 lc rgb "blue" tit "zoom on small step region"


set terminal postscript eps blacktext color enh    # size 8.3cm,4cm "Times-Roman" 12
 set output "gnuplot_scanStepSize.eps"
 replot
 set terminal X11
 unset output

pause 44
quit
```



Obviously, accuracy requires very small step size. This stems from the momentum change (integration in magnetic fields allows much greater step size).