Identification of Motor Imagery Movements from EEG Signals Using Dual Tree Complex Wavelet Transform

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***Abstract*—In this paper, Dual Tree Complex Wavelet Transform (DTCWT) domain based feature extraction method has been proposed to identify left and right hand motor imagery movements from electroencephalogram (EEG) signals. After first performing auto-correlation of the EEG signals to enhance the weak brain signals and reduce noise, the EEG signals are decomposed into several bands of real and imaginary coefficients using DTCWT. The energy of the coefficients from relevant bands have been extracted as features and from the one way ANOVA analysis, scatter plots, box plots and histograms, this features are shown to be promising to distinguish various kinds of EEG signals. Publicly available benchmark BCI-competition 2003 Graz motor imagery dataset is used for this experiment. Among different types of classifiers developed such as support vector machine (SVM), probabilistic neural network (PNN), adaptive neuro fuzzy inference system (ANFIS) and K-nearest neighbor (KNN), KNN classifiers have been shown to provide a good mean accuracy of 91.07% which is better than several existing techniques.**

Keywords*—Electroencephalogram (EEG); BCI; Dual Tree Complex Wavelet Transform (DTCWT); Auto correlation; Energy; KNN classifier.

I. INTRODUCTION

Brain computer interfacing (BCI) allows to control and operate computer aided systems by intent alone. It involves detection, analysis and classification of different types of motor imagery movements to implement real time control and communication. Electroencephalogram (EEG) signals are often used for BCI purpose since it can be implemented as a non-invasive system. One major category of BCI is the detection of motor imagery movements such as left and right hand movements. Many methods have been developed so far in the literature for classifying various types of arm movements. Separability of EEG signals using adaptive auto regressive parameter based method was discussed in [1] and [2]. In [3], wavelet coefficients, power spectral density and average power obtained from EEG signals have been used as features to classify left and right hand movements with different classifiers such as linear discriminant analysis (LDA), support vector machine (SVM) and KNN. In [4] wavelet packet along with Fourier transform has been applied to classify four arm movements whereas in [5], wavelet packet entropy of EEG signal is used as feature and SVM as the classifier.

Since EEG data for research purpose can be acquired with varying experimental setup and conditions, BCI competition was held providing standard data sets to evaluate and compare different algorithms. Different approaches have been studied to classify motor imagery movements in BCI competition 2003 Graz motor imagery dataset. Band passed EEG signals and power spectral density based LDA was described in [6] while in [7], raw EEG signal incorporated with Hidden Markov Model (HMM) was presented by the same author. Adaptive Auto Regressive (AAR) model based features with Bayesian Graphical Network (BGN) and Multi Layer Perceptron were reported in [8]. Morlet wavelet was used to extract features from mu rhythms with Bayes quadratic classifiers in [9]. Recently, multiple auto-correlation based feature extraction method along with learning vector quantization (LVQ) has been proposed in [10].

The objective of this paper is to identify imagery hand movements by extracting suitable features from pre-processed EEG signal in the dual tree complex wavelet transform (DTCWT) domain. DTCWT has been widely used for image and video processing [11], and recently, it has been used in the area of bio-medical signals [12], [13], [14]. Since motor imagery movements occur in low frequency EEG bands [15], to perform detail analysis on specific bands, DTCWT has been used since it is a rich level analysis tool than traditional discrete wavelet transform (DWT). First auto correlation has been performed on the EEG signal to enhance it and reduce the effect of noise. After that energy of the real and imaginary DTCWT coefficients of the auto-correlated EEG signal have been calculated and finally, KNN classifier has been employed for the purpose of classification.

II. DESCRIPTION OF THE EEG DATABASE

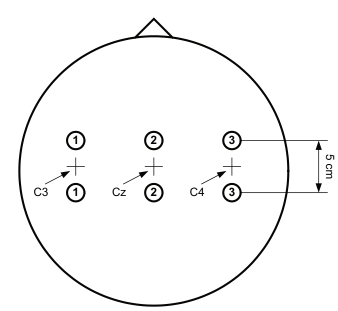

Fig. 1: Electrode positions.

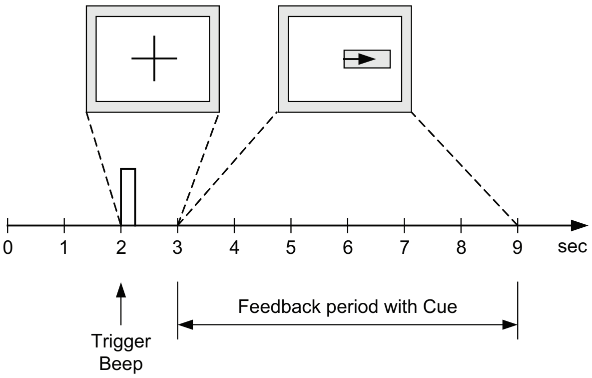

Fig. 2: Timing scheme of the experiment.

In this paper we have used BCI competition 2003 data set (motor imagery III) provided by Technical University of Graz. This data was recorded when a normal subject sitting in a chair with arm rests and was trying to control a feedback bar with imagery movements of left or right hands. The Left and right cues were in random order [16]. The experimental procedure consists of 7 runs with 40 trials each. During each trial at t= 2s an acoustic stimulus indicates the beginning of the trial and a cross '+' was displayed for 1 second. After the fixation cross, an arrow indicating left or right, was displayed at t= 3s as the cue. At the same time the subject was asked to move a bar into the direction of the cue which was controlled by adaptive auto-regressive (AAR) parameters of channel C3 and C4. The electrode positions and experimental setup are given in Fig. 1 and Fig. 2, respectively. The EEG signal was filtered between 0.5 and 30 Hz with 128 Hz sampling rate. A detail description of the experimental procedure up can be found in [17].

III. Dual Tree Complex Wavelet Transform

Dual tree complex wavelet transform (DTCWT) is a recent enhancement to the discrete wavelet transform (DWT) which has additional properties including nearly shift invariant and directionally selective in two and higher dimensions [18]. DTCWT is 2^d times redundant for any d dimensional signal as compared to DWT and offers directional information in six directions. Thus is it more efficient in time frequency localization of EEG signal.

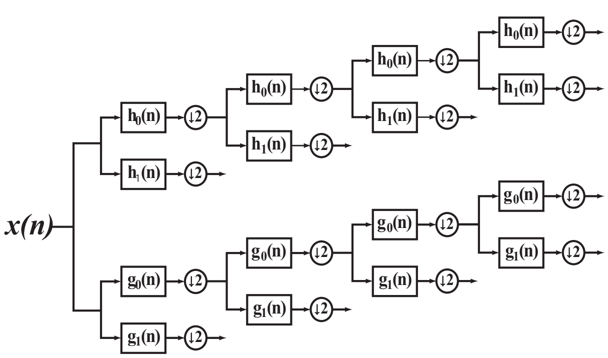

Fig. 3: Analysis filter bank of DTCWT.

Similar to positive/negative post-filtering of real subband signals, the idea behind dual tree approach is quite simple. DTCWT employs two real DWTs where the first DWT gives the real part of the transform while the second DWT gives the imaginary part. The analysis filter bank structure used to implement DTCWT is given in Fig. 3. Two real wavelet transforms use two different sets of filters which satisfy perfect reconstruction conditions.

If square matrices $\mathbf{H}_h$ and $\mathbf{H}_g$ denote the two real DWTs, then the DTCWT can be represented as follows:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_h \\ \mathbf{H}_g \end{bmatrix} \tag{1}$$

The inverse transform of $\mathbf{H}$ is given as,

$$\mathbf{H}^{-1} = \frac{1}{2} \begin{bmatrix} \mathbf{H}_h^{-1} & \mathbf{H}_g^{-1} \end{bmatrix} \tag{2}$$

If the vector $\mathbf{x}$ represents a real signal, then $\mathbf{w}_h = \mathbf{H}_h \mathbf{x}$ represents the real part and $\mathbf{w}_g = \mathbf{H}_g \mathbf{x}$ represents the imaginary part of the DTCWT. When the dual-tree CWT is applied to a real signal, the output of the upper and lower filter banks gives the real and imaginary parts of the complex coefficients respectively. Among several available wavelets, Farras [19] wavelets are used to perform DTCWT.

IV. Analysis in Dual Tree Complex Wavelet Transform Domain

The proposed method has three main steps: first is the auto-correlation of the EEG signals to attenuate noise effects; second is the decomposition of the EEG signal of each trial with dual tree complex wavelet transform into three levels and last step is to extract suitable features from different bands. After extracting suitable features, KNN classifier is deployed for the classification of hand movements.

A. Feature Extraction in DTCWT Domain

BCI 2003 Graz dataset was recorded with low pass filtering of 30 Hz and for this, no preprocessing of the data was needed to discard the unnecessary high frequency components. However, since EEG signal is very weak and corrupted with noise, auto-correlation has been performed on each trial of the EEG signals so that we can analyze the desired low passed signal in noise reduced space. Auto correlation enhances the EEG signal [10] and makes it suitable for analysis. Since motor imagery activity occurs in low frequency EEG band signal [15], so to have in-depth view, dual tree complex wavelet transform has been applied to the acquired EEG signal. Since forward transform of DTCWT results two branches containing real and imaginary coefficients, it gives a rich way of analyzing EEG signals than discrete wavelet transform (DWT). The experiment was carried out by taking feedback from C3 and C4 channel while using Cz as the reference. As a result, for applying DTCWT, EEG signals acquired from both C3 and C4 channel for a single trial is decomposed into three levels. If the original low passed signal is denoted by X which has 0.5 to 30 Hz frequency components, after first level decomposition, it provides Y_1 (16-30 Hz) and Z_1(0.5-15 Hz). After second level decomposition, Z_1 leads to Y_2(7.5-15 Hz) and Z_2(0.5-7.5 Hz). So after three levels of DTCWT, the four frequency bands are Y_1(16-30 Hz), Y_2(7.5-15 Hz), Y_3(3.75-7.5 Hz) and Z_3(0.5-3.75

Hz). Reconstructions of these components using the inverse DTCWT approximately correspond to the physiological EEG sub-bands delta, theta, alpha and beta respectively [20]. Since each frequency band gives both real and imaginary coefficients, we have total four real and four imaginary coefficient bands. From now on, the real and imaginary bands and associated levels will be denoted as RB_x and IB_x where 'x' is the level index. Fig. 4 represents the recorded EEG signals from C3 channel for left and right hand imagery movements and the auto-correlated signal. Fig. 5 shows the corresponding real and imaginary coefficients of the first and second level DTCWT.

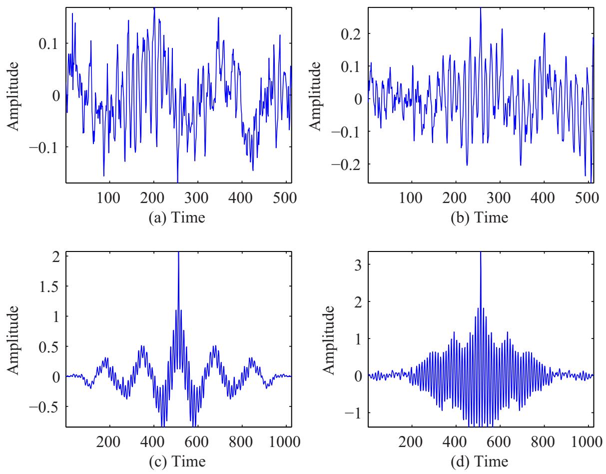

Fig. 4: (a)-(b) EEG signal from C3 channels for left and right hand movement; (c)-(d) auto correlated signal of (a) and (b) respectively.

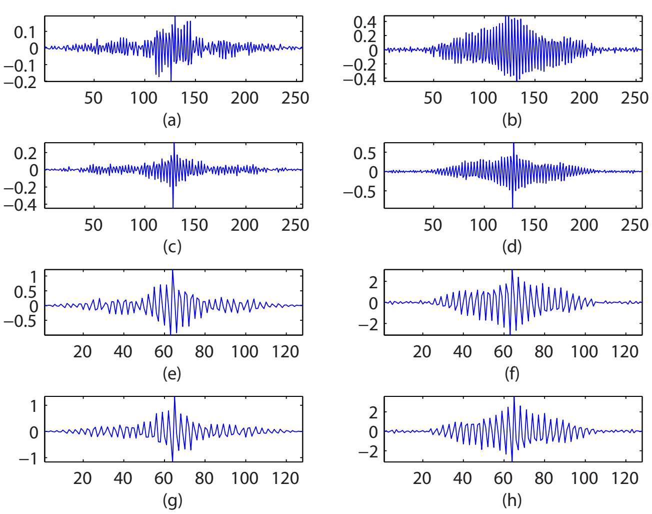

Fig. 5: (a)-(b) Real coefficients of level 1 for left and right imagery movements respectively; (c)-(d) imaginary coef. of level 1 for left and right movements respectively; (e)-(f) real coef. of level 2 for left and right movements respectively; (g)-(h) imaginary coef. of level 2 for left and right imagery movements respectively.

The underlying dynamics of EEG signals is spread over various sub-bands in the frequency domain. To classify motor imagery movements, we need to extract more information in lower frequency bands of EEG signals and mu rhythms (7.5-12.5 Hz) [9]. For analyzing the wavelet coefficients, we have plotted the square magnitude of Fourier transform (FFT) of the wavelet coefficients which is the indicator of power spectral density (PSD). Fig. 6 presents the square magnitude of FFT of $RB_3(C3)$ for left and right hand imagery movements, respectively and from this figure, it can be said that PSD of wavelet coefficients differs during two specific movements. So for feature extraction, we have used wavelet coefficient energy of different bands since energy is related to PSD.

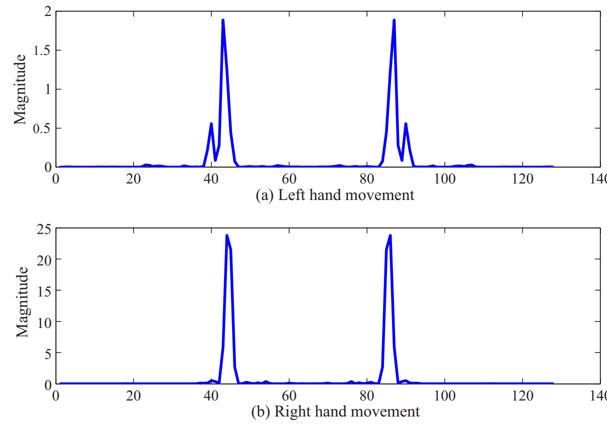

Fig. 6: (a)-(b) Square magnitude of FFT of $RB_3(C3)$ for left and right hand imagery movements, respectively.

Wavelet energy is a well-known feature in literature [21]-[22] and very simple to calculate yet effective in performance. Energy (E) of a distribution or a dataset containing N samples can be calculated as:

$$E = \sum_{i=1}^{N} |x_i|^2 \tag{3}$$

Table I gives the p-values of one way ANOVA analysis of wavelet coefficient energy for different bands for train data. The hypothesis about the p-values is that the value p<0.05 indicates that at least one sample mean is significantly different than the other sample means statistically [23]. From Table I, it is clear that energy values of 2^{nd} and 3^{rd} level coefficients have very small p-values since these corresponds to alpha wave and mu rhythms of EEG signals which differ during motor imagery hand movements. So wavelet coefficient energy of these two bands can be used as good features.

TABLE I: Results of ANOVA Analysis

Features bands	**P-value from ANOVA**
RB_2(C3)	2.8306e-11
IB_2(C3)	1.8924e-07
RB_3(C3)	4.8041e-11
IB_3(C3)	3.9698e-07
RB_2(C4)	1.7077e-06
IB_2(C4)	1.1957e-06
RB_3(C4)	2.7888e-06
IB_3(C4)	2.8517e-06

Apart from ANOVA analysis, scatter plots, box plots and histograms are provided to further illustrate the classification capability of the features.

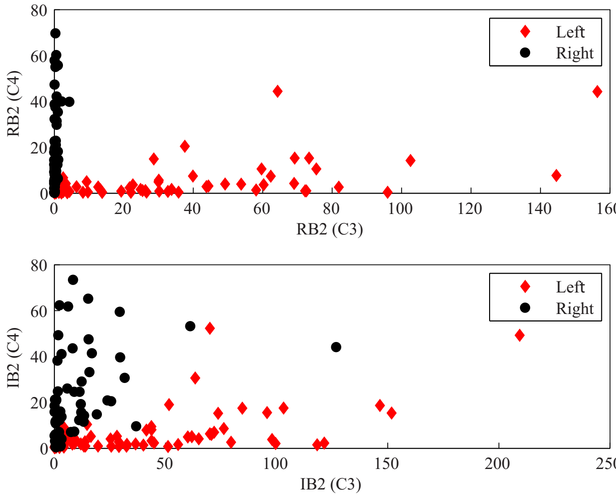

Fig. 7: (a)-(b) Scatter plots of $RB_2(C3)$, $RB_2(C4)$ and $IB_2(C3)$, $IB_2(C4)$ for energy feature during left and right hand imagery movements, respectively (train data only).

Fig. 7 represents the scatter plots of $RB_2(C3)$, $RB_2(C4)$ and $IB_2(C3)$, $IB_2(C4)$ respectively in the two subplots for coefficient energy on train data set whereas the two subplots of Fig. 8 show the scatter plots of $RB_3(C3)$, $RB_3(C4)$ and $IB_3(C3)$, $IB_3(C4)$ respectively. In Fig. 7 and Fig. 8, red diamonds and black circles indicate the corresponding feature values during left and right hand imagery movements, respectively. From the figures it can be noted that the red and black markers have significantly different values indicating their variation during different imagery hand movements and as a result, it can be concluded that these features may result good classification.

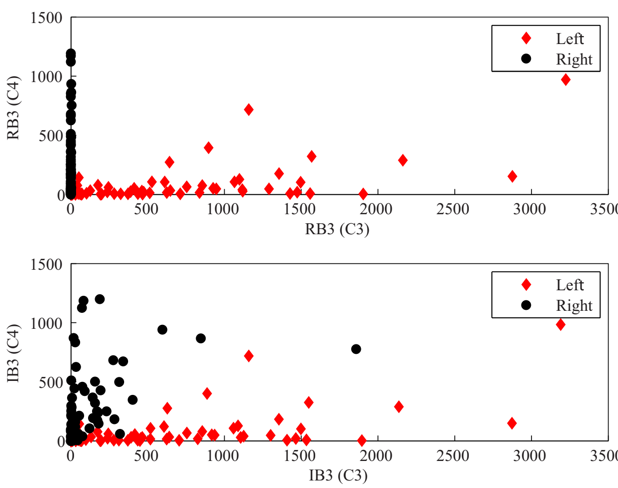

Fig. 8: (a)-(b) Scatter plots of $RB_3(C3)$, $RB_3(C4)$ and $IB_3(C3)$, $IB_3(C4)$ for energy feature during left and right hand imagery movements, respectively (train data only).

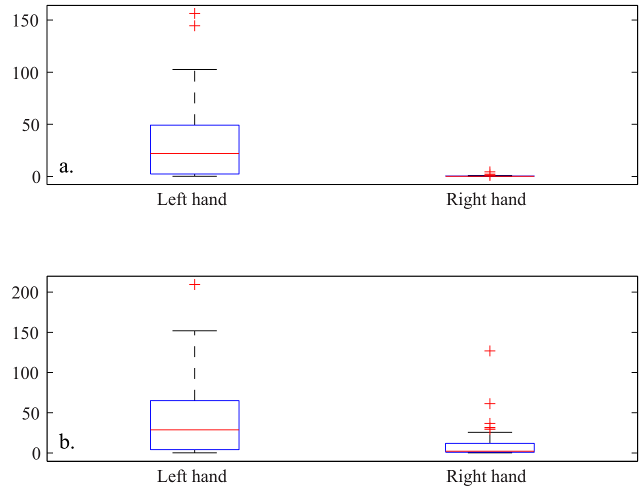

Fig. 9: (a)-(b) Box plots of $RB_2(C3)$ and $IB_2(C3)$ for left and right hand imagery movements, respectively (train data only).

Fig. 9 and Fig. 10 show the box plots of $RB_2(C3)$, $IB_2(C3)$, $RB_3(C4)$ and $IB_3(C4)$ respectively for left and right hand imagery movements. The box plots indicate that the features have distinct values for the two specific movements.

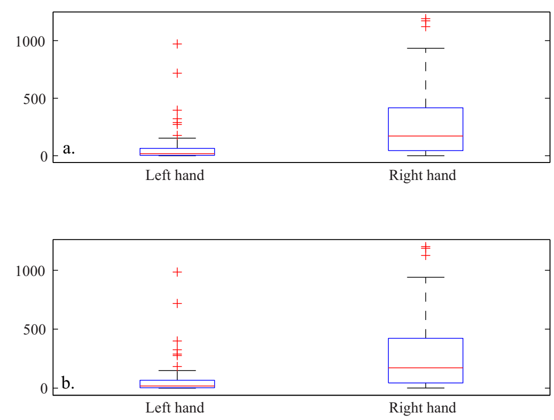

Fig. 10: (a)-(b) Box plots of $RB_3(C4)$ and $IB_3(C4)$ for left and right hand imagery movements, respectively (train data only).

Fig. 11 and Fig. 12 present the histograms of $IB_3(C3)$, $IB_3(C4)$, $RB_3(C4)$ and $RB_3(C3)$ for wavelet coefficient energy where green and red color indicates left and right hand imagery movements respectively. From the histograms, it can be said that the corresponding histograms have peaks in different regions which is the evidence that the features show different values during two specified motor imagery movements. For example, $IB_3(C3)$ has peak in 0-250 region for right hand imagery movement while for left hand imagery movements, it is distributed in 0-1000 region.

The scatter plots, box plots, histograms and the p-values of one way ANOVA indicate that the energy of real and imaginary coefficients from different bands of DTCWT of low frequency EEG signals have distinguishable values for left and right hand motor imagery movements. In other words, the features have

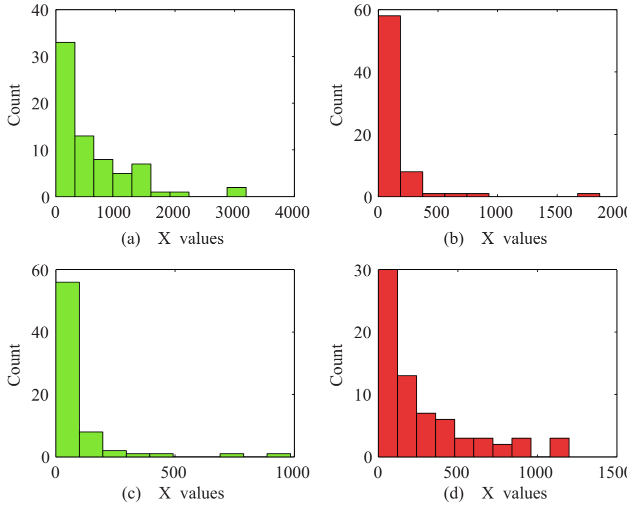

Fig. 11: (a)-(b) Histograms of $IB_3(C3)$ for left and right hand imagery movements, respectively; (c)-(d) histograms of $IB_3(C4)$ for left and right hand imagery movements, respectively (train data only).

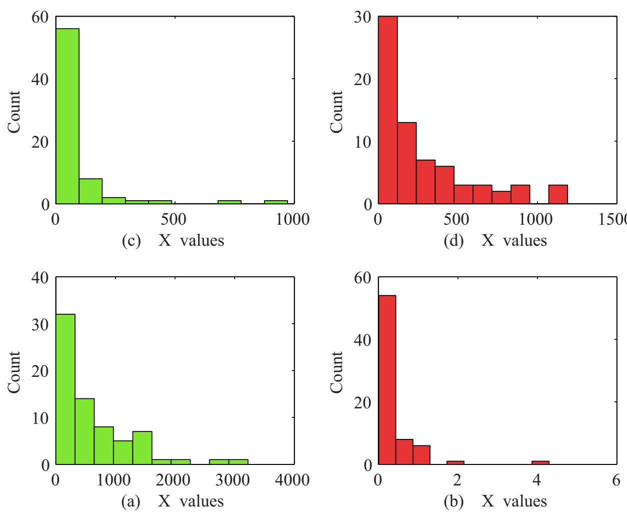

Fig. 12: (a)-(b) Histograms of $RB_3(C4)$ for left and right hand imagery movements, respectively; (c)-(d) histograms of $RB_3(C3)$ for left and right hand imagery movements, respectively (train data only).

good between-class distance and small within-class variance in the feature vector space [24] and as a result, they can be used as suitable features to classify two hand movements.

B. Classification Using KNN Classifier

For any classification problem, there are two main parts - first one is feature extraction and the second one is classification. If suitable features can be extracted, then a simple classifier can provide the desired outcome. Among different classifiers, KNN classifier has served best in our study.

k-Nearest Neighbors algorithm (KNN) is a non-parametric learning algorithm method used for classification. Among the various methods of supervised statistical pattern recognition, the Nearest Neighbor rule achieves consistently high performance, without a priori assumptions about the distributions from which the training examples are drawn. A new sample is classified by calculating the distance to the nearest training case; the sign of that point then determines the classification of the sample. The K-NN classifier extends this idea by taking the k nearest points and assigning the sign of the majority [25] . The number "K" decides how many neighbors influence the classification. If k = 1, then the algorithm is simply called the nearest neighbor algorithm.

V. Proposed Method and Results

Since it has been shown that energy in different sub-bands of DTCWT of auto-correlated EEG signals varies significantly, the proposed method extracts energy of wavelet coefficients as feature from RB_2, IB_2, RB_3 and IB_3 bands for both C3 and C4 channels. So, the feature vector (FV) of total length eight can be expressed as follows:

$$\begin{aligned} FV = [&RB_2(C3), IB_2(C3), RB_3(C3), IB_3(C3),\\ &RB_2(C4), IB_2(C4), RB_3(C4), IB_3(C4)]^T \end{aligned} \tag{4}$$

where T denotes transpose operation.

The EEG dataset has 140 trials each for left and right hand (total 280 trials) of 9 seconds length. Since the cue was given at t= 3 second, data segment after 3 seconds from C3 and C4 channels were used for classification. As a result, a total of eight features (four from each C3 and C4 channels) extracted during 3 to 8 seconds form the final feature vector. The train and test feature matrix has dimension of 140×8 which is fed to KNN classifier. It is to be noted that we have used both training and testing data set for our experiment. For validation leave one out method (one fold cross validation) is used. The experiment was carried out using MATLAB 2013b [23] on Windows-7 32 bit platform having 1 GB RAM and 2.93 GHz Intel Core 2 Duo processor.

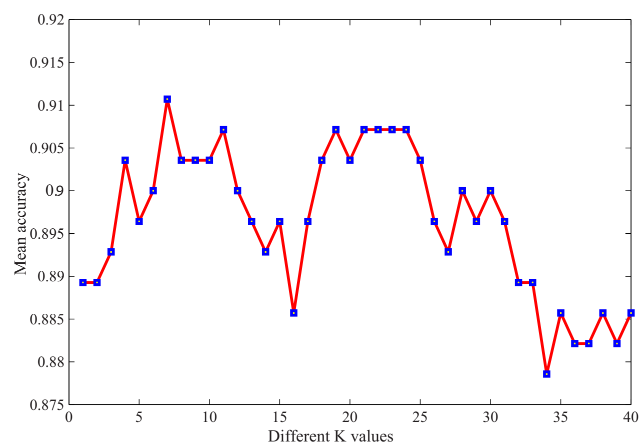

Fig. 13: Mean accuracy(%) vs different "K" values.

KNN classifier has primarily two parameters to adjust - distance parameter and K value. The distance parameter is selected as "cosine" which means one minus the cosine of the included angle between points [23]. Another parameter of KNN classifier is the value of K. For choosing the optimum K value, we have varied the value of K. Fig. 13 shows the variation of mean accuracy (%) for different K values and from this figure it is noted that for K= 7 the classifier provides highest mean accuracy of 91.07%.

In this study, along with proposing a suitable feature extraction scheme in the dual tree complex wavelet transform domain, we have also conducted performance comparison of different classifiers namely probabilistic neural network (PNN), generalized regression neural network (GRNN), adaptive neuro fuzzy inference system (ANFIS), support vector machine (SVM), linear discriminant analysis (LDA), Naive Bayes and k-Nearest Neighbor (KNN)with various parameters for the extracted features in Graz motor imagery database. Table II provides the outcome of different classifiers and from the table, it is clear that for our problem, KNN classifier with "cosine" distance gives better mean accuracy than other classifiers with varying parameters.

TABLE II: Performance Evaluation of Different Classifiers

Classifier Name	**Classifier Parameter**	**Mean Accuracy (%)**
Naive Bayes	Gaussian Distribution	75.71
Discriminant Analysis	Mahalanobis	76.43
	Linear	81.07
	Diaglinear	83.21
SVM	Multilayer Perceptron Kernel	73.93
	Linear Kernel	82.50
	Radial Basis Kernel	84.29
PNN	Radial Basis Network	83.93
GRNN	Radial Basis Network	87.86
ANFIS	Hybrid Method	89.29
KNN	Euclidean Distance	89.29
	Cityblock Distance	89.64
	Correlation Distance	90.71
	Cosine Distance	91.07

Finally table III compares the mean accuracy of the proposed method with several other methods. It can easily be concluded that using simple KNN classifier, our method provides good mean accuracy to classify left and right hand motor imagery movements than all the methods listed in table III.

VI. Conclusion

In this paper, a robust comprehensive method has been proposed to classify the left and right motor imagery hand movements which offers a promising support for an important application in BCI. EEG signals have been successfully classified by extracting the energy of the coefficients of auto-correlated EEG signals in the dual tree complex wavelet transform domain. The EEG signal is enhanced as well as the noise is suppressed for auto-correlation and as a result, the proposed method is more robust. Justification of using wavelet energy as feature has been provided with a number of scatter plots, box plots, histograms and one-way ANOVA analysis. Among various types of classifiers used, KNN provides the satisfactory accuracy (91.07%). Finally the mean accuracy of the proposed feature extraction scheme has been compared with several other recent methods available in EEG literature and has shown to be superior than others.

TABLE III: Comparison of mean accuracy of different methods.

Method	**Proposed by**	**Classifier**	**Mean Accuracy (%)**
PSD	Solhjoo et al. [6]	Mahalanobis distance	63.1
		Gaussian classifiers	65.4
		LDA	65.6
Raw EEG	Solhjoo et al. [7]	HMM	77.50
AAR	Tavakolian et al. [8]	Bayes Quadratic	82.86
		BGN	83.57
		MLP	84.29
Morlet wavelet	Lemm et al. [9]	Bayes Quadratic	89.29
Multiple auto correlation	Wang et al. [10]	LVQ	90.00
Proposed method (DTCWT)		KNN	91.07

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