Prof. Günter M. Ziegler
Freie Universität Berlin

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Polytopes – Extremal Examples and Combinatorial Parameters

Exercise Sheet 3

Problem 1

(a) Work out the combinatorics of $C_3(n)$ from the Gale evenness criterion.

(b) Show that one can construct this type of polytope from an $(n-1)$-gon by "vertex splitting".

(c) Work out the combinatorics of the dual polytope $C_3(n)^*$. Why is this called a "wedge"?

Problem 2

(a) How large integer vertex coordinates do you need to realize an n-gon?

(b) Can you realize $C_3(n)$ with small integer coordinates?

(c) How about $C_3(n)^*$?

Problem 3

Work out the combinatorics of the facets of $C_4(n)^*$ for small/all n.

Why are they called "wedges"?

Problem 4

(a) Show that the *Carathéodory curve* $c(t) := (\cos t, \sin t, \cos 2t, \sin 2t)$ is a curve of order 4 in $\mathbb{R}^4$.

(b) If you realize the cyclic polytope $C_4(n)$ with points at $t_k = \frac{k}{n}2\pi$ $(0 \leq k < n)$ spaced equally on the Carathéodory curve, show that there the polytope has a dihedral group of symmetries.

Problem 5

Work out the combinatorics of $C_d(d+2)$ for small/all d.

Problem 6

Work out the combinatorics of a product of two triangles, $\Delta_2 \times \Delta_2$.

What do the facets look like? Which facets are adjacent?

Problem 7

(a) Show that the Euler equation $f_0 - f_1 + f_2 = 2$ is the *only* linear equation valid on the set of f-vectors of 3-polytopes.

(b) Show that the Euler-Poincaré equation $f_0 - f_1 + f_2 - f_3 = 0$ is the *only* linear equation valid on the set of f-vectors of 4-polytopes.

Problem 8

Prove Steinitz's lemma: Show that for every pair of integers (f_0, f_2) with $f_2 \leq 2f_0 - 4$ and $f_0 \leq 2f_2 - 4$, there is a 3-polytope with these face numbers.