

# Simulation of CBETA BNL-CORNELL ERL Using Field Maps

## 1. Periodic Orbits and Optical Functions Along the Permanent Magnet Return Loop

The interest of using field maps is that it brings greatest accuracy in the modeling of the optical elements (of their magnetic field). Allied with stepwise ray-tracing techniques, it brings greatest accuracy on the resolution of the equation of particle motion through these fields.

We will lean on Ref. <sup>1</sup> for the details concerning CBETA design and its permanent magnet return loop, and on Refs. <sup>2 3</sup> concerning the present field map based optical methods. In particular, the theoretical arc cell is displayed in [1] Fig. 2.2.3, p. 24 or in [2] Fig. 3; the geometry of the ERL and of its return loop are shown in [1] Figs. 2.1.1, p. 18 and 2.1.2, p. 19 respectively. The OPERA model of the permanent magnet cell is shown in [2] Figs. 6 and 11.

There is no quadrupole knob over the FA-TA-ZA-ZB-TB-FB return loop, on the other hand there is corrector dipole windings (Fig. 11 in [2]) but we will ignore them here. In these conditions the reference orbits and the optical functions (betatron functions and dispersion) along the loop are fully determined by their values at the start of the FA arc. This exercise deals with the determination of the latter. Tests will be performed by propagating the initial orbit and optical function values so determined, all the way along the  $\approx 50$  m loop.

### Working hypotheses:

- For this exercise we will work in the folder

[pathTo]/CBETA/FFAGLoop/FA

and will essentially be dealing with the input data file FA-042\_1stcell\_FITInitialBta.dat. The field maps of the permanent magnets can be found in the folder [pathTo]/CBETA/fieldmaps.

- There are two subtleties in the handling of the periodic optical functions of the permanent magnet return loop.

- First, while the QF-BD doublet FA cell repeats itself 16 times over the upstream FA arc of the return loop (Fig. 2.1.2, p. 19, in [1]), the arc actually starts with a half BD magnet. This is for the purpose of optical matching at the connection between S1 and the FA arc.

Thus, the optical functions upstream of that half-BD, at the location of the downstream end of S1, are determined by the periodic functions of the FA cell.

- Next, an aspect proper to the use of field maps: the S1-FA arc connection happens to be located within the extent of the half-BD field map, thus getting them from there requires a numerical method.

We choose here to use a FIT procedure as a convenient and simple way to achieve that.

### Numerical experiments:

1/ Find the 4 design energy orbits in the FA cell, namely: reproduce the plot of Fig. 5-left in [2].

Plot the magnetic field along the 4 orbits.

2/ Find the periodic functions at the end of the FA cell, for the 4 design energies, using TWISS.

3/ Add the half-BD section that ensures the connection to S1, upstream of the FA cell. Using the optical functions at the downstream end of that sequence as the constraints, find the optical functions at the S1-FA connection. FIT [2] can be used for that.

4/ Check your results:

- inject the initial orbit coordinates at the start of the FFAG loop (file FFAGLoop.dat), plot them, all 4 energies (in a similar manner to the 42 MeV orbit in [2], Fig. 13-left)

- inject the initial optical functions at the start of the FFAG loop, for instance for the 42 MeV case, and verify that they propagate correctly all the way.

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<sup>1</sup>CBETA Design Report, J. Barley et als., Jan 27, 2017.

<sup>2</sup>A Full Field-Map Modeling of Cornell-BNL CBETA 4-Pass Energy Recovery Linac, F. Méot et als., ICAP18, Key West.

<sup>3</sup>Beam dynamics validation of the Halbach Technology FFAG Cell for Cornell-BNL Energy Recovery Linac, F. Mot et als., NIM A, vol. 896, pp. 60-67, 2018.