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Polytopes – Extremal Examples and Combinatorial Parameters

Exercise Sheet 2

Problem 1

Compute the f-vectors of
(a) the 5-cube,
(b) the 10-cube.

Problem 2

Assume that d is large, what is the "shape" of the f-vector of the d-cube?
Which entry is the largest?

Problem 3

What combinatorial type of polytope is $\Delta_3(2)$?
Is it a regular polytope?

Problem 4

Show that all 2-simple 2-simplicial 4-polytopes have symmetric a f-vector
(that is, the number of vertices is equal to the number of facets).

Problem 5

Let the d-dimensional "half-cube" be obtained as the subpolytope of the d-cube $[0,1]^d$ given as the convex hull of every second vertex.
(a) Draw and identify the 3-dimensional half-cube.
(b) Show that the half-cubes are 3-simplicial, but not 4-simplicial.
(c) Can you compute their f-vectors, for $d = 3, 4, 5$.
(d)* Can you find formulas for their f-vectors, for general d?

Problem 6

(a) Compute the f-vectors of $\Delta_d(k)$, as far as you get.
(b)* Can you come up with formulas?