Physics of electron rings

Zgoubi workshop

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Overview

- Electron rings past and present
- How to store an electron
- Fast overview of symplectic transverse and longitudinal dynamics
- Addition of Synchrotron radiation in modelling and its impact

Some electron storage rings past and present

Colliders:

- LEP (1989)
- HERA (1992-2007)
- PEP-II (1998-2008)
- KEK-B (2003-2010)

Synchrotron light sources:

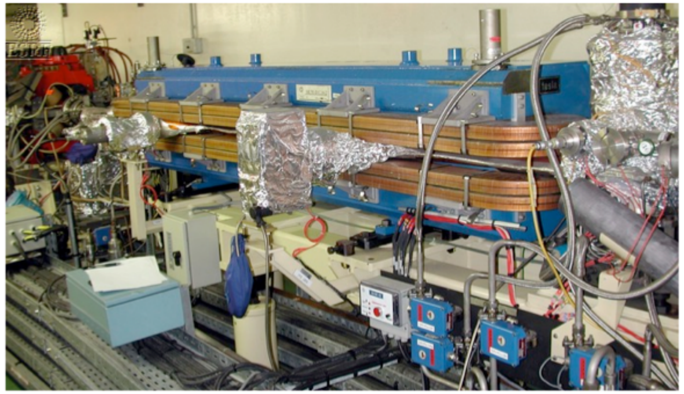
> 50 such facilities! (see https://lightsources.org/)

New round of upgrades decreasing emittance <200pm,
largely based on hybrid MBA lattice from ESRF
(MBA first done at Max-IV)
Pushes towards diffraction limit

How to store a high energy electron bunch?

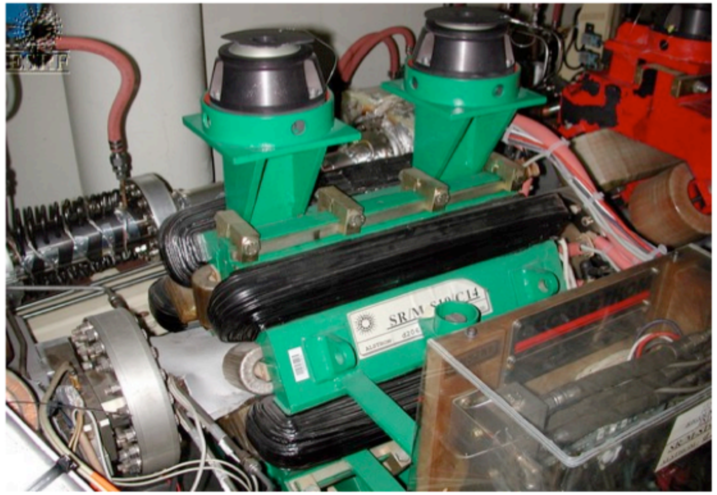
- Electrons accelerator with linac+booster
- Inject electrons into a ring
- Dipole magnets bend the trajectory
- Quadrupole magnets focus the trajectory
- Sextupole magnets fix “chromaticity”
- RF cavities replenish synchrotron radiation energy lost and provide longitudinal focussing

Storage ring components (pre-EBS ESRF)



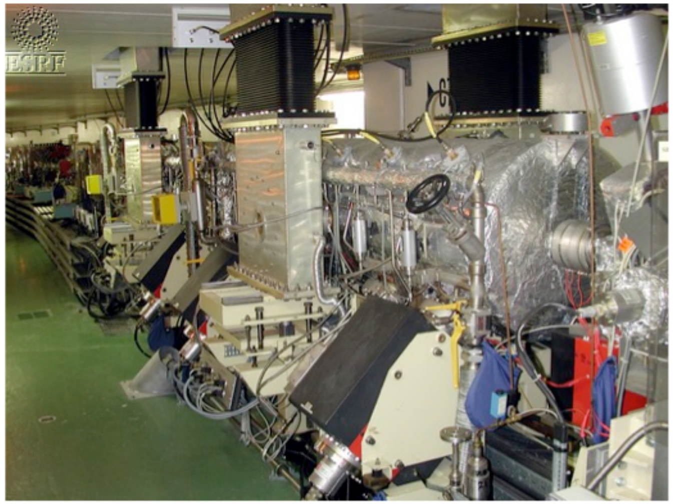
dipole

quadrupole



sextupole

RF cavity



Transverse dymamics

Equations of motion

Apply Newton's 2nd law for Lorentz force:

$$\dot{\vec{P}} = e(v\hat{z} \times \vec{B}) = ev(-B_1\hat{x} + B_1\hat{y})$$

divide by P_0 and change time derivative to s- derivative, take v=c, and we have:

$$x'' + k_x(s)x = 0$$

$$y'' + k_y(s)y = 0$$

$$k_x = -\frac{B_1}{B\rho}$$

$$k_y = \frac{B_1}{B\rho}$$

recall $B\rho = \dfrac{P}{e}$

Can be derived from this Hamiltonian

$$H(x,x',y,y') =$$

$$\frac{1}{2}\left(k_x x^2 + k_y y^2 + x'^2 + y'^2\right)$$

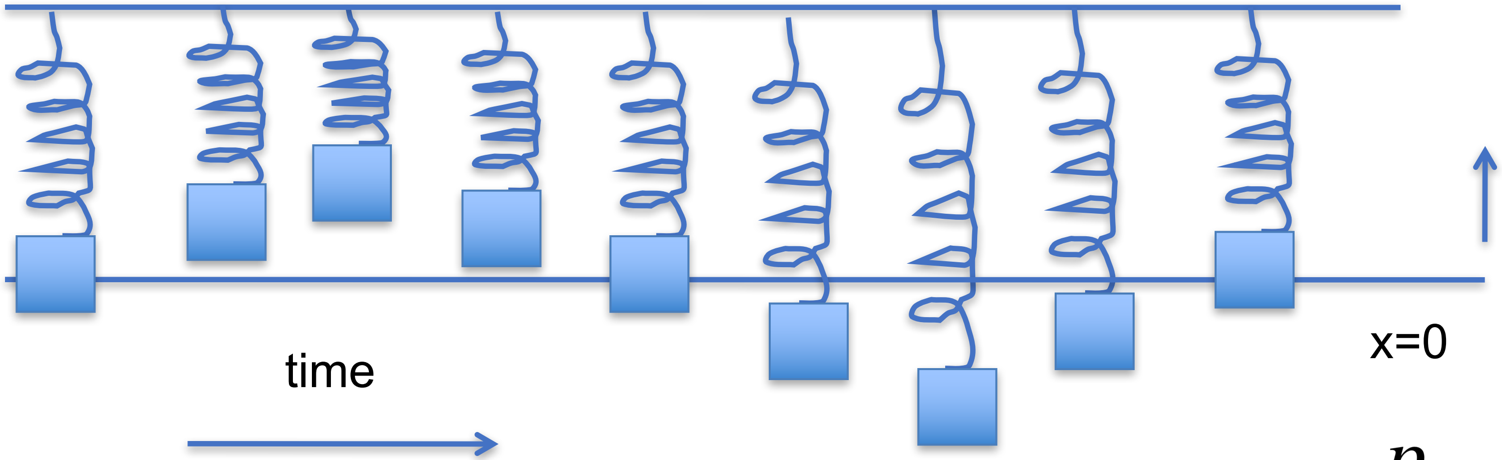
$$x' = \frac{p_x}{P_0} = \frac{dx}{ds}$$

$$y' = \frac{p_y}{P_0} = \frac{dy}{ds}$$

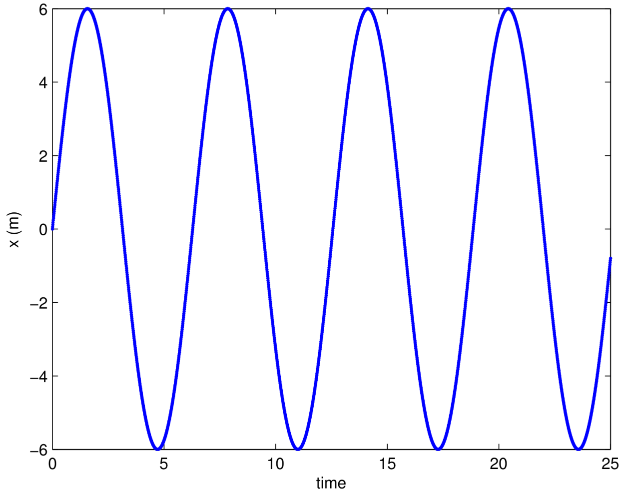
Harmonic oscillator with s-dependent, periodic spring constant. Known as Hill's equation.

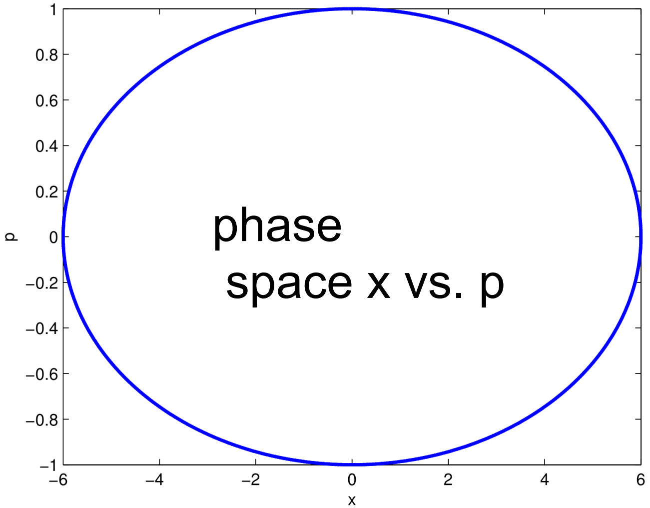
$$k_{x,y}(s) = k_{x,y}(s + C)$$

Phase space



configuration
space x vs. time





$$p_{x,y} = \gamma m v_{x,y}$$

for electron, we normalize with

$$P_0 = \gamma m v_s$$

and use

$$x' = \frac{p_x}{P_0} = \frac{dx}{ds}$$

2-D Matrix Analysis
Courant-Snyder transformation

Given one turn map matrix, can we transform it into a rotation?

$$A^{-1}MA = R$$

$$R = \begin{pmatrix} \cos\mu & \sin\mu \\ -\sin\mu & \cos\mu \end{pmatrix}$$

again, $\mu = 2\pi\nu$

One option for A:

$$A = \begin{pmatrix} \frac{1}{\sqrt{\beta}} & 0 \\ \frac{\alpha}{\sqrt{\beta}} & \sqrt{\beta} \end{pmatrix}$$

together with

$$\gamma = \frac{1+\alpha^2}{\beta}$$

$$\boxed{J_x = \gamma x^2 + 2\alpha x x' + \beta x'^2}$$

is Courant-Snyder invariant

also, one can show that $\alpha = -\frac{1}{2}\frac{d\beta}{ds}$

Twiss Parameters

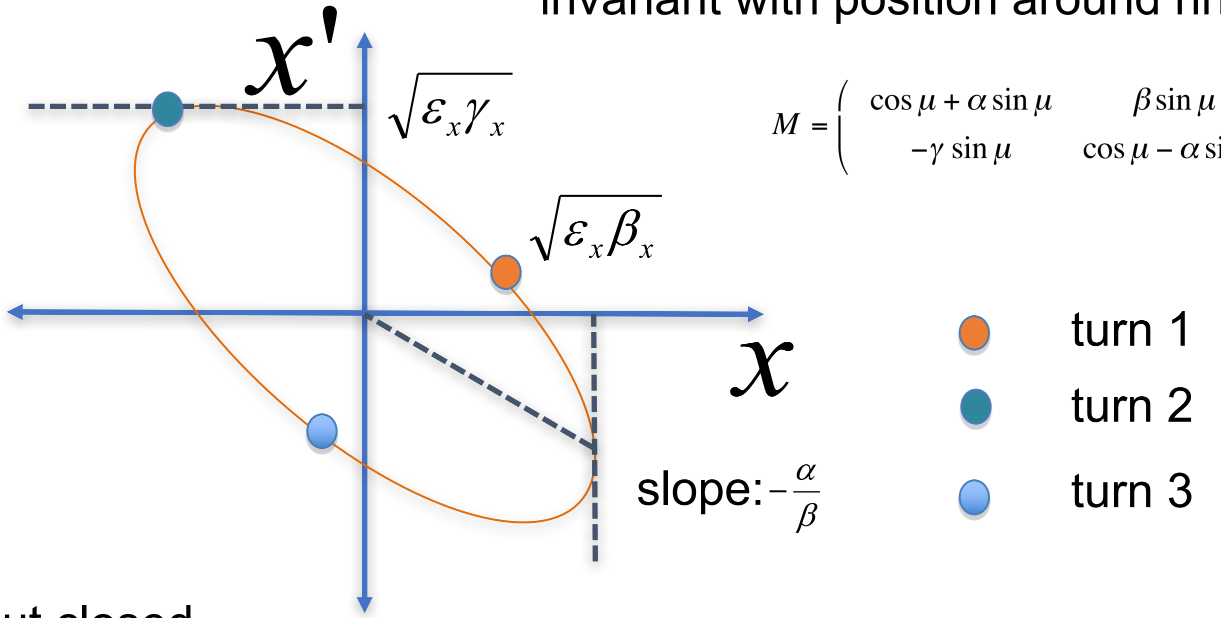
$$\begin{pmatrix} \alpha(s) \\ \beta(s) \\ \gamma(s) \end{pmatrix}$$ are known as 'Twiss Parameters'

$$\gamma_x \eta_x^2 + 2\alpha_x \eta_x \eta_x{}' + \beta_x \eta_x{}'^2$$

invariant with position around ring!

$$M = \begin{pmatrix} \cos\mu + \alpha\sin\mu & \beta\sin\mu \\ -\gamma\sin\mu & \cos\mu - \alpha\sin\mu \end{pmatrix}$$

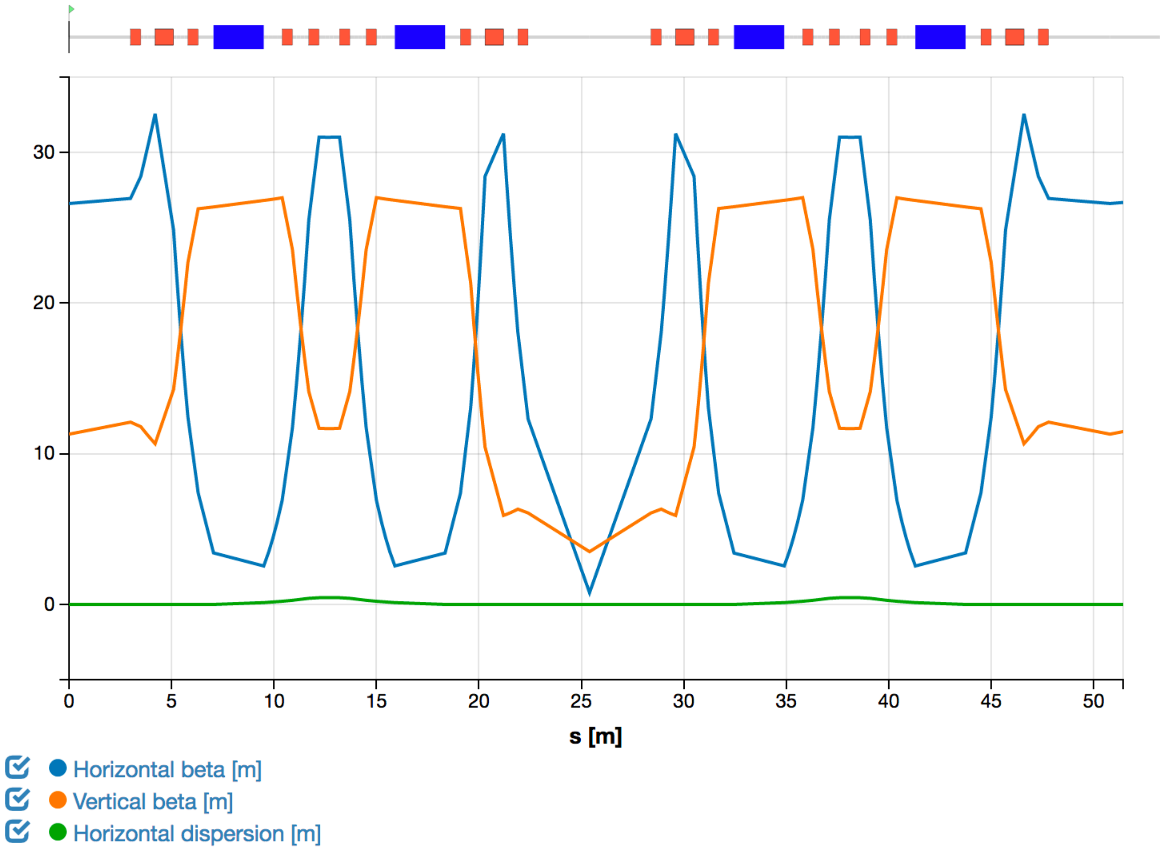
measuring the position over time, it will oscillate



tune is defined by
number of oscillations about closed orbit over 1 turn around ring.
Note that the matrix only captures fractional part.

This is at one position in the ring.

Results for our model ESRF lattice



nu_x=36.197428 nu_y=11.202545

Longitudinal Dynamics

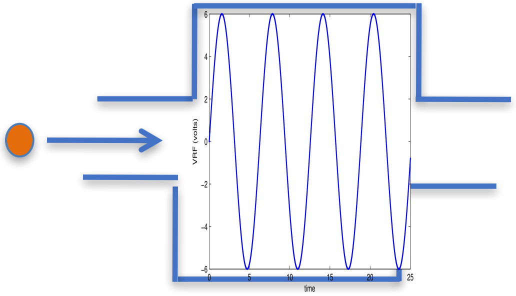
radiation with electrons, but not protons

energy loss per turn is $$U_0 = \frac{C_\gamma \beta^3 E_0^4}{\rho}$$ $$C_\gamma = 8.85 \times 10^{-5} \frac{m}{GeV^3}$$

$$U_0 = 4.88\,MeV$$ for present ESRF

We need to provide this energy back, and also focus longitudinally.

RF cavity dynamics



$$V(t) = \varepsilon \sin(\phi_{RF}(t) + \phi_0)$$

$$\phi_{RF}(t) = h\omega_0 t$$

To energy constant, we need

$$\Delta E = U_0$$

energy loss from
synchrotron radiation

$$\Delta E = e\varepsilon g T \sin\phi_s = eV_{RF} \sin\phi_s$$

g=gap
T=transit time

(here we assume just one cavity)

Longitudinal dynamics (2)

Now consider small variations in energy and arrival time (phase)

matrix for cavity

$$\begin{pmatrix} \delta \\ ct \end{pmatrix}_2 = \begin{pmatrix} 1 & 0 \\ \dfrac{heV_{RF}\cos\phi_s}{C2\pi\beta^2 E_0} & 1 \end{pmatrix} \begin{pmatrix} \delta \\ ct \end{pmatrix}_1$$

$$\phi_s = \pi - \arcsin\left(\frac{U_0}{eV_{RF}}\right)$$

synchronous phase is phase needed
to recover energy lost, U_0

Longitudinal dynamics (3) Momentum compaction

Now, we need to consider the rest of the ring.

Nominal energy E_0

$$\frac{\Delta\tau}{\tau} = \frac{\Delta C}{C} - \frac{\Delta\beta}{\beta} = \eta_s \frac{dE}{E}$$

Change in orbit length vs. energy has two terms:

momentum slip factor

$$\eta_s = \alpha_C - \frac{1}{\gamma^2}$$

$$\alpha_C \equiv \frac{1}{C}\frac{d\Delta C}{d\delta}$$

$$\alpha_C = \frac{1}{\eta_s C}\oint \frac{\eta(s)}{\rho(s)} ds$$

$$\alpha_C = 1.78 \times 10^{-4}$$

$$\frac{1}{\gamma^2} = 7.2 \times 10^{-9}$$

for ESRF

momentum compaction factor

transfer matrix is:

$$\eta_s = 0$$

transition energy:

$$\gamma_T = \frac{1}{\sqrt{\alpha_c}} = 75$$

Above transition: negative mass!

$$T = \begin{pmatrix} 1 & -C\eta_s \\ 0 & 1 \end{pmatrix}$$

Synchrotron tune

Multiplying cavity matrix with matrix for rest of ring, we find

$$\begin{pmatrix} \delta \\ ct \end{pmatrix}_2 = \begin{pmatrix} 1 & -C\eta_s \\ \dfrac{heV_{RF}\cos\phi_s}{C2\pi\beta^2 E_0} & 1 \end{pmatrix} \begin{pmatrix} \delta \\ ct \end{pmatrix}_1$$

One can now analyze this analogously as to the transverse

We derive the synchrotron tune:

$$\upsilon_s = \sqrt{\frac{h\ e\ V_{RF}\ \eta\ cos\phi_s}{2\pi\beta^2 E_0}}$$

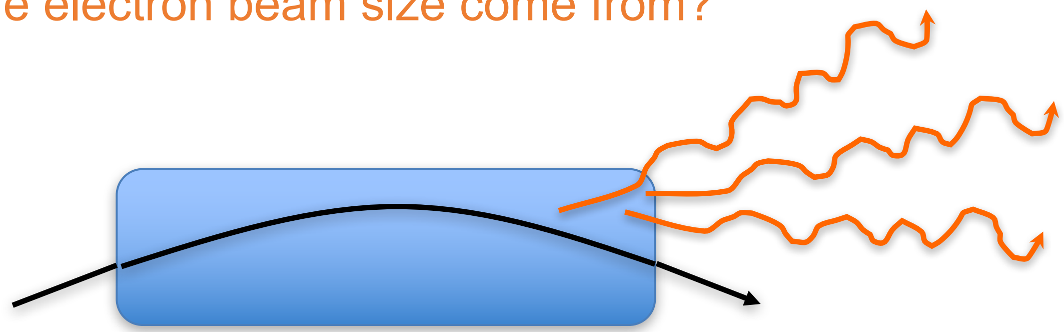
for V_{RF}=8MV

$$\nu_z = 6 \times 10^{-3}$$

or 1 oscillation every 166 turns

and a longitudinal ‘beta function’

$$\beta_z = \frac{C\alpha_C}{\mu_z}$$



The power radiated by a charged particle undergoing a bend:

$$P_\gamma = \frac{cC_\gamma}{2\pi}\frac{E^4}{\rho^2} \qquad C_\gamma = \frac{4\pi}{3}\frac{r_e}{(mc^2)^3} = 8.85 * 10^{-5}\ \frac{m}{GeV^3}$$

recall:

$$B\rho = \frac{P}{e}$$

Why do electrons radiate so much more than protons?

mass of proton is 1836 times mass of electron

At ESRF, the undulator radiation is typically 15-20% of the radiation emitted by the dipoles. Max 30%. Here, we focus on dipole radiation.

Radiation from electrons

Add this up for all the dipoles around the ring, and we get

$$U_0 = \int P_\gamma dt = \frac{C_\gamma E^4}{2\pi} \oint \frac{ds}{\rho^2}$$

$$U_0 = 4.88\,MeV$$

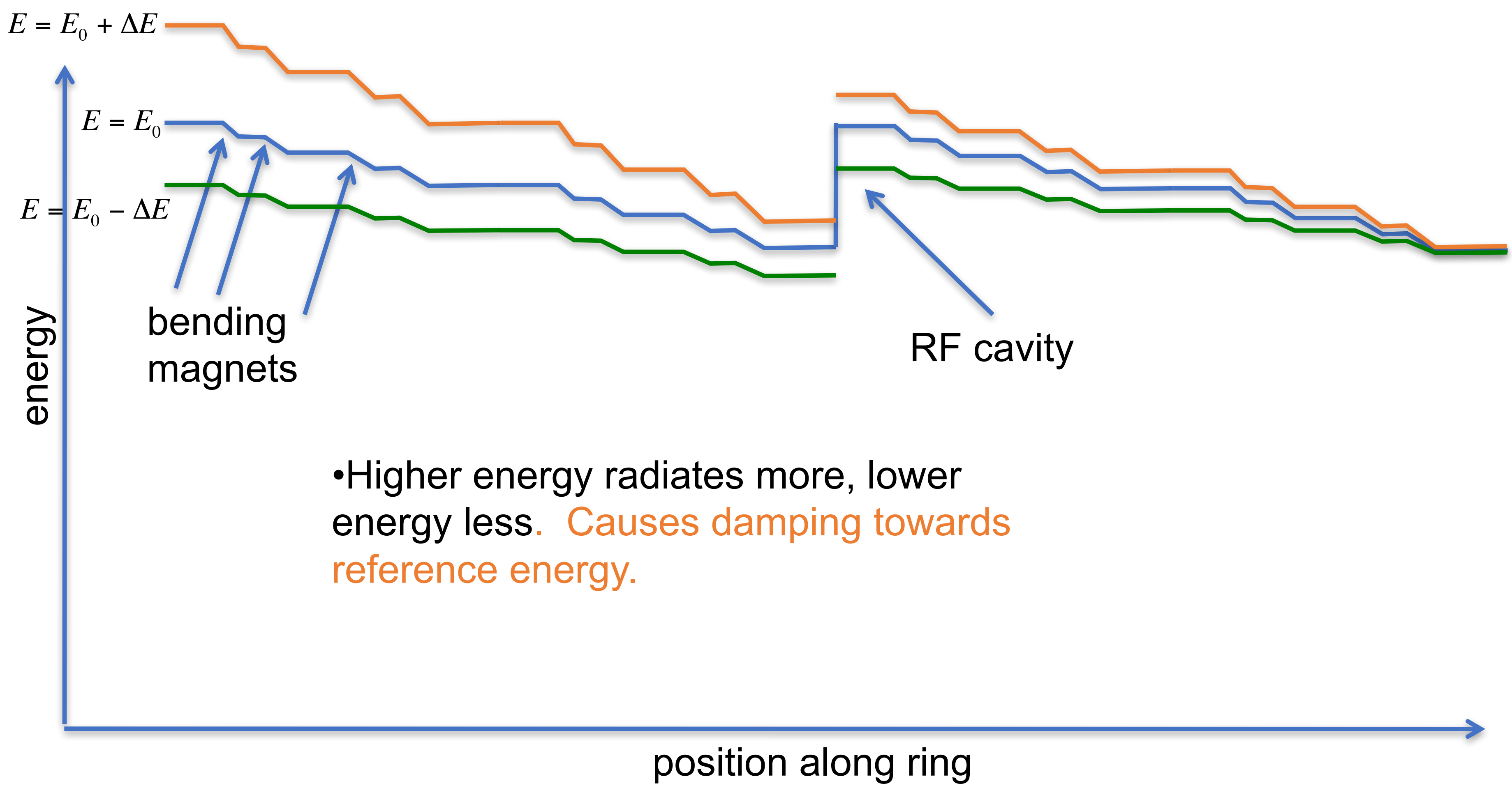
for current ESRF

New lattice will have decreased energy loss.
More, weaker dipoles with larger bending radius.

$$U_0 = 2.6\,MeV$$

for upgrade ESRF

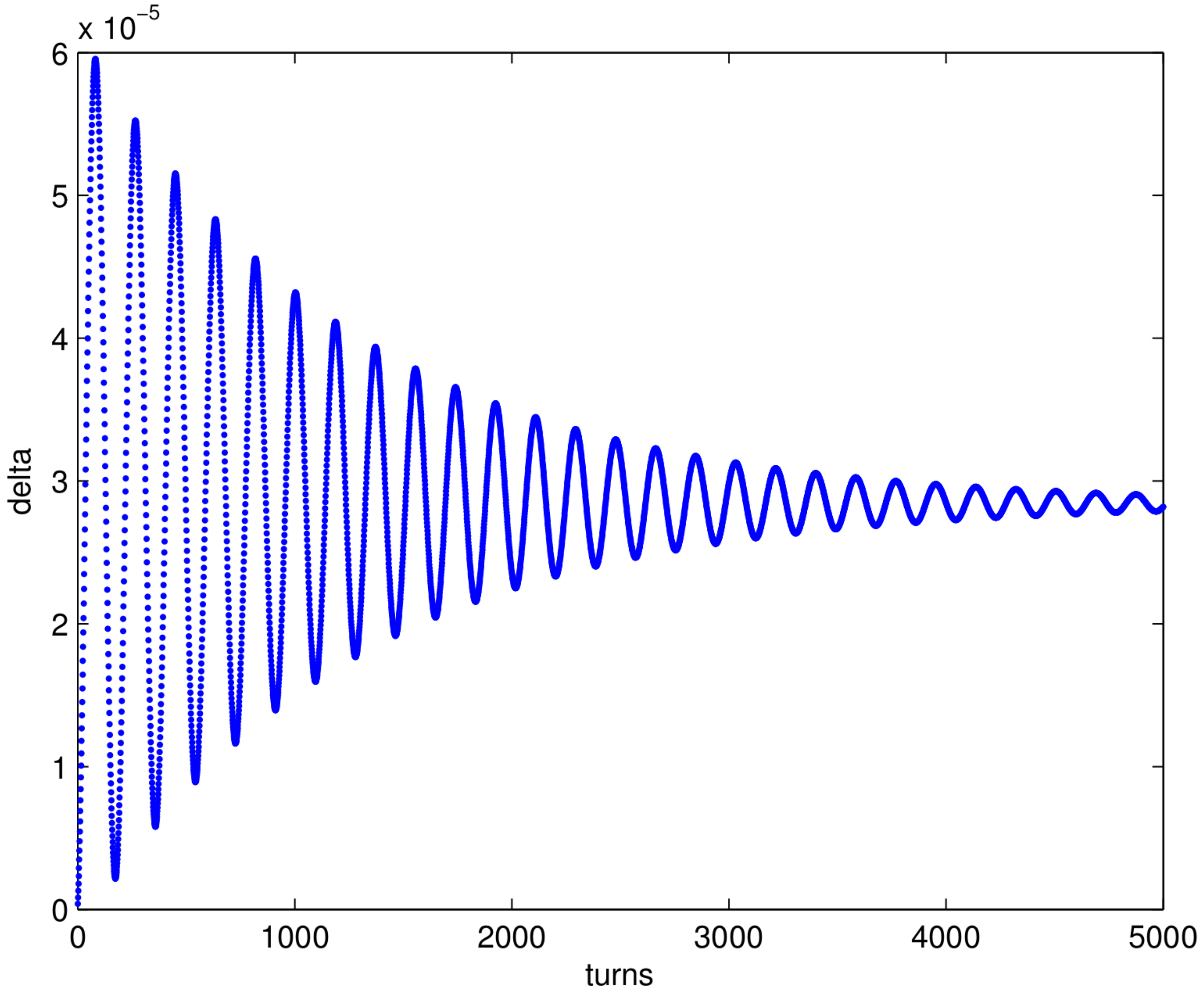
Radiation effect on Longutidinal dynamics



•Higher energy radiates more, lower energy less. Causes damping towards reference energy.

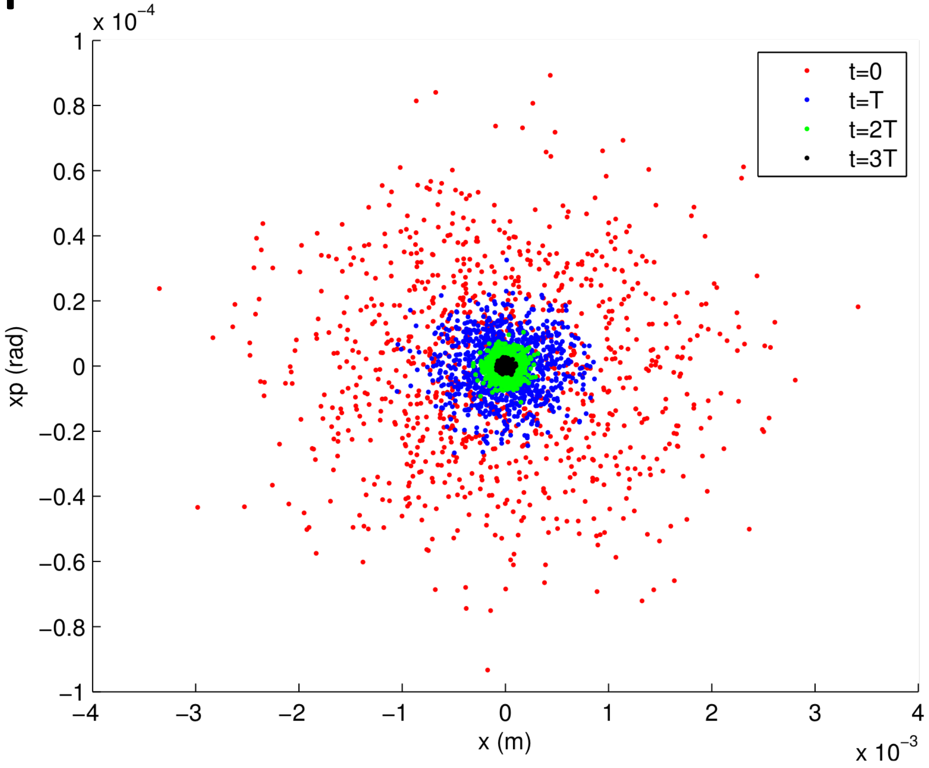
Radiation damping effect on energy over many turns

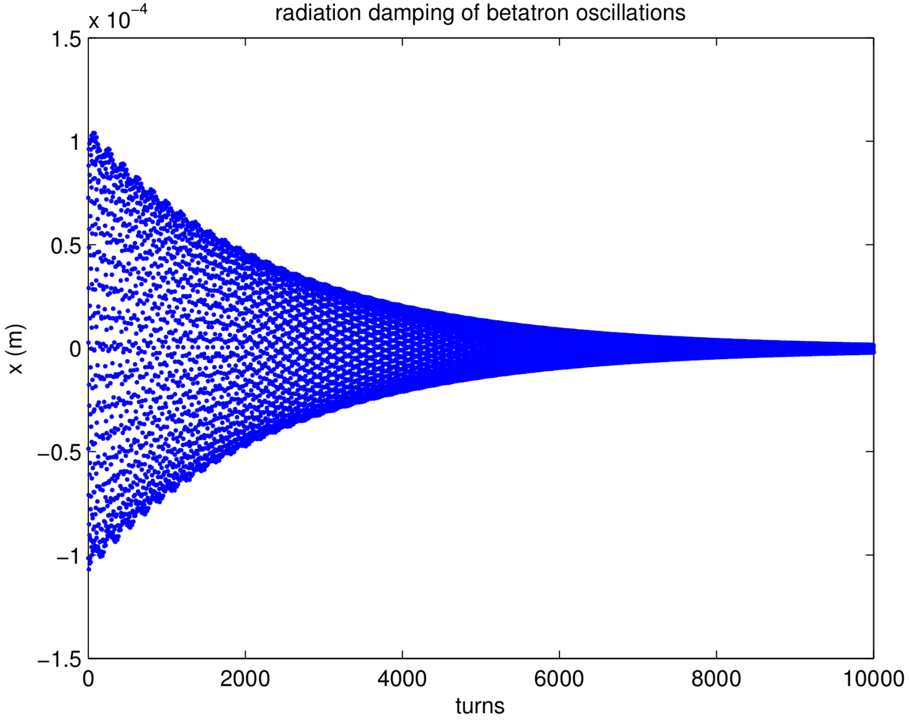
$$\delta = \frac{E - E_0}{E_0}$$



(ESRF current ring)

A question arises





What prevents the beam from collapsing to a point?

What is the physics that sets the size of the electron beam??

Look more closely at the radiation process

(follow Sands, Slac report 121 (1970))

The radiation power spectrum coming out of a dipole is given by:
(first computed by Schwinger (1948))

$$\wp(\omega) = \frac{P_\gamma}{\omega_c} S(\frac{\omega}{\omega_c}) \qquad \omega_c = \frac{3}{2}\frac{c\gamma^3}{\rho}$$

(critical frequency)

(18.8 KeV for current ESRF)

$$S(\xi) = \frac{9\sqrt{3}}{8\pi}\xi\int_\xi^\infty K_{5/3}(\bar{\xi})d\bar{\xi}$$

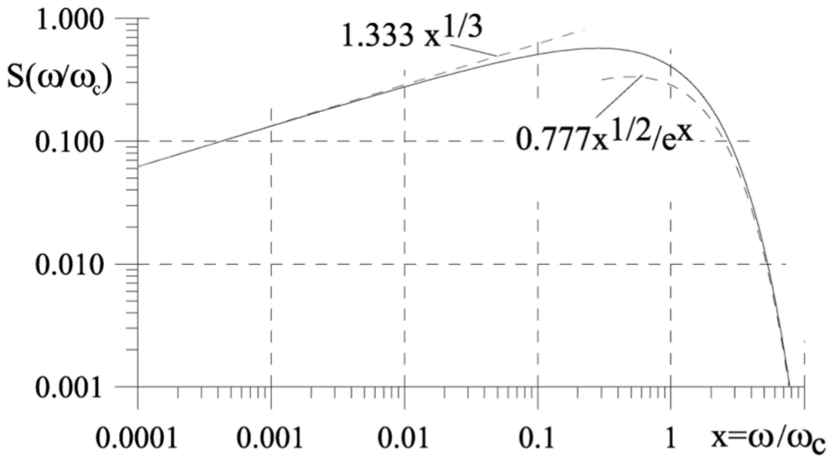


Fig. 22.11. Universal function: $S(\xi) = \frac{9\sqrt{3}}{8\pi}\xi\int_\xi^\infty K_{5/3}(x)\,dx$, with $\xi = \omega/\omega_c$

In fact, this radiation will be emitted from the electron as photons

We relate the power spectrum to the distribution of the number of emitted photons per unit time as follows:

$$un(u)du = \wp(u/\hbar)du/\hbar$$

$$u = \hbar\omega$$

The total emission rate is given by:

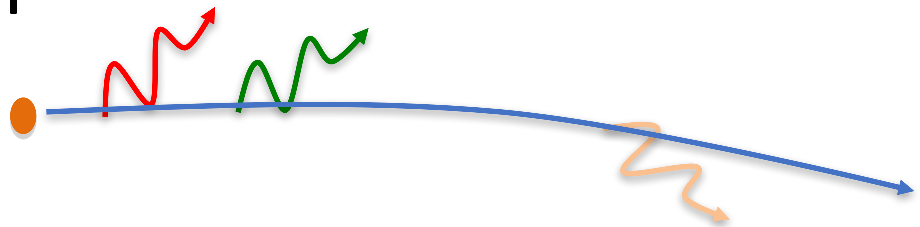
$$\dot{N} = \int_0^\infty n(u)du = \frac{15\sqrt{3}}{8}\frac{P_\gamma}{u_c}$$

One may compute the average # of photons emitted per radian:

$$= \frac{5}{2\sqrt{3}}\frac{\gamma}{137}$$

For 6.04 GeV electrons, this results in 756 photons per turn.
Or, for ESRF, an average of about 1 photon emitted per meter!
(approx. 12 photons per dipole)

Graininess of photon emission



Two sources of randomness:
emission time of photons are random: Poisson process

Energy emitted is also a random process, with the power spectrum as the probability distribution for each photon.

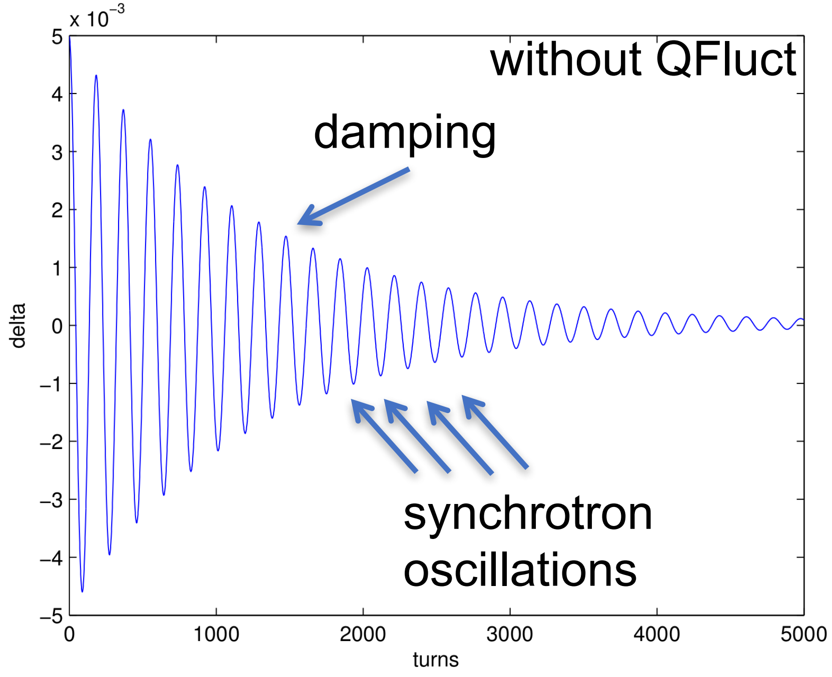
We need to compute the diffusion coefficient for this random walk.

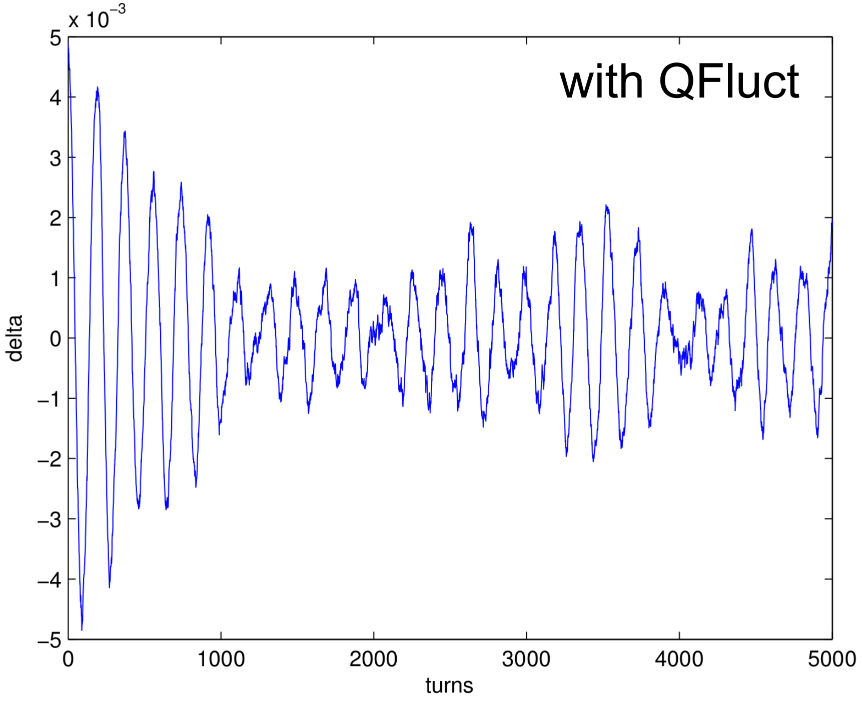
$$d = \dot{N}\langle u^2 \rangle = \int_0^\infty u^2 n(u)\, du$$

The result of this calculation is:

$$d = \frac{55}{24\sqrt{3}} \alpha \left(\frac{\hbar}{mc} \right) \frac{\gamma^5}{|\rho|^3}$$

Quantum fluctuation effect on longitudinal dynamics





Tracking with and without quantum fluctuations

Implementation of Monte Carlo method in Zgoubi

– probability of emission of one or more photons, by a particle with rigidity $B\rho$ (energy E), over an integration step Δs, under the effect of $1/\rho$ curvature,

$$p(k) = \frac{\Lambda^k}{k!} e^{-\Lambda} \quad \text{with} \quad \Lambda = < k > = < k^2 > \tag{2.1}$$

with $\Lambda = \frac{5er_0}{2\hbar\sqrt{3}} B\rho \frac{\Delta s}{\rho}$ the average number of photons radiated over Δs ($r_0 = e^2/4\pi\epsilon_0 m_0 c^2$ is the classical radius of the electron, e the elementary charge, m_0 the electron rest mass, $\epsilon_0 = 1/36\pi 10^9$, $\hbar$ is the Plank constant),

– energy ϵ of the emitted photon(s), following the probability

$$\mathscr{P}(\epsilon/\epsilon_c) = \frac{3}{5\pi} \int_0^{\epsilon/\epsilon_c} \frac{d\epsilon}{\epsilon_c} \int_{\epsilon/\epsilon_c}^{\infty} K_{5/3}(x)dx \tag{2.2}$$

with $K_{5/3}$ the modified Bessel function, $\epsilon_c = 3\hbar\gamma^3 c/2\rho$ the critical energy of the radiation ($\gamma = E/E_0$ with $E_0 = m_0 c^2$ the rest energy).

From F. Méot, JINST, 2015,
“Simulation of radiation damping in rings, using stepwise ray-tracing methods”

Equilibrium Energy spread

$$\frac{\partial f(z,\delta)}{\partial t} = \left[H, f\right] - b\frac{\partial f}{\partial \delta} + \frac{d}{2}\frac{\partial^2 f}{\partial \delta^2}$$

Equilibrium is a Gaussian

$$f(z,\delta) = \frac{1}{2\pi\sigma_z\sigma_\delta} e^{-\frac{z^2}{2\sigma_z^2}-\frac{\delta^2}{2\sigma_\delta^2}}$$

bunch length related to energy spread

$$\sigma_z = \frac{C\alpha_c}{2\pi\nu_s}\sigma_\delta$$

Calculation gives

$$\sigma_\delta^2 \approx \frac{C_q\gamma^2}{2\rho} \qquad C_q = \frac{55}{32\sqrt{3}}\frac{\hbar}{mc} = 3.84\times 10^{-13}\ m$$

$$(\gamma = 11{,}800 \quad \rho = 23\ m) \qquad \Rightarrow \sigma_\delta = 1.1\times 10^{-3}$$

for ESRF

Energy scaling of ESRF SR dynamics quantities

	Energy loss U_s (MeV/turn)	$\epsilon_{l,\mathrm{eq}}$ (μeV.s)	σ_l (mm)	τ_l (ms)	$\epsilon_{x,\mathrm{eq}}$ (nm)	τ_x (ms)	τ_y (ms)
Scaling:	γ^4	$\gamma^{3/2}$	$1/\gamma^{1/2}$	$1/\gamma^3$	γ^2	$1/\gamma^3$	$1/\gamma^3$
6 GeV	4.5956 *[4.5956]*	196 *[192]*	9.37 *[9.309]*	1.769 *[1.769]*	6.90 *[6.83]*	3.547 *[3.546]*	3.501 *[3.540]*
9 GeV	23.263 *[23.263]*	358 *[352]*	7.67 *[7.601]*	0.548 *[0.524]*	15.87 15.60 *[15.37]*	1.020 *[1.051]*	1.040 *[1.049]*
12 GeV	73.518 *[73.518]*	554 *[542]*	6.67 *[6.582]*	0.225 *[0.221]*	28.18 28.04 *[27.32]*	0.447 *[0.443]*	0.439 *[0.443]*
18 GeV	372.16 *[372.16]*	1022 *[996]*	5.42 *[5.375]*	0.068 *[0.066]*	65.77 63.24 *[61.46]*	0.132 *[0.131]*	0.130 *[0.131]*
Theory	$\frac{\mathscr{C}_\gamma}{2\pi}E_s^4 I_2 = \mathscr{C}_\gamma \frac{E_s^4}{\rho}$	$\frac{\alpha E_s}{2\Omega_s}\sigma^2_{\widehat{\frac{\Delta E}{E}}} = \frac{\alpha E_s}{\Omega_s}\frac{\mathscr{C}_q\gamma^2}{J_l\rho}$	$\frac{\alpha c}{\Omega_s}\sigma_{\frac{\Delta E}{E}}$	$\frac{3T_{\mathrm{rev}}}{2r_0\gamma^3(2I_2+I_4)} = \frac{T_{\mathrm{rev}}E_s}{U_s J_l}$	$\frac{\mathscr{C}_q\gamma^2}{J_x}\frac{I_5}{I_2} = \frac{\mathscr{C}_q\gamma^2}{J_x\rho}\bar{\mathscr{H}}$	$\frac{3T_{\mathrm{rev}}}{2r_0\gamma^3(I_2-I_4)} = \frac{T_{\mathrm{rev}}E_s}{U_s J_x}$	$\frac{3T_{\mathrm{rev}}}{2r_0\gamma^3 I_2} = \frac{T_{\mathrm{rev}}E_s}{U_s J_y}$

(ref. F. Méot, 2015)

Conclusions

- Electron storage rings are used for colliders, synchrotron light sources damping rings, and more!
- Transverse and longitudinal symplectic dynamics same as for hadron rings, but synchrotron radiation breaks this
- SR causes both damping and diffusion, resulting in equilibrium emittance
- Zgoubi (and other codes) implements SR with a Monte Carlo model. Both damping and diffusion result from single photon emission statistics!

Thanks for your attention!!